EECS 598 Motion Planning HW #1

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1 Questions

1. (10 points) Chapter 4 exercise 5
A rigid body in $\mathcal{W} = \mathbb{R}^3$ has 6 degrees of freedom and a configuration space of $\mathbb{R}^3 \times \mathbb{RP}^3$. The $\mathbb{RP}^3$ part of the manifold comes from the following equation where the quaternion representing rotation in 3D space has the constraint that $a^2 + b^2 + c^2 + d^2 = 1$ ($\mathbb{S}^3 \cong \mathbb{RP}^3$). We now add the following three constraints: $b \neq v_1 \sin\frac{\theta}{2}$ and $c \neq v_2 \sin\frac{\theta}{2}$ and $d \neq v_3 \sin\frac{\theta}{2}$ at the same time. v represents the vector for which rotations about its axis does not change the configuration of the cylinder and is a 1D manifold. However, not including those combinations, the degrees of freedom of the cylinder is not reduced. Thus the C-space is still $\mathbb{R}^3 \times \mathbb{RP}^3$.

2. (15 points) Chapter 4 exercise 8
The robot can translate and rotate in 2D. Given the identification of the top and bottom of the screen and the sides of the screen, the 2D-manifold that represents the space in which the robot can translate is the torus (T^2). The robot can also rotate on the 1D manifold of a circle (with 0 and 2π identified) respresented as S^1 or T^1. Thus the entire C-space for the robot can be represented by $T^2 \times T^1 = T^3$.

3. (10 points) Chapter 4 exercise 14

(a)

$$v = [\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}], \theta = \frac{-\pi}{2}$$
$$a = \cos\frac{\theta}{2}, \cos(\frac{-\pi}{4}) = \frac{\sqrt{2}}{2}$$
$$b = v_1 \sin\frac{\theta}{2} = \frac{1}{\sqrt{3}} \sin(\frac{-\pi}{4}) = \frac{-1}{\sqrt{6}}$$
$$c = v_2 \sin\frac{\theta}{2} = \frac{1}{\sqrt{3}} \sin(\frac{-\pi}{4}) = \frac{-1}{\sqrt{6}}$$
$$d = v_3 \sin\frac{\theta}{2} = \frac{1}{\sqrt{3}} \sin(\frac{-\pi}{4}) = \frac{-1}{\sqrt{6}}$$
$$\boxed{h_1 = \frac{\sqrt{2}}{2} - \frac{1}{\sqrt{6}}i - \frac{1}{\sqrt{6}}j - \frac{1}{\sqrt{6}}k}$$

(b)

$$
\begin{aligned}
&v = [0,1,0], \theta = \pi \\
&a = \cos\frac{\theta}{2} = \cos\frac{\pi}{2} = 0 \\
&b = 0 \\
&c = v_2 \sin\frac{\theta}{2} = \sin\frac{\pi}{2} = 1 \\
&d = 0 \\
&\boxed{h_2 = 0 + 0 + j + 0 = j}
\end{aligned}
$$

(c) Quaternion multiplication can be calculated with vector multiplication, a dot product and a cross product.

$$
\begin{aligned}
&h_3 = h_2 h_1 = a_3 + b_3 i + c_3 j + d_3 k \\
&a_3 = a_2 a_1 - v_2 \cdot v_1 = \frac{1}{\sqrt{6}} \\
&[b_3 c_3 d_3]^\intercal = a_2 v_1 + a_1 v_2 + v_2 \times v_1 = [\frac{-1}{\sqrt{6}} \frac{1}{\sqrt{2}} \frac{1}{\sqrt{6}}]
\end{aligned}
$$

Finally, taking the quaternion and interpreting it as a vector in a direction plus a rotation can be done with the equations on page 151 and 152 in the Planning Algorithms book.

$$
\begin{aligned}
&\theta = 2 \arccos a_3 \\
&v_1 = \frac{b_3}{\sin\frac{\theta}{2}} \\
&v_2 = \frac{c_3}{\sin\frac{\theta}{2}} \\
&v_3 = \frac{d_3}{\sin\frac{\theta}{2}} \\
&\boxed{\theta = 2.3rad, \begin{bmatrix} v_1 & v_2 & v_3 \end{bmatrix} = \begin{bmatrix} -0.4472 & 0.7746 & 0.4472 \end{bmatrix}}
\end{aligned}
$$

4. (10 points) Chapter 4 exercise 16

There are 5 bodies and each body can move independently. If the world in which the bodies can move in is $\mathcal{W} = \mathbb{R}^3$, then the configuration space for the robot defined as a composite of all 5 bodies is as follows:

$$
\begin{aligned}
\mathbb{C} &= \mathbb{C}_1 \times \mathbb{C}_2 \times \mathbb{C}_3 \times \mathbb{C}_4 \times \mathbb{C}_5 \\
&= \boxed{\mathbb{R}^{15} \times \mathbb{RP}^3 \times \mathbb{RP}^3 \times \mathbb{RP}^3 \times \mathbb{RP}^3}
\end{aligned}
$$

2 Software

1. (15 points) Include a screenshot showing that the fastgrasping example runs successfully.

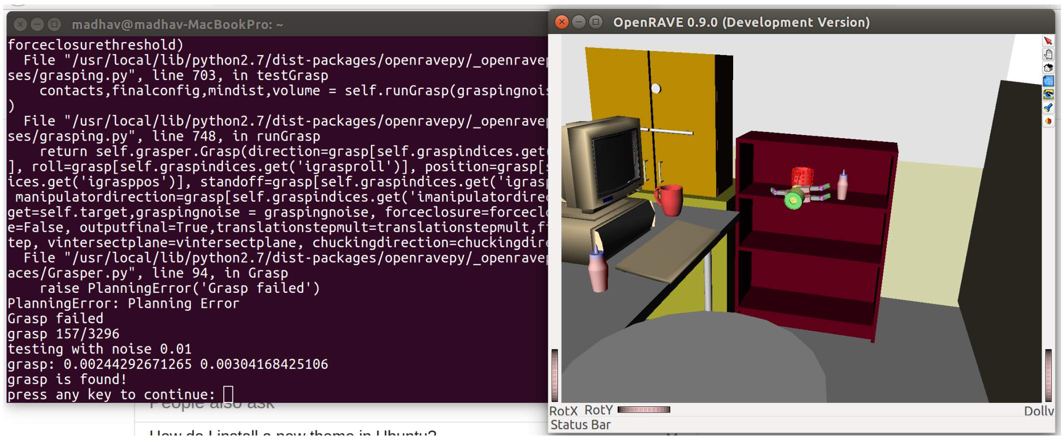

Figure 1: Screenshot after running `openrave.py --example fastgrasping`

3 Implementation

1. (5 points) Include a screenshot of the all the tables moved on the PR2's side of the room.

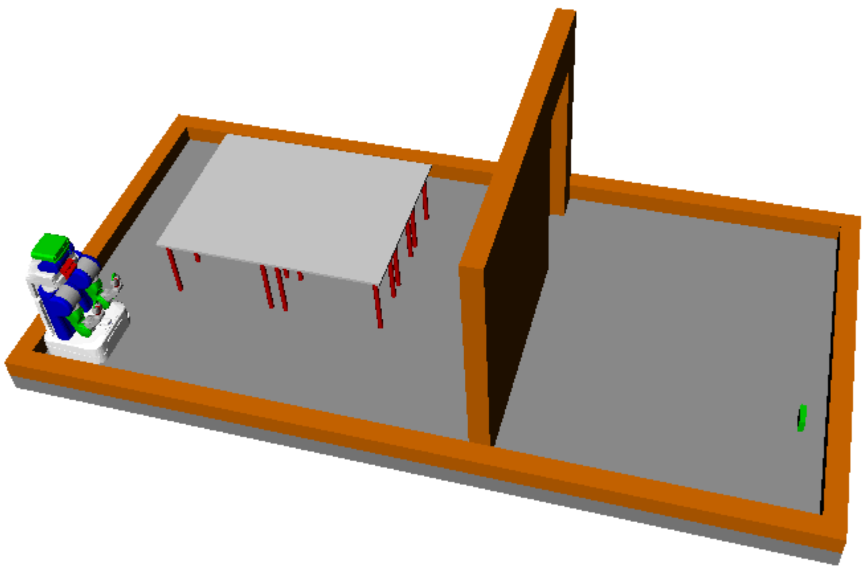

Figure 2: Screenshot after setting the initial poses of all the tables to the PR2 side of the room

2. (20 points) Include a screenshot of the Puma robot arm and the PR2 colliding. The PR2 robot's left arm should be extended causing it to collide with the PUMA.

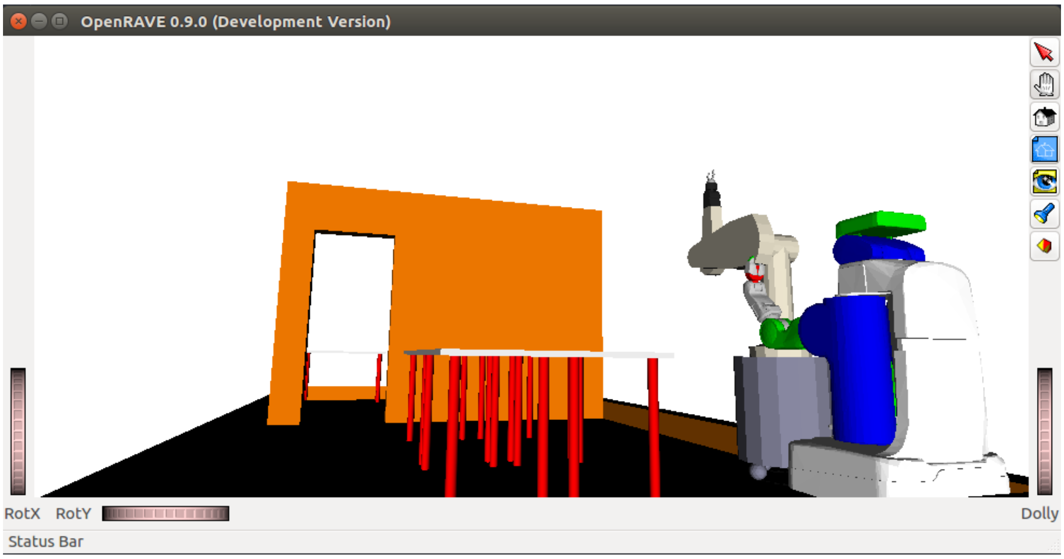

Figure 3: Screenshot of the left arm of the PR2 robot colliding with the PUMA gripper

3. (10 points) Draw a series of 35 blue points in a circle that encompasses the scene. In addition, draw red bounding boxes around all of the tables.

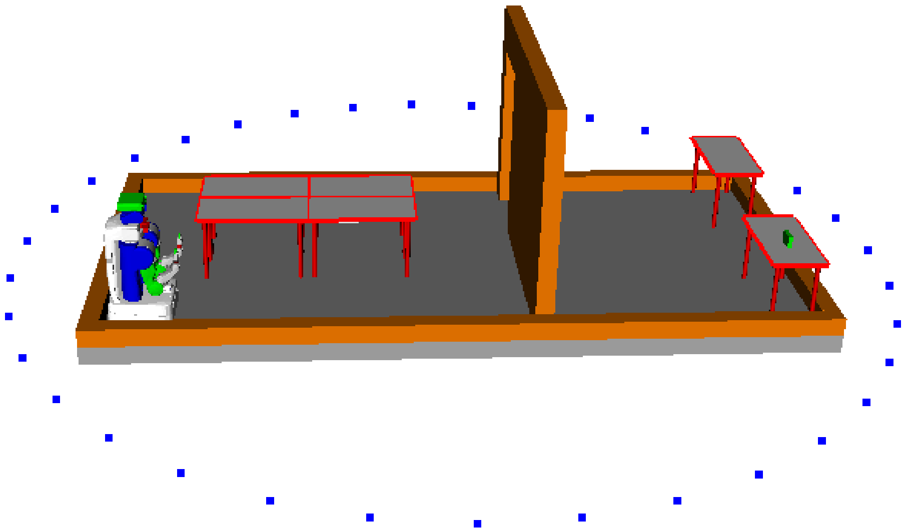

Figure 4: Screenshot of 35 blue points drawn around the scene along with bounding boxes around tables drawn in red

4. (15 points) Move the PR2 close to the wall and extend it's right arm. Use the SetDesired() function on the robot to comand the robot to the new configuration.

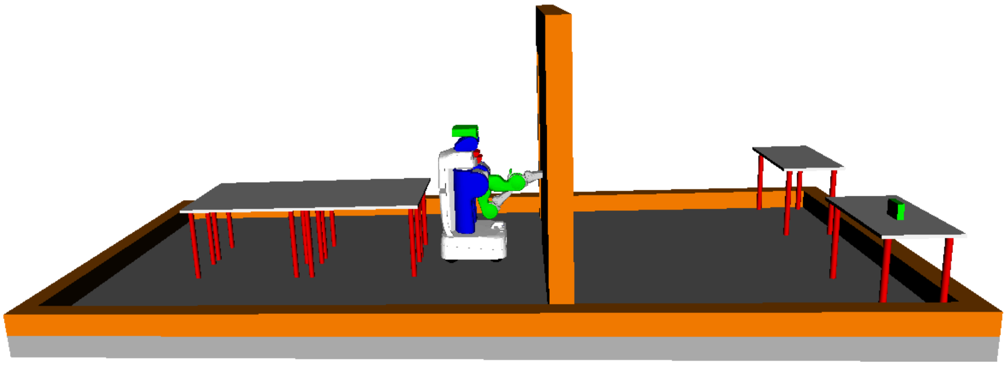

Figure 5: Screenshot of right arm of PR2 extending and colliding with wall in the center of the scene

Explain why you placed waitrobot() inside/outside the with env:. What would happen if it were placed in the other location and why?

Similar to the code in the drawing.py template, I placed the `waitrobot(robot)` outside of the `with env:` block. Placing the `waitrobot(robot)` function inside the `with env:` block causes the software to hang. This could be due to the the fact that in the `with` block, the thread running acquires a lock on changing the environment. The controller needs the lock to be removed to change the environment, but has to wait until the software exits the `with` block. This causes a deadlock and the result is that the controller never finishes executing the desired motion.