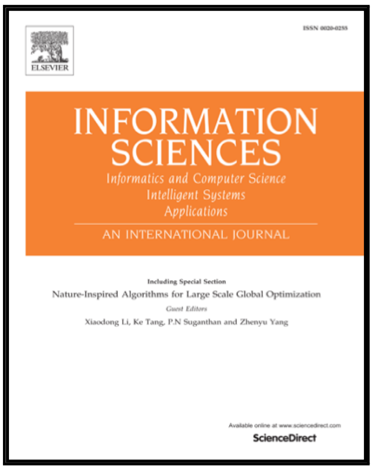

Weighted fuzzy interpolated reasoning based on ranking values of polygonal fuzzy sets and new scale and move transformation techniques

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Weighted fuzzy interpolated reasoning based on ranking values of polygonal fuzzy sets and new scale and move transformation techniques

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Abstract

In this paper, we propose a new transformation-based weighted fuzzy interpolative reasoning (FIR) method based on ranking values of polygonal fuzzy sets (PFSs) and the proposed new scale and move transformation techniques. The proposed weighted FIR method gets more reasonable FIR results than the ones of the existing methods, where the weight of each antecedent variable and the weight of each fuzzy rule are generated automatically. Moreover, the proposed new scale and move transformation techniques can deal with FIR using singleton fuzzy sets and PFSs. We also apply the proposed weighted FIR method to predict the diarrheal disease rates in remote villages. The proposed weighted FIR method provides us with a very useful way for weighted FIR in sparse fuzzy rule-based systems.

***Keywords*:** Fuzzy interpolative reasoning; Interpolated fuzzy rules; Polygonal fuzzy sets; Ranking values; Scale and move transformation techniques.

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1. Introduction

In a fuzzy rule-based system, a complete fuzzy rule base is essential to obtain reasonable fuzzy reasoning results. However, the conventional fuzzy reasoning methods may produce unreasonable fuzzy reasoning results when the fuzzy rule base is incomplete or sparse. In order to overcome this limitation, some fuzzy interpolative reasoning (FIR) methods have been presented for sparse fuzzy rule-based systems. The existing FIR methods can be classified into two categories, i.e., the non-transformation-based FIR methods [3], [6]-[10], [11]-[13], [16], [18], [21], [22], [30], [34] and the transformation-based FIR methods [4], [5], [17]-[20], [28], [33]. The non-transformation-based FIR methods directly interpolate the consequence according to the given observations of fuzzy sets (FSs) [36]. The most typical non-transformation-based FIR method is proposed by Kóczy and Hirota [21]. They presented a FIR method by the linear rule interpolation, where the consequence FSs are constructed by the exploitation of α-cuts operations [36] of FSs, where $\alpha \in [0, 1]$. The existing non-transformation-based FIR methods are analyzed as follows:

1) The FIR methods presented in [11], [16], [18], [21] and [30] deal with FIR with two fuzzy rules.
2) The FIR methods presented in [3], [8]-[13], [22] and [34] deal with FIR with multiple fuzzy rules.
3) The FIR methods presented in [3] and [8]-[13] deal with FIR with polygonal fuzzy sets (PFSs) or bell-shaped FSs.
4) The FIR methods presented in [8], [9] and [12] deal with weighted FIR based on the weighted FIR scheme.
5) The FIR method presented in [11] deals with FIR using interval type-2 FSs [24].
6) The FIR method presented in [10] deals with FIR using rough-FSs.
7) The FIR methods presented in [6] and [13] deal with adaptive FIR.

The transformation-based FIR methods are based on the analogical reasoning mechanism [1]. Firstly, they interpolate an intermediate fuzzy rule, then they modify the intermediate fuzzy rule by performing the transformation techniques, and then the FIR result is derived by calculating the average of the degrees of similarity between the modified antecedent FSs of the obtained intermediate fuzzy rule and the observation FSs. The first transformation-based FIR method was proposed by Huang and Shen [17]. They proposed a FIR method using the concept of the "center of gravity" of FS and the scale and move transformation techniques to ensure that the FIR results are normal and convex FSs. The scale and move transformation techniques have the following advantages: 1) it can handle interpolation of multiple antecedent variables and 2) it guarantees the normality and convexity of the FIR results. The existing transformation-based FIR methods are analyzed as follows:

1) The FIR methods presented in [17]-[20], [28] and [33] are based on FSs.
2) The FIR method presented in [5] deals with FIR using interval type-2 FSs.
3) The FIR method presented in [4] deals with FIR using rough-FSs.
4) The FIR method presented in [33] deals with adaptive FIR.
5) The FIR method presented in [19] deals with backward FIR.

Yang and Shen [34] presented a FIR method based on the extension principle [36]. Cheng *et al.* [12] proposed a FIR method based on the ranking values (RVs) of polygonal fuzzy sets (PFSs), where their method deals with FIR by applying the α-cuts operations [36] of PFSs to get the characteristic points (CPs) of PFSs, where $\alpha \in [0, 1]$. In [12], Cheng *et al.* pointed out that the limitation of the method presented in [34] is that it cannot hold the normality and the convexity of the FIR results. In this paper, we also find that the methods presented in [3], [8], [12], [16]-[18], [21] and [34] have the following limitations:

1) The methods presented in [16], [18], [21] and [34] only can deal with FIR using triangular FSs or trapezoidal FSs.

2) The methods presented in [3], [8], [12], [16]-[18], [21] and [34] cannot preserve the normality and the convexity of FIR results and they produce abnormal FSs in some situations.
3) The scale and move transformation techniques presented in [17] is not flexible enough when dealing with FIR using PFSs.
4) The method presented in [16], [18] and [21] cannot deal with FIR with multiple antecedent variables and multiple fuzzy rules.

Therefore, we need to develop a new method to overcome the limitations of the methods presented in [3], [8], [12], [16]-[18], [21] and [34].

In this paper, we propose a new transformation-based weighted fuzzy interpolative reasoning (FIR) method for sparse fuzzy rule-based systems, which is based on the ranking values (RVs) of PFSs and the proposed new scale and move transformation techniques, where the weight of each antecedent variable and the weight of each fuzzy rule are generated automatically. In order to compare the FIR results of the proposed method with the ones of the existing methods [3], [8], [12], [16]-[18], [21], [34], we utilize PFSs in the proposed weighted FIR method. Moreover, the proposed new transformation techniques can deal with weighted FIR using singleton FSs and PFSs. We also apply the proposed weighted FIR method to predict the diarrheal disease rates in remote villages [12], [34]. The proposed weighted FIR method gets more reasonable FIR results than the ones of the existing FIR methods [3], [8], [12], [16]-[18], [21], [34].

The remaining sections of this paper are organized as follows. In Section 2, we briefly review the definitions of FSs [36], the α-cuts [36] of FSs, the characteristic points (CPs) of PFSs [12], the ranking values (RVs) [11] of PFSs and the distance measure [12] between PFSs. In Section 3, we propose a new weighted FIR method based on the RVs of PFSs and the proposed new scale and move transformation techniques. In Section 4, we compare the experimental results of the proposed weighted FIR method with the ones of the methods presented in [3], [8], [12], [16]-[18], [21] and [34]. The conclusions are discussed in Section 5.

2. Preliminaries

In this section, we review the definitions of fuzzy sets (FSs) [36], the α-cuts [36] of FSs, the characteristic points (CPs) of PFSs [12], the ranking values (RVs) [11] of PFSs and the distance measure [12] between PFSs.

***Definition 2.1* [36]:** A FS A in the universe of discourse $U = \{x_1, x_2, \ldots, x_n\}$ can be represented as follows:

$$A = \mu_A(x_1)/x_1 + \mu_A(x_2)/x_2 + \ldots + \mu_A(x_n)/x_n, \tag{1}$$

where $\mu_A(x_j)$ represents the membership degree of element x_j belonging to the FS A, the symbol "/" represents the separator, the symbol "+" represents the union operator, $0 \leq \mu_A(x_j) \leq 1$ and $1 \leq j \leq n$.

***Definition 2.2* [36]:** The α-cut A_α of the FS A in the universe of discourse U is defined as follows:

$$A_\alpha = \{x_j \mid \mu_A(x_j) \geq \alpha \text{ and } x_j \in U\}, \tag{2}$$

where $\alpha \in [0, 1]$.

Let A be a PFS shown in Fig. 1 represented by $A = (a_0, a_1, \ldots, a_{s-2}, a_{s-1}; \mu_A(a_0), \mu_A(a_1), \ldots, \mu_A(a_{s-2}), \mu_A(a_{s-1}))$ [12], where a_0, a_1, …, a_{s-2} and a_{s-1} represent the CPs of the PFS A, $\mu_A(a_t)$ represents the membership degree of the CP a_t belonging to the PFS A, $\mu_A(a_t) \in [0, 1]$, $0 \leq t \leq s-1$ and s is an even number. In Fig. 1, the CPs a_0, a_c and a_{s-1} are called the "left extreme CP", the "center CP" and the "right extreme CP", respectively, where $h = \left\lfloor \frac{s-1}{2} \right\rfloor$, $a_c =$

$\frac{a_0 + a_{s-1}}{2}$, the membership degrees of the CPs a_h and a_{h+1} belonging to the PFS A are equal to 1, respectively, and the CPs a_h and a_{h+1} do not need to be different. By performing the α-cuts operations [36] to the PFS A shown in Fig. 1, we can get $A_{\alpha_0} = [a_0, a_{s-1}]$, $A_{\alpha_1} = [a_1, a_{s-2}]$, $A_{\alpha_2} = [a_2, a_{s-3}]$, …, $A_{\alpha_{r-1}} = [a_{h-1}, a_{h+2}]$ and $A_{\alpha_r} = [a_h, a_{h+1}]$, where $\alpha_0 < \alpha_1 < \alpha_2 < \cdots < \alpha_{r-1} < \alpha_r$, $\alpha_0 = 0$ and $\alpha_r = 1$.

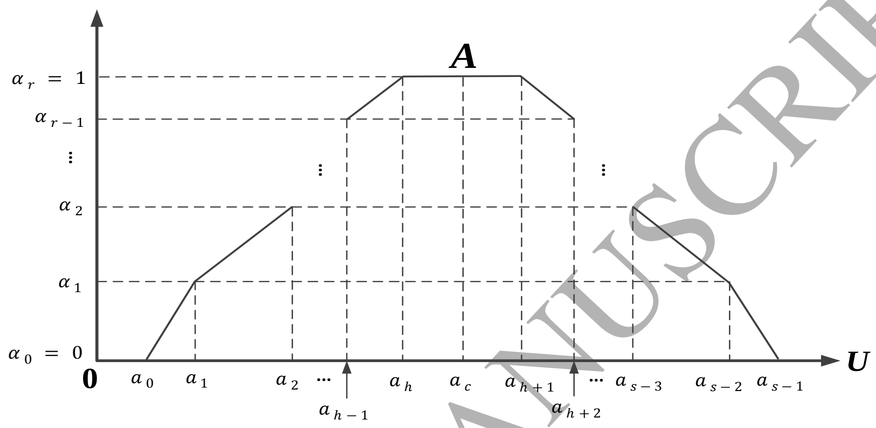

Fig. 1. A PFS and the membership degrees of its CPs [12].

Definition 2.3 **[11]:** The RV $Rank(A)$ of the PFS $A = (a_0, a_1, \ldots, a_{s-2}, a_{s-1}; \mu_A(a_0), \mu_A(a_1), \ldots, \mu_A(a_{s-2}), \mu_A(a_{s-1}))$ is defined as follows:

$$Rank(A) = M_0(A) + M_1(A) + \cdots + M_{s-2}(A) + \mu_A(a_1) + \mu_A(a_2) + \cdots + \mu_A(a_{s-2}) - \frac{1}{s}(S_0(A) + S_1(A) + \cdots + S_{s-2}(A) + S_s(A)), \quad (3)$$

where $M_p(A) = \frac{a_p + a_{p+1}}{2}$, $0 \le p \le s-2$, $S_q(A) = \sqrt{\frac{1}{2}\sum_{t=q}^{q+1}\left(a_t - \frac{a_t + a_{t+1}}{2}\right)^2}$, $0 \le q \le s-2$, $S_s(A) = \sqrt{\frac{1}{s}\sum_{t=0}^{s-1}\left(a_t - \frac{1}{s}\sum_{t=0}^{s-1} a_t\right)^2}$, $0 \le t \le s-1$ and s is an even number.

Definition 2.4 **[12]:** The distance $D(Rank(A), Rank(B))$ between the RVs $Rank(A)$ and $Rank(B)$ is defined as follows:

$$D(Rank(A), Rank(B)) = |Rank(A) - Rank(B)|. \quad (4)$$

3. A new weighted FIR method based on the RVs of PFSs and the proposed new scale and move transformation techniques

In this section, we propose a new transformation-based weighted FIR method based on the ranking values (RVs) of polygonal fuzzy sets (PFSs) and the proposed new scale and move transformation techniques. Consider the following weighted FIR scheme with m antecedent variables and n fuzzy rules [12]:

Rule R_1: **IF** X_1 is $A_1^1(w_{11})$ **and** X_2 is $A_2^1(w_{12})$ **and** … **and** X_m is $A_m^1(w_{1m})$ **THEN** Y is $B_1(W_1)$
Rule R_2: **IF** X_1 is $A_1^2(w_{21})$ **and** X_2 is $A_2^2(w_{22})$ **and** … **and** X_m is $A_m^2(w_{2m})$ **THEN** Y is $B_2(W_2)$
⋮
Rule R_n: **IF** X_1 is $A_1^n(w_{n1})$ **and** X_2 is $A_2^n(w_{n2})$ **and** … **and** X_m is $A_m^n(w_{nm})$ **THEN** Y is $B_n(W_n)$
Observation: X_1 is A_1^* **and** X_2 is A_2^* **and** … **and** X_m is A_m^*

Conclusion: Y is B^*

where the weighted FIR scheme with m antecedent variables and n fuzzy rules using PFSs is illustrated in Fig. 2 [12], X_i represents the ith antecedent variable of the fuzzy rules; Y represents the consequence variable of the fuzzy rules; PFS A_i^j of the ith antecedent variable X_i of fuzzy rule R_j is characterized by $A_i^j = (a_{i,0}^j, a_{i,1}^j, \ldots, a_{i,s-2}^j, a_{i,s-1}^j; \mu_{A_i^j}(a_{i,0}^j), \mu_{A_i^j}(a_{i,1}^j), \ldots, \mu_{A_i^j}(a_{i,s-2}^j), \mu_{A_i^j}(a_{i,s-1}^j))$, where $\mu_{A_i^j}(a_{i,0}^j)$, $\mu_{A_i^j}(a_{i,1}^j)$, …, $\mu_{A_i^j}(a_{i,s-2}^j)$ and $\mu_{A_i^j}(a_{i,s-1}^j)$) denote the membership degrees of the CPs $a_{i,0}^j$, $a_{i,1}^j$, …, $a_{i,s-2}^j$ and $a_{i,s-1}^j$ belonging to the PFS A_i^j, respectively, $\mu_{A_i^j}(a_{i,t}^j) \in [0, 1]$, $1 \le j \le n$, $1 \le i \le m$, $0 \le t \le s-1$ and s is an even number; PFS B^j of the consequence variable Y of fuzzy rule R_j is represented by $B^j = (b_0^j, b_1^j, \ldots, b_{s-2}^j, b_{s-1}^j; \mu_{B^j}(b_0^j), \mu_{B^j}(b_1^j), \ldots, \mu_{B^j}(b_{s-2}^j), \mu_{B^j}(b_{s-1}^j))$, where $\mu_{B^j}(b_0^j)$, $\mu_{B^j}(b_1^j)$, …, $\mu_{B^j}(b_{s-2}^j)$ and $\mu_{B^j}(b_{s-1}^j)$ denote the membership degrees of the CPs b_0^j, b_1^j, …, b_{s-2}^j and b_{s-1}^j belonging to the PFS B^j, respectively, $\mu_{B^j}(b_t^j) \in [0, 1]$, $1 \le j \le n$ and $0 \le t \le s-1$; the ith observation PFS A_i^* is represented by $A_i^* = (a_{i,0}^*, a_{i,1}^*, \ldots, a_{i,s-2}^*, a_{i,s-1}^*; \mu_{A_i^*}(a_{i,0}^*), \mu_{A_i^*}(a_{i,1}^*), \ldots,$

$\mu_{A_i^*}(a_{i,s-2}^*)$, $\mu_{A_i^*}(a_{i,s-1}^*)$), where $\mu_{A_i^*}(a_{i,0}^*)$, $\mu_{A_i^*}(a_{i,1}^*)$, …, $\mu_{A_i^*}(a_{i,s-2}^*)$ and $\mu_{A_i^*}(a_{i,s-1}^*)$ denote the membership degrees of the CPs $a_{i,0}^*$, $a_{i,1}^*$, …, $a_{i,s-2}^*$ and $a_{i,s-1}^*$ belonging to the PFS A_i^*, respectively, $\mu_{A_i^*}(a_{i,t}^*) \in [0, 1]$, $1 \leq i \leq m$ and $0 \leq t \leq s-1$; PFS B^* of the FIR result is represented as B^* = (b_0^*, b_1^*, …, b_{s-2}^*, b_{s-1}^*; $\mu_{B^*}(b_0^*)$, $\mu_{B^*}(b_1^*)$, …, $\mu_{B^*}(b_{s-2}^*)$, $\mu_{B^*}(b_{s-1}^*)$), where $\mu_{B^*}(b_0^*)$, $\mu_{B^*}(b_1^*)$, …, $\mu_{B^*}(b_{s-2}^*)$ and $\mu_{B^*}(b_{s-1}^*)$ denote the membership degrees of the CPs b_0^*, b_1^*, … , b_{s-2}^* and b_{s-1}^* belonging to the PFS B^*, respectively, $\mu_{B^*}(b_t^*) \in [0, 1]$ and $0 \leq t \leq s-1$; w_{ji} represents the weight of the ith antecedent variable X_i of fuzzy rule R_j, where $w_{ji} \in [0, 1]$, $1 \leq j \leq n$, $1 \leq i \leq m$ and $\sum_{j=1}^{n} w_{ji} = 1$; W_j represents the weight of fuzzy rule R_j, where $W_j \in [0, 1]$, $\sum_{j=1}^{n} W_j = 1$ and $1 \leq j \leq n$.

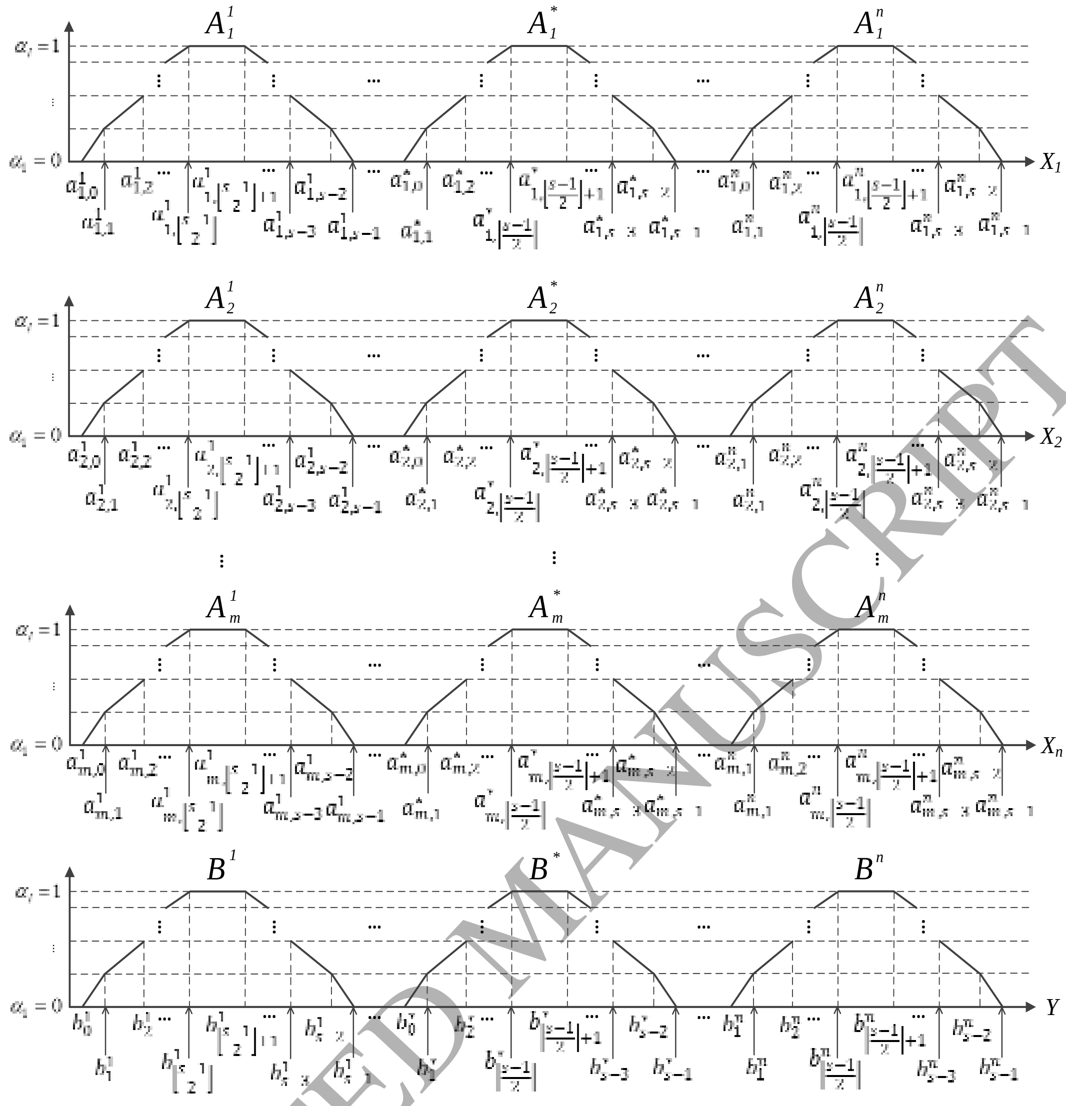

Fig. 2. The FIR scheme using PFSs [12].

The proposed transformation-based weighted FIR method based on the proposed new scale and move transformation techniques to get the FIR result B^* = (b_0^*, b_1^*, …, b_{s-2}^*, b_{s-1}^*; $\mu_{B^*}(b_0^*)$, $\mu_{B^*}(b_1^*)$, …, $\mu_{B^*}(b_{s-2}^*)$, $\mu_{B^*}(b_{s-1}^*)$) is presented as follows:

Step 1: Calculate the RV $Rank(A_i^j)$ of PFS $A_i^j = (a_{i,0}^j, a_{i,1}^j, \ldots, a_{i,s-2}^j, a_{i,s-1}^j;$ $\mu_{A_i^j}(a_{i,0}^j), \mu_{A_i^j}(a_{i,1}^j), \ldots, \mu_{A_i^j}(a_{i,s-2}^j), \mu_{A_i^j}(a_{i,s-1}^j))$ of the *i*th antecedent variable X_i of fuzzy rule R_j and calculate the RV $Rank(A_i^*)$ of the *i*th observation PFS A_i^* = ($a_{i,0}^*$, $a_{i,1}^*$, …, $a_{i,s-2}^*$, $a_{i,s-1}^*$; $\mu_{A_i^*}(a_{i,0}^*)$, $\mu_{A_i^*}(a_{i,1}^*)$, …, $\mu_{A_i^*}(a_{i,s-2}^*)$, $\mu_{A_i^*}(a_{i,s-1}^*)$), shown as follows [11]:

$$Rank(A_i^j) = M_1(A_i^j) + M_2(A_i^j) + \cdots + M_{s-1}(A_i^j) + \mu_{A_i^j}(a_{i,1}^j) + \mu_{A_i^j}(a_{i,2}^j) + \cdots + \mu_{A_i^j}(a_{i,s-2}^j) - \frac{1}{s}\left(S_1(A_i^j) + S_2(A_i^j) + \cdots + S_s(A_i^j)\right), \quad (5)$$

$$Rank(A_i^*) = M_1(A_i^*) + M_2(A_i^*) + \cdots + M_{s-1}(A_i^*) + \mu_{A_i^*}(a_{i,1}^*) + \mu_{A_i^*}(a_{i,2}^*) + \cdots + \mu_{A_i^*}(a_{i,s-2}^*) - \frac{1}{s}\left(S_1(A_i^*) + S_2(A_i^*) + \cdots + S_s(A_i^*)\right), \quad (6)$$

where $\mu_{A_i^j}(a_{i,1}^j)$, $\mu_{A_i^j}(a_{i,2}^j)$, …, and $\mu_{A_i^j}(a_{i,s-2}^j)$ denote the membership degrees of the CPs $a_{i,0}^j$, $a_{i,1}^j$, …, $a_{i,s-2}^j$ and $a_{i,s-1}^j$ belonging to the PFS A_i^j, respectively, $\mu_{A_i^*}(a_{i,1}^*)$, $\mu_{A_i^*}(a_{i,2}^*)$, …, and $\mu_{A_i^*}(a_{i,s-2}^*)$ represent the membership degrees of the CPs $a_{i,0}^*$, $a_{i,1}^*$, …, $a_{i,s-2}^*$ and $a_{i,s-1}^*$ belonging to the PFS A_i^*, respectively, $\mu_{A_i^j}(a_{i,t}^j) \in [0,1]$, $\mu_{A_i^*}(a_{i,t}^*) \in [0,1]$, $1 \le i \le m$, $1 \le j \le n$, n denotes the number of fuzzy rules, m denotes the number of antecedent variables of fuzzy rules and $0 \le t \le s-1$.

Step 2: Calculate the weight w'_{ji} of PFS $A_i^j = (a_{i,0}^j, a_{i,1}^j, \ldots, a_{i,s-2}^j, a_{i,s-1}^j; \mu_{A_i^j}(a_{i,0}^j), \mu_{A_i^j}(a_{i,1}^j), \ldots, \mu_{A_i^j}(a_{i,s-2}^j), \mu_{A_i^j}(a_{i,s-1}^j))$ of the ith antecedent variable X_i of fuzzy rule R_j, shown as follows [12]:

$$w'_{ji} = 1 - \frac{D\left(Rank(A_i^*),\ Rank(A_i^j)\right)}{\sum_{j=1}^{n} D\left(Rank(A_i^*),\ Rank(A_i^j)\right)}, \quad (7)$$

where $1 \le i \le m$, $1 \le j \le n$ and "$D\left(Rank(A_i^*),\ Rank(A_i^j)\right)$" is calculated as follows [12]:

$$D\left(Rank(A_i^*), Rank(A_i^j)\right) = \left|Rank(A_i^*) - Rank(A_i^j)\right|, \quad (8)$$

where $1 \le i \le m$ and $1 \le j \le n$. Calculate the normalized weight w_{ji} of PFS $A_i^j = (a_{i,0}^j, a_{i,1}^j, \ldots, a_{i,s-2}^j, a_{i,s-1}^j; \mu_{A_i^j}(a_{i,0}^j), \mu_{A_i^j}(a_{i,1}^j), \ldots, \mu_{A_i^j}(a_{i,s-2}^j), \mu_{A_i^j}(a_{i,s-1}^j))$ of the ith antecedent variable X_i of fuzzy rule R_j, shown as follows:

$$w_{ji} = \frac{w'_{ji}}{\sum_{j=1}^{n} w'_{ji}}, \tag{9}$$

where $w_{ji} \in [0,1]$, $1 \le j \le n$, $1 \le i \le m$ and $\sum_{j=1}^{n} w_{ji} = 1$.

Step 3: Calculate the weight W'_j of fuzzy rule R_j, shown as follows:

$$W'_j = \sum_{i=1}^{m} \frac{w_{ji} \times \frac{1}{1 + D\left(Rank(A_i^*),\ Rank(A_i^j)\right)}}{\sum_{j=1}^{n} \frac{1}{1 + D\left(Rank(A_i^*),\ Rank(A_i^j)\right)}}, \tag{10}$$

where $W'_j \in [0,1]$ and $1 \le j \le n$. Calculate the normalized weight W_j of fuzzy rule R_j as follows:

$$W_j = \frac{W'_j}{\sum_{j=1}^{n} W'_j}, \tag{11}$$

where $W_j \in [0,1]$, $1 \le j \le n$ and $\sum_{j=1}^{n} W_j = 1$.

Step 4: Calculate the ranking displacement factor λ_i^{jk} for the RV $Rank(A_i^j)$ of PFS $A_i^j = (a_{i,0}^j, a_{i,1}^j, \ldots, a_{i,s-2}^j, a_{i,s-1}^j;\ \mu_{A_i^j}(a_{i,0}^j), \mu_{A_i^j}(a_{i,1}^j), \ldots, \mu_{A_i^j}(a_{i,s-2}^j), \mu_{A_i^j}(a_{i,s-1}^j))$ of the ith antecedent variable X_i of fuzzy rule R_j and the RV $Rank(A_i^k)$ of PFS $A_i^k = (a_{i,0}^k, a_{i,1}^k, \ldots, a_{i,s-2}^k, a_{i,s-1}^k;\ \mu_{A_i^k}(a_{i,0}^k), \mu_{A_i^k}(a_{i,1}^k), \ldots, \mu_{A_i^k}(a_{i,s-2}^k), \mu_{A_i^k}(a_{i,s-1}^k))$ of the ith antecedent variable X_i of fuzzy rule R_k with respect to the RV $Rank(A_i^*)$ of the ith observation PFS $A_i^* = (a_{i,0}^*, a_{i,1}^*, \ldots, a_{i,s-2}^*, a_{i,s-1}^*;\ \mu_{A_i^*}(a_{i,0}^*), \mu_{A_i^*}(a_{i,1}^*), \ldots, \mu_{A_i^*}(a_{i,s-2}^*), \mu_{A_i^*}(a_{i,s-1}^*))$, shown as follows:

$$\lambda_i^{jk} = \frac{Rank(A_i^*) - Rank(A_i^j)}{Rank(A_i^k) - Rank(A_i^j)}, \tag{12}$$

where $1 \le j < k \le n$ and $1 \le i \le m$. Calculate the average ranking displacement factor λ^{jk} of the pair of fuzzy rules R_j and R_k, shown as follows:

$$\lambda^{jk} = \frac{\sum_{i=1}^{m} \lambda_i^{jk}}{m}, \tag{13}$$

where $1 \le j < k \le n$ and $1 \le i \le m$.

Step 5: Calculate the CP $a_{i,t}^{jk}$ of PFS $A_i^{jk} = (a_{i,0}^{jk}, a_{i,1}^{jk}, \ldots, a_{i,s-2}^{jk}, a_{i,s-1}^{jk};\ \mu_{A_i^{jk}}(a_{i,0}^{jk}),$

$\mu_{A_i^{jk}}(a_{i,1}^{jk})$, …, $\mu_{A_i^{jk}}(a_{i,s-2}^{jk})$, $\mu_{A_i^{jk}}(a_{i,s-1}^{jk})$) of the *i*th antecedent variable X_i of the interpolated fuzzy rule R_{jk} generated between fuzzy rule R_j and fuzzy rule R_k based on the CPs of PFS $A_i^j = (a_{i,0}^j,\ a_{i,1}^j,\ \dots,\ a_{i,s-2}^j,\ a_{i,s-1}^j;\ \mu_{A_i^j}(a_{i,0}^j), \mu_{A_i^j}(a_{i,1}^j), \dots,$ $\mu_{A_i^j}(a_{i,s-2}^j), \mu_{A_i^j}(a_{i,s-1}^j))$ of the *i*th antecedent variable X_i of fuzzy rule R_j and the CPs of PFS $A_i^k = (a_{i,0}^k,\ a_{i,1}^k,\ \dots,\ a_{i,s-2}^k,\ a_{i,s-1}^k;\ \mu_{A_i^k}(a_{i,0}^k),\ \mu_{A_i^k}(a_{i,1}^k), \dots,\ \mu_{A_i^k}(a_{i,s-2}^k),$ $\mu_{A_i^k}(a_{i,s-1}^k))$ of the *i*th antecedent variable X_i of fuzzy rule R_k, shown as follows:

$$a_{i,t}^{jk} = \left(1 - \lambda^{jk}\right) \times a_{i,t}^{j} + \lambda^{jk} \times a_{i,t}^{k}, \tag{14}$$

where $1 \le j < k \le n$, $1 \le i \le m$ and $0 \le t \le s-1$. Calculate the CP b_t^{jk} of PFS $B^{jk} = (b_0^{jk},\ b_1^{jk},\ \dots,\ b_{s-2}^{jk},\ b_{s-1}^{jk};\ \mu_{B^{jk}}(b_0^{jk}),\ \mu_{B^{jk}}(b_1^{jk}),\ \dots,$ $\mu_{B^{jk}}(b_{s-2}^{jk}), \mu_{B^{jk}}(b_{s-1}^{jk}))$ of the consequence variable Y of the interpolated fuzzy rule R_{jk} generated between fuzzy rule R_j and fuzzy rule R_k based on the CPs of PFS $B^j = (b_0^j, b_1^j,\ \dots, b_{s-2}^j, b_{s-1}^j;\ \mu_{B^j}(b_0^j),\ \mu_{B^j}(b_1^j),\ \dots,\ \mu_{B^j}(b_{s-2}^j), \mu_{B^j}(b_{s-1}^j))$ of the consequence variable Y of fuzzy rule R_j and the CPs of PFS $B^k = (b_0^k, b_1^k,\ \dots,$ $b_{s-2}^k,\ b_{s-1}^k;\ \mu_{B^k}(b_0^k),\ \mu_{B^k}(b_1^k),\ \dots,\ \mu_{B^k}(b_{s-2}^k),\ \mu_{B^k}(b_{s-1}^k))$ of the consequence variable Y of fuzzy rule R_k, shown as follows:

$$b_t^{jk} = \left(1 - \lambda^{jk}\right) \times b_t^{j} + \lambda^{jk} \times b_t^{k}, \tag{15}$$

where $1 \le j < k \le n$ and $0 \le t \le s-1$.

Step 6: Construct the intermediate fuzzy rule R_λ "IF X_1 is A_1^λ and X_2 is A_2^λ and … and X_m is A_m^λ THEN Y is B^λ", described as follows. Calculate the CP $a_{i,t}^\lambda$ of PFS $A_i^\lambda = (a_{i,0}^\lambda, a_{i,1}^\lambda,\ \dots, a_{i,s-2}^\lambda,\ a_{i,s-1}^\lambda;\ \mu_{A_i^\lambda}(a_{i,0}^\lambda), \mu_{A_i^\lambda}(a_{i,1}^\lambda),\ \dots,\ \mu_{A_i^\lambda}(a_{i,s-2}^\lambda), \mu_{A_i^\lambda}(a_{i,s-1}^\lambda))$ of the *i*th antecedent variable X_i of the intermediate fuzzy rule R_λ based on the CPs of PFS $A_i^{jk} = (a_{i,0}^{jk},\ a_{i,1}^{jk},\ \dots,\ a_{i,s-2}^{jk},\ a_{i,s-1}^{jk};\ \mu_{A_i^{jk}}(a_{i,0}^{jk}),\ \mu_{A_i^{jk}}(a_{i,1}^{jk}),\ \dots,$

$\mu_{A_i^{jk}}(a_{i,s-2}^{jk}), \mu_{A_i^{jk}}(a_{i,s-1}^{jk}))$ of the ith antecedent variable X_i of the interpolated fuzzy rule R_{jk} generated between fuzzy rule R_j and fuzzy rule R_k, where

$$a_{i,t}^{\lambda} = \frac{\sum_{1 \le j < k \le n} a_{i,t}^{jk} \times (W_j + W_k)}{\sum_{1 \le j < k \le n} (W_j + W_k)}, \tag{16}$$

$1 \le j < k \le n$, $1 \le i \le m$ and $0 \le t \le s-1$. Calculate the CP b_t^{λ} of PFS $B^{\lambda} = (b_0^{\lambda}, b_1^{\lambda}, \ldots, b_{s-2}^{\lambda}, b_{s-1}^{\lambda}; \mu_{B^{\lambda}}(b_0^{\lambda}), \mu_{B^{\lambda}}(b_1^{\lambda}), \ldots, \mu_{B^{\lambda}}(b_{s-2}^{\lambda}), \mu_{B^{\lambda}}(b_{s-1}^{\lambda}))$ of the consequence variable Y of the intermediate fuzzy rule R_{λ} based on the CPs of PFS $B^{jk} = (b_0^{jk}, b_1^{jk}, \ldots, b_{s-2}^{jk}, b_{s-1}^{jk}; \mu_{B^{jk}}(b_0^{jk}), \mu_{B^{jk}}(b_1^{jk}), \ldots, \mu_{B^{jk}}(b_{s-2}^{jk}), \mu_{B^{jk}}(b_{s-1}^{jk}))$ of the consequence variable Y of the interpolated fuzzy rule R_{jk} generated between fuzzy rule R_j and fuzzy rule R_k, where

$$b_t^{\lambda} = \frac{\sum_{1 \le j < j \le n} b_t^{jk} \times (W_j + W_k)}{\sum_{1 \le j < k \le n} (W_j + W_k)}, \tag{17}$$

$1 \le j < k \le n$ and $0 \le t \le s-1$.

Step 7: Calculate the distance ratio $\overline{a_{i,s-1}^{\lambda} a_{i,0}^{\lambda}}$ between the right extreme CP $a_{i,s-1}^{\lambda}$ and the left extreme CP $a_{i,0}^{\lambda}$ of PFS $A_i^{\lambda} = (a_{i,0}^{\lambda}, a_{i,1}^{\lambda}, \ldots, a_{i,s-2}^{\lambda}, a_{i,s-1}^{\lambda}; \mu_{A_i^{\lambda}}(a_{i,0}^{\lambda}), \mu_{A_i^{\lambda}}(a_{i,1}^{\lambda}), \ldots, \mu_{A_i^{\lambda}}(a_{i,s-2}^{\lambda}), \mu_{A_i^{\lambda}}(a_{i,s-1}^{\lambda}))$ of the ith antecedent variable X_i of the intermediate fuzzy rule R_{λ} with respect to the right extreme CP $a_{i,s-1}^{*}$ and the left extreme CP $a_{i,0}^{*}$ of the ith observation PFS $A_i^{*} = (a_{i,0}^{*}, a_{i,1}^{*}, \ldots, a_{i,s-2}^{*}, a_{i,s-1}^{*}; \mu_{A_i^{*}}(a_{i,0}^{*}), \mu_{A_i^{*}}(a_{i,1}^{*}), \ldots, \mu_{A_i^{*}}(a_{i,s-2}^{*}), \mu_{A_i^{*}}(a_{i,s-1}^{*}))$, shown as follows:

$$\overline{a_{i,s-1}^{\lambda} a_{i,0}^{\lambda}} = \frac{a_{i,s-1}^{*} - a_{i,0}^{*}}{a_{i,s-1}^{\lambda} - a_{i,0}^{\lambda}}, \tag{18}$$

where $1 \le i \le m$. Calculate the center CP $a_{i,c}^{\lambda}$ of PFS $A_i^{\lambda} = (a_{i,0}^{\lambda}, a_{i,1}^{\lambda}, \ldots, a_{i,s-2}^{\lambda}, a_{i,s-1}^{\lambda}; \mu_{A_i^{\lambda}}(a_{i,0}^{\lambda}), \mu_{A_i^{\lambda}}(a_{i,1}^{\lambda}), \ldots, \mu_{A_i^{\lambda}}(a_{i,s-2}^{\lambda}), \mu_{A_i^{\lambda}}(a_{i,s-1}^{\lambda}))$ of the ith antecedent variable X_i of the intermediate fuzzy rule R_{λ}, shown as follows:

$$a_{i,c}^{\lambda} = \frac{a_{i,s-1}^{\lambda} + a_{i,0}^{\lambda}}{2}, \tag{19}$$

where $1 \leq i \leq m$. Calculate the center CP b_c^λ of PFS $B^\lambda = (b_0^\lambda, b_1^\lambda, \ldots, b_{s-2}^\lambda, b_{s-1}^\lambda; \mu_{B^\lambda}(b_0^\lambda), \mu_{B^\lambda}(b_1^\lambda), \ldots, \mu_{B^\lambda}(b_{s-2}^\lambda), \mu_{B^\lambda}(b_{s-1}^\lambda))$ of the consequence variable Y of the intermediate fuzzy rule R_λ, shown as follows:

$$b_c^\lambda = \frac{b_{s-1}^\lambda + b_0^\lambda}{2}. \tag{20}$$

Modify the left extreme CP $a_{i,0}^\lambda$ and the right extreme CP $a_{i,s-1}^\lambda$ of PFS $A_i^\lambda = (a_{i,0}^\lambda, a_{i,1}^\lambda, \ldots, a_{i,s-2}^\lambda, a_{i,s-1}^\lambda; \mu_{A_i^\lambda}(a_{i,0}^\lambda), \mu_{A_i^\lambda}(a_{i,1}^\lambda), \ldots, \mu_{A_i^\lambda}(a_{i,s-2}^\lambda), \mu_{A_i^\lambda}(a_{i,s-1}^\lambda))$ of the ith antecedent variable X_i of the intermediate fuzzy rule R_λ, respectively, shown as follows:

$$a_{i,0}^\lambda = a_{i,c}^\lambda - \frac{\left(a_{i,s-1}^\lambda - a_{i,0}^\lambda\right) \times \overline{a_{i,s-1}^\lambda a_{i,0}^\lambda}}{2}, \tag{21}$$

$$a_{i,s-1}^\lambda = a_{i,c}^\lambda + \frac{\left(a_{i,s-1}^\lambda - a_{i,0}^\lambda\right) \times \overline{a_{i,s-1}^\lambda a_{i,0}^\lambda}}{2}, \tag{22}$$

where $1 \leq i \leq m$. Calculate the left extreme CP b_0^* and the right extreme CP b_{s-1}^* of PFS $B^* = (b_0^*, b_1^*, \ldots, b_{s-2}^*, b_{s-1}^*; \mu_{B^*}(b_0^*), \mu_{B^*}(b_1^*), \ldots, \mu_{B^*}(b_{s-2}^*), \mu_{B^*}(b_{s-1}^*))$ of the FIR result, respectively, shown as follows:

$$b_0^* = b_c^\lambda - \frac{\frac{1}{m}\sum_{i=1}^{m}\left[\left(a_{i,s-1}^\lambda - a_{i,0}^\lambda\right) \times \overline{a_{i,s-1}^\lambda a_{i,0}^\lambda}\right]}{2}, \tag{23}$$

$$b_{s-1}^* = b_c^\lambda + \frac{\frac{1}{m}\sum_{i=1}^{m}\left[\left(a_{i,s-1}^\lambda - a_{i,0}^\lambda\right) \times \overline{a_{i,s-1}^\lambda a_{i,0}^\lambda}\right]}{2}, \tag{24}$$

where $1 \leq i \leq m$.

For $i = 1$ **to** m **Do**

For $u = 0$ **to** $\left\lfloor \frac{s-1}{2} \right\rfloor - 1$ **Do**

Calculate the distance ratio $\overline{a_{i,u+1}^\lambda a_{i,u}^\lambda}$ between the CPs $a_{i,u+1}^\lambda$ and $a_{i,u}^\lambda$ of PFS $A_i^\lambda = (a_{i,0}^\lambda, a_{i,1}^\lambda, \ldots, a_{i,s-2}^\lambda, a_{i,s-1}^\lambda; \mu_{A_i^\lambda}(a_{i,0}^\lambda), \mu_{A_i^\lambda}(a_{i,1}^\lambda), \ldots, \mu_{A_i^\lambda}(a_{i,s-2}^\lambda), \mu_{A_i^\lambda}(a_{i,s-1}^\lambda))$ of the ith antecedent variable X_i of the intermediate fuzzy rule R_λ with respect to the CPs $a_{i,u+1}^*$ and $a_{i,u}^*$ of the ith observation

PFS $A_i^* = (a_{i,0}^*, a_{i,1}^*, \ldots, a_{i,s-2}^*, a_{i,s-1}^*; \mu_{A_i^*}(a_{i,0}^*), \mu_{A_i^*}(a_{i,1}^*), \ldots, \mu_{A_i^*}(a_{i,s-2}^*), \mu_{A_i^*}(a_{i,s-1}^*))$, shown as follows:

$$\overline{a_{i,u+1}^\lambda a_{i,u}^\lambda} = \frac{a_{i,u+1}^* - a_{i,u}^*}{a_{i,u+1}^\lambda - a_{i,u}^\lambda}, \tag{25}$$

where $0 \le u \le \left\lfloor \frac{s-1}{2} \right\rfloor - 1$. Modify the CP $a_{i,u+1}^\lambda$ of PFS $A_i^\lambda = (a_{i,0}^\lambda, a_{i,1}^\lambda, \ldots, a_{i,s-2}^\lambda, a_{i,s-1}^\lambda; \mu_{A_i^\lambda}(a_{i,0}^\lambda), \mu_{A_i^\lambda}(a_{i,1}^\lambda), \ldots, \mu_{A_i^\lambda}(a_{i,s-2}^\lambda), \mu_{A_i^\lambda}(a_{i,s-1}^\lambda))$ of the *i*th antecedent variable X_i of the intermediate fuzzy rule R_λ, shown as follows:

$$a_{i,u+1}^\lambda = a_{i,0}^\lambda + (a_{i,u+1}^\lambda - a_{i,u}^\lambda) \times \overline{a_{i,u+1}^\lambda a_{i,u}^\lambda}, \tag{26}$$

where $0 \le u \le \left\lfloor \frac{s-1}{2} \right\rfloor - 1$

End;

For $u = \left\lfloor \frac{s-1}{2} \right\rfloor + 2$ **to** $s - 1$ **Do**

Calculate the distance ratio $\overline{a_{i,u}^\lambda a_{i,u-1}^\lambda}$ between the CPs $a_{i,u}^\lambda$ and $a_{i,u-1}^\lambda$ of PFS $A_i^\lambda = (a_{i,0}^\lambda, a_{i,1}^\lambda, \ldots, a_{i,s-2}^\lambda, a_{i,s-1}^\lambda; \mu_{A_i^\lambda}(a_{i,0}^\lambda), \mu_{A_i^\lambda}(a_{i,1}^\lambda), \ldots, \mu_{A_i^\lambda}(a_{i,s-2}^\lambda), \mu_{A_i^\lambda}(a_{i,s-1}^\lambda))$ of the *i*th antecedent variable X_i of the intermediate fuzzy rule R_λ with respect to the CPs $a_{i,u}^*$ and $a_{i,u-1}^*$ of the *i*th observation PFS $A_i^* = (a_{i,0}^*, a_{i,1}^*, \ldots, a_{i,s-2}^*, a_{i,s-1}^*; \mu_{A_i^*}(a_{i,0}^*), \mu_{A_i^*}(a_{i,1}^*), \ldots, \mu_{A_i^*}(a_{i,s-2}^*), \mu_{A_i^*}(a_{i,s-1}^*))$, shown as follows:

$$\overline{a_{i,u}^\lambda a_{i,u-1}^\lambda} = \frac{a_{i,u}^* - a_{i,u-1}^*}{a_{i,u}^\lambda - a_{i,u-1}^\lambda}, \tag{27}$$

where $\left\lfloor \frac{s-1}{2} \right\rfloor + 2 \le u \le s - 1$. Modify the CP $a_{i,u-1}^\lambda$ of PFS $A_i^\lambda = (a_{i,0}^\lambda, a_{i,1}^\lambda, \ldots, a_{i,s-2}^\lambda, a_{i,s-1}^\lambda; \mu_{A_i^\lambda}(a_{i,0}^\lambda), \mu_{A_i^\lambda}(a_{i,1}^\lambda), \ldots, \mu_{A_i^\lambda}(a_{i,s-2}^\lambda), \mu_{A_i^\lambda}(a_{i,s-1}^\lambda))$ of the *i*th antecedent variable X_i of the intermediate fuzzy rule R_λ, shown as follows:

$$a_{i,u-1}^{\lambda} = a_{i,s-1}^{\lambda} - \left(a_{i,u}^{\lambda} - a_{i,u-1}^{\lambda}\right) \times \overline{a_{i,u}^{\lambda} a_{i,u-1}^{\lambda}}, \tag{28}$$

where $\left\lfloor \frac{s-1}{2} \right\rfloor + 2 \le u \le s-1$

End

End;

For $u = 0$ **to** $\left\lfloor \frac{s-1}{2} \right\rfloor - 1$ **Do**

Calculate the CP b_{u+1}^* of PFS $B^* = (b_0^*, b_1^*, \ldots, b_{s-2}^*, b_{s-1}^*;$ $\mu_{B^*}(b_0^*), \mu_{B^*}(b_1^*), \ldots, \mu_{B^*}(b_{s-2}^*), \mu_{B^*}(b_{s-1}^*))$ of the FIR result, shown as follows:

$$b_{u+1}^* = b_0^* + \frac{\frac{1}{m}\sum_{i=1}^{m}\left[\left(a_{i,u+1}^{\lambda} - a_{i,u}^{\lambda}\right) \times \overline{a_{i,u+1}^{\lambda} a_{i,u}^{\lambda}}\right]}{2}, \tag{29}$$

where $0 \le u \le \left\lfloor \frac{s-1}{2} \right\rfloor - 1$ and $1 \le i \le m$

End;

For $u = \left\lfloor \frac{s-1}{2} \right\rfloor + 2$ **to** $s-1$ **Do**

Calculate the CP b_{u-1}^* of PFS $B^* = (b_0^*, b_1^*, \ldots, b_{s-2}^*, b_{s-1}^*;$ $\mu_{B^*}(b_0^*), \mu_{B^*}(b_1^*), \ldots, \mu_{B^*}(b_{s-2}^*), \mu_{B^*}(b_{s-1}^*))$ of the FIR result, shown as follows:

$$b_{u-1}^* = b_{s-1}^* - \frac{\frac{1}{m}\sum_{i=1}^{m}\left[\left(a_{i,u}^{\lambda} - a_{i,u-1}^{\lambda}\right) \times \overline{a_{i,u}^{\lambda} a_{i,u-1}^{\lambda}}\right]}{2}, \tag{30}$$

where $\left\lfloor \frac{s-1}{2} \right\rfloor + 2 \le u \le s-1$ and $1 \le i \le m$

End.

Step 8: Calculate the CPs $a_{i,0}^{\lambda}$, $a_{i,1}^{\lambda}$, … , $a_{i,s-2}^{\lambda}$ and $a_{i,s-1}^{\lambda}$ of PFS $A_i^{\lambda} = (a_{i,0}^{\lambda}, a_{i,1}^{\lambda}, \ldots, a_{i,s-2}^{\lambda}, a_{i,s-1}^{\lambda}; \mu_{A_i^{\lambda}}(a_{i,0}^{\lambda}), \mu_{A_i^{\lambda}}(a_{i,1}^{\lambda}), \ldots, \mu_{A_i^{\lambda}}(a_{i,s-2}^{\lambda}), \mu_{A_i^{\lambda}}(a_{i,s-1}^{\lambda})$ of the *i*th antecedent variable X_i of the intermediate fuzzy rule R_λ and calculate the CPs b_0^*, b_1^*, …, b_{s-2}^* and b_{s-1}^* of the PFS $B^* = (b_0^*, b_1^*, \ldots, b_{s-2}^*, b_{s-1}^*;$ $\mu_{B^*}(b_0^*), \mu_{B^*}(b_1^*), \ldots, \mu_{B^*}(b_{s-2}^*), \mu_{B^*}(b_{s-1}^*))$ of the FIR result, respectively, shown as follows:

For $i = 1$ **to** m **Do**

For $t = 0$ **to** $s-1$ **Do**

Calculate the move rate $\phi_{a_{i,t}^{\lambda}}$ of the CP $a_{i,t}^{\lambda}$ of PFS $A_i^{\lambda} = (a_{i,0}^{\lambda}, a_{i,1}^{\lambda}, \ldots, a_{i,s-2}^{\lambda}, a_{i,s-1}^{\lambda}; \mu_{A_i^{\lambda}}(a_{i,0}^{\lambda}), \mu_{A_i^{\lambda}}(a_{i,1}^{\lambda}), \ldots, \mu_{A_i^{\lambda}}(a_{i,s-2}^{\lambda}), \mu_{A_i^{\lambda}}(a_{i,s-1}^{\lambda})$ of the ith antecedent variable X_i of the intermediate fuzzy rule R_{λ} with respect to the CP $a_{i,t}^{*}$ of the ith observation PFS $A_i^{*} = (a_{i,0}^{*}, a_{i,1}^{*}, \ldots, a_{i,s-2}^{*}, a_{i,s-1}^{*}; \mu_{A_i^{*}}(a_{i,0}^{*}), \mu_{A_i^{*}}(a_{i,1}^{*}), \ldots, \mu_{A_i^{*}}(a_{i,s-2}^{*}), \mu_{A_i^{*}}(a_{i,s-1}^{*}))$, shown as follows:

$$\phi_{a_{i,t}^{\lambda}} = a_{i,t}^{*} - a_{i,t}^{\lambda}, \tag{31}$$

where $0 \le t \le s-1$. Modify the CP $a_{i,t}^{\lambda}$ of PFS $A_i^{\lambda} = (a_{i,0}^{\lambda}, a_{i,1}^{\lambda}, \ldots, a_{i,s-2}^{\lambda}, a_{i,s-1}^{\lambda}; \mu_{A_i^{\lambda}}(a_{i,0}^{\lambda}), \mu_{A_i^{\lambda}}(a_{i,1}^{\lambda}), \ldots, \mu_{A_i^{\lambda}}(a_{i,s-2}^{\lambda}), \mu_{A_i^{\lambda}}(a_{i,s-1}^{\lambda})$ of the ith antecedent variable X_i of the intermediate fuzzy rule R_{λ}, shown as follows:

$$a_{i,t}^{\lambda} = a_{i,t}^{\lambda} + \phi_{a_{i,t}^{\lambda}}, \tag{32}$$

where $0 \le t \le s-1$.

End

End;

For $t = 0$ **to** $s-1$ **Do**

Calculate the move rate $\phi_{b_t^{*}}$ of the CP b_t^{*} of PFS $B^{*} = (b_0^{*}, b_1^{*}, \ldots, b_{s-2}^{*}, b_{s-1}^{*}; \mu_{B^{*}}(b_0^{*}), \mu_{B^{*}}(b_1^{*}), \ldots, \mu_{B^{*}}(b_{s-2}^{*}), \mu_{B^{*}}(b_{s-1}^{*}))$ of the FIR result, shown as follows:

$$\phi_{b_t^{*}} = \frac{1}{m}\sum_{i=1}^{m} \phi_{a_{i,t}^{\lambda}}, \tag{33}$$

where $1 \le i \le m$ and $0 \le t \le s-1$. Modify the CP b_t^{*} of PFS $B^{*} = (b_0^{*}, b_1^{*}, \ldots, b_{s-2}^{*}, b_{s-1}^{*}; \mu_{B^{*}}(b_0^{*}), \mu_{B^{*}}(b_1^{*}), \ldots, \mu_{B^{*}}(b_{s-2}^{*}), \mu_{B^{*}}(b_{s-1}^{*}))$ of the FIR result, shown as follows:

$$b_t^* = b_t^* + \phi_{b_t^*}, \tag{34}$$

where $0 \le t \le s-1$

End.

Therefore, we can get the final FIR result B^* represented by a PFS, where $B^* = (b_0^*, b_1^*, \ldots, b_{s-2}^*, b_{s-1}^*; \mu_{B^*}(b_0^*), \mu_{B^*}(b_1^*), \ldots, \mu_{B^*}(b_{s-2}^*), \mu_{B^*}(b_{s-1}^*))$.

4. Experimental results

In this section, we use six examples to compare the FIR results of the proposed method with the ones of the methods presented in [3], [8], [12], [16]-[18], [21] and [34].

Example 4.1 **(Diarrheal Disease Prediction Problem) [12], [34]:** The diarrheal disease rate of a remote village is directly affected by two factors as shown in Fig. 3 [12], [34], i.e., (1) a low social connectedness tends to failure in creating an adequate water and sanitation infrastructure, which is usually resulting in a higher diarrheal disease rate and (2) more frequent contact between the residents within the village and those outside the village tends to increase the rate of the introduction of pathogens, which raises the diarrheal disease rate. From Fig. 3, we can see that there are five variables in the diarrheal disease prediction problem, i.e., “Contact Outside of the Village”, “Reintroduction of Pathogenic Strains”, “Social Connectedness”, “Hygiene and Sanitation Infrastructure”, and “Diarrheal Disease Rate”, which are represented as the fuzzy variables X_1, X_2, X_3, X_4 and X_5, respectively. The fuzzy rules in the sparse fuzzy rule base are shown as follows [12], [34]:

R_1: IF X_1 is A_1 THEN X_2 is B_1,
R_2: IF X_1 is A_2 THEN X_2 is B_2,
R_3: IF X_1 is A_3 THEN X_2 is B_3,
R_4: IF X_3 is C_4 THEN X_4 is D_4,
R_5: IF X_3 is C_5 THEN X_4 is D_5,
R_6: IF X_2 is B_6 and X_4 is D_6 THEN X_5 is E_6,

R_7: IF X_2 is B_7 and X_4 is D_7 THEN X_5 is E_7,

R_8: IF X_2 is B_8 and X_4 is D_8 THEN X_5 is E_8.

The fuzzy variables and their object values represented by trapezoidal FSs are shown in Table 1 [12], [34].

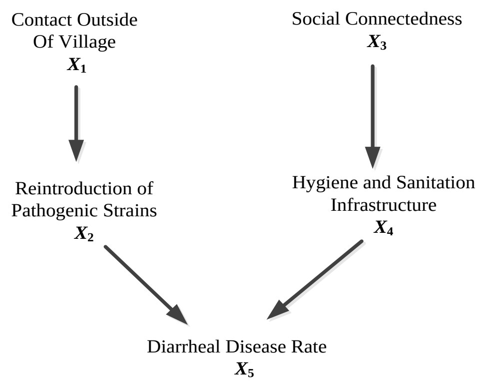

Fig. 3. Causal diagram of the application problem [12], [34].

Table 1
Variables and their object values of ***Example 4.1*** [12], [34].

Variables	Object values
X_1	A_1 = (0.38, 0.40, 0.42, 0.44; 0, 1, 1, 0); A_2 = (0.70, 0.72, 0.74, 0.76; 0, 1, 1, 0); A_3 = (0.90, 0.92, 0.94, 0.96; 0, 1, 1, 0)
X_2	B_1 = (0.46, 0.48, 0.50, 0.52; 0, 1, 1, 0); B_2 = (0.65, 0.67, 0.69, 0.71; 0, 1, 1, 0); B_3 = (0.76, 0.78, 0.80, 0.82; 0, 1, 1, 0); B_6 = (0.30, 0.32, 0.34, 0.36; 0, 1, 1, 0); B_7 = (0.60, 0.62, 0.64, 0.66; 0, 1, 1, 0); B_8 = (0.92, 0.94, 0.96, 0.98; 0, 1, 1, 0)
X_3	C_4 = (0.28, 0.30, 0.32, 0.34; 0, 1, 1, 0); C_5 = (0.55, 0.57, 0.59, 0.61; 0, 1, 1, 0)
X_4	D_4 = (0.26, 0.28, 0.30, 0.32; 0, 1, 1, 0); D_5 = (0.61, 0.63, 0.65, 0.67; 0, 1, 1, 0); D_6= (0.36, 0.38, 0.40, 0.42; 0, 1, 1, 0); D_7 = (0.58, 0.60, 0.62, 0.64; 0, 1, 1, 0); D_8 = (0.88, 0.90, 0.92, 0.94; 0, 1, 1, 0)
X_5	E_6= (0.18, 0.20, 0.22, 0.24; 0, 1, 1, 0); E_7 = (0.68, 0.70, 0.72, 0.74; 0, 1, 1, 0); E_8 = (0.76, 0.78, 0.80, 0.82; 0, 1, 1, 0)

Fig. 4 shows the hierarchical component-based diagram [12], [34], where the FIR component F_1 takes the observation FS $A^* = (0.69,\ 0.75,\ 0.76,\ 0.77;\ 0,\ 1,\ 1,\ 0)$ as its input and produces the FIR result B^* as its output; the FIR component F_2 takes the observation FS $C^* = (0.50,\ 0.51,\ 0.53,\ 0.54;\ 0,\ 1,\ 1,\ 0)$ as its input and produces the FIR result D^* as its output; the FIR component F_3 takes the obtained FIR result B^* of the FIR component F_1 and the obtained FIR result D^* of the FIR component F_2 as its inputs and produces the FIR result E^* as its output.

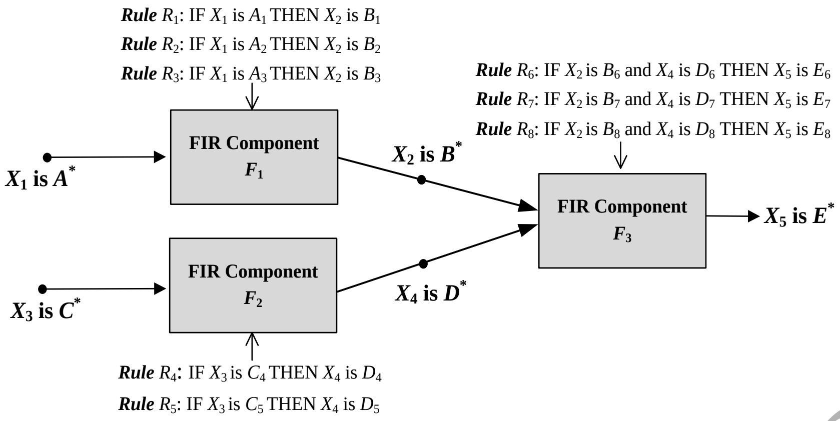

Fig. 4. Hierarchical component-based diagram [12].

Fig. 5 shows the FIR results B^*, D^* and E^* of Yang and Shen's method [34], which produced abnormal FIR results B^*, D^* and E^*. Fig. 6 shows the FIR results of Cheng *et al.*'s method [12], where the FIR results B^*, D^* and E^* obtained by Cheng *et al.*'s method [12] are normal and convex represented by trapezoidal FSs on the X-axis. Fig. 7 shows the FIR results of the proposed method, where the FIR results B^*, D^* and E^* obtained by the proposed method are normal and convex represented by trapezoidal FSs on the X-axis. Moreover, from Fig. 6 and Fig. 7, we also can see that Cheng *et al.*'s method [12] and the proposed method can maintain the shape of the FIR results B^*, D^* and E^* to be the same as the shape of the given observations A^* and C^* represented by trapezoidal FSs. Therefore, the proposed method and Cheng *et al.*'s method [12] can overcome the limitation of Yang and Shen's method [34] in this situation.

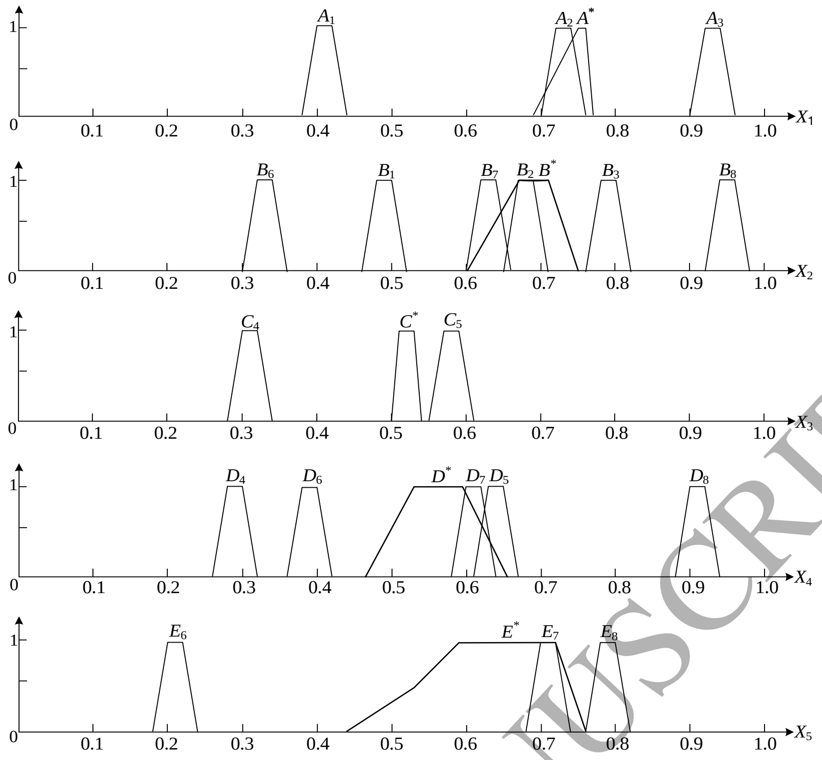

Fig. 5. FIR results B^*, D^* and E^* of Yang and Shen's method [34] for ***Example 4.1***.

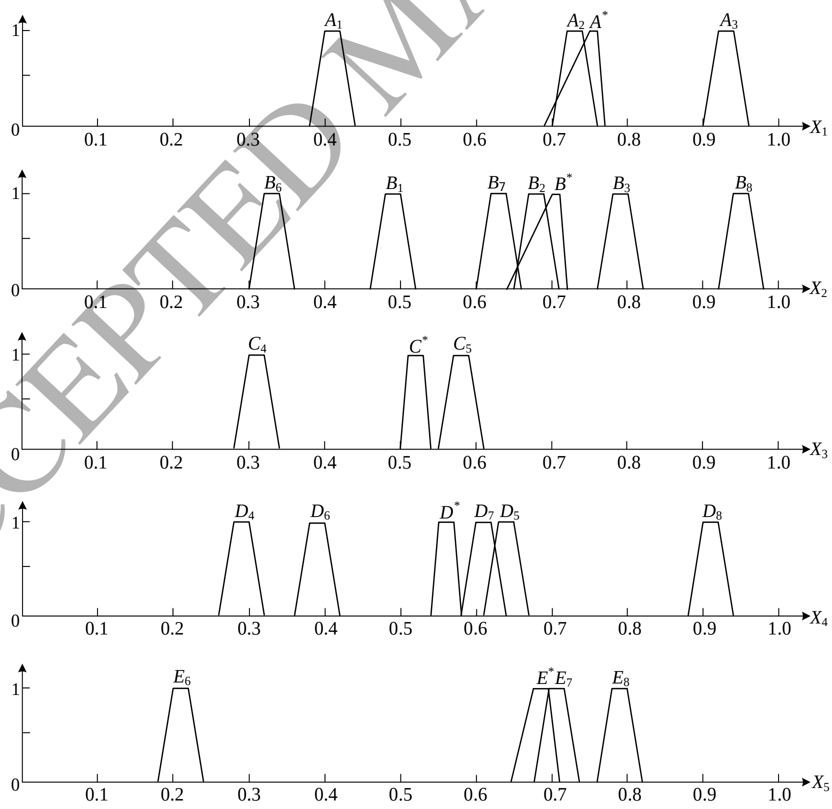

Fig. 6. FIR results B^*, D^* and E^* of Cheng *et al*.'s method [12] for ***Example 4.1***.

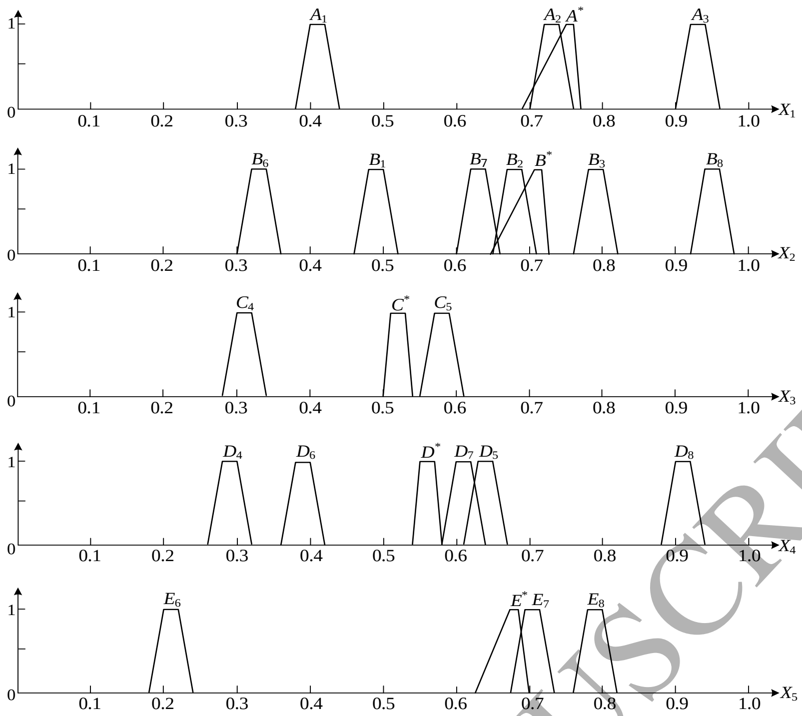

Fig. 7. FIR results B^*, D^* and E^* of the proposed method for ***Example 4.1***.

Example 4.2. Let us consider the eight fuzzy rules for predicting the diarrheal disease prediction rates shown in ***Example 4.1***. Let us consider the variables and their object values represented by trapezoidal FSs shown in Table 2.

Table 2
Variables and their object values of ***Example 4.2***.

Variables	Object values
X_1	A_1 = (0.38, 0.40, 0.42, 0.44; 0, 1, 1, 0); A_2 = (0.70, 0.72, 0.74, 0.76; 0, 1, 1, 0); A_3 = (0.90, 0.92, 0.94, 0.96; 0, 1, 1, 0)
X_2	B_1 = (0.46, 0.48, 0.50, 0.52; 0, 1, 1, 0); B_2 = (0.65, 0.67, 0.69, 0.71; 0, 1, 1, 0); B_3 = (0.76, 0.78, 0.80, 0.82; 0, 1, 1, 0); B_6 = (0.30, 0.32, 0.34, 0.36; 0, 1, 1, 0); B_7 = (0.60, 0.62, 0.64, 0.66; 0, 1, 1, 0); B_8 = (0.92, 0.94, 0.96, 0.98; 0, 1, 1, 0)
X_3	C_4 = (0.28, 0.34, 0.37, 0.38; 0, 1, 1, 0); C_5 = (0.55, 0.56, 0.62, 0.63; 0, 1, 1, 0)
X_4	D_4 = (0.26, 0.28, 0.30, 0.32; 0, 1, 1, 0); D_5 = (0.61, 0.63, 0.65, 0.67; 0, 1, 1, 0); D_6= (0.36, 0.38, 0.40, 0.42; 0, 1, 1, 0); D_7 = (0.58, 0.60, 0.62, 0.64; 0, 1, 1, 0); D_8 = (0.88, 0.90, 0.92, 0.94; 0, 1, 1, 0)
X_5	E_6= (0.18, 0.20, 0.22, 0.24; 0, 1, 1, 0); E_7 = (0.68, 0.70, 0.72, 0.74; 0, 1, 1, 0); E_8 = (0.76, 0.78, 0.80, 0.82; 0, 1, 1, 0)

Fig. 8 shows the FIR results B^*, D^* and E^* of the method presented in [34], which produced abnormal FIR results B^*, D^* and E^*. Fig. 9 shows the FIR results B^*,

D^* and E^* of Cheng *et al*.'s method [12], which produced an abnormal twisted FS D^* of the FIR result and produced an abnormal triangular FS E^* of the FIR result, which both are not reasonable due to the fact that the abnormal twisted FS D^* is not a convex FS and the width of the abnormal triangular FS E^* is too wide. Therefore, Cheng *et al.*'s method [12] cannot preserve the normality and the convexity of the FIR results in this situation. On the other hand, from Fig. 10, we can see that the proposed method produced normal and convex trapezoidal FSs B^*, D^* and E^* of the FIR results. Moreover, from Fig. 10 we also can see that the proposed method can maintain the shape of the FIR results B^*, D^* and E^* to be the same as the shape of the given observation FSs A^* and C^* represented by trapezoidal FSs.

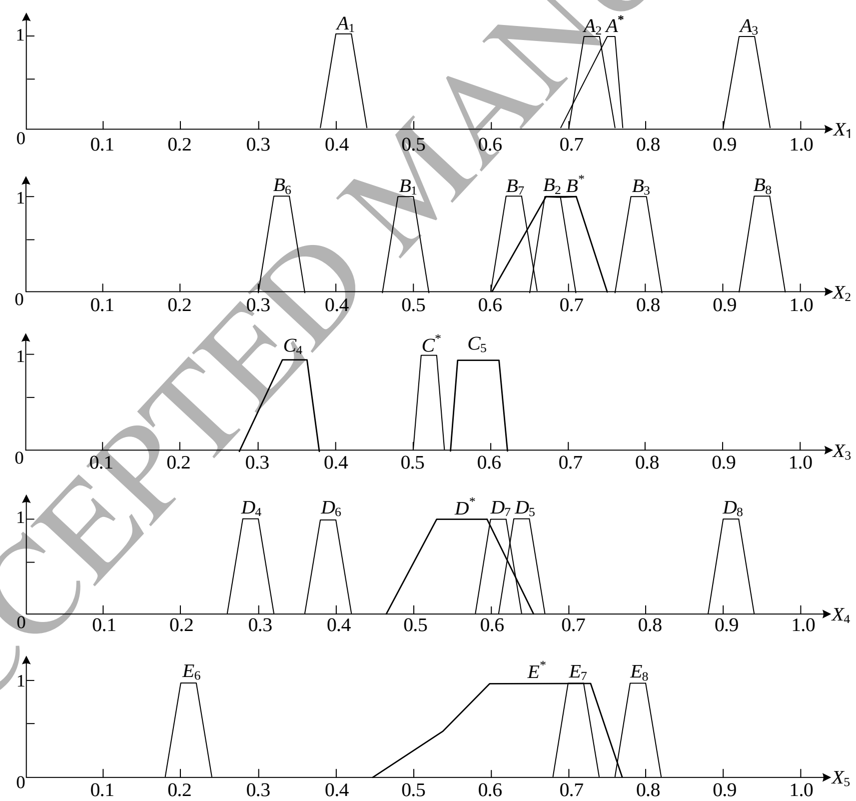

Fig. 8. FIR results B^*, D^* and E^* of Yang and Shen's method [34] for ***Example 4.2***.

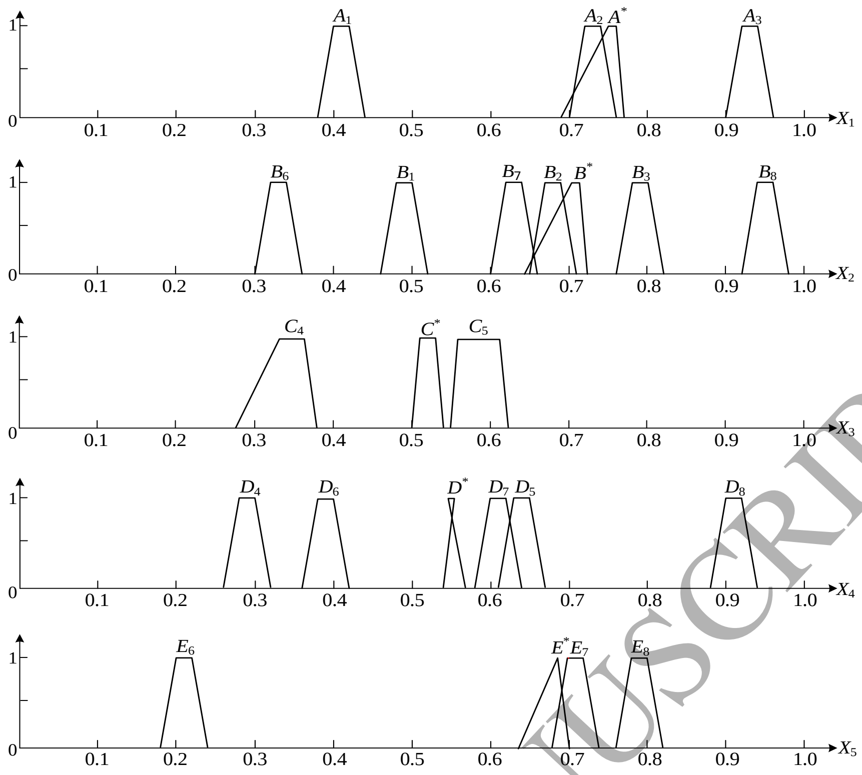

Fig. 9. FIR results B^*, D^* and E^* of Cheng *et al.*'s method [12] for ***Example 4.2***.

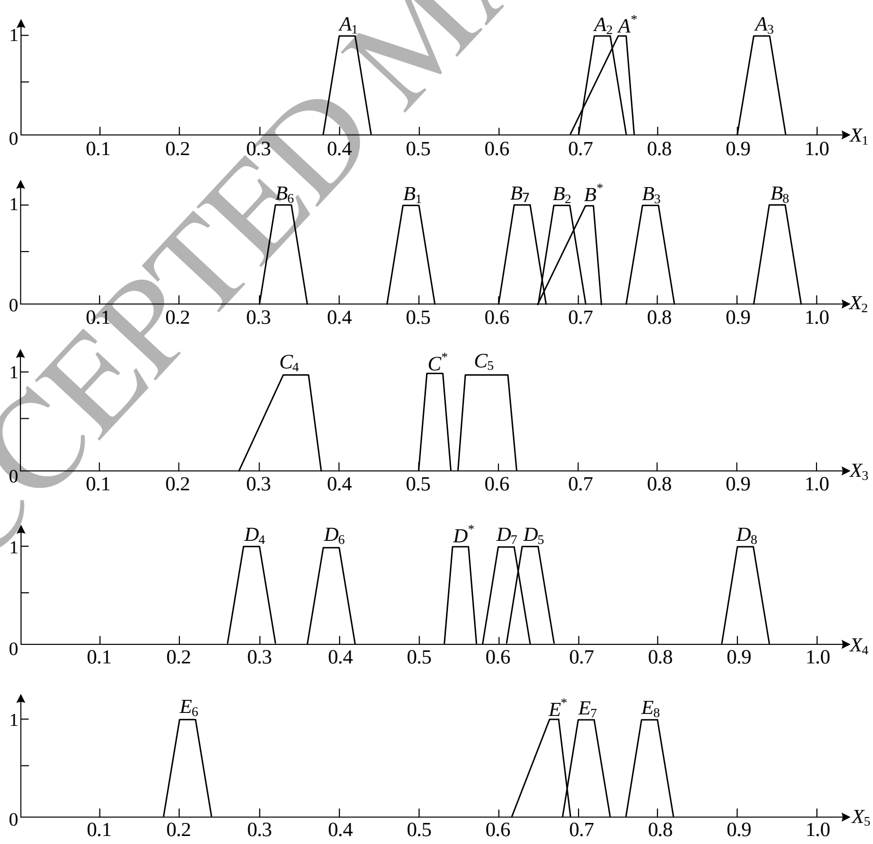

Fig. 10. FIR results B^*, D^* and E^* of the proposed method for ***Example 4.2***.

***Example 4.3* [3], [12]:** Let us consider the following FIR scheme:

Rule R_1: IF X is A_1 THEN Y is B_1

Rule R_2: IF X is A_2 THEN Y is B_2

Observation: IF X is A^*

Conclusion: Y is B^*

where A_1, A_2, B_1 and B_2 are triangular FSs, A_1 = (0, 5, 6; 0, 1, 0), A_2 = (11, 13, 14; 0, 1, 0), B_1 = (0, 2 , 4; 0 ,1, 0), B_2 = (0, 11, 13; 0, 1, 0), A^* is a singleton FS and A^* = (8, 8, 8; 0, 1, 0) = 8. According to [12], the triangular FSs A_1, A_2, B_1, B_2 and A^* can be represented by trapezoidal fuzzy sets using an even number of CPs, respectively, where A_1 = (0, 5, 5, 6; 0, 1, 1, 0), A_2 = (11, 13, 13, 14; 0, 1, 1, 0), B_1 = (0, 2, 2, 4; 0, 1, 1, 0), B_2 = (10, 11, 11, 13; 0, 1, 1, 0) and A^* = (8, 8, 8, 8; 0, 1, 1, 0). A comparison of the membership function curves of the FIR result B^* for different methods is shown in Fig. 11. From Fig. 11, we can see that the method presented in [21] produced an abnormal FIR result B^*, the method presented in [16] produced an abnormal rectangular FIR result B^* and the method presented in [34] produced a triangular FIR result B^*, whereas the methods presented in [3], [8], [12], [17] and [18] produced the singleton FIR result B^*. From Fig. 11, we also can see that the proposed method also produced the singleton FIR result B^*. Therefore, in this situation, the proposed method can overcome the limitations of methods presented in [16], [21] and [34] in terms of the shape of the FIR result B^*.

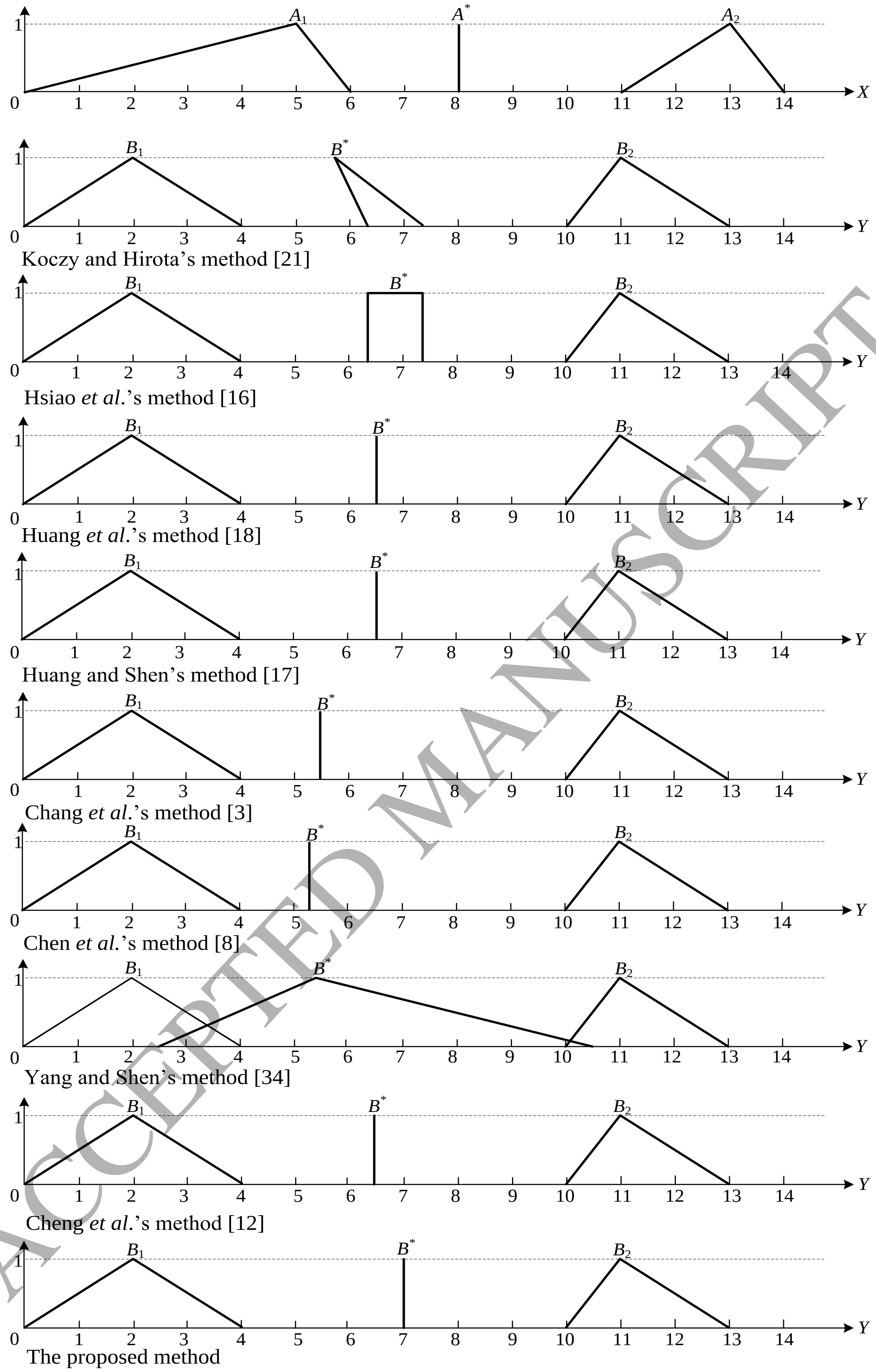

Fig. 11. A comparison of the membership function curves of the FIR result B^* for different methods for ***Example 4.3***.

***Example 4.4* [3], [12]:** Let us consider the following FIR scheme:

Rule R_1: IF X is A_1 THEN Y is B_1

Rule R_2: IF X is A_2 THEN Y is B_2

Observation: IF X is A^*

Conclusion: Y is B^*

where A_1 is a hexagonal FS represented by A_1 = (0, 1, 3, 4, 5, 5.5; 0, 0.5, 1, 1, 0.5, 0), A_2 is a trapezoidal FS represented by A_2 = (11, 12, 13, 14; 0, 1, 1, 0), B_1 is a pentagonal FS represented by B_1 = (0, 1, 3, 4, 4.5; 0, 1, 1, 0.5, 0), B_2 is a pentagonal FS represented by B_2 = (10.5, 11, 12, 13, 14; 0, 1, 1, 0.5, 0), A^* is a pentagonal FS represented by A^* = (6, 7, 9, 10, 10.5; 0, 1, 1, 0.5, 0). According to [12], the trapezoidal FS A_2, the pentagonal FS B_1, the pentagonal FS B_2 and the pentagonal FS A^* can be represented by hexagonal FSs characterized by six CPs, respectively, where A_2 = (11, 11.5, 12, 13, 13.5, 14; 0, 0.5, 1, 1, 0.5, 0), B_1 = (0, 0.5, 1, 3, 4, 4.5; 0, 0.5, 1, 1, 0.5, 0), B_2 = (10.5, 11, 12, 13, 13.5, 14; 0, 0.5, 1, 1, 0.5, 0) and A^* = (6, 6.5, 7, 9, 10, 10.5; 0, 0.5, 1, 1, 0.5, 0). In this example, the methods presented in [16] and [18] cannot produce the FIR result B^* due to the fact that they cannot handle FIR using PFSs. Fig. 12 shows a comparison of the membership function curves of the FIR result B^* of the proposed method with the ones of the methods presented in [3], [8], [12], [17], [21] and [34]. From Fig. 12, we can see that the FIR result B^* of the methods presented in [21] and [34] produced an abnormal FIR result B^*; the methods presented in [8] and [12] produced convex FSs of the FIR result B^*, but they have different shapes from the observation FS A^*. Therefore, from Fig. 12, we can see that the FIR result B^* obtained by the proposed method and the methods presented in [3] and [17] are more reasonable than the ones obtained by the methods presented in [8], [12], [16], [18], [21] and [34] in terms of the shape of the FIR result B^*.

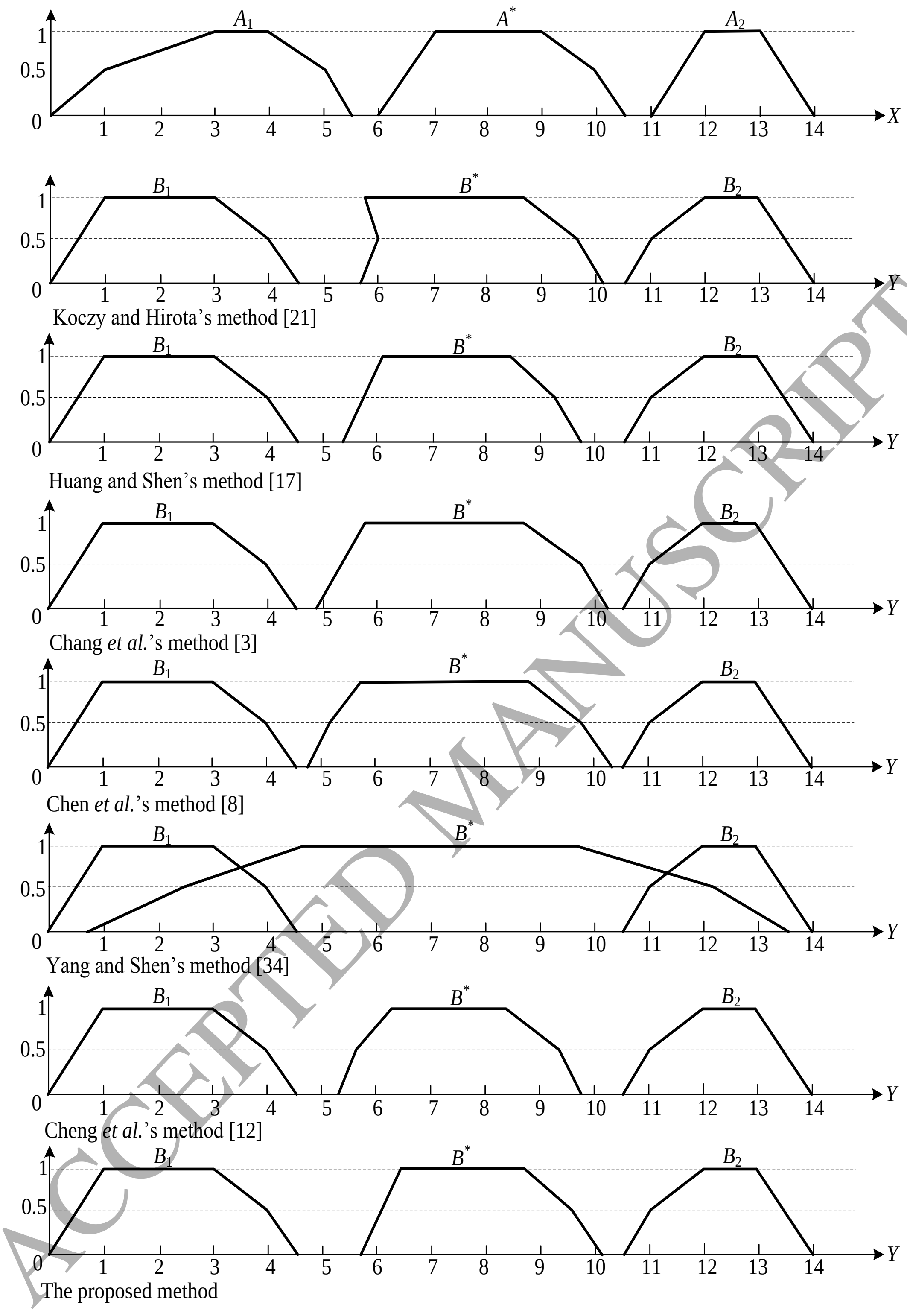

Fig. 12. A comparison of the membership function curves of the FIR result B^* for different methods for ***Example 4.4***.

Example 4.5 **[3], [12]:** Let us consider the following FIR scheme:

Rule R_1: IF X is A_1 THEN Y is B_1

Rule R_2: IF X is A_2 THEN Y is B_2

Observation: IF X is A^*

Conclusion: Y is B^*

where A_1, A_2, B_1, B_2 and A^* are trapezoidal FSs, A_1 = (0, 4, 5, 6; 0, 1, 1, 0), A_2 = (11, 12, 13, 14; 0, 1, 1, 0), B_1 = (0, 2, 3, 4; 0, 1, 1, 0), B_2 = (10, 11, 12, 13; 0, 1, 1, 0) and A^* = (6, 6, 9, 10; 0, 1, 1, 0). In this example, the method presented in [16] cannot produce the FIR result B^* due to the fact that the slopes of the antecedent FSs of fuzzy rules cannot be calculated. A comparison of the membership function curves of the FIR result B^* for different methods is shown in Fig. 13. From Fig. 13, we can see that the method presented in [21] produced an abnormal FIR result B^*, the method presented in [18] produced an abnormal twisted FIR result B^* and the method presented in [34] produced an unreasonable FIR result B^*, whereas the FIR result B^* of the methods presented in [3], [8], [12], [17], [18] and the proposed method produced a convex and normal FIR result B^*.

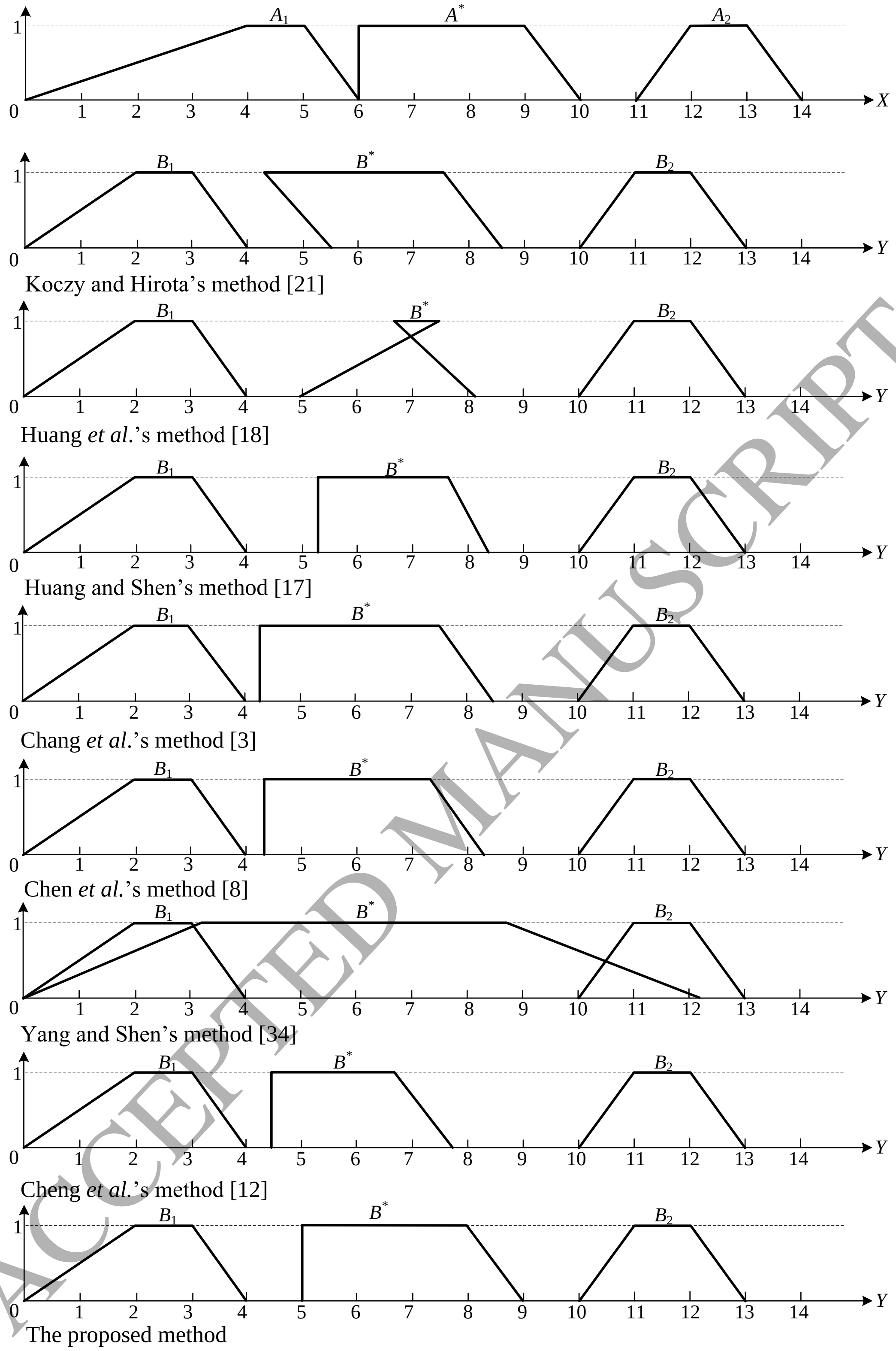

Fig. 13. A comparison of the membership function curves of the FIR result B^* for different methods for ***Example 4.5***.

***Example 4.6* [3], [12]:** Let us consider the following FIR scheme:

Rule R_1: IF X_1 is A_{11} and X_2 is A_{12} THEN Y is B_1

Rule R_2: IF X_1 is A_{21} and X_2 is A_{22} THEN Y is B_2

Observation: IF X_1 is A_1^* and X_2 is A_2^*

Conclusion: Y is B^*

where A_{11}, A_{12}, A_{21}, A_{22}, B_1, B_2, A_1^* and A_2^* are trapezoidal FSs, A_{11} = (0, 4, 5, 6; 0, 1, 1, 0), A_{12} = (1, 2, 3, 4; 0, 1, 1, 0), A_{21} = (11, 12, 13, 14; 0, 1, 1, 0), A_{22} = (12, 14, 15, 16; 0, 1, 1, 0), B_1 = (0, 2, 3, 4; 0, 1, 1, 0), B_2 = (10, 11, 12, 13; 0, 1, 1, 0), A_1^* = (6, 7, 9, 11; 0, 1, 1, 0) and A_2^* = (6, 8, 10, 12; 0, 1, 1, 0). In this example, the methods presented in [16] and [18] cannot produce the FIR result B^* due to the fact that they cannot deal with FIR with multiple antecedent variables. Fig. 14 shows a comparison of the membership function curves of the FIR result B^* for different methods. From Fig. 14, we can see that the FIR result B^* of the method presented in [34] produced an unreasonable FIR result B^*, whereas the methods presented in [3], [8], [12] and [17] and the proposed method produced a normal and convex FIR result B^*.

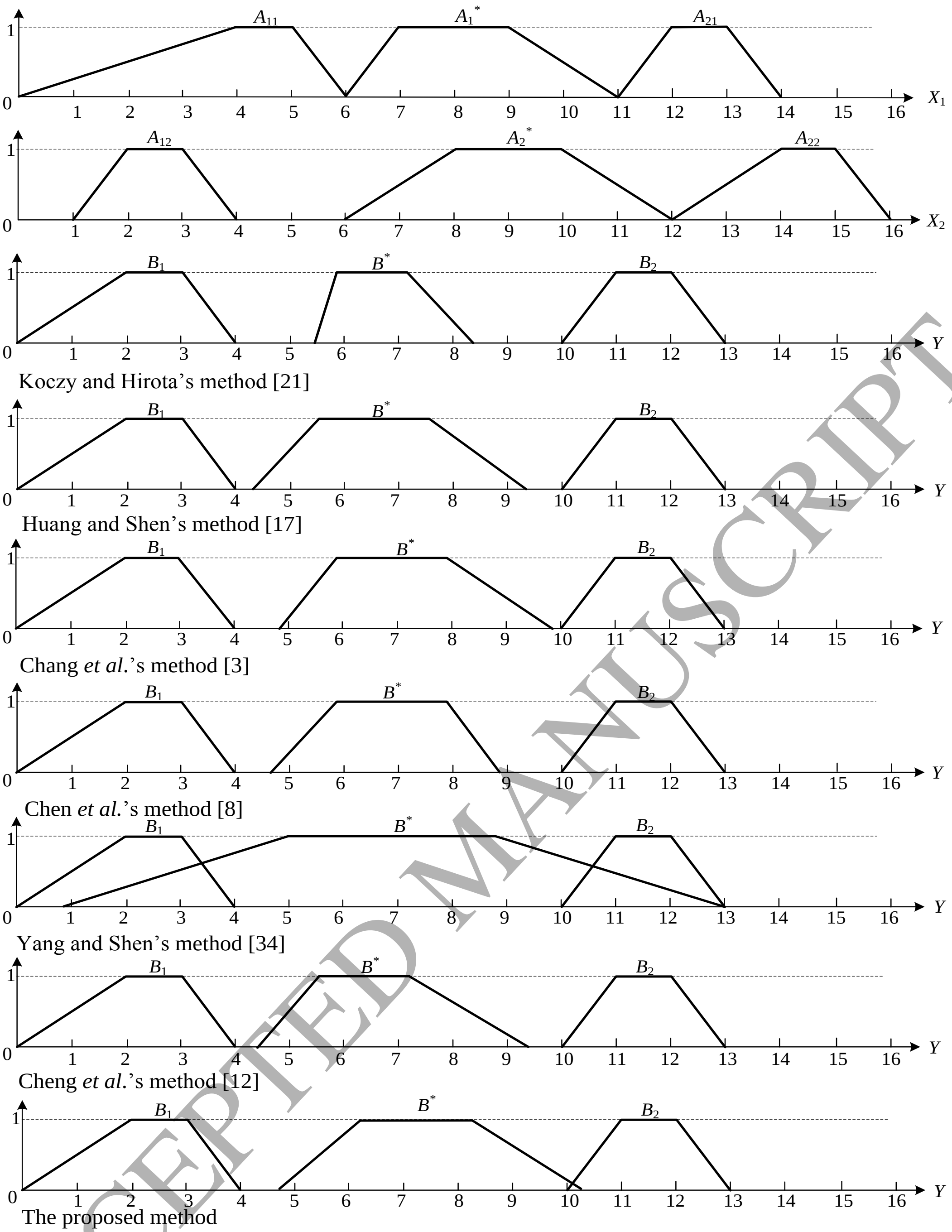

Fig. 14. A comparison of the membership function curves of the FIR result B^* for different methods for ***Example 4.6***.

5. Conclusions

In this paper, we have proposed a new transformation-based weighted FIR method based on the ranking values (RVs) of polygonal fuzzy sets (PFSs) and the proposed new scale and move transformation techniques. The proposed weighted FIR method gets more reasonable FIR results than the existing methods [3], [8], [12], [16]-[18], [21], [34], where the weight of each antecedent variable and the weight of each fuzzy rule are generated automatically. Moreover, the proposed new scale and move transformation techniques can deal with weighted FIR using singleton FSs and PFSs. The differences between Cheng *et al*.'s method [12] and the proposed method are as follows:

1) Cheng *et al*.'s method [12] is categorized as a non-transformation-based FIR method, whereas the proposed method is categorized as a transformation-based FIR method due to the fact that the proposed method is based on the proposed scale and move transformation techniques.
2) When performing the FIR, Cheng *et al*.'s method [12] only considers the two closest fuzzy rules, whereas the proposed method considers all the fuzzy rules due to the fact that the proposed method deals with the FIR with multiple antecedent variables and multiple fuzzy rules.
3) Cheng *et al*.'s method [12] only can automatically calculate the weight of the antecedent variables, whereas the proposed method can automatically calculate the weight of the antecedent variables and the weight of the fuzzy rules.
4) To maintain the normality and the convexity of the FIR results, Cheng *et al*.'s method [12] uses the concept of the ratio of fuzziness, whereas the proposed method uses the proposed scale and move transformation techniques.

The main contribution of this paper is that we propose a new weighted FIR method and develop new scale and move transformation techniques to overcome the limitations of the methods presented in [3], [8], [12], [16]-[18], [21] and [34]. In the future, we will apply granular computing techniques [2], [14], [15], [23], [25], [27], [29], [31], [32], [35] to deal with FIR in sparse fuzzy rule-based systems.

Acknowledgements

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