Mechanical Exam Prep FE

How to Pass the FE Exam

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Study Guide & Exam Problems

Engineering Pro Guides

- Learn key concepts and skills required for the exam
- Simplify and focus your studying
- Covers all exam topics (Mathematics, Probability & Statistics, Computational Tools Tools, Ethics, Economics, Electricity & Magnatism, Statics, Dynamics, Kinematics, Vibration, Mechanics of Materials, Material Properties/Processing, Fluid Mechanics, Thermodynamics, Heat Transfer, Measurement, Instrumentation, Controls, Mechanical Design & Analysis)
- Over 150 exam level difficulty practice problems

Mechanical FE Practice Exam & Technical Study Guide
How to pass the FE Mechanical exam

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INTRODUCTION

Section 1.0 - Introduction

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1.0 INTRODUCTION

One of the most important steps in an engineer's career is obtaining the professional engineering (P.E.) license. It allows an individual to legally practice engineering in the state of licensure. This credential can also help to obtain higher compensation and develop a credible reputation. The first step towards obtaining your P.E. is passing the Fundamentals of Engineering (F.E.) Exam. Both tests are administered by the National Council of Examiners for Engineering and Surveying (NCEES). The FE Exam is a year round computer based test that can be taken as early as your senior year in college or with at least 3 years of engineering-related work experience. Once passed, the FE Exam will certify you as an Engineering in Training (EIT). With enough experience after passing the EIT, you will become eligible for the PE Exam. Engineering Pro Guides focuses on helping engineers pass the NCEES exam through the use of free content on the website, http://www.engproguides.com and through the creation of books like sample exams and guides that outline how to pass the PE exam.

In the FE exam you will not be able to bring in any outside reference material. You will be given the *NCEES FE Exam Reference Handbook*, which contains all the necessary equations, tables, and graphs that you will need to solve each problem. The *NCEES FE Exam Reference Handbook* will be provided as a searchable electronic pdf during the test. The key to passing the FE exam is understanding the key concepts and skills that are tested on the exam and becoming familiar with using this handbook to solve each problems in approximately 2-3 minutes. Although the NCEES handbook provides the necessary equations for the exam, knowing how to apply them and which equations to use requires an understanding of the concepts and practice of the skills. The FE Exam is available for 6 disciplines plus a generic engineering discipline. This technical guide teaches you the key concepts and skills required to pass the Mechanical F.E. Exam in a single document.

1.1 EXAM FORMAT

How is the exam formatted?

The FE exam format and additional exam day information can be found on the NCEES Examinee Guide (https://ncees.org/exams/examinee-guide/). The entire exam period is about 6 hours, with 2 minutes for signing agreements, 8 minutes for tutorials, and one break up to 25 minutes. You will have a total of 5 hours 20 minutes of actual exam time to solve 110 problems, which equates to about 2.9 minutes per problem if spread out evenly. The test is broken up into two sessions. The length of each session is determined by the number of problems, 55 problems per session, and not the time. So, you could spend more or less than half the time on the first session, and the remaining 5 hours 20 minutes will be allotted for the second session. Since the first session doesn’t have a halfway time limit, it is very important to keep watch of the clock to make sure you have enough time for the second session. Before each session is completed, you are allowed to go back to problems that you may have skipped or want to check in that session. However, once the first session is completed and submitted, you are no longer allowed to revisit the questions in that session. There is a 25 minute break in between the sessions. You are allowed to take less than a 25 minute break or no break at all, but this does not increase the time you have to answer exam questions. No points are deducted for incorrect answers, so be sure to provide an answer for all questions, even if it is a guess. The final results are scaled based on the exam difficulty. There are five types of question formats that could be presented on the exam.

1. Multiple Choice (4 choices) – Select one option, *majority of questions in the exam*
2. Multiple Answers – Select multiple answers that are correct
3. Select by Clicking – Click on a point on a graph, etc
4. Drag and Drop – Matching, sorting, labeling, etc
5. Fill in the Blank – Type in the answer

The types of questions and number of questions per topic will be based on the outline provided by NCEES, discussed in the next section. These topics will not be labeled on the test. Finally, the NCEES Examinee Guide states that there will be some questions that will not be scored in the exam. These are questions that are tested for their quality and possible use in future exams. Your final results will be given to you 7-10 days after you take the exam.

1.2 Key Concepts and Skills

How are the key concepts and skills determined?

The key concepts and skills tested in the sample exams and taught in this technical study guide were first developed through an analysis of the topics and information presented by NCEES. The above factors related to timing is considered. The Mechanical FE exam will focus on the following topics as indicated by NCEES. (https://ncees.org/engineering/fe/):

1 **Mathematics - (6-9 questions)**
   - i) *Analytic Geometry*
   - ii) *Calculus*
   - iii) *Linear Algebra*
   - iv) *Vector Analysis*
   - v) *Differential Equations*
   - vi) *Numerical Methods*

2 **Probability and Statistics - *(4-6 questions)***
   - i) *Probability Distributions*
   - ii) *Regression and Curve Fitting*

3 **Computational Tools - *(3-5 questions)***
   - i) *Spreadsheets*
   - ii) *Flow Charts*

4 **Ethics and Professional Practice - *(3-5 questions)***
   - i) *Codes of Ethics*
   - ii) *Agreements and Contracts*
   - iii) *Ethical and Legal Considerations*
   - iv) *Professional Liability*
   - v) *Public Health, Safety, and Welfare*

5 **Engineering Economics - *(3-5 questions)***
   - i) *Time Value of Money*
   - ii) *Cost: incremental, average, sunk, estimating*
   - iii) *Economic Analyses*
   - iv) *Depreciation*

SECTION 1
MATHEMATICS

Section 1.0 – Mathematics

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1.0 INTRODUCTION

Mathematics accounts for approximately 6 to 9 questions on the Mechanical FE exam. The topics covered in this section include Analytic Geometry, Calculus, Linear Algebra, Vector Analysis, Differential Equations and Numerical Methods. At first glance, these topics seem very vast and daunting. But you should not be worried because you will most likely only need to know the equations that are shown in the NCEES FE Reference Handbook as they relate to Mechanical Engineering.

Section 14.0 Measurement, Instrumentation and Controls (5 to 8 Problems)		
NCEES Outline Value	**Engineering Pro Guides**	
	Section 1.0	Introduction
1A	Section 2.0	Analytic Geometry
1B	Section 3.0	Calculus
1C	Section 4.0	Linear Algebra
1D	Section 5.0	Vector Analysis
1E	Section 6.0	Differential Equations
1F	Section 7.0	Numerical Methods
	Section 8.0	Practice Exam Problems

2.0 ANALYTIC GEOMETRY

Analytic geometry uses algebra to characterize various geometric objects such as shapes, lines and points.

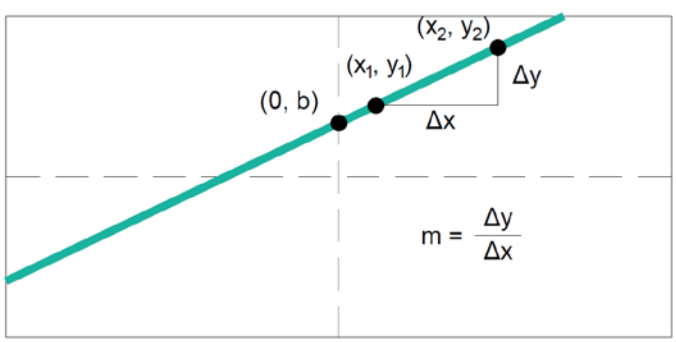


Figure 1: The slope of a line can be found with the difference between the y-values and x-values of two points.

2.1 FIND LINE EQUATION GIVEN TWO POINTS

If you are given two points, then you can follow the below process to solve for the equation of the line.

First solve for slope, "m", where the slope is equal to the change in y over the change in x.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Next, solve for the y-intercept, "b", using the following equation for a line. To solve for "b", plug in the value of the slope, "m", and one of the (x, y) points along the line. Then solve for the y-intercept, "b".

$$y - intercept\ equation\ for\ line \rightarrow\ y = mx + b$$

$$y_1 = \left(\frac{y_2 - y_1}{x_2 - x_1}\right) * x_1 + b$$

Finally, replace "b" in the equation and you will have found the equation of the line.

2.2 Parallel and Perpendicular Lines

Another skill you should have is being able to calculate the equation for lines that are parallel or perpendicular to each other. Parallel lines share the same slope.

$$Parallel\ Lines \rightarrow m_1 = m_2$$

You can then find the equation of one line that is parallel to another by determining the offset between the lines. Add the offset to the y-intercept of one line to find the equation of the other line.

$$Line\ 1 \rightarrow y = m_1 x + b$$

$$Line\ 2 \rightarrow y = m_2 x + (b + offset); where\ m_1 = m_2$$

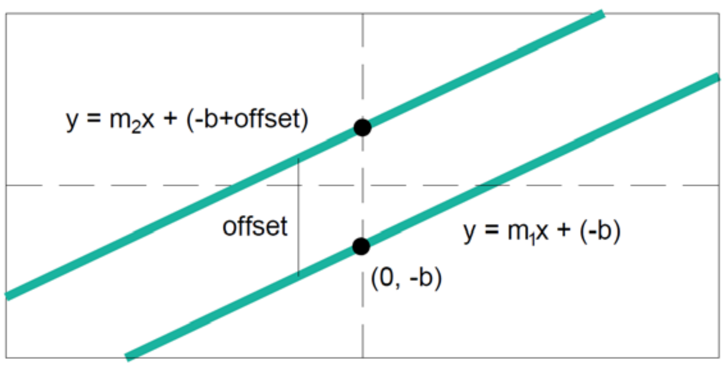


Figure 2: Parallel lines have the same slope and never intersect.

Perpendicular lines have inverse, negative slopes.

$$Perpendicual\ Lines \rightarrow -\frac{1}{m_1} = m_2$$

Find the slope of the perpendicular line, then solve for the new y-intercept, "b_2", by substituting one of the (x, y) coordinates on the line.

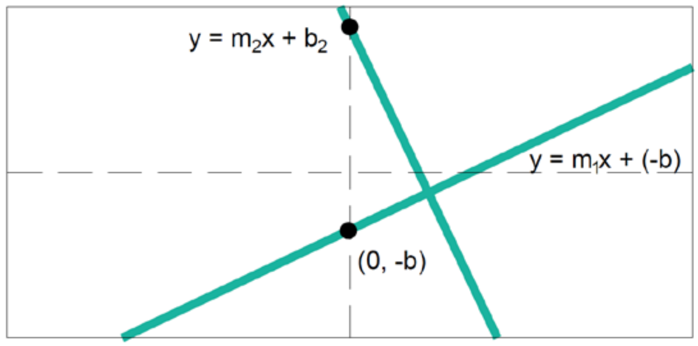


Figure 3: This figure shows the perpendicular intersection of two lines.

2.4 Distance Between Two Points

The distance between two points can be calculated with the quadratic equation.

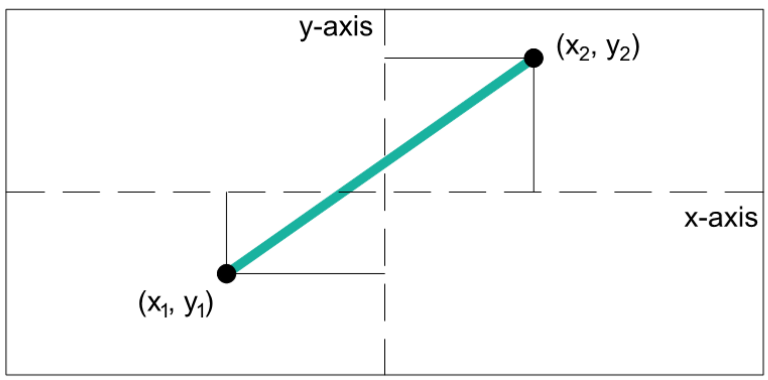


Figure 4: This figure sets up the calculation for the distance between two points.

First, find the total x-distance and the total y-distance. Then you use those values in the quadratic equation.

$$Quadratic\ Equation \rightarrow c^2 = a^2 + b^2$$

$$Distance^2 = |x_1 - x_2|^2 + |y_1 - y_2|^2$$

$$Integral \rightarrow \int_{x_1}^{x_2} f(x)$$

You should be familiar with calculating the integral for all the functions shown in the *NCEES FE Reference Handbook* like exponential equations, logarithmic equations, natural log equations, trigonometry equations, etc.

4.0 LINEAR ALGEBRA

The linear algebra required for mechanical engineering includes being able to define lines with equations and matrices.

4.1 LINEAR EQUATIONS

Linear equations can be used in statics and dynamics, especially when you want to find the intersection of two functions. In these types of problems you will be given two equations and two unknowns. Then, you can use the skills in linear equations to find the unknown values that satisfy both equations. A very simple example is shown below. The actual questions will most likely be more difficult and will be in the context of mechanical engineering.

$$y = 2x + 5; y = -5x + 10;$$

First, substitute one of the y-values into the other equation, then solve for x.

$$(-5x + 10) = 2x + 5;$$

$$-7x = -5 \rightarrow x = \frac{5}{7}$$

Then plug in the x-value to either equation.

$$y = 2 * \left(\frac{5}{7}\right) + 5$$

$$y = \frac{45}{7}$$

The correct answer is, (x,y) = (5/7, 45/7).

Linear equations can extend to multiple unknowns and multiple equations. To simplify the process, matrices are used.

4.2 MATRICES

Matrices are another way to solve linear equations. Matrices are an array of values that can be used to represent multiple linear equations.

For example, the following matrices will represent the below equations.

$$y = x + z - 10; y = 2x + 80; y = -5x + 4z - 45;$$

Rearrange the equation so that the variables follow the same order, constant times x plus constant times y plus constant times z equal to a constant.

$$1st\ Equation \rightarrow x + (-1)y + z = 10;$$

$$2nd\ Equation \rightarrow 2x + (-1)y + (0)z = -80;$$

$$3rd\ Equation \rightarrow (-5)x + (-1)y + (4)z = 45;$$

The constants for each variable are placed in the first matrix and the variables are placed in the second matrix. The third matrix shows the other side of each equation. The constants in each same row all belong to the same equation and the constants in each column all belong to the same variable.

$$\begin{bmatrix} 1 & -1 & 1 \\ 2 & -1 & 0 \\ -5 & -1 & 4 \end{bmatrix} * \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10 \\ -80 \\ 45 \end{bmatrix}$$

The values for x, y and z that satisfy all three equations can be found with matrix functions.

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & -1 & 1 \\ 2 & -1 & 0 \\ -5 & -1 & 4 \end{bmatrix}^{-1} * \begin{bmatrix} 10 \\ -80 \\ 45 \end{bmatrix}$$

The rules for adding, multiplying, taking the transpose, and finding the inverse of a matrix is described in the *NCEES FE Reference Handbook.* For this test, you should be able to quickly solve for matrices on your calculator. The steps for a typical calculator are shown below. Read through your calculator instruction manual for any variations for your specific model.

Calculator Button	**Description**
2ND	Press 2nd function
$Matrix$	Access matrix function
$A \rightarrow Edit$	Choose matrix A and edit.
Row/Column	Input 3 rows by 3 columns. Sets dimensions of matrix
Enter Values	You will be able to input the values for the matrix
$Enter/Clear$	Now the matrix is stored as matrix A.
2ND	Press 2nd function
$Matrix$	Access matrix function
$B - Edit$	Choose matrix B and edit
Row/Column	Input 3 rows by 1 columns. Sets dimensions of matrix
Enter Values	You will be able to input the values for the matrix
$Enter/Clear$	Now the matrix is stored as matrix A.

Now that you have input and stored each matrix, the next step is to use the matrices in functions.

Calculator Button	Description
$2nd \rightarrow Matrix \rightarrow A$	Press 2nd function, matrix, then select matrix A.
$[A] \rightarrow Enter$	With matrix A selected (not edit), press enter and matrix A will appear on your screen.
$2nd \rightarrow Matrix$	Go back to the matrix menu
$Path \rightarrow Inverse$	Select the inverse function and press enter. This will apply the inverse function to matrix A.
$Multioly$	Insert the standard multiplication function.
$2nd \rightarrow Matrix$	Go back to the matrix menu
$[B] \rightarrow Enter$	With matrix B selected (not edit), press enter and matrix B will appear on your screen.
$Enter$	Complete the function, $[A]^{-1}$ * [B]

$$\begin{bmatrix} 1 & -1 & 1 \\ 2 & -1 & 0 \\ -5 & -1 & 4 \end{bmatrix}^{-1} = \begin{bmatrix} 4/3 & -1 & -1/3 \\ 8/3 & -3 & -2/3 \\ 7/3 & -2 & -1/3 \end{bmatrix}$$

$$\begin{bmatrix} 4/3 & -1 & -1/3 \\ 8/3 & -3 & -2/3 \\ 7/3 & -2 & -1/3 \end{bmatrix} * \begin{bmatrix} 10 \\ -80 \\ 45 \end{bmatrix} = \begin{bmatrix} 235/3 \\ 710/3 \\ 505/3 \end{bmatrix}$$

The correct answer is shown below.

$$(x, y, z) = (78.33, 236.67, 168.33)$$

You should also be able to calculate determinants, dot products, and cross products of matrices on your calculator.

5.0 Vector Analysis

Vector analysis is used heavily in analyzing forces in Section 7 Statics, Section 8 Dynamics, Kinematics and Vibrations, Section 9 Mechanics of Materials, and Section 15 Mechanical Design and Analysis. One of the main skills that you need is the ability to break down a vector into its x and y components.

Vectors are used to visualize forces and moments in Engineering Mechanics. Vectors describe the magnitude and direction of force or moment. On the FE exam you must be able to translate words or diagrams into force/moment vectors and you must be able to add/subtract vectors and multiply/divide vectors by scalars.

Vectors can either be represented in a rectangular form or a polar form. The rectangular form consists of an x-component and a y-component. These values are used to represent the magnitude in the x and y directions. In real applications, there will also be a z-component, but for the purposes of the exam you most likely will only need the x and y components. The polar form is shown as a magnitude and an angle. The magnitude describes the length of the vector while the angle determines the direction.

5.1 Rectangular Form

The rectangular form is shown as, x plus the y component. The y component is shown as j and the x component is shown as i.

$$Rectangular\ form:\ xi + yj$$

$$x = x-\ component; y = y-\ component$$

$$Example\ A\ \rightarrow 5i + 2j\ or\ Example\ B \rightarrow\ -1i - 5j$$

The rectangular form is used when adding and subtracting vectors and follows the same rules as normal addition and subtraction, where only like terms can be added and subtracted. For example, "Example A" plus "Example B", is solved with the following process.

$$Example\ A + Example\ B = (5i + 2j) + (-1i - 5j)$$

$$(5i - 1i) + (2j - 5j) \rightarrow 4i - 3j$$

The rectangular form can also be understood via a graphical format, where the x-axis represents the real component and the y-axis represents the imaginary component.

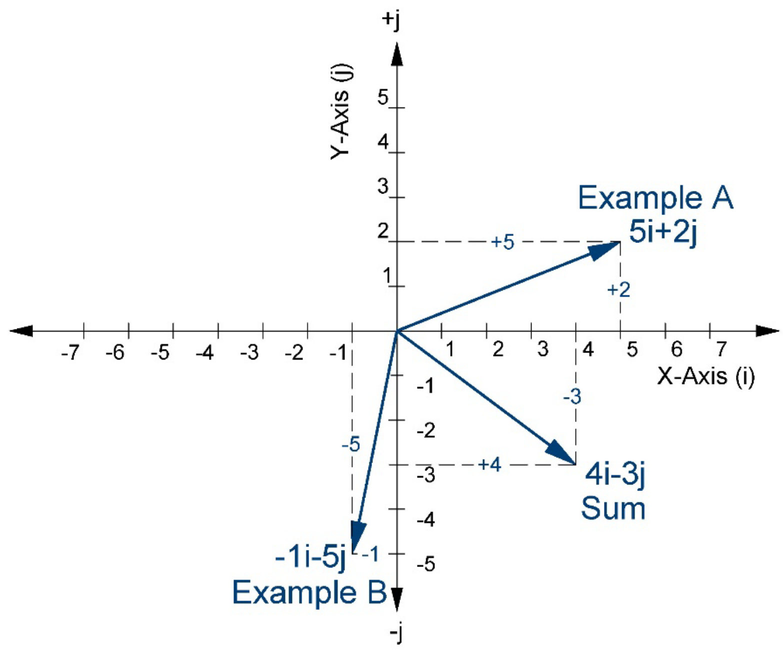


Figure 11: Example "A" vector, example "B" vector and the sum of the two vectors is shown in the above graph.

5.2 Polar Form

The polar form is best understood in its graphical format. The format consists of a phasor magnitude at a phasor angle relative to the x-axis.

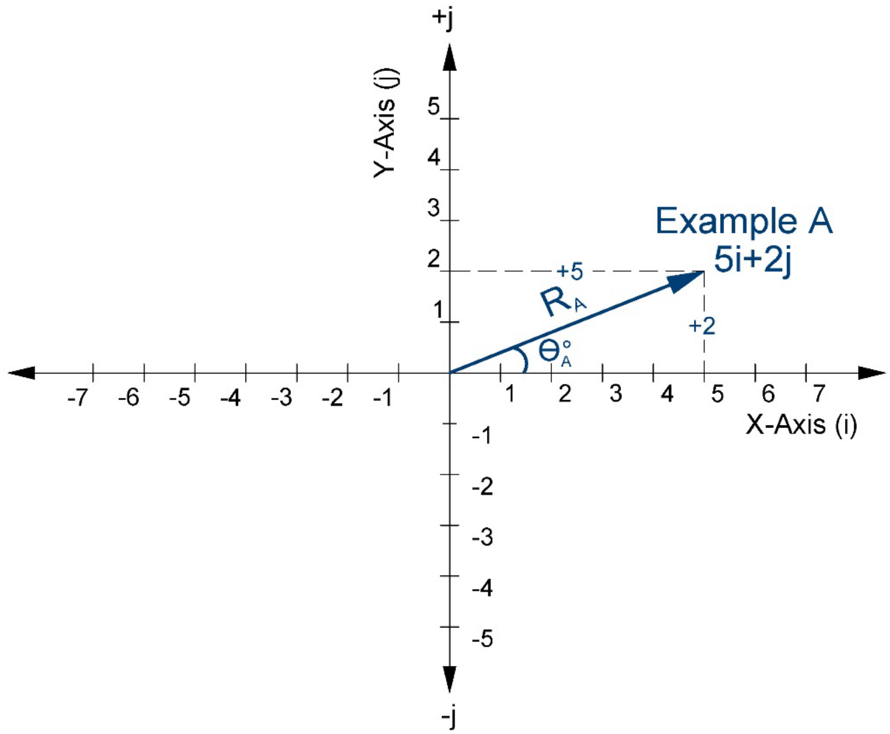


Figure 12: A phasor shown in both polar and rectangular form.

$$Polar\ form: R\angle\theta^\circ\ ;\ R = phasor\ magnitude;\ \theta = phasor\ angle\ relative\ to\ x - axis$$

$$Example\ A: R_A = \sqrt{5^2 + 2^2} = 5.4;\ \theta_A = \tan^{-1}\left(\frac{2}{5}\right) = 21.8^\circ$$

In the above example, the polar form is converted from the rectangular form by using the Pythagorean Theorem to find the radius (i.e. the magnitude) and the inverse tangent to find the angle. The polar form is not typically used for adding or subtracting, but it is used for multiplication and division. When multiplying or dividing two polar forms, you must multiple/divide the radiuses and add or subtract the angles. If the polar forms are being multiplied, then you must add the angles and if you are dividing one polar form from another then you subtract the divisor angle from the dividend angle.

$$5.5\angle 45^\circ\ \div\ 1.1\angle 85^\circ\ \rightarrow\ \left(\frac{5.5}{1.1}\right)\angle(45 - 85^\circ)\ \rightarrow\ 5\angle - 40^\circ$$

5.3 Converting Polar and Rectangular Forms - Calculator

During the exam, you will need to convert from polar form to rectangular form and vice versa. You will need to convert between the two forms in order to carry out multiplication/division or addition/subtraction. You should be able to quickly convert between the forms with your calculator. This will help to save you time for more difficult tasks during the exam.

You should use the most advanced calculator allowed by NCEES. The calculators allowed by the NCEES are shown below.

- Casio: All fx-115 and fx-991 models (Any Casio calculator must have “fx-115” or “fx-991” in its model name.)
- Hewlett Packard: The HP 33s and HP 35s models, but no others
- Texas Instruments: All TI-30X and TI-36X models (Any Texas Instruments calculator must have “TI-30X” or “TI-36X” in its model name.)

Casio FX-115, Casio FX-991, HP35 and TI-36X are all decent calculators that can do multiple calculation steps without having to use the memory function in the calculator or without having to write down numbers in between steps. However, the Casio and TI models are the cheapest. Past test takers seem to prefer the TI-36X Pro for its interface and its ability to save previous calculations when it is shut off and the Casio FX-991EX ClassWiz for its faster processor and its spreadsheet option. Whichever calculator you decide on using, make sure that you use the calculator during your studying so that you can be well versed in the use of your calculator prior to taking the exam.

The following are the basic steps in converting from polar form to rectangular form. The above calculators also have the option of adding/subtracting and multiplying/dividing multiple polar or rectangular numbers. Please be sure to learn this skill, because you will use the skill throughout the exam.

Convert Polar $(R\angle\theta°)$ to Rectangular $xi + yj$

Calculator Button	**Description**
R	Press your value for “R”
$P \rightarrow R$	Press the Polar to Rectangular Button
θ	Press your value for “θ”
=	Press the equal sign
	The first value shown will be your x-component
$x \rightarrow y$	Press this button to show the y-component
	The second value shown will be your y-component

- Convert Rectangular $(xi + yj)$ to Polar $R\angle\theta°$

Calculator Button	**Description**

"x" value	Press your value for "x"
$R \rightarrow P$	Press the Rectangular to Polar Button
"y" value	Press your value for "y"
=	Press the equal sign
	The first value shown will be your R -component
$R \rightarrow \theta$	Press this button to show the y-component
	The second value shown will be your θ -component

The Casio FX-115ES Plus/Casio FX-991EX ClassWiz and TI-36X Pro also has a complex mode which allows you to add/multiply complex numbers in various forms and easily converting the results between rectangular and polar. The stored lines of calculations and the various functions, including matrices and integrals make this calculator one of the favored among test takers.

5.4 Vector Mathematical Operations

Addition & Subtraction of Vectors: Vectors can be added or subtracted from one another by simply adding or subtracting their real components together and doing the same for their imaginary components. It is very easy to do this when vectors are presented in their Rectangular coordinates, however it is more difficult to do this by hand when the vectors are in polar form. For example, if you are adding 3 and 3j, then the answer is simply 3 + 3j. Another example would be if you were to add the vectors "2+2j" and "4+4j", then the answer would be "6+6j".

1st Example: You can complete this problem by inputting the polar vectors and adding them. If you are doing the equation by hand, then you should follow the recommended method by converting polar to rectangular and then adding the like terms to one another (real + real and imaginary + imaginary).

$$Convert\ to\ Rectangular \rightarrow 3\angle 0° + 3\angle 90° \rightarrow 3 + 3j$$

$$Convert\ to\ Polar \rightarrow 3 + 3j \rightarrow 3\angle 45°$$

2nd Example: This example is a little less intuitive, but since both vectors have the same angle, they can be added directly or with your calculator. If you are doing the equation by hand, then you should follow the recommended method by converting polar to rectangular and then adding the like terms to one another (real + real and imaginary + imaginary).

$$Convert\ to\ Rectangular \rightarrow 2\angle 45° + 4\angle 45° \rightarrow 2 + 2j + 4 + 4j$$

$$Convert\ to\ Polar \rightarrow 6 + 6j \rightarrow 6\angle 45°$$

3rd Example: In this example, the answer cannot be seen as intuitively, but as long as you either use your calculator to add polar coordinates or convert to rectangular and then add the like terms, then you will be able to complete this problem.

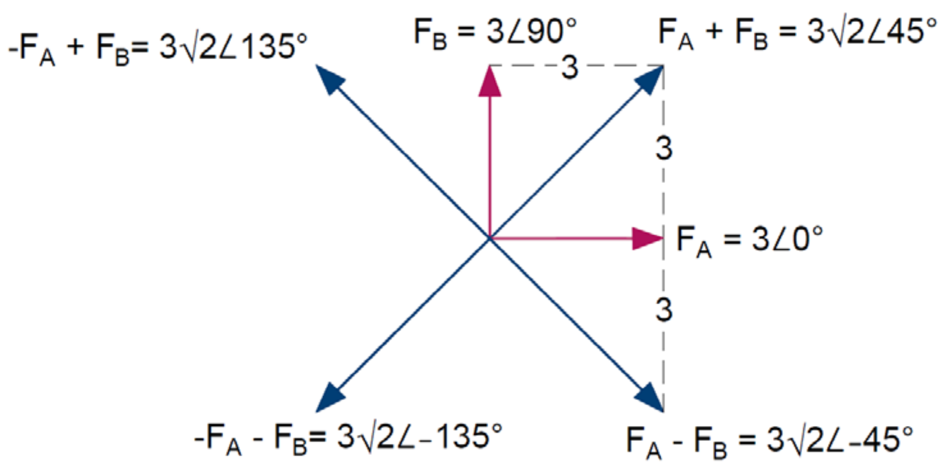
$$3\angle 35° + 5\angle 60°$$

$$Convert\ to\ Rectangular \rightarrow 3\angle 35° + 5\angle 60° \rightarrow (2.46 + 1.72j) + (2.5 + 4.33j)$$

$$Convert\ Sum\ to\ Polar \rightarrow (4.96 + 6.05j) \rightarrow 7.82\angle 50.65°$$

$$Convert\ to\ Polar \rightarrow 6 + 6j \rightarrow 6\angle 45°$$

This figure below shows how vectors can be added and subtracted. This figure also corresponds to the first example and it also shows how the answer would change if vectors A



Multiplication & Division of Vectors: The multiplication and division of vectors is easy for vectors in polar form but difficult for vectors in rectangular form. Your calculator should be able to divide and subtract in both rectangular form, but if you are not familiar with this function on your calculator, then you can also complete this by hand. In order to multiply or divide vectors by hand you must convert the vector to polar form and then multiply or divide the radius of the vectors and then add or subtract the angles. You add the vector angles for multiplication and subtract the vectors angles for division.

1st Example: In this example, the vectors are already in polar form, so you can simply multiply the radiuses and add the vector angles.

$$3\angle 0° \times 3\angle 0° = 9\angle 0°$$

2nd Example: Again, the vectors are already in polar form, so multiply the radiuses and add the vector angles.

$$3\angle 0° \times 3\angle 45° = 9\angle 45°$$

3rd Example: In this example, the vectors must first be converted to polar and then the radiuses can be divided and the vector angles can be subtracted.

$$(2+2j)\times(4+4j)\rightarrow 2\angle 45^\circ \div 4\angle 45^\circ$$

$$\frac{1}{2}\angle 0^\circ \rightarrow 0.5+0j$$

6.0 DIFFERENTIAL EQUATIONS

Differential equations are used in mechanical engineering to curve fit data. This type of math typically uses software, so it is unlikely that you will have a question on differential equations on the FE exam.

7.0 NUMERICAL METHODS

Numerical methods are used with differential equations in mechanical engineering to curve fit data. This type of math typically uses software, so it is unlikely that you will have a question on differential equations on the FE exam.

8.0 PRACTICE EXAM PROBLEMS

8.1 PRACTICE PROBLEM 1 – CALCULUS

What is the integral of the function below from 0 to 90 degrees?

$$\int_{0\,deg}^{90\,deg} 10 * \sin(\theta)\, d\theta$$

(a) 0

(b) 1

(c) π

(d) 10

8.1.1 Solution 1 – Calculus

Use the integral equations from the NCEES FE Reference Handbook.

$$\int_{0\,deg}^{90\,deg} 10 * \sin(\theta)\, d\theta = -10\cos(\theta)|_{0\,deg}^{90\,deg}$$

$$= -10 * \cos(90\text{deg}) - (-10 * \cos(0\text{ deg}))$$

$$= 0 - (-10 * 1) = 10$$

The correct answer is most nearly, (d) 10.

8.2 PRACTICE PROBLEM 2 – CALCULUS

What is the integral of the function below from 1 to 2?

$$\int_{1}^{2} \ln(x) dx$$

(a) 0.39

(b) 1.01

(c) 1.77

(d) 2.00

8.2.1 Solution 2 – Calculus

$$\int_1^2 \ln(x)\,dx = x(\ln(x) - 1)|_1^2$$

$$= 2 * (\ln(2) - 1) - 1 * (\ln(1) - 1)$$

$$= -0.614 + 1 = 0.386$$

The correct answer is most nearly, (a) 0.39

8.3 Practice Problem 3 - Calculus

An object's speed is governed by the below equation. How much distance is covered between 0 and 3 seconds? Assume the location at time t = 0 sec is 0 ft.

$$v(t) = 10\left(\frac{ft}{s^2}\right) * t + 2\left(\frac{ft}{s}\right)$$

(a) 35 ft

(b) 50 ft

(c) 65 ft

(d) 90 ft

8.3.1 Solution 3 - Calculus

First, integrate the function.

$$\int v(t)\,dt = \frac{1}{2}t^2 * 10 + 2t$$

$$x(t) = \int v(t)\,dt = 5t^2 + 2t$$

Then evaluate from 0 to 3 seconds.

$$x(0) = 5(0)^2 + 2(0) = 0\;ft$$

$$x(3) = 5(3)^2 + 2(3) = 51\;ft$$

$$\Delta x = x(3) - x(0) = 51ft$$

The correct answer is most nearly (b) 50 ft.

SECTION 2
PROBABILITY AND STATISTICS

Section 2.0 – Probability and Statistics

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1.0 INTRODUCTION

Probability and Statistics accounts for approximately 5 to 8 questions on the Mechanical FE exam. Statistics is primarily used in Machine Design for statistical quality control, which is covered under Section 15.0 Mechanical Design and Analysis, under the topics Quality & Reliability. This section focuses on the following NCEES Outline topics, Probability Distributions and Regression Curve Fitting.

Probability Distribution involves applying a mathematical formula to describe the probability of a measured variable occurring at a certain value. This is useful for characterizing the measured output of any mechanical system property when you are taking a sample of a larger number. For example, you measure the weight of 100 products, but this is only a sample of the 10,000 products that are produced. A probability distribution will help to characterize all 10,000 products.

Regression curve fitting involves measuring a variable as a function of another variable, then plotting the data points and assigning a mathematical formula to approximate the function. This is useful in predicting how a change in one variable will affect another.

Section 2.0 Probability and Statistics (4 to 6 Problems)		
NCEES Outline Value	**Engineering Pro Guides**	
	Section 1.0	Introduction
2A	Section 2.0	Probability Distributions
2B	Section 3.0	Regression Curve Fitting
	Section 4.0	Practice Exam Problems

2.0 PROBABILITY DISTRIBUTIONS

Before you get to probability distributions, you need to understand some of the basic topics in probability like the difference between samples and population, mean, mode, standard deviation. As you go through these topics, you should remember that probability is used in mechanical engineering to measure the reliability of a set of data points.

2.1 MEAN OR AVERAGE

The mean of a set of data points is calculated by summing up all the values and dividing by the total number of data points. The mean is also known as the average.

$$\bar{x} = \frac{\sum_1^n x_i}{n}$$

$$A, B, C \,\&\, D \text{ are constants that are automatically determined by the software}$$

The software program then will output an R^2 value. This value is called the coefficient of determination and it describes the fit of the curve as compared to the data points. Please see more in the goodness of fit topic below.

3.1 Goodness of Fit

After you input data into your software program and apply a curve fit to approximate the data with a formula, you will be given an equation and an R^2 value.

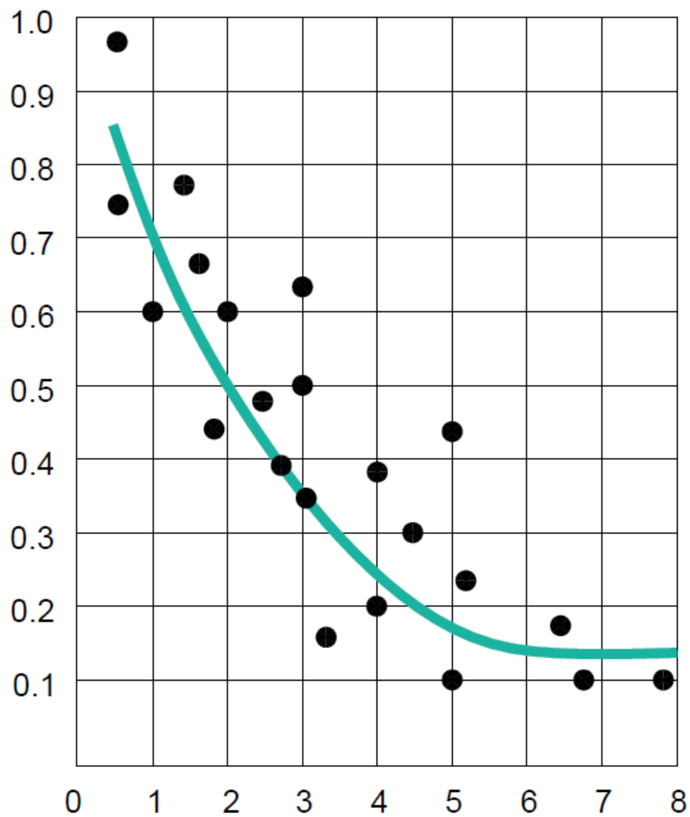


Figure 7: This figure shows data points in black and a mathematical curve fitted to that data.

For a linear fit, the coefficient of determination (R^2) value can be found through the below equation. In this equation you need the sum of squares. The sum of squares describe how far apart individual data points are from the mean. The mean is calculated on the right hand side of the equation. There are two sum of squares required for the coefficient of determination, first the sum of squares for the x-values and the sum of squares for the product of the y-values.

$$S_{xx} = \sum_{i=1}^{n} (x_i)^2 - \frac{(\sum_1^n x_i)^2}{n}$$

$$S_{yy} = \sum_{i=1}^{n} (y_i)^2 - \frac{(\sum_1^n y_i)^2}{n}$$

$$S_{xy} = \sum_{i=1}^{n} (x_i * y_i)^2 - \frac{(\sum_1^n x_i) * (\sum_1^n y_i)}{n}$$

You also should understand that for a linear fit, the equation can be found by using the above sum of squares too.

$$slope = \frac{S_{xy}}{S_{xx}}; y - intercept = y_{mean} - slope * x_{mean}$$

Finally, the coefficient of determination can be determined with the following equation.

$$R^2 = \frac{SS_{xy}}{SS_{xx} * SS_{yy}}$$

For all other fits, the FE exam will have to give you the following values to calculate the R2 value. However, this equation is not shown in the NCEES FE Reference Handbook, so it is unlikely that the exam will test you on this topic.

$$R^2 = 1 - \frac{SS_{res}}{SS_{tot}}$$

$$SS_{res} = sum\ of\ residual\ squares = \sum_1^n (y_i - f_i)^2$$

$$SS_{tot} = sum\ of\ squares = \sum_1^n (y_i - \bar{y})^2$$

$$f_i = y - value\ from\ curve\ fit\ function; y_i = value\ from\ data\ point;$$

$$n = total\ number\ of\ data\ points$$

4.0 Practice Exam Problems

4.1 Practice Problem 1 – Binomial Distribution

A product has shown to have a satisfactory probability of 0.5. If there are 10 samples, then what is the probability that only 3 of the samples will pass? The table shown below is a cumulative binomial probability table.

		P=0.5
n=10	x=0	0.001
	1	0.0107
	2	0.0547
	3	0.1719
	4	0.377
	5	0.623
	6	0.8281
	7	0.9453
	8	0.9893
	9	0.999

(A) 0.1%

(B) 1.07%

(C) 5.47%

(D) 11.7%

4.1.1 Solution 1 – Binomial Distribution

In order to complete this problem, you need to understand that the probability of 0 out of 10 successes is shown as 0.001. Then the probability of 0 and 1 out of 10 successes is shown as 0.0107. Then the probability of 0 and 1 and 2 out of 10 successes is shown as 0.0547. Then the probability of 0 and 1 and 2 and 3 out of 10 successes is shown as 0.1719. This means that in order to just get 3 successes you need to subtract 0.0547 from 0.1719.

$$only\ 3\ sucesses \rightarrow 0.1719 - 0.0547 = 0.1172$$

The correct answer is most nearly, (D) 11.7%.

4.2 PRACTICE PROBLEM 2 – STANDARD DEVIATION

A manufacturing plant produces 500 parts per day. Five parts are sampled and measured with the following tolerances: 1mm, 0.5mm, 0.4mm, 0.4mm, 0.3mm. What is the sample standard deviation of the measured tolerances?

(A) 0.14 mm

(B) 0.25 mm

(C) 0.28 mm

(D) 0.31 mm

4.2.1 Solution 2 – Standard Deviation

Use the sample standard deviation equation. Note: be sure to use the equation for sample standard deviation, s, and not population standard deviation, σ.

$$s = \sqrt{\frac{\sum_{i=1}^{n}(X_i - \bar{X})^2}{n-1}}$$

First find the mean of the sample set, where the sample size, n = 5.

$$\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i = \frac{(1 + 0.5 + 0.4 + 0.4 + 0.3)mm}{5} = 0.52mm$$

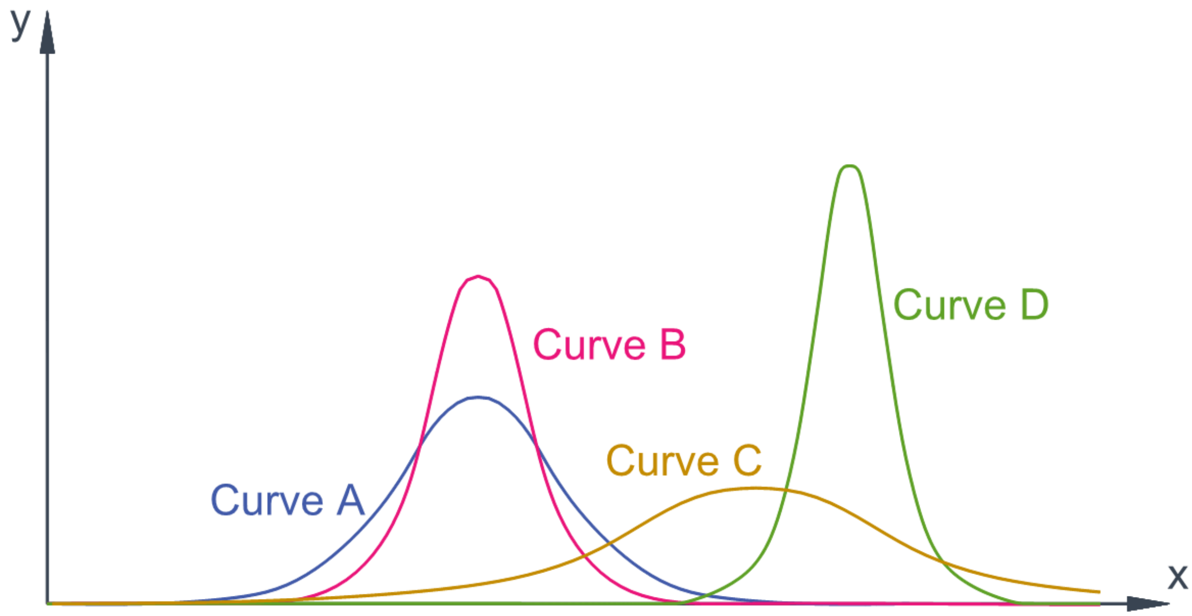
Then, solve for the sample standard deviation.

$$s = \sqrt{\frac{(1-0.52)^2 + (0.5-0.52)^2 + (0.4-0.52)^2 + (0.4-0.52)^2 + (0.3-0.52)^2}{5-1}} = 0.277mm$$

The answer is (C) 0.28 mm.

4.3 Practice Problem 3 – Probability Distribution

Answer the questions with the following options: *Curve A, Curve B, Curve C, Curve D, Unknown*



Which curve has the smallest variance? ________

Which curve has the largest mean? _______

Which curve has the largest standard deviation? ________

Which curve has the smallest mode? ________

SECTION 3
COMPUTATIONAL TOOLS

Section 3.0 – Computational Tools

Table of Contents

1.0 INTRODUCTION

Computational Tools accounts for approximately 3 to 5 questions on the Mechanical FE exam. This section covers the basic tools for performing engineering calculations. These are engineering computational and Boolean logic skills that are required for Microsoft Excel® and other programs. Flow charts can be found in the "Electrical and Computer Engineering" section of the *NCEES FE Reference Handbook*.

Section 3.0 Computational Tools (3 to 5 Problems)		
NCEES Outline Value	**Engineering Pro Guides**	
	Section 1.0	Introduction
3A	Section 2.0	Spreadsheets
3B	Section 3.0	Flow Charts
	Section 6.0	Practice Exam Problems

2.0 SPREADSHEETS

Spreadsheets refer to commonly used formulas that are used in Microsoft Excel® or other similar computational programs. The basic operations and commands are not included in the *NCEES FE Reference Handbook*, but rather are understood to be used in everyday practice. The intent of spreadsheets is to simplify the calculations, increase computational efficiency, be able to quickly change variables and outputs, and to output data in a presentable format. The other benefit of spreadsheets is being able to compute results for multiple input data either in list or array format.

2.1 CELLS

A spreadsheet consists of an array of cells labeled by a letter for each column and a number for each row. A specific cell is identified by a letter-number combination. For example, the cells with an x is labeled A1, the cell with xx is labeled B2, and the cell with xxx is labeled D5.

	A	B	C	D	E	F
1	x					
2		xx				
3						
4						
5				xxx		
6						

Within each cell, a text, number, or formula can be inserted. A text is a string of letters, a number is a numerical value, and a formula is used for mathematical or logic operations.

2.2 Common Operations

The following are common mathematical formulas and operations used in spreadsheets. A formula use numbers or reference cells in an operation.

Common Operations			
Operation	**Description**	**Example**	**Note**
=	Equals	=A1+B2	Required at the beginning of a formula in a cell
+	Addition	=5+5	
-	Subtraction	=10-5	
*	Multiplication	=3*5	
/	Division	=100/4	
%	Percent	80%	Equivalent to 0.8
sqrt(…)	Square root	=sqrt(100)	Square root calculated for the number inside the parenthesis
^	Exponent	=7^2	Square of 7 = 49.
E	Power of 10	=5E3	= 5*10^3 = 5,000
pi()	Pi	=pi()	Pi = 3.412159…
(…)	Order of Operation	=((5+4)*20/4)^2*sqrt(4)	Result equals 4050
=SUM(…)	Summation	=SUM(5,6,4)	Sum of all values in the parenthesis. Result is 15. Comma separates the values to be added.
=AVERAGE(…)	Average	=AVERAGE(5,2,9)	Finds the average of all values in the parenthesis. Result is 5.33
=COUNT(…)	Count	=COUNT(A1:A5)	Counts the number of cells within A1 thru A5 that has numbers.
=COUNTIF(…,…)	Count if	=COUNTIF(A1:A5,>2)	Counts the number of cells within A1 thru A5 that are greater than 2. =countif(range, criteria)
"…"	Text	"abc"	Identifies texts within formulas
&	And	=5 & 4 ="x"&"y"	First result is 54, second result is xy. Combines two statements.
>	Greater than	Used in Logic Formulas	See if, then example
>=	Greater than or equal	"	
<	Less than	"	
<=	Less than or equal	"	
=	Equal	"	
<>	Not Equal	"	
=IF(.., …, …)	If-Then, true or false statement	=if(5>4, 3, 1)	Result is 3 =if(logic_test, value_if_true, value_if_false)

4.5 PRACTICE PROBLEM 5 – CRITICAL PATH

Determine the length of the critical path for the following machining process.

Machining Process	Duration (minutes)	Dependency
A	5	Start
B	2	Start
C	3	A & B
D	5	Start
E	6	Start
F	4	D & E
G	2	C & F
H	5	G
I	7	G
J	3	H
Finish	0	I & J

(A) 17 minutes

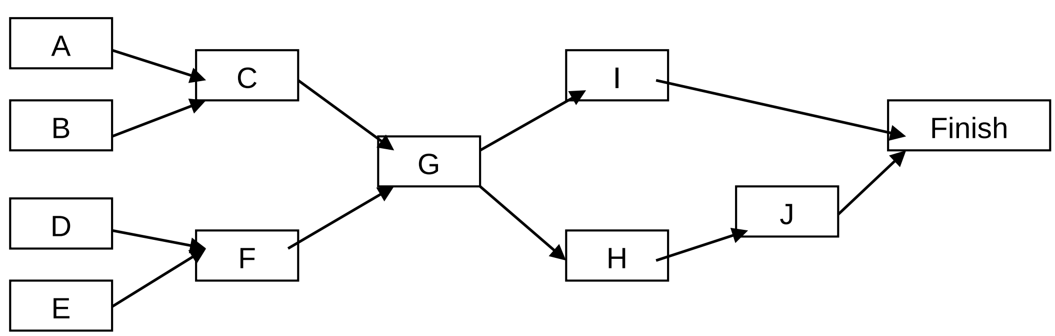
(B) 18 minutes

(C) 19 minutes

(D) 20 minutes

4.5.1 Solution 5 – Critical Path

Determine the length of the critical path for the following machining process.



A->C->G->I-> 17 minutes

A->C->G->H->J-> 18 minutes

B->C->G->I-> 14 minutes

B->C->G->H->J-> 15 minutes

D->F->G->I-> 18 minutes

D->F->G->H-> J -> 19 minutes

E->F->G->I-> 19 minutes

E->F->G->H-> J -> 20 minutes

The critical path is most nearly, **(D) 20 minutes**.

SECTION 4
ETHICS & PROFESSIONAL PRACTICE

Section 4.0 – Ethics and Professional Practice

Table of Contents

1.0 INTRODUCTION

Ethics and Professional Practice accounts for approximately 3 to 5 questions on the Mechanical FE exam. The NCEES outline provides the following topics on its outline, Codes of Ethics, Agreements and Contracts, Ethical and Legal Considerations, Professional Liability, Public Health, Safety and Welfare. There is no readily available or commonly used content that covers these topics, except for what is shown on the NCEES website and the *NCEES FE Reference Handbook*. In addition, the topics cover concepts that are open for interpretation. These two facts make it very difficult to fairly test this topic.

Section 4.0 Ethics and Professional Practice (3 to 5 Problems)		
NCEES Outline Value	**Engineering Pro Guides**	
	Topic 1.0	Introduction
4A	Topic 2.0	Codes of Ethics
4B	Topic 3.0	Agreements and Contracts
4C	Topic 4.0	Ethical and Legal Considerations
4D	Topic 5.0	Professional Liability
4E	Topic 6.0	Public Health, Safety and Welfare
	Topic 7.0	Practice Exam Problems

The only preparation for these problems can be found in the *NCEES FE Reference Handbook*. Please read through the handbook, so that you may more quickly answer these types of problems on the actual FE exam. The following practice problems will help you to become familiar with the problems that may appear on the exam.

2.0 PRACTICE EXAM PROBLEMS

2.1 PRACTICE PROBLEM 1 – CODES OF ETHICS

An engineer has submitted a large donation to a public official to secure a state funded condominium project. Which section of the *Model Rules* code of ethics does this violate?

(A) Licensee's Obligation to the Public

(B) Licensee's Obligation to Other Licensees

(C) Licensee's Obligation to Employers and Clients

(D) Licensee's Obligation to Engineering Etiquette

2.1.1 Solution 1 – Codes of Ethics

Engineers are expected to follow a code of ethics for the expertise and work they perform as a professional. One of these codes is the Model Rules. Refer to the Ethics section of the *NCEES FE Reference Manual.* Answers A, B, and C are major sections of the Model Rules, answer D is irrelevant. In reviewing the sections, paragraph 2 under the Licensee's Obligation to Other Licensees states that a licensed engineer shall not give gifts to secure work, nor shall they give political gifts with the expectation of receiving work for a public official.

The answer is (B) Licensee's Obligation to Other Licensees.

2.2 PRACTICE PROBLEM 2 – AGREEMENTS AND CONTRACTS

You are seeking to secure a contract to design a new fuel facility and distribution system for the local gas company. Based on the Model Rules, which of the following would not allow you to proceed with the project?

(A) You sit on the board of directors for the local gas company, but you have disclosed all affiliations with the gas company.

(B) You are unfamiliar with the particular intricacies required for this fuel design, but you have a licensed engineer within your firm that will oversee, stamp, and sign for the plans and documents.

(C) With the agreement of the local gas company, your firm will use the information gathered from this design to upgrade facilities in other states.

SECTION 5
ENGINEERING ECONOMICS

Section 5.0 – Engineering Economics

Table of Contents

1.0 INTRODUCTION

Engineering Economics accounts for approximately 3 to 5 questions on the Mechanical FE exam. As an engineer, you will be tasked with determining the course of action for a design. Often times this will entail choosing one alternative instead of several other design alternatives. You need to be able to present engineering economic analysis to their clients in order to justify why a certain alternative is more financially sound than other alternatives. The following topics will present only the engineering economic concepts that you need for the FE exam and does not present a comprehensive look into the study of engineering economics. For the FE exam you are required to know the following concepts shown in the table below. Applicable equations for these topics can be found in the Engineering Economics section of the *NCEES FE Reference Handbook*.

Section 5.0 Engineering Economics (3 to 5 Problems)		
NCEES Outline Value	**Engineering Pro Guides**	
	Section 1.0	Introduction
5A	Section 2.0	Time Value of Money
5B	Section 3.0	Cost, Including Incremental, Average, Sunk and Estimating
5C	Section 4.0	Economic Analyses
5D	Section 5.0	Depreciation
	Section 6.0	Practice Exam Problems

2.0 TIME VALUE OF MONEY

2.1 FUTURE AND PRESENT VALUE

Before discussing interest rates, it is important that the engineer understands that money today is worth more than that same value of money in the future, due to factors such as inflation and interest. This is the time value of money concept. For example, if you were given the option to have $1,000 today or to have $1,000 ten years from now, most people will choose $1,000 today, without understand why this option is worth more. The reason $1,000 today is worth more is because of what could have done with that money; in the financial world, this means the amount of interest that could have been earned with that money. If you took $1,000 today and invested it at 4% per year, you would have $1,040 dollars at the end of the first year.

$$\$1{,}000\ x\ (1 + .04) = \$1{,}040$$

- If you kept the $1,040 in the investment for another year, then you would have $1,081.60.

$$\$1{,}040\ x\ (1 + .04) = \$1{,}081.60$$

- At the end of the 10 years the investment would have earned, $1,480.24.

$$\$1{,}000\ x\ (1+.04)x(1.04)x(1.04)\ldots\ldots = \$1{,}000\ x\ (1.04)^{10} = \$1{,}480.24$$

- An important formula to remember is the Future Value (FV) is equal to the Present Value (PV) multiplied by (1+interest rate), raised to the number of years.

$$PV\ x\ (1+i)^{10} = FV$$

- As an example, what would be the present value of $1,000, 10 years from now, if the interest rate is 4%.

$$PV\ x\ (1+.04)^{10} = \$1{,}000$$

$$PV\ = \$675.46$$

- Thus in the previous example, receiving $1,000, 10 years from now, is only worth $675.46 today.

It is important to understand present value because when analyzing alternatives, cash values will vary with time and the best way to make a uniform analysis is to *first convert all values to consistent terms, like present value.*

For example, if you were asked whether you would like $1,000 today or $1,500 in ten years (interest rate at 4%), then it would be a much more difficult question than the previous question. But with an understanding of present value, the "correct" answer would be to accept $1,500 ten years from now, because the $1000 today at 4% interest is only worth $1,480 ten years from now. In this example, the $1,000 today was converted to its future value 10 years from now. Once this value was converted, it was then compared to the $1,500, which was presented as future value in 10 years. Notice how all values were converted to *future value* for comparison.

2.2 Annual Value or Annuities

The previous section described the difference between present value and future value. It also showed how a lump sum given at certain times are worth different amounts in present terms. In engineering, there are often times when annual sums are given in lieu of one time lump sums. An example would be annual energy savings due to the implementation of a more efficient system. Thus, it is important for the engineer to be able to determine the present/future value of future **annual** gains or losses.

For example, let's assume that a solar hot water project, provides an annual savings of $200. Using the equations from the previous section, each annual savings can be converted to either present or future value. Then these values can be summed up to determine the future and present value of annual savings of $200 for four years at an interest rate of 4%.

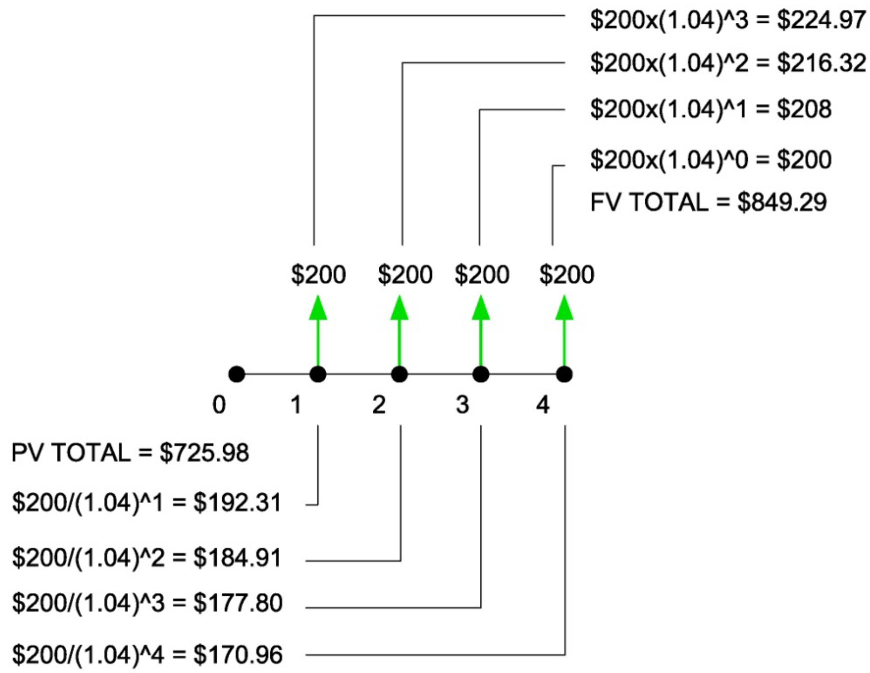


Figure 1: Annual value shown on a cash flow diagram

- For longer terms, this method could become tedious. Luckily there is a formula that can be used to speed up the process in converting annuities (A) to present value and future value.

$$\mathrm{FV} = \mathrm{A} * \left(\frac{(1+\mathrm{i})^{\mathrm{n}} - 1}{\mathrm{i}}\right)$$

$$FV = 200 * \left(\frac{(1+.04)^4 - 1}{.04}\right) = \$849.29$$

$$\mathrm{PV} = \mathrm{A} * \left(\frac{1 - (1+\mathrm{i})^{-\mathrm{n}}}{\mathrm{i}}\right)$$

$$PV = 200 * \left(\frac{1 - (1+.04)^{-4}}{.04}\right) = \$725.98$$

- Reverse Equations, where annual value is solved:

$$\mathrm{A} = \mathrm{PV} * \left(\frac{\mathrm{i} * (1+\mathrm{i})^{\mathrm{n}}}{(1+\mathrm{i})^{\mathrm{n}} - 1}\right)$$

$$\mathrm{A} = \mathrm{FV}\left(\frac{\mathrm{i}}{(1+\mathrm{i})^{\mathrm{n}} - 1}\right)$$

5.0 Depreciation

Depreciation is the value that an asset decreases over time. For example, as a building or an equipment gets older, it starts to gradually deteriorate and reduce in useful life over time. Depreciation values can be represented as either a straight line or accelerated form.

5.1 Straight Line

Straight line depreciation distributes the depreciation values evenly over the life of the asset. This is the simplest method for calculating depreciation and is represented by the following equation.

$$Depreciation\ (\$/Year) = \frac{Capital\ Cost\ (\$) - Salvage\ Value\ (\$)}{Useful\ Life\ (Years)}$$

For example, a machine is purchased at $100,000 and has a salvage value of $10,000. If the machine has a useful life of 10 years, then the straight line depreciation value is:

$$Depreciation\ = \frac{\$100{,}000 - \$10{,}000}{10\ Years} = \$9{,}000/Year$$

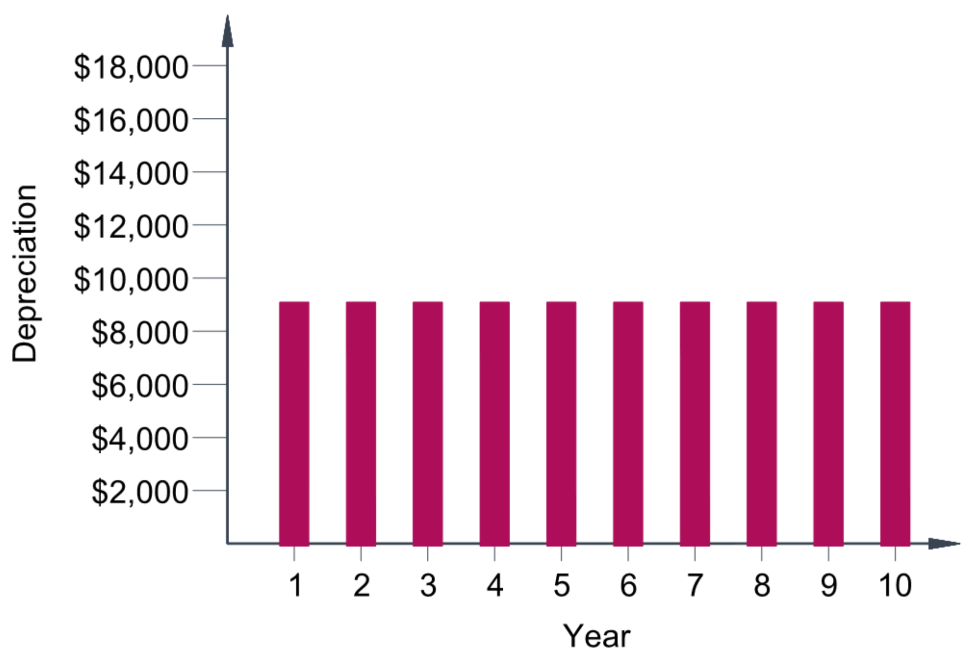


Figure 4: Example of Straight Line Depreciation for an asset with ten years of usable life

5.2 Modified Accelerated Cost Recovery System (MACRS)

The modified accelerated cost recovery depreciation system distributes the depreciation to be heavily weighted in the earlier years of the asset's usable life and less weighted in the later years. In other words, it accelerates the depreciation to earlier in the lifetime of the asset. This system is used for taxes in the United States. It allows for the company to take larger depreciation credits in the earlier years, thereby deferring taxes to later in the asset's lifetime.

There are two main differences between this depreciation method and the straight line method. First, the depreciation occurs over n+1 years, where "n" is the lifetime of the asset. In addition, there is no salvage value for MACRS depreciation. At the end of the "n+1" years, the asset will have a salvage value of $0.

$$Depreciation_{year,j}\ (\$\ at\ Year\ j) = Recovery\ Rate\ (\%)_{year,j} * Capital\ Cost\ (\$)$$

Use the table in the *NCEES FE Reference Handbook* for the recovery rate based on the recovery period. The same equipment used in the straight line example above ($100,000 initial cost, 10 year lifespan) will have a recovery rate at year 2 of 18%, and a recovery rate at year 8 at 6.55%.

$$Depreciation_{year\ 2} = 18\% * \$100{,}000 = \$18{,}000$$

$$Depreciation_{year\ 8} = 6.55\% * \$100{,}000 = \$6{,}550$$

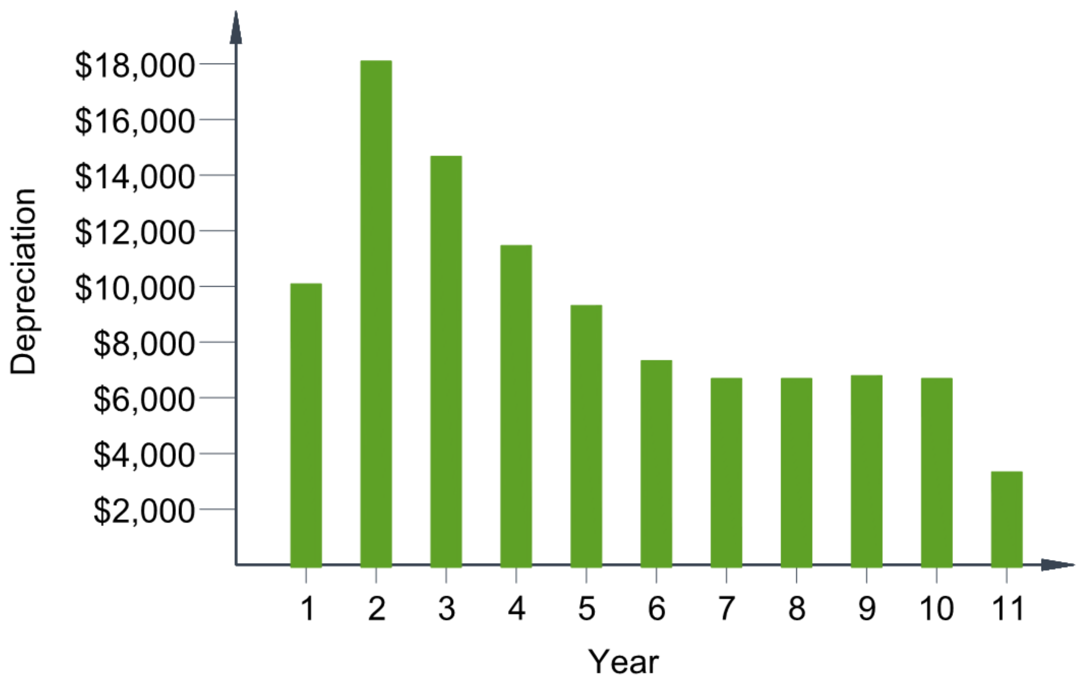


Figure 5: Example of MACRS Depreciation for an asset with ten years of usable life

6.6 PRACTICE PROBLEM 6

A new chiller was purchased for $50,000 in 2016. Using MACRS depreciation, what is the depreciation amount of the chiller at year 9? Assume the equipment has a life of 10 years.

(a) $1,640

(b) $3,280

(c) $4,920

(d) $46,720

6.6.1 Solution 6

A new chiller was purchased for $50,000 in 2016. Using MACRS depreciation, what is the depreciation amount of the chiller at the end of its 10 year life?

In order to solve this problem you need the MACRS depreciation schedule for 10 years, which is found in the *NCEES FE Reference Handbook*. This table shows that the depreciation at year 9 is as shown below.

$$D_{10} = 6.56\%$$

$$Depreciation\, Value\ @\ Year\ 10 = \$50{,}000 * (6.55\%) = \$3{,}280$$

SECTION 6
ELECTRICITY & MAGNETISM

Section 6.0 – Electricity and Magnetism

Table of Contents

1.0 Introduction

Electricity and Magnetism accounts for approximately 3 to 5 questions on the Mechanical FE exam. The mechanical discipline intersects with the electrical when mechanical energy is converted to electrical and vice versa. Hence, the mechanical engineer should have a basic understanding of electricity and magnetism. The most common application is the production of power via a generator and the reception of power through equipment via motors. Other applications include control circuits, variable frequency drives, and the effects of mechanical equipment on power quality. Remember that only the basics of the "Electricity and Computer Engineering" section of the *NCEES FE Reference Handbook* will be tested during the exam. The questions must also relate to the topics below.

Section 6.0 Electricity & Magnetism (3 to 5 Problems)		
NCEES Outline Value	**Engineering Pro Guides**	
	Section 1.0	Introduction
6A	Section 2.0	Charge, Current, Voltage, Power and Energy
6B	Section 3.0	Current and Voltage Laws (Kirchoff, Ohm)
6C	Section 4.0	Equivalent Circuits (Series, Parallel)
6D	Section 5.0	AC Circuits
6E	Section 6.0	Motors and Generators
	Section 7.0	Practice Exam Problems

2.0 Charge, Current, Voltage, Power and Energy

2.1 Charge

An electric charge, Q, describes the number of electrons or protons there are. It can be positive (protons) or negative (electrons) and is measured in Coulombs (C). For example, one electron has $-1.6\text{x}10^{-19}$ C of charge and one proton has $+1.6\text{x}10^{-19}$ C. The movement of electrons is the foundation of how electricity works. It is unlikely that charge itself will be tested. It is more important to understand how charge is used to describe other concepts like current, power, voltage, and energy.

2.2 Current

Current, I, is the movement of charge and is more specifically defined as the rate at which charge flows. It is represented in terms of Amps, where one amp is equal to the movement of one Columb of charge per second.

$$Current, I(Amps) = \frac{Charge\ (C)}{Time\ (\text{sec})}$$

$$I = \frac{dq}{dt} \quad \rightarrow \quad Charge, Q = \int_{t1}^{t2} i(t)\, dt$$

For steady flow, current can be calculated as:

$$I\ (Amps) = \frac{\Delta Q}{\Delta t}$$

One characteristic to distinguish is that current flows in the opposite direction of electrons. Current flows from positive to negative, see the green arrow in the figure below, start at the positive end of the battery, loop around the circuit and end at the negative end. Electrons on the other hand are attracted to positive charge, so it will flow from negative to positive, as shown in red below.

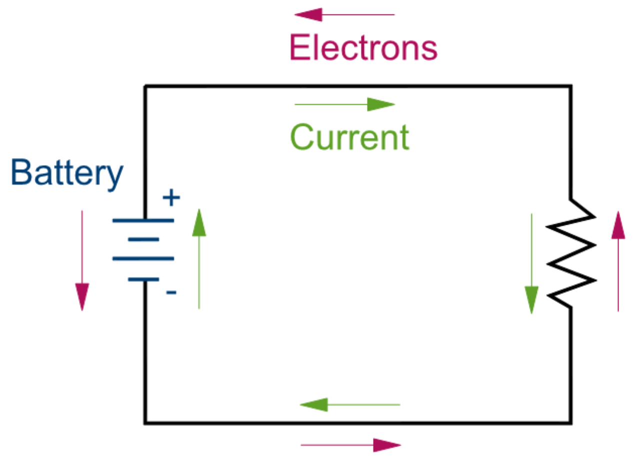


Figure 1: Current flows in a circuit from the positive end of the battery to the negative, as shown in green, while electrons flow from negative to positive.

Direct current (DC) is the supply of current in one direction. As mentioned previously, current flows from the positive voltage terminal to the negative terminal in a circuit. Current is deemed positive when it flows in this direction. Current is considered negative when it flows from a negative terminal to a positive terminal. DC current is a constant source and does not switch between negative and positive. The simplest example of a DC source is a battery.

Alternating current (AC) is able to supply current in both directions, positive to negative and negative to positive. This is shown in the graph below, where the current can be positive (above the 0-axis) or negative (below the 0-axis). Alternating current is what is supplied by the electric company to buildings. Alternating current is further discussed in the Alternating Circuits topic.

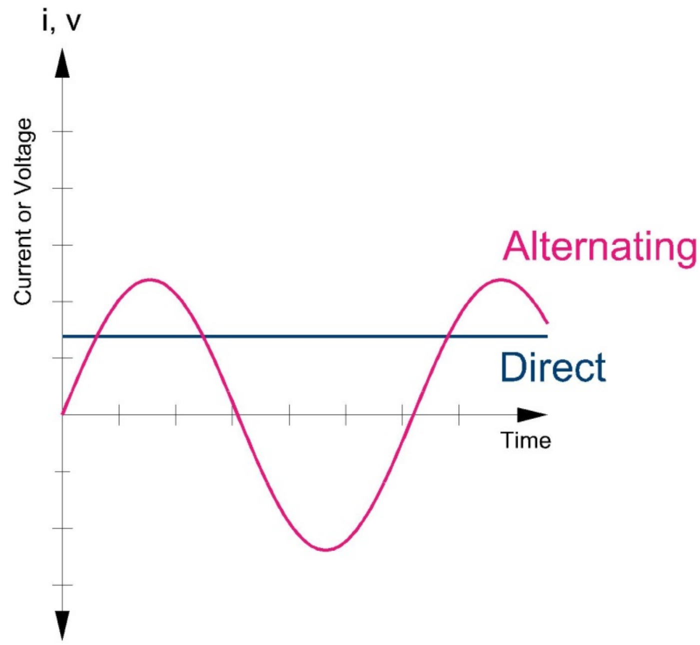


Figure 2: In an AC circuit, current can alternate its flow from positive to negative. In a DC circuit, current is constant.

2.3 VOLTAGE

Voltage, V is the potential energy in electricity; it is the amount of energy held in one charge and is given in units of volts.

$$Voltage\ (V) = \frac{Potential\ Energy\ (J)}{Charge\ (C)}$$

Voltage is measured between two points because potential energy is the difference in energy. This potential energy in a circuit is what drives the flow of electrons, and therefore the current, from one point to the next. A battery is typically a source voltage, supplying energy into a circuit. A voltage can also be measured across a load or a resistor, this is known as voltage drop since the energy is being absorbed by the load.

2.4 RESISTORS

To complete a circuit, wires are connected from a voltage source to a load, which is represented by the resistance. Resistance is measured in units of ohms (Ω). It is the opposition to the flow of current. One ohm is described as the level of resistance that will allow 1 ampere to flow when 1 volt is applied to a circuit. The following figure depicts a basic circuit with current flowing from a voltage source, then through a resistor.

7.4 Practice Problem 4 – Parallel Circuit

Background: A new electrical circuit has 2 resistors in parallel. One resistor has a resistance of 4 Ω. The resistance of the other resistor is unknown. The circuit is powered by a 12 V battery.

Problem: If the total current through the circuit is 8 amps, what is the resistance of the 2nd resistor?

(a) 1.2

(b) 1.6

(c) 2.0

(d) 2.4

7.4.1 Solution 4 – Parallel Circuit

Background: A new electrical circuit has 2 resistors in parallel. One resistor has a resistance of 4 Ω. The resistance of the other resistor is unknown. The circuit is powered by a 12 V battery.

Problem: If the total current through the circuit is 8 amps, what is the resistance of the 2nd resistor?

Determine the current through the 1st resistor.

$$I = \frac{12}{4} = 3\ amp$$

Determine the current through the 2nd resistor.

$$Total\ current - current\ through\ 1st\ resistor = current\ through\ 2nd\ resistor$$

$$8\ amps - 3\ amps = 5\ amps$$

Determine the resistance of the 2nd resistor.

$$R = \frac{12}{5} = 2.4$$

Correct Answer: (B) 2.4 Ω

7.5 Practice Problem 5 – AC Circuit

There is a 480V and 20 A, single phase, load. What is the real power of this load, if the power factor is 0.85? Assume the voltages and currents are root mean square values.

(A) 5.1 kW

(B) 8.2 kW

(C) 11.9 kW

(D) 25.2 kW

7.5.1 Solution AC Circuit

There is a 480V and 20 A, single phase, load. What is the real power of this load, if the power factor is 0.85? Assume the voltages and currents are root mean square values.

For this question, you must use the real power equation which is shown below.

$$P = PF * V_{rms} * I_{rms}$$

$$P = 0.85 * 480\ V * 20\ A$$

$$P = 8{,}160\ W$$

The correct answer is most nearly, **(B) 8.2 kW**.

SECTION 7
STATICS

Section 7.0 – Statics

Table of Contents

1.0 INTRODUCTION

Statics accounts for approximately 8 to 12 questions on the Mechanical FE exam. These questions can cover statics, but not dynamics. The statics topic on the NCEES exam is similar to a common statics college engineering class. Statics is the study of components at equilibrium, which means the components are at rest or at zero acceleration. This topic includes vectors, free body diagrams, moments, reaction forces, first moment of area, static friction and second moment of area. These concepts and skills are used to solve problems on pulleys, cables, springs, beams, trusses, frames, etc.

The *NCEES FE Reference Handbook* Statics section has some basic equations for the topics below, but it does not explain the skills and concepts necessary to use these equations. You should learn the skills and concepts presented in this section and go through the handbook to confirm that you know how to use the basic equations. You may also need to know some of the Mathematics equations like law of cosines and other trigonometry equations presented in the Mathematics section. The handbook also presents screw threads but this is covered more in *Section 15.0 Mechanical Design and Analysis* in this book.

Section 7.0 Statics (8 to 12 Problems)		
NCEES Outline Value	**Engineering Pro Guides**	
	Section 1.0	Introduction
7A	Section 2.0	Resultants of Force Systems
7B	Section 3.0	Concurrent Force Systems
7C	Section 4.0	Equilibrium of Rigid Bodies
7D	Section 5.0	Frames and Trusses
7E	Section 6.0	Centroids
7F	Section 7.0	Moments of Inertia
7G	Section 8.0	Static Friction
	Section 9.0	Practice Exam Problems

2.0 RESULTANTS OF FORCE SYSTEMS

As previously stated, statics is the study of mechanical components at equilibrium, which means the components have zero acceleration or are at rest. The material presented on statics focuses on the key equations and skills necessary to complete the possible problems within this topic on the FE exam.

2.1 VECTORS

Vectors are used to visualize forces and moments in this section and in *Section 9 Mechanics of Materials* and they are used to visualize movement in *Section 8.0 Dynamics, Kinematics and Vibration*. This section will focus on vectors being used to describe the magnitude and direction of force or moment. On the FE exam you must be able to translate words or diagrams into force/moment vectors and you must be able to add/subtract vectors and multiply/divide vectors by scalars.

Vectors can either be represented in a rectangular form or a polar form. The rectangular form consists of an x-component and a y-component. These values are used to represent the magnitude in the x and y directions. In real applications, there will also be a z-component, but for the purposes of the exam you most likely will only need the x and y components. The polar form is shown as a magnitude and an angle. The magnitude describes the length of the vector while the angle determines the direction.

Vector forms are discussed in the Mathematics section of this book, but is repeated here for completeness.

2.1.1 Rectangular Form

The rectangular form is shown as, x plus the y component. The y component is shown as j and the x component is shown as i.

$$Rectangular\ form\text{:}\ xi + yj$$

$$x = x -\ component; y = y -\ component$$

$$Example\ A\ \rightarrow 5i + 2j\ or\ Example\ B \rightarrow\ -1i - 5j$$

The rectangular form is used when adding and subtracting vectors and follows the same rules as normal addition and subtraction, where only like terms can be added and subtracted. For example, “Example A” plus “Example B”, is solved with the following process.

$$Example\ A + Example\ B = (5i + 2j) + (-1i - 5j)$$

$$(5i - 1i) + (2j - 5j) \rightarrow 4i - 3j$$

The rectangular form can also be understood via a graphical format, where the x-axis represents the real component and the y-axis represents the imaginary component.

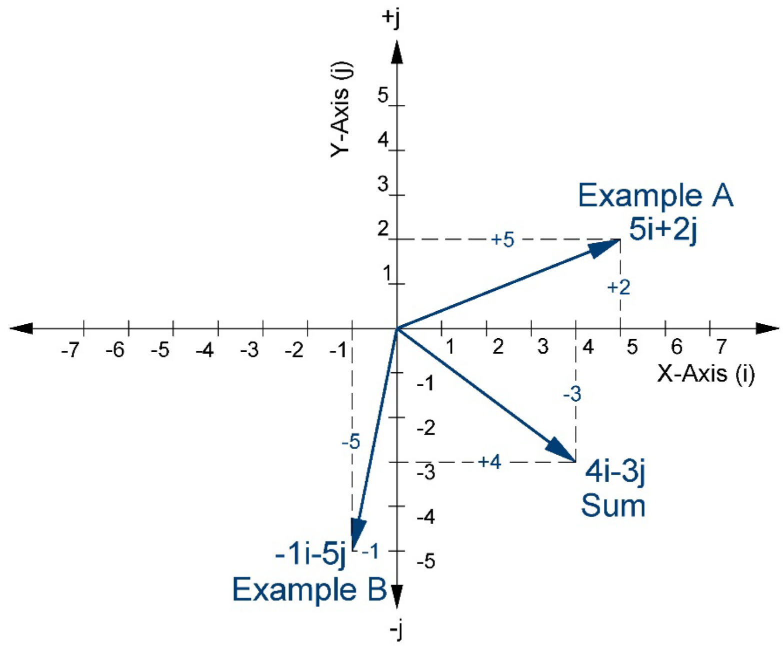


Figure 1: Example "A" vector, example "B" vector and the sum of the two vectors is shown in the above graph.

2.1.2 Polar Form

The polar form is best understood in its graphical format. The format consists of a phasor magnitude at a phasor angle relative to the x-axis.

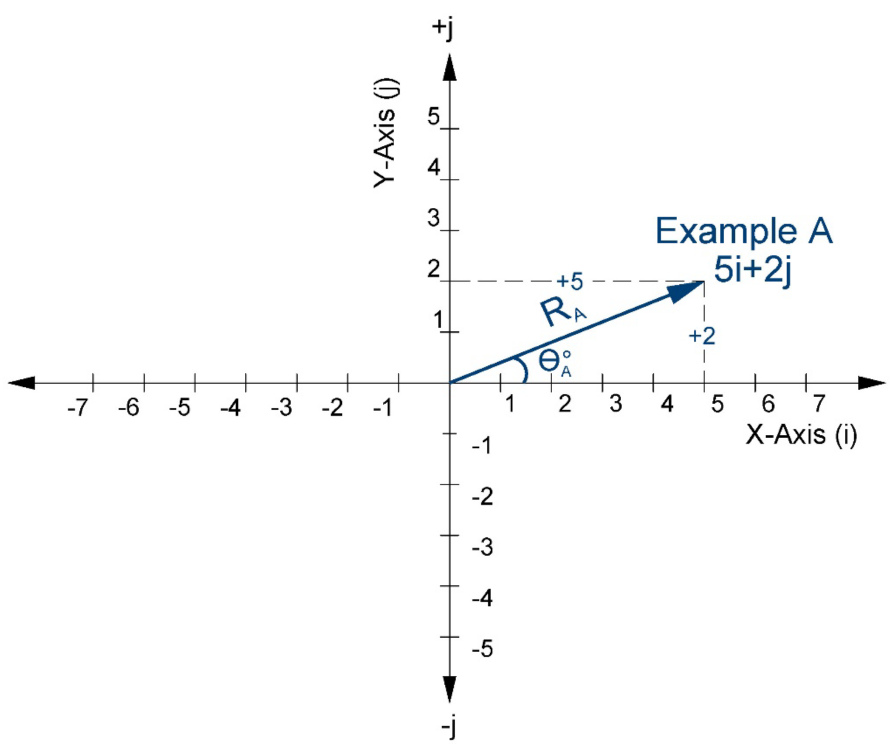


Figure 2: A phasor shown in both polar and rectangular form.

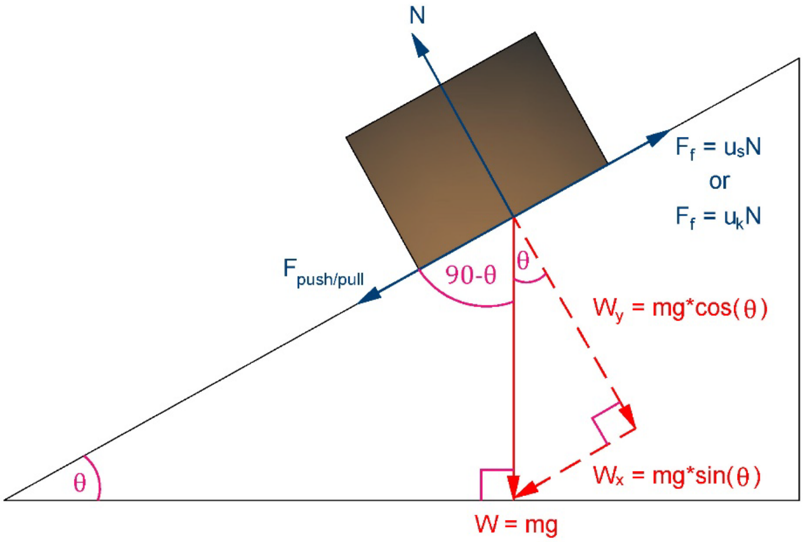
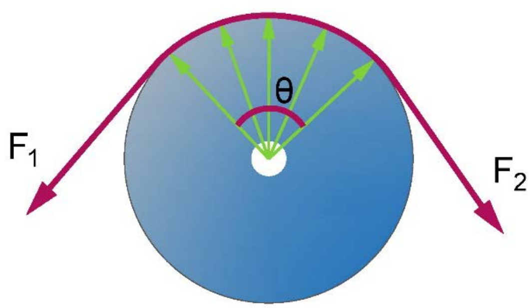


Figure 31: A sliding block with an external force acting upon the block

X-Direction (F_x)	Y-Direction (F_y)
$Impending\ Motion$ $\mu_{static}N = mg * sin(\theta) + F_{push\ or\ pull};$	$Impending\ Motion$ $N = mg * cos(\theta);$

8.3 BELTS AND PULLEYS FRICTION

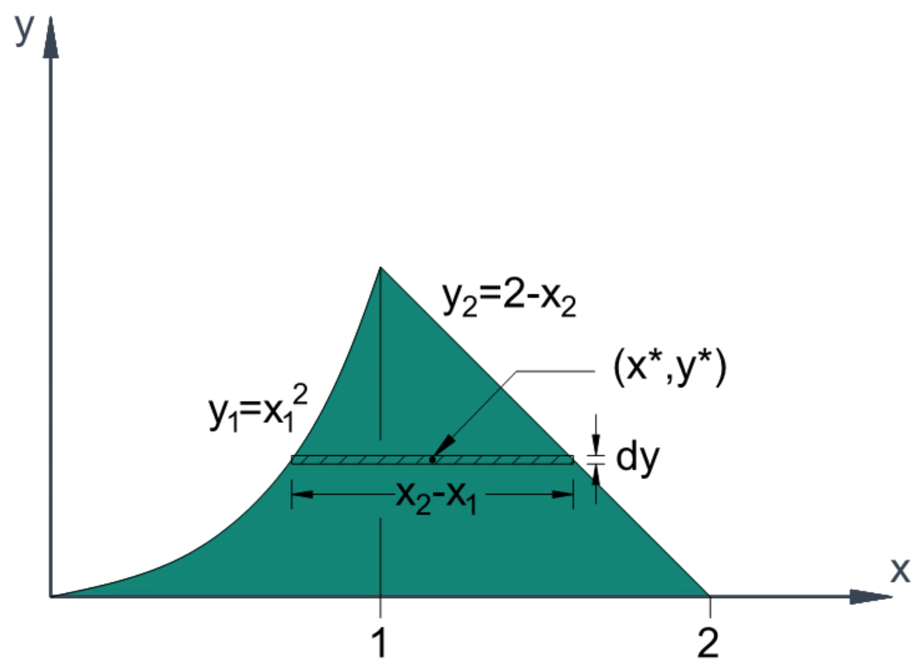
Unlike the previous discussions of friction, the friction at belts and pulleys is dependent on the amount of contact with the rotating surface. At each point of contact, there will be a normal force from the center of the rotating part to the point of contact. In addition there will be a frictional force at that point of contact, perpendicular to the normal force. A simple moment or force balance equation will not suffice for this situation. You must use the following formulas to solve these types of problems.



$$Pulley, Flat\ Belt\ Friction \rightarrow F_2 = F_1e^{\mu\theta}$$

$$\theta = Contact\ surface;\ \mu = static\ friction$$

You should always make sure that your angle values are in radians, when using this equation.



Then, the two curves become:

$$x_1 = \sqrt{y_1} \qquad and \qquad x_2 = 2 - y_2$$

Since y_1 and y_2 are now the independent variable, then y_1 is equal to y_2. The shaded area between the two curves becomes:

$$x_2 - x_1 = (2 - y) - \sqrt{y}$$

Then, find the incremental area of each strip, dA.

$$dA = [x_2 - x_1] * dy = \left[(2 - y) - \sqrt{y}\right] * dy$$

Next, find the center point of each incremental area dA. For this orientation, where the shaded region is taken along the y axis, dy, the center points are defined as the following:

$$x^* = \frac{x_2 - x_1}{2} \qquad and \qquad y^* = y$$

Remark:: if the shaded region were taken along the x-axis, dx, then $x^* = x$ *and* $y^* = y/2$.

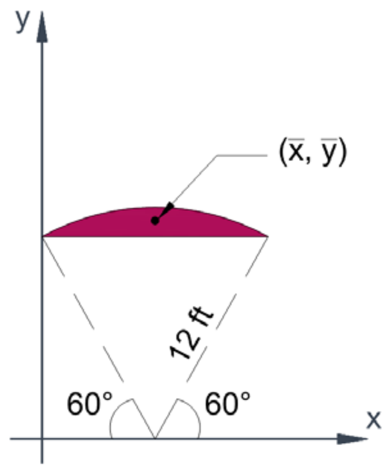
Finally, replace the expressions in the initial equation, where the integral along the y-axis should be taken from 0 to 1, as this is the extent of the shaded region.

$$\bar{y} = \frac{\int_A (y^*)dA}{\int_A \, dA} = \frac{\int_0^1 y * (2 - y - \sqrt{y})dy}{\int_0^1 (2 - y - \sqrt{y})dy} = \frac{\int_0^1 (2y - y^2 - y^{\frac{3}{2}})dy}{\int_0^1 (2 - y - \sqrt{y})dy}$$

The correct answer is (B).

9.10 Practice Problem 10 – Centroid

Find the centroid coordinates, $(\bar{x}, \bar{y})$ for the shaded region of the following figure.



(A) (3ft, 11ft)
(B) (4ft, 10ft)
(C) (4ft, 11.5ft)
(D) (5ft, 11ft)

9.10.1 Solution 10 - Centroid

Find the equation for the centroid of a circular segment in the Statics section of the *NCEES FE Reference Handbook.* The figure is rotated 90 degrees, so switch the x and y equations and shift the center point of the x centroid. The equations become

$$\bar{x} = \frac{12ft * \cos(60°)}{2} = 3ft$$

$$\bar{y} = \frac{2 * a}{3} \frac{\sin^3 \theta}{\theta - sin\theta * cos\theta}$$

In the above equation, a=12ft and θ = 90º-60º = 30º = π/6.

$$\bar{y} = \frac{2 * 12ft}{3} * \frac{\sin^3(30°)}{\pi/6 \ - \sin(30°) * \cos(30°)} = 11.04ft$$

So, the centroid, $(\bar{x}, \bar{y})$ = (3ft, 11ft)

The correct answer is (A) (3ft, 11ft)

SECTION 8
DYNAMICS, KINEMATICS & VIBRATIONS

Section 8.0 – Dynamics, Kinematics and Vibrations

Table of Contents

1.0 INTRODUCTION

Dynamics, Kinematics and Vibrations accounts for approximately 9 to 14 questions on the Mechanical FE exam. This section is first broken down into the two topics, (1) particles and (2) rigid bodies. Particles are a single mass subject and rigid bodies consist of a collection of particles into a solid body that does not deform. The kinematics, work, energy, impulse and momentum and kinetics equations first focus on a single mass subject. You may be familiar with problems that had a block moving down a hill or a ball being thrown. These types of problems covered particles. Problems on rigid bodies are those that contain mechanical components like cams where one part of the body moves relative to another part of the body.

Kinematics covers the movement, speed and acceleration of particles and rigid bodies. This includes radial movement and movement due to gravity. Kinetics builds upon kinematics by including force and energy, which also transitions into the work-energy topic. Lastly, this section covers friction and impulse-momentum.

Section 8.0 Dynamics, Kinematics & Vibrations (9 to 14 Problems)		
NCEES Outline Value	Engineering Pro Guides	
	Section 1.0	Introduction
8A	Section 2.0	Kinematics of Particles
8B	Section 3.0	Kinetic Friction
8C	Section 4.0	Newton's Second Law for Particles
8D	Section 5.0	Work-Energy of Particles
8E	Section 6.0	Impulse-Momentum of Particles
8F	Section 7.0	Kinematics of Rigid Bodies
8G	Section 8.0	Kinematics of Mechanisms
8H	Section 9.0	Newton's Second Law for Rigid Bodies
8I	Section 10.0	Work-Energy of Rigid Bodies
8J	Section 11.0	Impulse-Momentum of Rigid Bodies
8K	Section 12.0	Free and Forced Vibrations
	Section 13.0	Practice Exam Problems

2.0 KINEMATICS OF PARTICLES

Kinematics type problems will be centered on finding one of these three variables, (1) the distance traveled by an object, (2) the velocity of an object or (3) the acceleration of an object at any given time or location. If a problem asks for one of these three variables and forces are not involved, then most likely the solution will be found using the equations presented in the kinematics topic.

Within the topic of kinematics you may encounter problems with either linear motion or angular motion. Linear motion is the movement of an object within the x-y-z plane in either a straight line or a curve. Curve type movement is typical of projectiles and straight line movement is typical of vehicles, sliding blocks, pistons and springs. Angular motion is the circular movement of an object about an axis, within the x-y-z plane. This type of movement is typical of gears, pumps, fans and any other equipment that rotates about an axis.

2.1 LINEAR MOTION

2.1.1 Linear Displacement

The first variable in linear motion is displacement.

$$x(t)$$

$$Constant\ velocity \rightarrow x_f = x_i + vt; a = 0;$$

On the FE exam, you can use the following equations to help you solve any kinematics problems for the distance (x_f) when there is constant acceleration.

Solving for Distance with Uniform Acceleration (a; V_i ; x_i = 0)			
$x_f = x_i + V_i t + \frac{1}{2}at^2$	$x_f = \frac{(V_f + V_i)t}{2}$	$x_f = \frac{(V_f^2 - V_i^2)}{2a}$	$x_f = V_f t - \frac{1}{2}at^2$

2.1.2 Linear Velocity

The instantaneous velocity at a time (t) is the derivative of the position.

$$v(t) = \frac{dx}{dt};\ \ Units \rightarrow \frac{m}{s}\ or \frac{ft}{s}$$

The average velocity of an object can be found by dividing the change in position over a specific time interval.

$$Average\ velocity = \frac{\Delta x}{\Delta t}$$

On the FE exam, you can use the following equations to help you solve any kinematics problems for the specific scenario of uniform acceleration.

Solving for Velocity with Uniform Acceleration (a; V_i ; x_i = 0)			
$V_f = V_i + at$	$V_f = \frac{2x_f}{t} - V_i$	$V_f = \frac{x_f}{t} + \frac{1}{2}at$	$V_f = \sqrt{V_i^2 + 2ax_f}$

2.1.3 Linear Acceleration

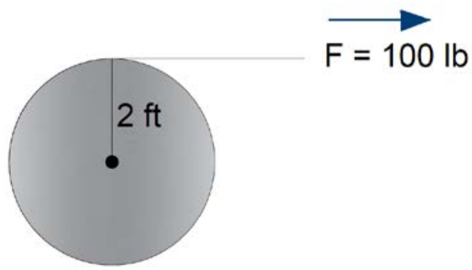
The instantaneous acceleration at a time (t) is the derivative of the velocity or the second derivative of the position.

$$v_b = 7\frac{ft}{min}$$

The correct answer is most nearly (C) 7 ft/min.

13.12 PRACTICE PROBLEM 12 – WORK-ENERGY

An average force of 100 lbs is used to pull a cable that is wrapped around a spool as shown in the figure below. Assume the initial velocity at the end of the cable where the force is applied is 0 ft/s. The moment of inertia of the spool and cable is 10 lb-ft-s^2. Assume the cable and spool surfaces are frictionless. After the cable has been pulled for 5 feet of horizontal length, what will be the velocity at the end of the cable where the pulling force is applied?



(a) 5 ft/s

(b) 10 ft/s

(c) 15 ft/s

(d) 20 ft/s

13.12.1 Solution 12 – Work-Energy

In order to solve this problem you must do a conservation of energy equation for the initial and final conditions. Initially the energy is equal to zero.

$$Initial\ Energy + Work = Final\ Energy$$

The force will conduct positive work that will cause a rotational energy in the spool/cable. You can use the below equation to solve for rotational velocity.

$$0 + 100\ lb * (5\ feet) = \frac{1}{2} I * \omega^2$$

$$500\ lb - ft = \frac{1}{2} * 10\ lb - ft - s^2 * \omega^2$$

$$\omega = 10\frac{rad}{s}$$

The next step is to convert the angular velocity to linear velocity.

$$v = \omega * r = 10\frac{rad}{s} * 2\,ft = 20\frac{ft}{s}$$

The correct answer is most nearly (D) 20 ft/s.

13.13 PRACTICE PROBLEM 13 – KINEMATICS

A motor shaft is rotating at 1750rpm. Power is cut to the motor and it takes 110 seconds for the shaft to come to rest. Assuming uniform acceleration, how many rotations does the shaft make before coming to rest?

(a) 1604 rotations

(b) 4812 rotations

(c) 9600 rotations

(d) 10,080 rotations

13.13.1 Solution 13 - Kinematics

Refer to the *NCEES FE Reference Handbook* and use the rigid body motion equation around a fixed axis with constant angular acceleration. First solve for the angular acceleration from the initial rotational speed to the time of rest using the following equation.

$$\omega = \omega_0 + \alpha_0 t$$

The shaft starts at 1750 rpm, then decelerates to 0 rpm. Convert the rotational velocity to radians per second.

$$\omega_0 = 1750\frac{rotation}{min} * 2\pi\frac{rad}{rotation} * \frac{1min}{60sec} = 183.3\frac{rad}{s}$$

Solve for acceleration

$$0 = 183.3\frac{rad}{s} + \alpha_0 * 110sec$$

$$\alpha_0 = -1.67\frac{rad}{s^2}$$

Finally, solve for the number of rotations.

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha_0 t^2$$

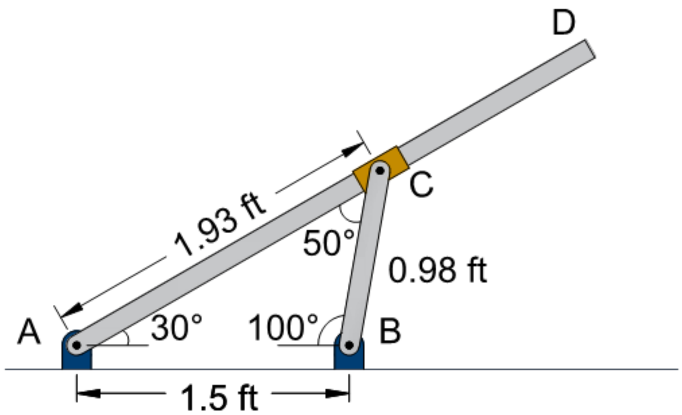
$$\theta = 0 + 183.3\frac{rad}{s} * 110s + \frac{1}{2} * \left(-1.67\frac{rad}{s^2}\right) * (110s)^2 = 10{,}079\,rad$$

$$\theta = 10{,}079\ rad * \frac{1\ rot}{2\pi\ rad} = 1604\ rotations$$

The answer is (A) 1604 rotations.

13.14 Practice Problem 14 – Kinematics of Mechanisms

In the figure below, rod BC rotates at a constant speed of 15 rad/s in a clockwise direction. Point C slides along rod AD and rotates with rod BC. What is the velocity of the slider at point C, relative to rod AD?



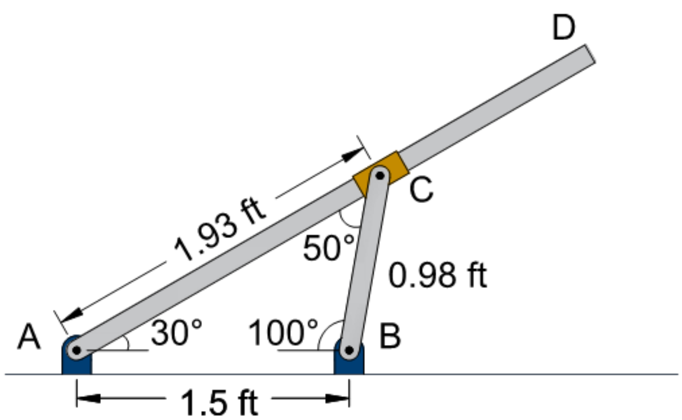
(a) 14.7 ft/s

(b) 16.9 ft/s

(c) 19.2 ft/s

(d) 22.8 ft/s

13.14.1 Solution 14 – Kinematics of Mechanisms



Since point C slides along rod AD, the linear velocity at point C will be perpendicular to rod AD. Therefore the linear velocity at point C is the angular velocity of rod AD times the distance AC.

$$V_c = AC * \omega_{AD}$$

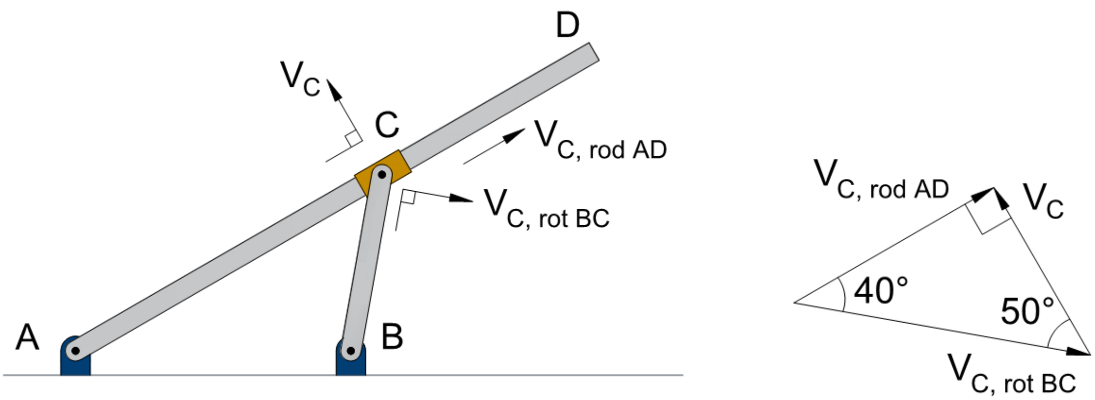
The linear velocity at point C can also be written as the linear velocity obtained from the rotational velocity from rod BC plus the linear velocity of point C, relative to rod AD

$$V_c = V_{C,rot\ BC} + V_{C,rod\ AD}$$

We can solve for $V_{C,\text{ rot BC}}$

$$V_{C,rot\ BC} = BC * \omega_{BC} = 0.98\ ft * 15\frac{rad}{s} = 14.69\ ft/s$$

The vector diagram for the velocities are shown below.



Use the vector diagram to solve for $V_{C,\text{ rod AD}}$

$$V_{C,rod\ AD} = \frac{V_{C,rot\ BC}}{\cos(40°)} = \frac{14.7\ ft/s}{\cos(40°)} = 19.2 ft/s$$

The answer is (C) 19.2 ft/s.

SECTION 9
MECHANICS OF MATERIALS

Section 9.0 – Mechanics of Materials

Table of Contents

1.0 INTRODUCTION

Mechanics of Materials accounts for approximately 8 to 12 questions on the Mechanical FE exam. Mechanics of Materials questions basically will cover calculating the stress or strain due to different loadings, like axial loads, bending loads, torsional loads and shear loads. In addition, you will have to calculate the displacement, shear force and moments for beams and the buckling forces and stresses in columns.

The equations shown in the *NCEES FE Reference Handbook* on Mechanics of Materials is comprehensive and all of these equations are fairly simple and easy to use. This means that the equations do not have any complex math. After going through this section, you should be familiar with the different types of loadings and all the equations within the handbook and should be able to quickly recognize when to use each equation for each problem.

Section 9.0 Mechanics of Materials (8 to 12 Problems)		
NCEES Outline Value	**Engineering Pro Guides**	
	Section 1.0	Introduction
9A	Section 2.0	Shear and Moment Diagrams
9B	Section 3.0	Stress Types (axial, bending, torsion, shear)
9C	Section 4.0	Stress Transformations
9D	Section 5.0	Mohr's Circle
9E	Section 6.0	Stress and Strain caused by Axial Loads
9F	Section 7.0	Stress and Strain caused by Bending loads
9G	Section 8.0	Stress and Strain caused by Torsion
9H	Section 9.0	Stress and Strain caused by Shear
9I	Section 10.0	Combined loading
9J	Section 11.0	Deformations
9K	Section 12.0	Columns
	Section 13.0	Practice Exam Problems

2.0 SHEAR AND MOMENT DIAGRAMS

Shear and moment diagrams are used to graphically show how forces, displacement and moments change depending on the location within a component. The components that you will most likely need to know are beams, since the shear and moment diagrams included in the *NCEES FE Reference Handbook* are only for beams.

However, before you use the diagrams you should understand the concepts behind the beam diagrams and the resulting equations. Once you have a good grasp on the concepts and how to use the equations, then you should be able to solve these types of problems.

There are three main types of beam diagrams, (1) Free Body Diagram, (2) Shear Diagram and (3) Moment Diagram. The first step (1) is to determine the forces acting upon the beam in order to construct the beam diagram. Beams can be loaded with a load at a point or a distributed load along the entire length of the beam. The loads are primarily downward and in order to create equilibrium there will be reaction forces upward at the supports. Equilibrium equations for both forces and moments about each of the supports are used to find the reaction forces. The forces must equal zero, since the beam is restricted from movement. Also the sum of the moments at each of the supports must be zero, since the beam is restricted from twisting. Once you have found all loads and reaction forces, then you can construct the free body diagram.

The next step (2) is to construct the shear diagram. The shear diagram describes the internal forces within the beam at any point of the beam. This diagram is created by splitting up each segment separated by a point force or reaction force. Then you must cover all external forces to the right of each point and sum up all external forces and this sum is the shear force at that point.

The final step (3) is to construct the moment diagram. The moment diagram is comprised of the moment at any location along the length of the beam. The moment diagram is constructed in a similar fashion as the shear diagram.

In practice, shear diagrams and moment diagrams are not often re-constructed, since the diagrams have already been documented. Please see the *NCEES FE Reference Handbook* for completed beam diagrams with equations. Beam diagrams will most likely be used on the FE exam in order to find the maximum stress in the beam or the maximum deflection in the beam. This can be done by using the given beam diagrams and using the corresponding equations. However, there may be conceptual type problems which will require you to have an understanding of how these diagrams are constructed. The next few pages will show a few diagrams and walk you through some points on the diagram.

In practice, beam supports can be classified based on how much the beam is restricted by the support.

- Loose support (simply supported): A loose support, which would be the same as the simply support diagram, is used to support the weight vertically. The beam can be moved in all other directions. Including along the length of the beam (longitudinal) and perpendicular to the direction of the pipe (transverse). The beam is only restricted from moving vertically (up and down).

- Loose support with longitudinal guide: This type of support includes the loose support but also has a restriction on the transverse. The beam will not be allowed to move perpendicular to the direction of the beam. This would include putting a clamp around a beam to stop the side to side movement.

- Loose support with transverse guide: This type of support includes the loose support but also has a restriction on the longitudinal. This would stop the beam from expanding and contracting in the longitudinal direction.

- Loose support with anchor: This type of support restricts movement in both the longitudinal and transverse direction.

2.2 Simple beam with a uniformly distributed load

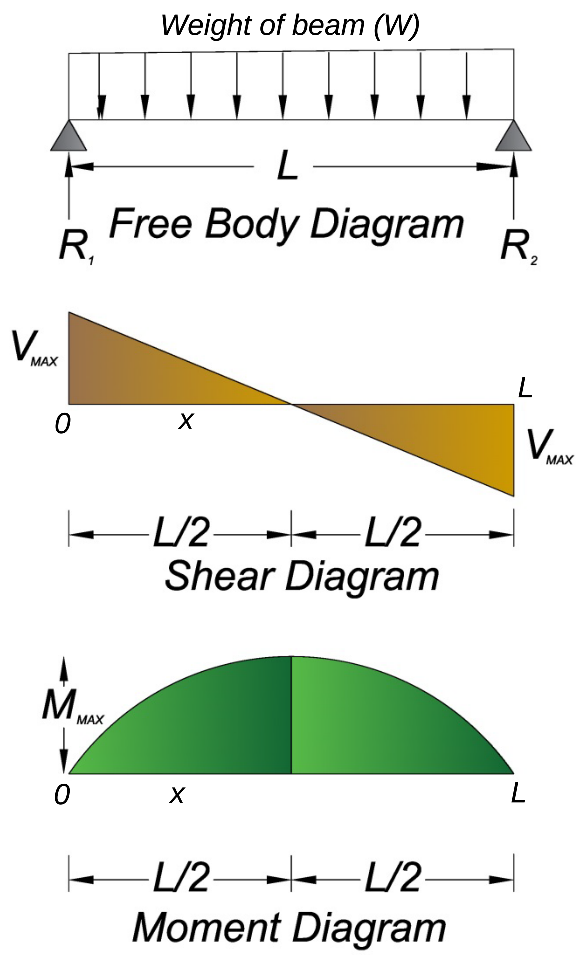


Figure 1: Simple beam with uniformed load diagrams

This diagram is typically used when a beam is supported at both ends for only the weight of the beam. The downward force is equal to the weight of the beam. The weight is evenly distributed evenly the entire length of the beam, which is why the vectors are of equal size.

$$Total\ Weight\ of\ Beam = W\ (lbf) = w\ (lbf/ft) * L\ (ft)$$

$$Total\ Weight\ of\ Beam = W\ (N) = w\ (N/m) * L\ (m)$$

There will be upward reaction forces at the support that will counteract the weight of the beam. The force at each support will be equal to one-half the entire weight.

$$R_1 = R_2 = \frac{W}{2}\ (N\ \ or\ \ lbf)$$

$$\tau_{shear\ stress} = \frac{T}{J}\ and\ \gamma = \frac{\Delta\theta}{L}$$

Then you can plug in the above equations back into Hooke's law.

$$\frac{T}{J} = G * \frac{\Delta\theta}{L}$$

$$\Delta\theta = \frac{TL}{GJ}$$

Now plug in the deformation amount back into the original strain energy equation.

$$Strain\ Energy\ (U) = \left(\frac{1}{2}\right) * T * \frac{TL}{GJ}$$

12.0 Columns

12.1 Buckling

Buckling occurs when an object is subject to compression and as the compressive load increases, there is a quick bowing movement outwards, perpendicular to the direction of the loading. This situation becomes very unstable and the support will most likely be either deformed or unable to carry the force. A material's stress during buckling will most likely be less than the stress required to fracture or crush the same material in compression, which is typically the ultimate tensile stress or yield stress.

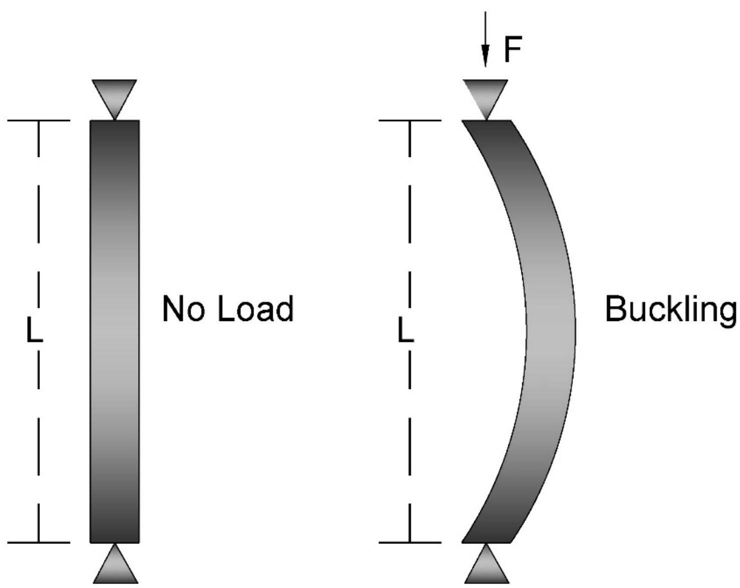


Figure 24: Buckling occurs when the length of a material is typically an order of magnitude greater than the width or diameter of the material.

13.13.1 Solution 13 – Axial Strain

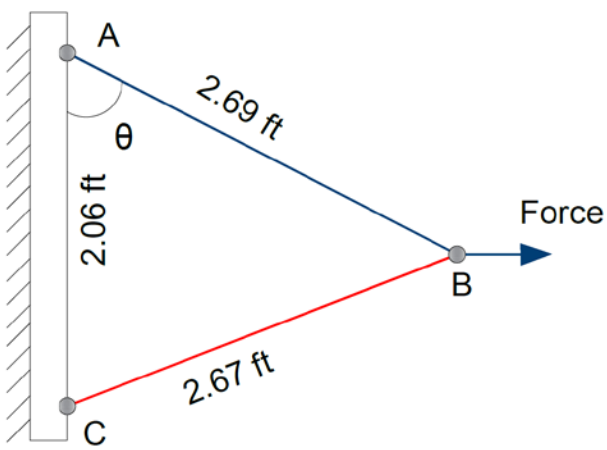
First, solve for the new lengths of AB and CB.

$$\varepsilon = \frac{\Delta L}{L}$$

$$\varepsilon_{AB} = .05\frac{in}{in} = \frac{\Delta L}{2.56\,ft * 12\,(\frac{in}{ft})} \rightarrow \Delta L = 1.536\,in; \rightarrow L_{AB,new} = 32.26\,in\ or\ 2.688\,ft$$

$$\varepsilon_{CB} = .09\frac{in}{in} = \frac{\Delta L}{2.45\,ft * 12\,(\frac{in}{ft})} \rightarrow \Delta L = 2.646\,in; \rightarrow L_{AB,new} = 32.05\,in\ or\ 2.6705\,ft$$

Now you can use the new triangle lengths to solve for an angle within the triangle with the Law of Cosines.

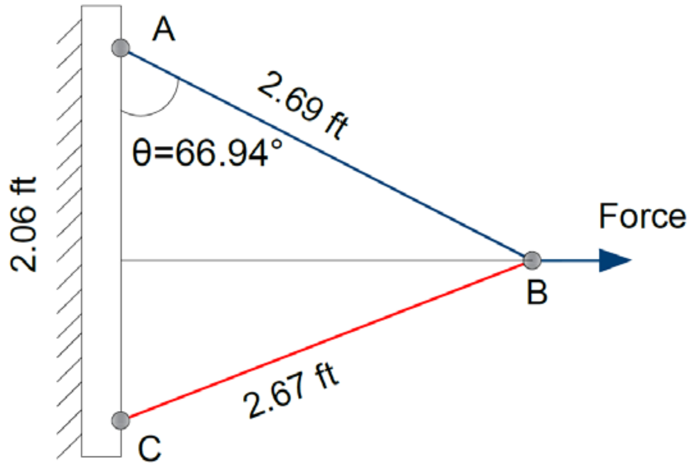


Law of Cosines

$$2.67^2 = 2.06^2 + 2.69^2 - 2 * 2.69 * 2.06 * \cos(\theta)$$

$$\theta = 66.94°$$

Now make a right triangle and solve for the new perpendicular distance between the wall and point B.



$$\sin(66.94) = \frac{Distance}{2.69\ ft}$$

$$Distance = 2.48\ ft$$

The correct answer is most nearly (C) 2.48 ft.

SECTION 10
MATERIAL PROPERTIES & PROCESSING

Section 10.0 – Material Properties and Processing

Table of Contents

1.0 INTRODUCTION

Material Properties and Processing accounts for approximately 8 to 12 questions on the Mechanical FE exam. This section works in conjunction with *Section 10 Mechanics of Materials*. The Material Properties part covers the mechanical properties covered in Section 10 and other material properties like chemical, electrical and physical types. This section then covers the material processing, material types, phase diagrams and other material issues like corrosion, fatigue and cracks.

There are a few pages on Material Properties & Processing in the *NCES FE Reference Handbook* that you should be familiar with in order to pass the FE exam. However, in order to use those pages you need to understand the concepts and skills presented in this section.

Section 10.0 Material Properties & Processing (8 to 12 Problems)		
NCEES Outline Value	**Engineering Pro Guides**	
	Section 1.0	Introduction
10A	Section 2.0	Material Properties (Chemical, Electrical, Mechanical, Physical, Thermal)
10B	Section 3.0	Stress-Strain Diagrams
10C	Section 4.0	Engineered Materials
10D	Section 5.0	Ferrous Metals
10E	Section 6.0	Nonferrous Metals
10F	Section 7.0	Manufacturing Processes
10G	Section 8.0	Phase Diagrams
10H	Section 9.0	Phase Transformation, Equilibrium, and Heat Treating
10I	Section 10.0	Materials Selection
10J	Section 11.0	Surface Conditions
10K	Section 12.0	Corrosion Mechanisms and Control
10L	Section 13.0	Thermal Failure
10M	Section 14.0	Ductile or Brittle Behavior
10N	Section 15.0	Fatigue
10O	Section 16.0	Crack Propagation
	Section 17.0	Practice Exam Problems

2.0 MATERIAL PROPERTIES (CHEMICAL, ELECTRICAL, MECHANICAL, PHYSICAL, THERMAL)

2.1 CHEMICAL

A chemical property is the property of a material that undergoes change in its chemical structure during a chemical reaction. An important chemical property is corrosion, which will be discussed later in this section.

2.2 ELECTRICAL

The important electrical properties of materials are conductivity and capacitance. The conductivity describes the material's ability to conduct electricity and capacitance describes the material's ability to hold a charge. Conductivity is often expressed as its inverse, which is called resistivity. This is the material's ability to resist an electrical current.

$$Conductivity(C) = \frac{1}{Resistivity\ (\rho)}$$

Resistivity is used to calculate the resistance of an object like a copper wire. Resistance is given in units of ohms and is a function of the material's resistivity, the cross sectional area of the material and the length of the material.

$$R\ (\Omega) = \frac{\rho L}{A}$$

$$A = area\ (m^2); l = length\ (m)$$

$$\rho = resistivity\ (\Omega - \mathrm{m}); C = conductivity\ (\frac{1}{\Omega - \mathrm{m}})$$

2.3 MECHANICAL

The mechanical properties of materials are found by a series of tests. These tests include the tension or compression test, which determines the Stress-Strain diagram. The Rockwell or Brinell tests determine the hardness of the material. There are also many more tests that determine the other properties like creep, thermal expansion, thermal conductive, specific heat capacity and visco-elasticity. Each of the mechanical material properties will be discussed in the next paragraphs.

2.3.1 Strength

Strength is covered in *Topic 3.0 Stress-Strain Diagrams*.

2.3.2 Hardness

Hardness is the property that describes a material's ability to withstand abrasion, scratching and indentation. Hardness is measured in terms of MPa or is a dimensionless value. The hardness measurements are useful when comparing one material to another.

Hardness is measured by any one of the following tests, (1) Rockwell, (2) Brinell, (3) Meyer, (4) Vickers, (5) Knoop or (6) Scleroscope. Each of these tests use a different object of varying shape and material to impact the subject material. The effect of this impact due to a known force is then measured and a hardness value is assigned. The two main tests that you should know for the FE exam are Rockwell and Brinell. You should also know that hardness values are not absolute, these tested values are relative to each other. Hardness values are used to compare the hardness between different materials.

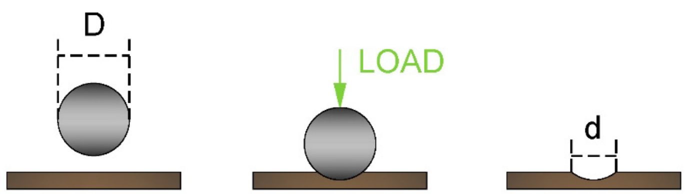


Figure 1: Hardness tests involve a known load impacting a material's surface and the measurement of the load and the result of the impact upon the material's surface.

2.3.1 Rockwell

The Rockwell hardness test is the most popular test. In this test, an indenter is first used as a pre-load to remove any effects of the surface finish and then the full load is used to indent the material. The depth of the indentation determines the Rockwell hardness value. This test has different scales based on the size of the indenter used to impact the test material. The scales range from A through G. Each scale is used for different types of materials. A summary of the scales are shown below.

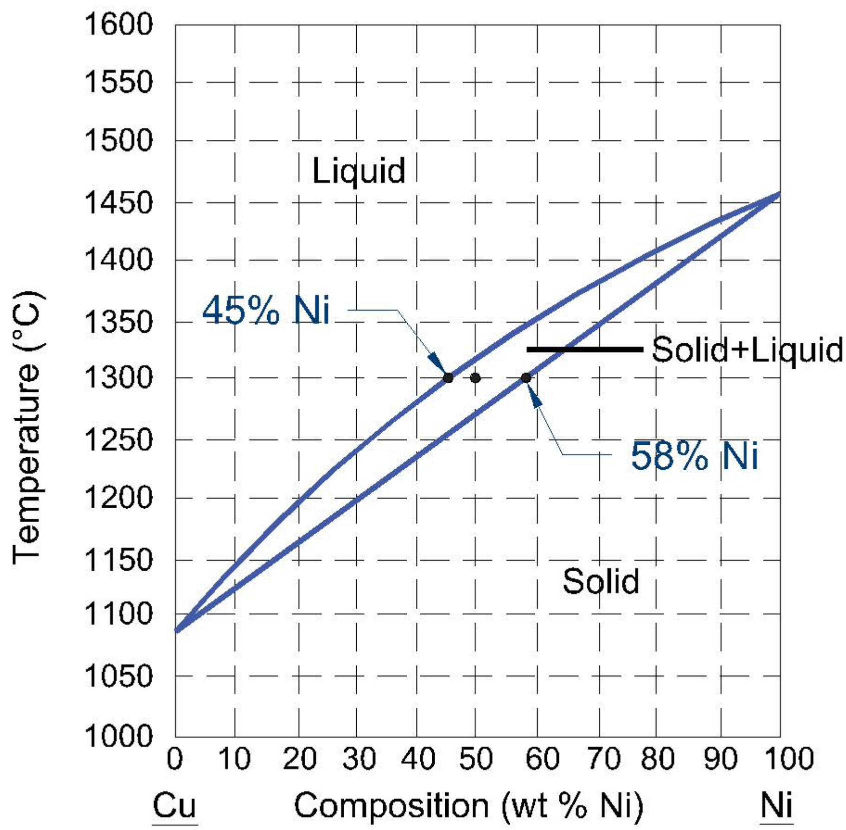
Rockwell Hardness Scale	Full Load Value (kg)	Indenter Size	Typical Use
A	60	120° cone, 0.2 mm radius diamond	Extremely hard materials
B	100	1/16"-D Steel Sphere	Soft steel
C	150	120° cone, 0.2 mm radius diamond	Hard steel, iron
D	100	120° cone, 0.2 mm radius diamond	Medium hardness
E	100	1/8"-D Steel Sphere	Medium Copper, aluminum
F	60	1/16"-D Steel Sphere	Soft Copper, aluminum
G	150	1/16"-D Steel Sphere	Softest materials

The hardness number is then determined by measuring the thickness of the indent made into the material and inserting this value into one of the following equations.

$$Rockwell\ A, C\ and\ D\ Hardness\ \# = 100 - 500t$$

17.6 PRACTICE PROBLEM 6 – ALLOYS

A copper-nickel alloy has 50% nickel and exists as a solid-liquid mixture at 1300ºC? What is the percentage by weight of solid at this temperature?



a) 38%

b) 50%

c) 62%

d) 63%

17.6.1 Solution 6 - Alloys

At 50% nickel and 1300ºC, the alloy exists within the solid + liquid state. Use the lever rule to determine what percentage of the mixture is solid.

$$\%Weight_{Solid} = \frac{Composition_x - Composition_{liquid}}{Composition_{solid} - Composition_{liquid}} x100\%$$

First, find the composition of the liquidus and solidus points at 1300ºC.

$$Composition_{solidus} = 58\%\ wt\ Nickel,\ \ Composition_{liquidus} = 45\%\ wt\ Nickel$$

SECTION 11
FLUID MECHANICS

Section 11.0 – Fluid Mechanics

Table of Contents

1.0 INTRODUCTION

Fluid mechanics accounts for 9 to 14 problems on the Mechanical FE Exam. The topics range from college fluid mechanics topics like fluid properties, fluid statics, energy, impulse, momentum, internal flow, external flow and compressible flow to the topics that are more often used in practice like incompressible flow, power, efficiency, performance curves and scaling laws for fans, pumps and compressors. As you go through this section, you should also check the fluids topics within the *NCEES FE Reference Handbook*.

Section 11.0 Fluid Mechanics (9 to 14 Problems)		
NCEES Outline Value	**Engineering Pro Guides**	
	Section 1.0	Introduction
11A	Section 2.0	Fluid Properties
11B	Section 3.0	Fluid Statics
11C	Section 4.0	Energy, Impulse and Momentum
11D	Section 5.0	Internal Flow
11E	Section 6.0	External Flow
11F	Section 7.0	Incompressible Flow
11G	Section 8.0	Compressible Flow
11H	Section 9.0	Power and Efficiency
11I	Section 10.0	Performance Curves
111J	Section 11.0	Scaling Laws for Fans, Pumps and Compressors
	Section 12.0	Practice Exam Problems

2.0 FLUID PROPERTIES

During the exam you will need to be able to find and use fluid properties to complete many problems. You should be very familiar with the *NCEES FE Reference Handbook* and where to find these fluid properties. As you go through these descriptions of the important fluid properties, look through the handbook and you will see that the only fluids mentioned in the handbook with all of these properties are air and water. This should give you an indication that most of the questions on fluids will revolve around air and water and if another fluid is given in a question, then all the properties for that fluid must be provided in the question, except for heat capacity for select fluids like air, water, ethane, methane, mercury, etc. Only select fluids have their densities shown in the handbook.

2.1 DENSITY

The density of a substance is its mass per unit volume. For example, density is typically shown as pound-mass per cubic foot or kilograms per cubic meter.

$$Density = \frac{lbm}{ft^3}[IP] \;\; or \;\; \frac{kg}{m^3}[metric]$$

The density of a fluid is measured as a weight per unit volume. Specific volume is the inverse of density and is measured as a volume per unit mass.

$$Specific\ Volume = \frac{ft^3}{lbm}[IP] \;\; or \;\; \frac{m^3}{kg}[metric]$$

2.1.1 IP Conversions

When performing calculations in English units, it is important to distinguish between pound mass (lbm) and pound force (lbf). The mass of an object is measured in pound-mass, similar to the English units of kilograms (kg). Pound-force, on the other hand, is a measurement of weight. It is a unit of force and is used to describe the mass of an object subject to gravity. Pound-force is comparable to the metric force of Newtons.

$$Weight\ [lbf] = mass * gravity = 1lbm * 32.174\frac{ft}{s^2}$$

$$\mathbf{1lbf = 32.174\frac{lbm \cdot ft}{s^2}}$$

To perform calculations between pound-mass and pound force, the conversion factor, g_c is used. Since g_c is merely a unit conversion factor, it can be multiplied or divided anywhere in an equation.

$$g_c\ [conversion\ factor\ ] = \frac{ma}{F}$$

$$\mathbf{g_c = 32.2\ \frac{lbm \cdot ft}{lbf \cdot s^2}}$$

Finally, mass can be represented in terms of slugs, which simplifies the force equations, essentially internalizing the g_c conversion factor.

$$\mathbf{1\ slug = 32.2lbm}$$

$$1lbf = \cancel{1lbm} * \frac{1slug}{\cancel{32.2lbm}} * \cancel{32.2}\frac{ft}{s^2} = 1slug * \frac{ft}{s^2}$$

$$\mathbf{1\ slug = \frac{1lbf \cdot s^2}{ft}}$$

Example: Density relationships in terms of lbm, slug and lbf.

$$Density, \rho = 1\frac{lbm}{ft^3} = \frac{lbm}{ft^3} * \frac{1}{g_c} = \frac{\cancel{lbm}}{ft^3} * \frac{lbf \cdot s^2}{32.2\cancel{lbm} \cdot ft} = \frac{1}{32.2}\frac{lbf \cdot s^2}{ft^4}$$

$$\mathbf{32.2\frac{lbm}{ft^3} = 1\frac{lbf \cdot s^2}{ft^4} = 1\frac{slug}{ft^3}}$$

Realize that the densities in the *NCEES FE Reference Handbook* properties table is given in terms of $\frac{lbf \cdot s^2}{ft^4}$ or $\frac{slug}{ft^3}$.

2.2 VISCOSITY

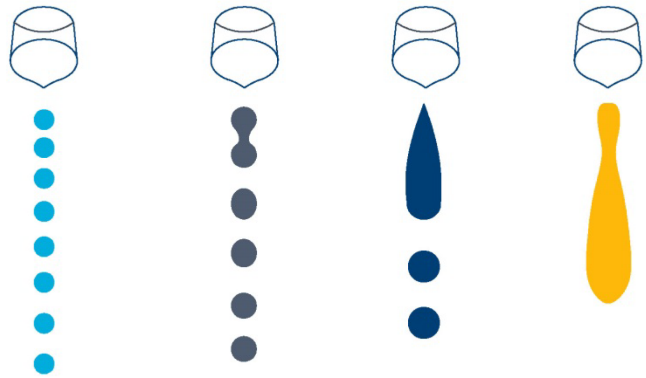


Figure 1: Varying liquids and their viscosities

The viscosity of a fluid describes the fluids resistance to flow. Viscosity is measured in cP or centipoises and is represented by the variable, μ or mu. Viscosity is measured with a device called a viscometer. There are many different types of viscometers, but each typically has the fluid moving past/through an object or it has the object moving through the fluid. The time of travel will vary based on the viscosity of the fluid. For example, water has a viscosity of ~1.00 cP (centipoises) at 68° F, while syrup has a viscosity of ~1400 cP and air has a viscosity of ~.01827 cP.

$$\mu\ (viscosity) = \left[\frac{g}{cm * s}\right]$$

The units described above are related to cP by a factor of 100. 100 cP is equal to 1 $[g/(cm * s)]$. The imperial units are$[lbm/(ft * s)]$ and are related to cP by the following conversion.

$$1 \text{ cP} = 6.72 \text{ x } 10^{-4} \left[\frac{lbm}{ft * s}\right]$$

There are two types of viscosities, dynamic (absolute) viscosity and kinematic viscosity. The previously discussed viscosity μ is dynamic viscosity. Kinematic viscosity describes the ratio of the fluids resistance to flow (dynamic viscosity) to the fluids density. Kinematic viscosity is indicated by the symbol, v or nu.

$$v(nu) = \frac{\mu \left[\frac{lbm}{ft * s}\right]}{\rho \left[\frac{lbm}{ft^3}\right]} = \left[\frac{ft^2}{s}\right]$$

$$where\ \rho = density, \mu = dynamic\ viscosity; v(nu) = kinematic\ viscosity$$

Kinematic viscosity has the units ft^2/s as shown above.

Viscosities of water have been included below for your convenience.

Table 1: This table shows how water's viscosity and density change as the temperature of water increases.

Temperature [F]	Fluid	Viscosity [cP]	Density [lbm/ft^3]	Density [lbf-s^2/ ft^4 =slugs/ft^3]	Kinematic Viscosity [ft^2/sec]
32	Water	1.792	62.42	1.940	1.9291E-05
50	Water	1.308	62.41	1.940	1.4083E-05
68	Water	1.002	62.32	1.937	1.0804E-05
86	Water	0.7978	62.15	1.932	8.6259E-06
104	Water	0.6531	61.94	1.925	7.0853E-06
122	Water	0.5471	61.68	1.917	5.9603E-06
140	Water	0.4668	61.38	1.908	5.1104E-06
158	Water	0.4044	61.04	1.897	4.4519E-06
176	Water	0.355	60.67	1.886	3.9319E-06
194	Water	0.315	60.26	1.873	3.5126E-06
212	Water	0.2822	59.83	1.860	3.1695E-06

2.3 Specific Gravity

Specific gravity is the term used to describe the ratio between a liquid's densities compared to the density of water. Water has a specific gravity of 1.0.

$$SG = \frac{\rho_{fluid}}{\rho_{water}}$$

$$\rho_{diesel} = 59.9\frac{lbm}{ft^3};\ \ \rho_{water} = 62.4\frac{lbm}{ft^3}$$

$$SG_{diesl} = \frac{59.9}{62.4} = 0.96$$

Specific gravity can also be used to describe the ratio between a gas's densities compared to the density of air. Air has a specific gravity of 1.0.

$$SG = \frac{\rho_{gas}}{\rho_{air}}$$

$$\rho_{methane} = 0.041\frac{lb}{ft^3};\ \ \rho_{air} = 0.075\frac{lb}{ft^3}$$

$$SG_{methane} = \frac{0.041}{0.075} = 0.55$$

system as a function of flow rate is shown as the system resistance curve, which is a squared, parabolic, upward sloping curve.

However, the pump cannot operate at any point along the system resistance curve, it can only operate along its pump curve. So in order to change the pump flow rate, you must change the system resistance curve. This is done by closing and opening valves (changes in the system operating conditions). If valves are opened, this reduces the restrictions, which causes the system resistance curve to change. Now, the intersection between the system resistance curve and the pump curve will be at a point of higher flow and lower pressure. If valves are closed, this increases restrictions, which causes the system resistance curve to change. Now the pump will operate at a point of lower flow and higher pressure.

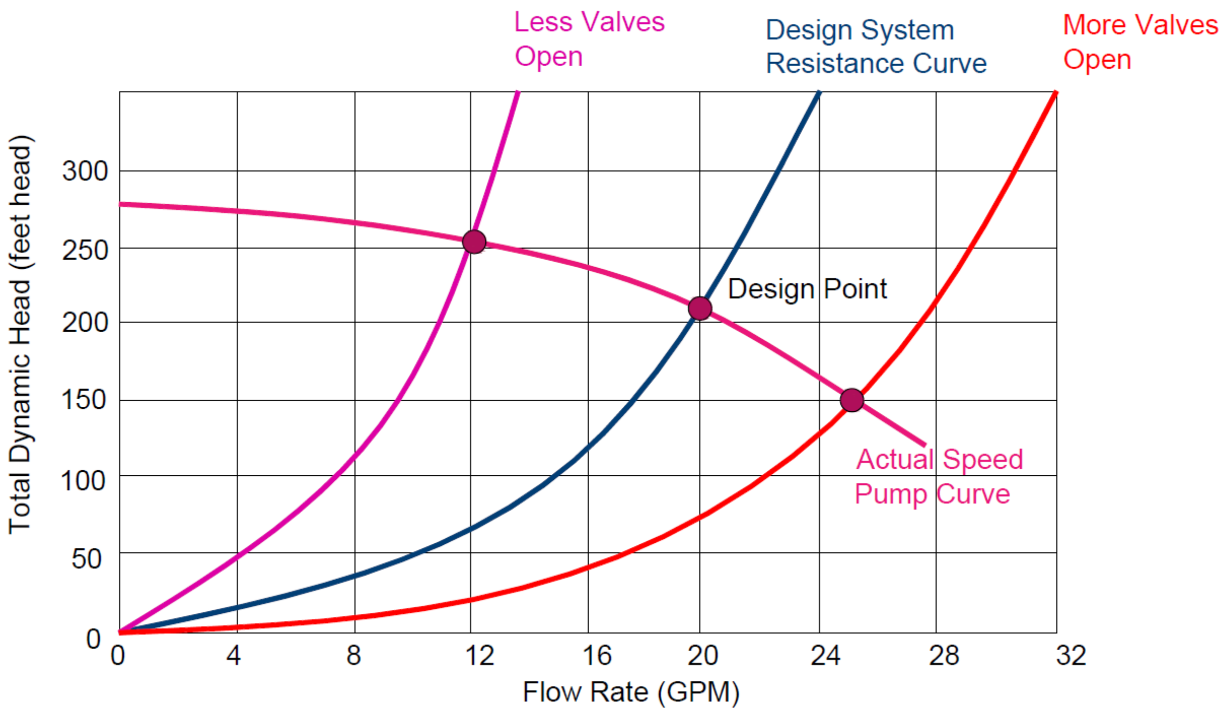


Figure 23: This figure shows that in order to change the flow rate and pressure produced from a constant flow pump, you must change the system resistance curve.

12.0 Scaling Laws for Fans, Pumps & Compressors

This section is in reference to the affinity and similarity laws for centrifugal fans, pumps and compressors. These laws do not apply to other types of fans, pumps and compressors. For example, they do not apply to positive displacement pumps or compressors, nor do they apply to propeller fans.

12.1 Affinity Laws for Fans, Pumps & Compressors

First, if the impeller diameter is held constant and the speed of the fan is changed, then flow rate varies directly with the speed, available pressure varies with the square of the speed and the power use varies with the cube of the speed.

$$\boldsymbol{CASE\ 1:\ N_{old} = N_{new}}$$

$$CFM_{new} = \left(\frac{RPM_{new}}{RPM_{old}}\right)^1 CFM_{old}$$

$$P_{new} = \left(\frac{RPM_{new}}{RPM_{old}}\right)^2 P_{old}$$

$$BHP_{new} = \left(\frac{RPM_{new}}{RPM_{old}}\right)^3 BHP_{old}$$

Second, if the speed is held constant and the impeller diameter of the fan is changed, then flow rate varies directly with the cube of the diameter, available pressure varies with the square of the diameter and the power use varies with the diameter raised to the fifth power.

$$\boldsymbol{CASE\ 2:\ RPM_{old} = RPM_{new}}$$

$$CFM_{new} = \left(\frac{D_{new}}{D_{old}}\right)^3 CFM_{old}$$

$$P_{new} = \left(\frac{D_{new}}{D_{old}}\right)^2 P_{old}$$

$$BHP_{new} = \left(\frac{D_{new}}{D_{old}}\right)^5 BHP_{old}$$

12.2 Similarity Laws for Fans, Pumps & Compressors

You may come across these formulas, if you encounter a question that compares two similar fans. These formulas are called the similarity laws. These laws compare similar fans within the same series of fans. The previous formulas compared the original condition and new condition of the same fan. These formulas compare two similar fans, with different diameters. In order to best understand what is meant by same series of fans, visit a manufacturer's website and you will see various fan series that have varying sizes within the same series. Within a series of fans, a fan with "x" diameter wheel and "y" diameter casing can be compared to another fan in the same series of fans, but with "2x" diameter wheel and "2y" diameter casing. The second pump is similar but has twice the diameter of the first fan.

$\frac{Q_1}{Q_2}$	=	$\left(\frac{D_1}{D_2}\right)^3 \left(\frac{N_1}{N_2}\right)$

13.15 PRACTICE PROBLEM 15 – EXTERNAL FLOW

Water flows around a 4cm diameter pipe that is placed across a large channel. Water flows in the 20m long, 10m deep channel at a rate of 10m/s and a temperature of 20^{o}C. What is the drag force on the pipe? Assume a drag coefficient of 0.4.

(a) 8 kN

(b) 16 kN

(c) 20 kN

(d) 25 kN

13.15.1 Solution 15 – External Flow

The find the drag force on the pipe, use the following equation for immersed objects in a large body of water.

$$F_D = \frac{C_D \rho v^2 A}{2}$$

From the NCEES FE Reference Handbook, the density of water at 20^{o}C is 998.2 kg/m^3 (997 at 25). Solve for the front face area of the pipe that the fluid flow will see.

$$A = D * Channel\ Width = 4cm * \frac{1m}{100cm} * 20m = 0.8m^2$$

Solve for the drag force.

$$F_D = \frac{0.4 * (998.2kg/m^3)(10m/s)^2(0.8m^2)}{2} * \frac{1N}{1kg * m/s^2}$$

$$F_D = 15{,}970N$$

The answer is most nearly (B) 16kN.

13.15 PRACTICE PROBLEM 16 – COMPRESSIBLE FLOW

Air flows through a duct at 21psig, 110ºF, and a Mach number of 0.4. What is the stagnation pressure of the air?

(a) 23 psig

(b) 25 psig

(c) 36 psig

(d) 38 psig

13.15.1 Solution 15 – Compressible Flow

The stagnation pressure is the static pressure of the air, when all the kinetic energy is isentropically converted to pressure energy. It is calculated from the pressure in the duct using the following isentropic relationship.

$$\frac{P_0}{P} = \left(1 + \frac{k-1}{2} * Ma^2\right)^{\frac{k}{k-1}}$$

Where k is the specific heat ratio, c_p/c_v and P_o is the stagnation pressure. The specific heat ration of air is 1.4. The pressure in the above equation is given in absolute values.

$$P = 21psig + 14.7 = 35.7psia$$

Solve for the stagnation pressure.

$$P_0 = P\left(1 + \frac{k-1}{2} * Ma^2\right)^{\frac{k}{k-1}} = 35.7psia * \left(1 + \frac{1.4-1}{2} * 0.4^2\right)^{\frac{1.4}{1.4-1}}$$

$$P_0 = 39.9psia$$

$$P_0 = 39.9psia - 14.7 = 25.2psig$$

The answer is most nearly (B) 25psig

SECTION 12
THERMODYNAMICS

Section 12.0 –Thermodynamics

Table of Contents

1.0 INTRODUCTION

Thermodynamics accounts for approximately 13 to 20 questions on the FE Mechanical exam. Thermodynamics is the largest topic on the FE Mechanical Exam. It contains so much information that most Mechanical Engineering college courses do not cover all of this information within their entire Mechanical Engineering degree course curriculum. 12G, 12I, 12J are most often left out of Thermodynamic college courses. In addition, many college courses cover a lot of theory and do not focus on the key concepts and skills that are necessary for practical application of the theory. The following list is the outline of the Thermodynamics topics that can appear on the exam. This section will go into detail on the key concepts and skills that are necessary under each topic. Following, the key concepts and skills there will be problems that you can use to test your understanding of the concepts and skills.

Section 12.0 Thermodynamics (13 to 20 Problems)		
NCEES Outline Value	**Engineering Pro Guides**	
	Topic 1.0	Introduction
12A	Topic 2.0	Properties of Ideal Gases and Pure Substances
12B	Topic 3.0	Energy Transfers
12C	Topic 4.0	Laws of Thermodynamics
12D	Topic 5.0	Processes
12E	Topic 6.0	Performance of Components
12F	Topic 7.0	Power Cycles, Thermal Efficiency and Enhancements
12G	Topic 8.0	Refrigeration and Heat Pump Cycles and Coefficients of Performance
12H	Topic 9.0	Non-reacting Mixtures of Gases
12I	Topic 10.0	Psychrometrics
12J	Topic 11.0	Heating, Ventilating and Air-Conditioning (HVAC) Processes
12K	Topic 12.0	Combustion and Combustion Products
	Topic 13.0	Practice Exam Problems

7.0 POWER CYCLES

7.1 IDEAL OPEN GAS TURBINE CYCLE

The two major application areas of the open gas turbine cycle are for aircrafts and electric power generation. In an open cycle the working fluid (air) only passes through the cycle once and is then exhausted to the atmosphere. In a closed cycle, the working fluid (air) is recycled through the cycle. One assumption that you should be aware of is that the mass flow rate through both the open and closed cycles are assumed to be constant. Although fuel does enter the cycle, it is assumed that the only mass flow rate to be considered in doing problems is the mass flow rate of the air. This is typically a safe assumption because the ratio of air to fuel is quite large, typically fuel can be around 2% of the total mass flow rate.

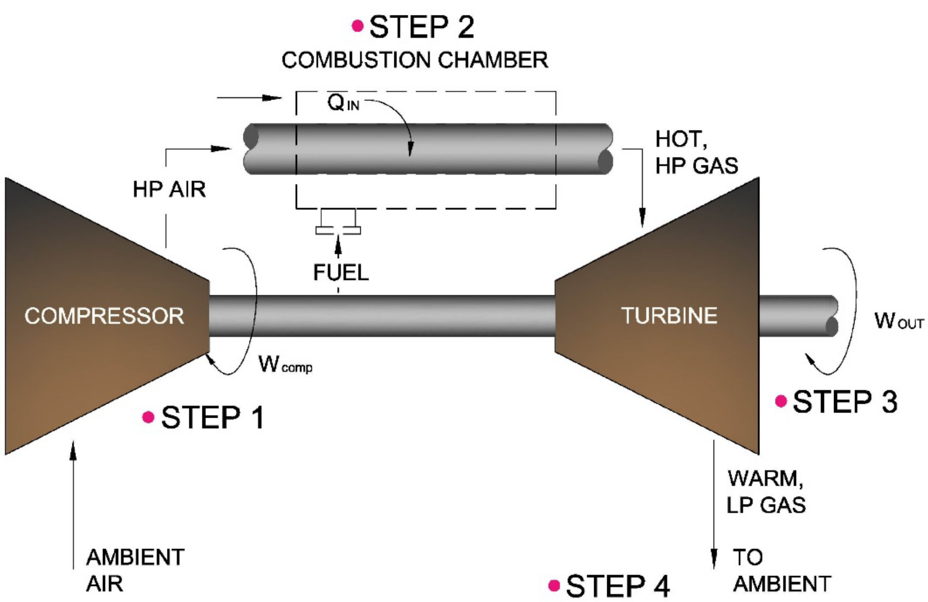
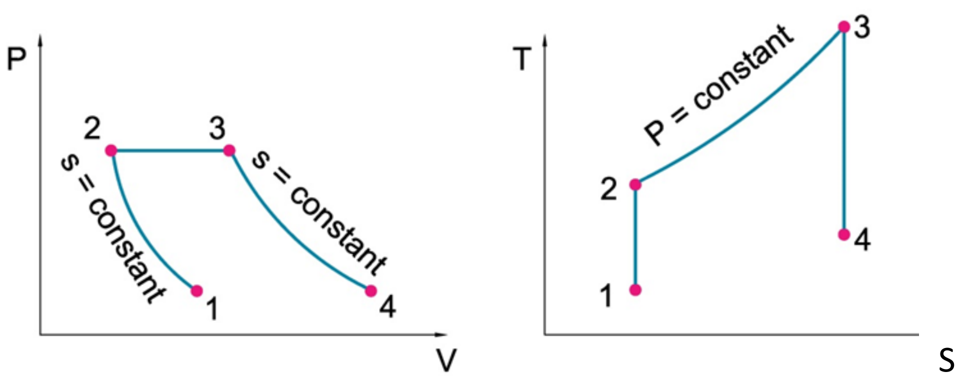


Figure 16: The above figure shows the components of an open Brayton cycle. Step 1 to 2 is the isentropic compressor. Step 2 to 3 is a constant pressure combustion chamber. Step 3 to 4 is the isentropic turbine.



There are two entering conditions to the expansion device shown on the following diagram. The first situation has 0 °F of sub-cooling [D''] and the second situation has 15 °F of sub-cooling [D'''].

The expansion device expands the high pressure refrigerant gas adiabatically to a low pressure liquid-vapor refrigerant mixture. Adiabatic expansion indicates that there is no change in enthalpy and is characterized by a downward vertical line as shown on the below graph.

Note on the graph below as the refrigerant moves from point D to point A, the refrigerant moves from the liquid phase of the graph to the vapor-liquid mixture region. The amount of gas that is formed during this expansion is called flash gas.

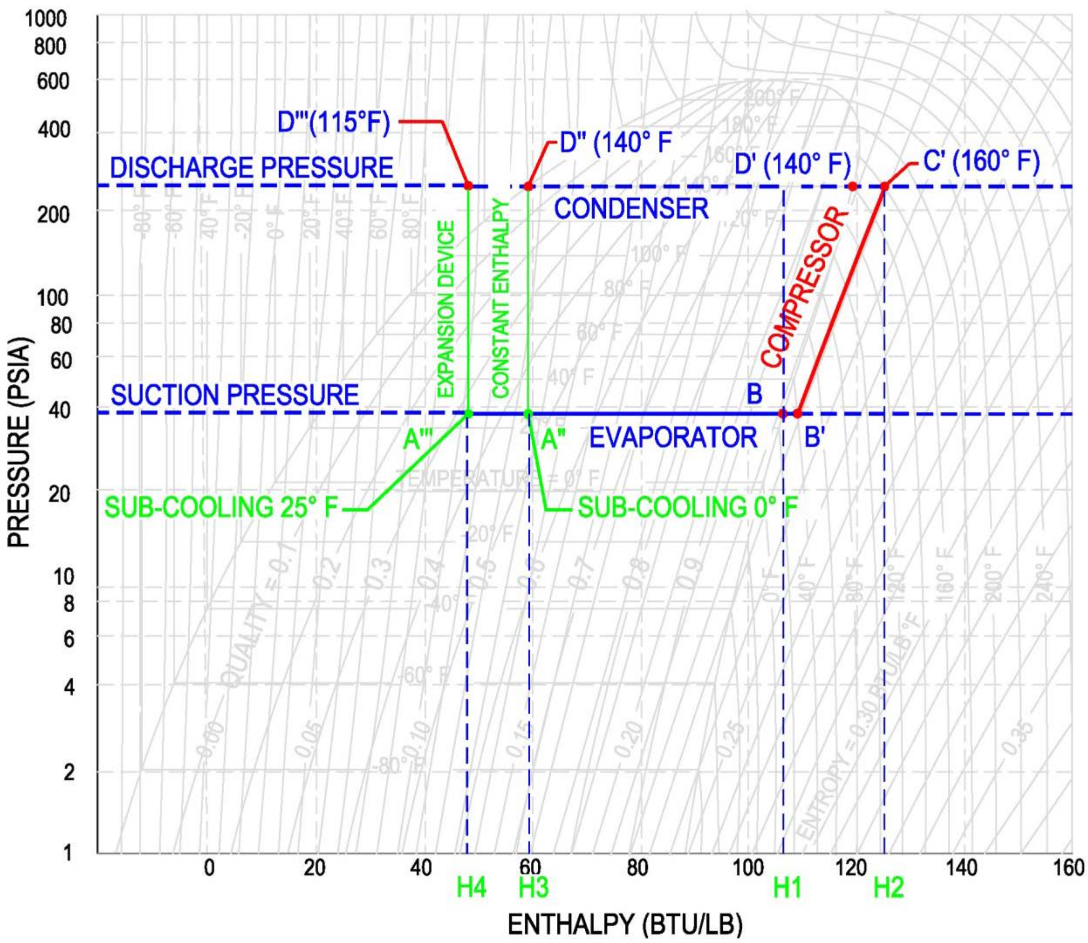


Figure 34: Pressure-enthalpy diagram – expansion device

13.8.1 Solution 8 – Expansion Device

If the amount of super heat is increasing from normal conditions,

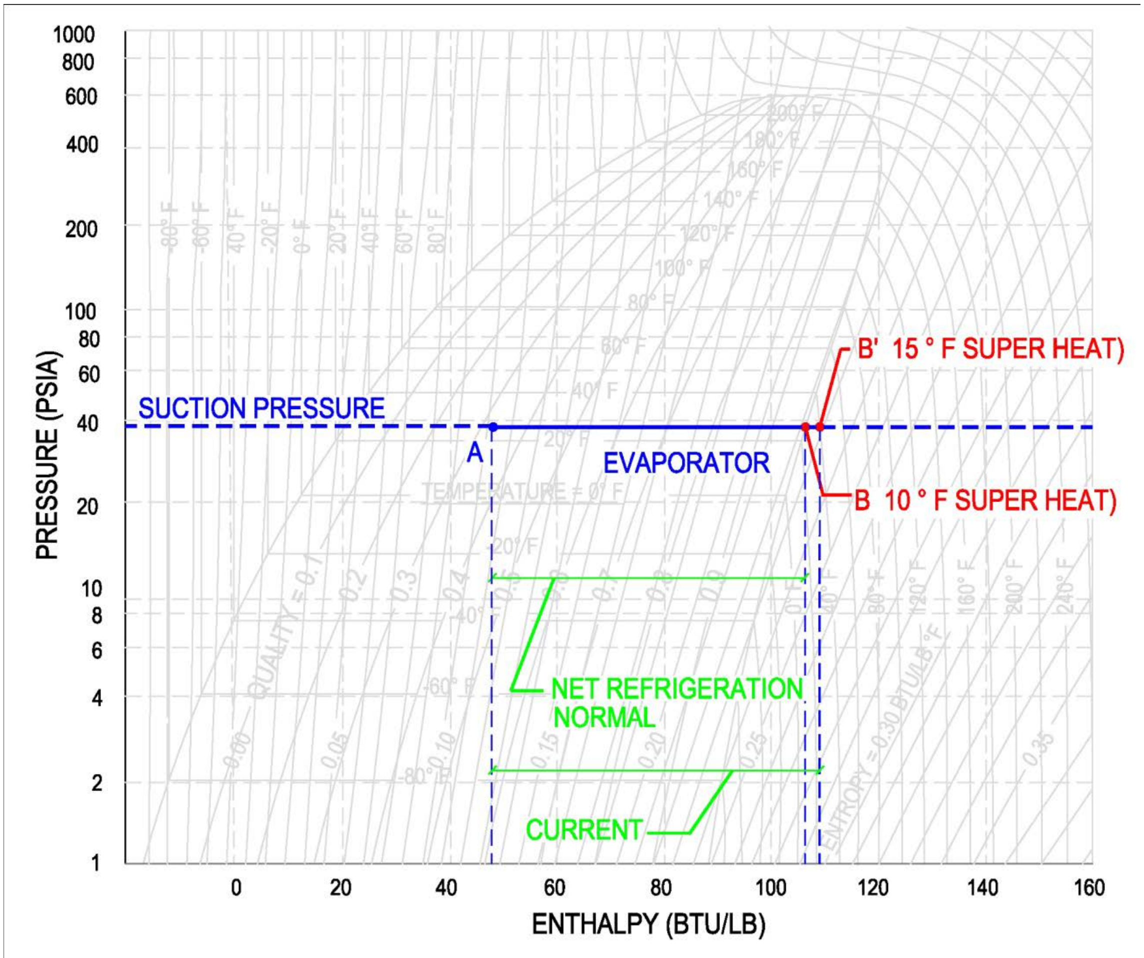
then the net refrigeration effect is increasing, as point B is moving further to the right.

Cooling load is increasing.

If the cooling load is increasing,

then the TXV should provide more refrigerant to increase net refrigeration effect

$$\overset{\uparrow}{Q}_{net\ refrigeration\ effect}\ [Btuh] = (H_1 - \overset{\downarrow}{H_4})\left[\frac{Btu}{lb}\right] * \overset{\uparrow}{(Refrig\ Flow\ Rate)}\left[\frac{lb}{min}\right] * (60)\left[\frac{min}{hr}\right]$$



SECTION 13
HEAT TRANSFER

Section 13.0 – Heat Transfer

Table of Contents

1.0 INTRODUCTION

Heat Transfer accounts for approximately 9 to 14 questions on the Mechanical FE exam. The heat transfer principles tested on the FE Mechanical exam are shown in the outline below. There are three main areas of heat transfer: conduction, convection and radiation. Conduction is the transfer of heat through contact. In this type of heat transfer, common skills needed include finding overall heat transfer coefficients, finding insulation values and temperature transitions through materials. Convection is the transfer of heat through a moving fluid. This is most commonly seen in heat exchangers as moving hot fluids transfer heat to cool fluids. The main skill needed in this area include finding the convective heat transfer coefficient through the Nusselt number, Reynolds number and/or Rayleigh number. The final type is radiation, which will require finding the radiative heat transfer coefficient and finding the radiative heat transfer between an object and its surroundings.

The last three topics are heat exchangers, boiling and condensation. For heat exchangers, you must be able to calculate the log mean temperature difference for various types of heat exchangers and the heat transfer between the hot and cold fluid.

Section 13.0 Heat Transfer (9 to 14 Problems)		
NCEES Outline Value	**Engineering Pro Guides**	
	Topic 1.0	Introduction
13A	Topic 2.0	Conduction
13B	Topic 3.0	Convection
13C	Topic 4.0	Radiation
13D	Topic 5.0	Thermal Resistance
13E	Topic 6.0	Transient Processes
13F	Topic 7.0	Heat Exchangers
13G	Topic 8.0	Boiling
13H	Topic 9.0	Condensation
	Topic 10.0	Practice Exam Problems

2.0 CONDUCTION

Conduction is the method of heat transfer through material(s) in physical contact. The driving force in conduction is a temperature difference on either side of the material(s). For example, if the end of a metal rod is placed in a fire, heat will be conducted through the metal rod to the other end. Heat transfer due to conduction is most commonly calculated for wall and roof heat loads. The outside of a wall or roof is heated by the outdoor conditions. Then the heat is conducted from the outside of the wall through the wall material and to the inside of the wall, where the heat is transferred to the space. The formula for calculating heat transfer due to conduction through a material is as follows:

$$Q = \frac{k * A * (T_{hot} - T_{cold})}{t}$$

$$where\ Q = quantity\ of\ heat\ transferred\ \left[W\ or\ \frac{Btu}{hr}\right]$$
$$k = thermal\ conductivity\ of\ material\ \left[\frac{W}{m * °K}\ or\ \frac{Btu}{hr * ft * °\mathrm{F}}\right]$$
$$T_{hot} - T_{cold} = temperature\ difference\ [°K\ or\ °\mathrm{F}]$$
$$t = thickness\ of\ material\ [m\ or\ ft]$$
$$A = area\ of\ heat\ transferl\ [m^2\ or\ ft^2]$$

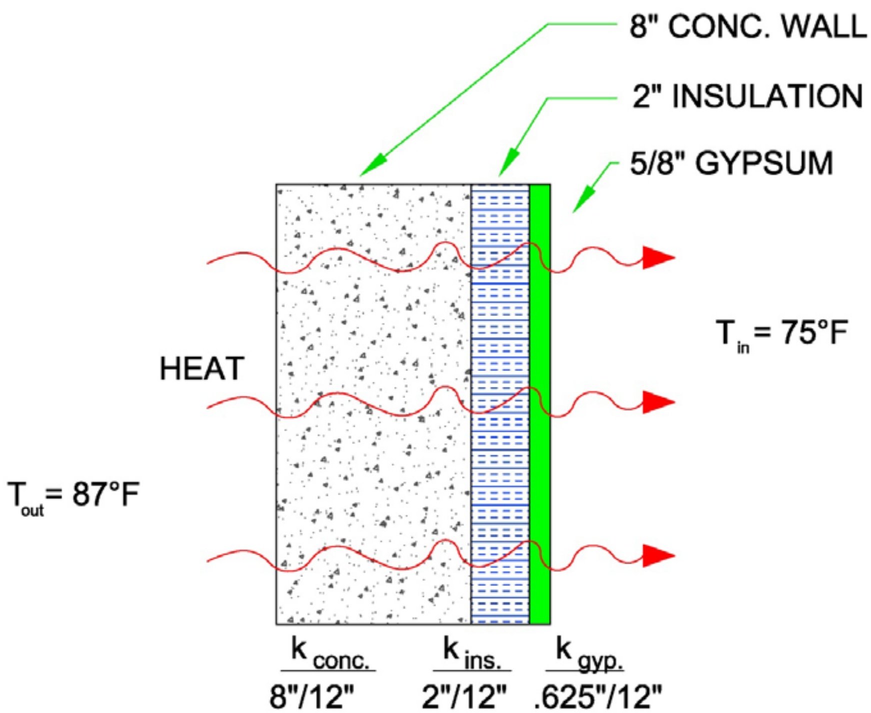


Figure 1: Conduction through a building wall

The amount of heat transferred is linearly dependent on the difference in temperature between the inside and outside surfaces of the wall. The conduction equation shows that as the temperature difference increases, the heat load also increases. The same is also true for the area available for heat transfer and the thermal conductivity. On the other hand, the amount of heat transferred is inversely related to the thickness of the wall or roof material.

For a pipe with multiple layers of different materials, the equation becomes:

$$Q = \frac{2k\pi L * (T_{inner} - T_{outer})}{\frac{\ln\left(\frac{r_2}{r_{inner}}\right)}{k_i} + \frac{\ln\left(\frac{r_3}{r_2}\right)}{k_{ii}} + \cdots + \frac{\ln\left(\frac{r_{outer}}{r_n}\right)}{k_m}}$$

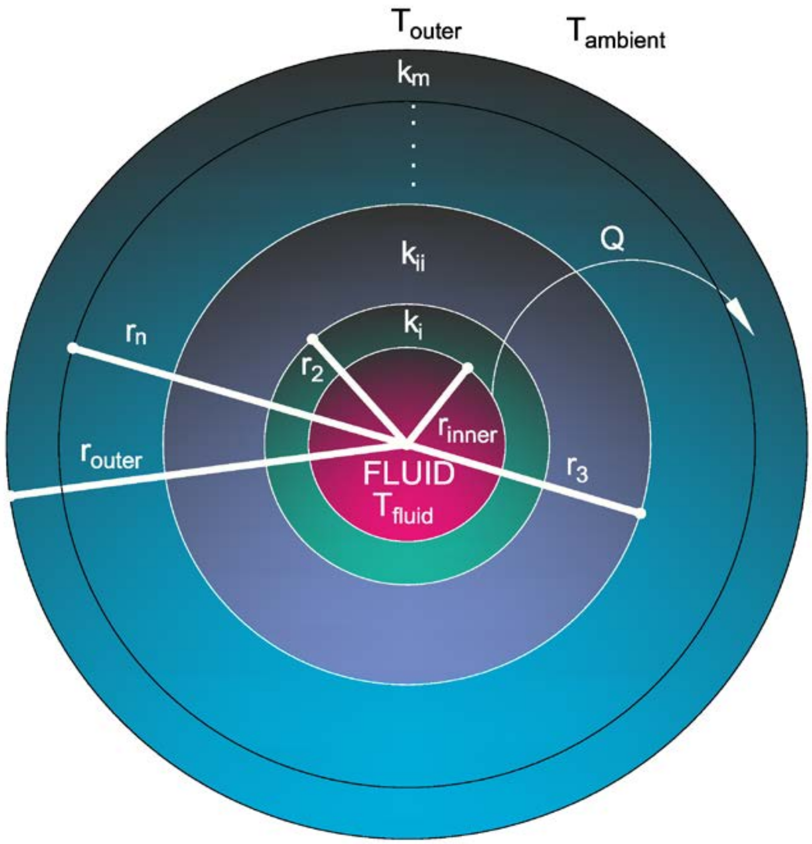


Figure 2: Heat transfer takes place from the fluid to the exterior through the various layers around the pipe.

10.12 PROBLEM 12 – RADIATION & CONVECTION

A 4" diameter copper pipe carries hot water from the heater to the equipment. The surface temperature of the pipe is 200 F. The ambient temperature is 75 F. Assume that the convective heat transfer coefficient is 1.85 Btu/(hr-ft2-F). What is the total heat loss per 100 feet of pipe? Assume black body radiation.

(A) 24,300 Btuh

(B) 31,900 Btuh

(C) 43,600 Btuh

(D) 67,100 Btuh

10.12.1 Solution 12 – Radiation & Convection

A 4" diameter copper pipe carries hot water from the heater to the equipment. The surface temperature of the pipe is 200 F. The ambient temperature is 75 F. Assume that the convective heat transfer coefficient is 1.85 Btu/(hr-ft2-F). What is the total heat loss per 100 feet of pipe? Assume black body radiation.

The main concept to understand in this problem is that the total heat loss from a pipe includes both radiation and convection. Since the problem does not indicate that the pipe is physically touching anything, then we can assume that there is no conduction taking place.

$$Q_{conv} = h_{conv} * A_{surface} * (T_{surface} - T_{ambient})$$

$$Q_{conv} = 1.85 \frac{Btu}{hr - ft^2 - °\text{F}} * A_{surface} * (200 - 75°\text{F})$$

$$A_{surface} = 100\, ft * \left(\pi * \frac{4"}{12"/ft}\right) = 104.72\, ft^2$$

$$Q_{conv} = 1.85 \frac{Btu}{hr - ft^2 - °\text{F}} * 104.72\, ft^2 * (200 - 75°\text{F}) = 24{,}216\, Btuh$$

$$\varepsilon = 1\ (black\ body\ radiation)$$

$$Q_{rad} = \varepsilon\sigma * A_{surface}({T^4}_{surface} - {T^4}_{ambient})$$

$$Q_{rad} = 1.714\, x\, 10^{-9} \frac{Btu}{hr - ft^2 - R^4} * 104.72\, ft^2 * ((200 + 460)^4 - (75 + 460)^4)$$

$$Q_{rad} = 19{,}353\, Btuh$$

SECTION 14
MEASUREMENTS, INSTRUMENTATION & CONTROLS

Section 14.0 – Measurement, Instrumentation & Controls

Table of Contents

1.0 INTRODUCTION

Measurement, Instrumentation and Controls accounts for approximately 5 to 8 questions on the Mechanical FE exam. This section covers the following topics, sensors, block diagrams, system response and measurement uncertainty. In the sensors topic, you must be familiar with the types of sensors that are used to measure, strain, temperature and pressure, since these are the properties that are most commonly used in mechanical engineering, unlike the pH and chemical sensors which are also shown in the *NCEES FE Reference Handbook.* Block diagrams are used to analyze a control system that consists of different functions in graphical format. You must be able to read and simplify these block diagrams for the FE exam. The system response topic focuses on how control systems respond to various inputs like step, ramp and parabolic inputs. This topic will teach you how to determine if a control system will be stable with the Routh test and how to determine the response error.

Section 14.0 Measurement, Instrumentation and Controls (5 to 8 Problems)		
NCEES Outline Value	**Engineering Pro Guides**	
	Section 1.0	Introduction
14A	Section 2.0	Sensors
14B	Section 3.0	Block Diagrams
14C	Section 4.0	System Response
14D	Section 5.0	Measurement Uncertainty
	Section 6.0	Practice Exam Problems

2.0 SENSORS

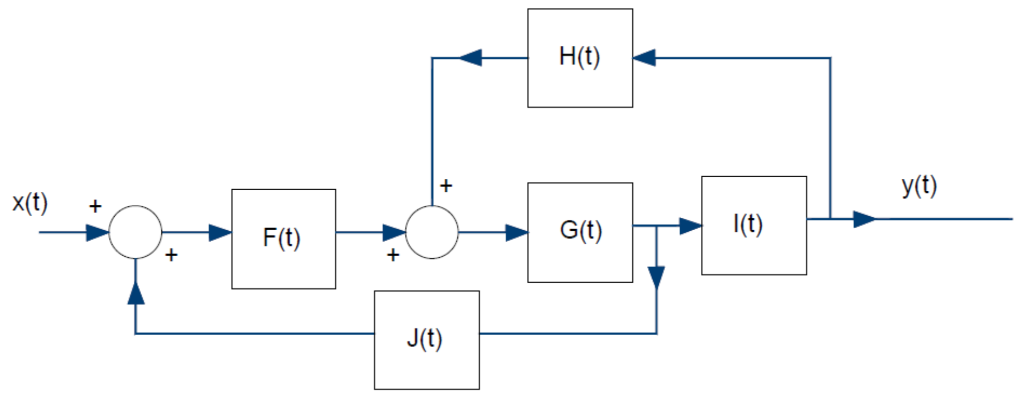
The sensors that you need to know for the FE exam are those sensors that convert a physical measurement into an electrical signal. Typically, the physical measurement changes a circuit's resistance, which in turn changes the measured voltage, assuming that current remains constant. Others change the voltage directly, like in a thermocouple.

2.1 TEMPERATURE SENSORS

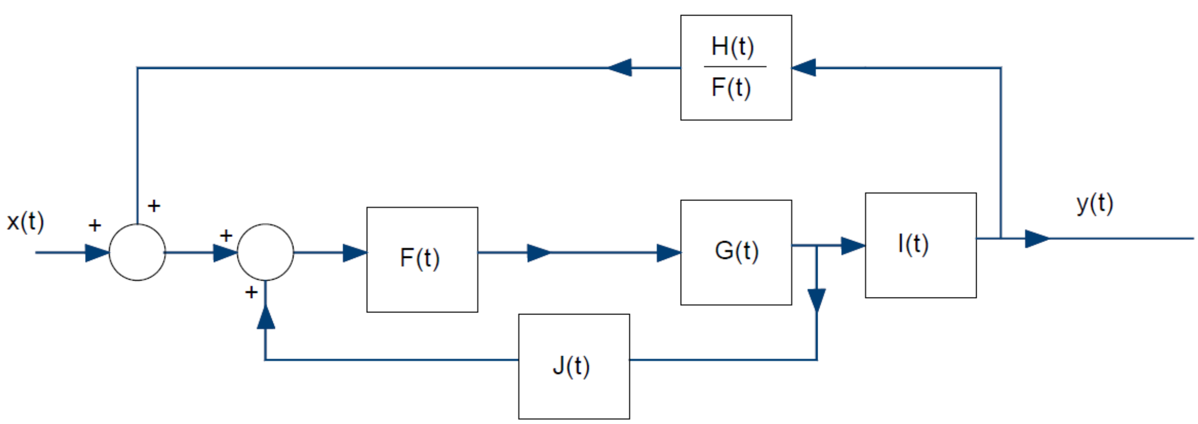
There are two main types of temperature sensors that you need to know for the FE exam, (1) Thermocouple and (2) Resistance Temperature Detector.

The thermocouple uses a composite of two dissimilar metals that creates a voltage as a function of temperature. As the temperature increases, the thermoelectric effect occurs and this effect

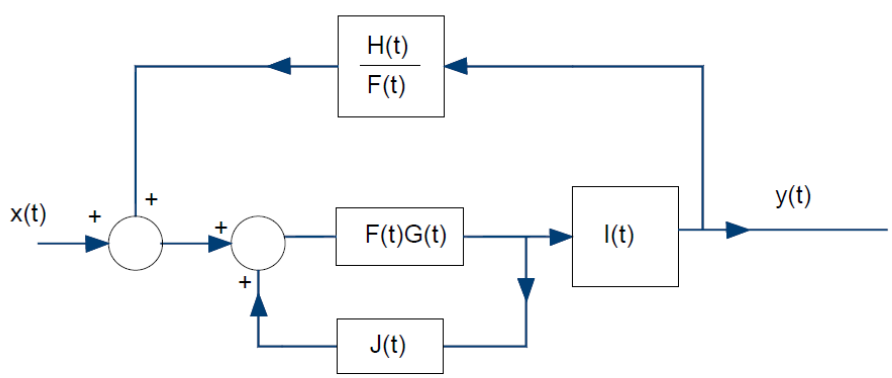
6.3.1 Solution 3 – Block Diagram



First, move H(t) outside of F(t).



Then combine F(t) and G(t)



Then combine the feedback loop J(t).

SECTION 15
MECHANICAL DESIGN & ANALYSIS

Section 15.0 – Mechanical Design and Analysis

Table of Contents

1.0 INTRODUCTION

Mechanical Design and Analysis accounts for approximately 9 to 14 questions on the Mechanical FE exam. This makes Mechanical Design and Analysis tied for second as the Sections with the most possible problems on the FE exam. This section primarily focuses on machine design mechanical components like springs, pressure vessels, beams, piping, bearings, power screws and transmissions. In addition, other topics that support machine design mechanical components are discussed in this section, like manufacturability, quality, and reliability. Then this section switches to topics that are common in Thermal & Fluids like hydraulics, pneumatics and electromechanical components. Finally, this section repeats topics like beams, piping, stress analysis, deformation, stiffness that are covered in previous sections. This section will point you to the correct section when these repeat topics are discussed.

A list of the topics are shown in the table below.

Section 15.0 Mechanical Design & Analysis (9 to 14 Problems)		
NCEES Outline Value	**Engineering Pro Guides**	
	Heading 1.0	Introduction
15A	Heading 2.0	Stress Analysis of Machine Elements *(repeat from previous Section 9.0 Mechanics of Materials)*
15B	Heading 3.0	Failure Theories and Analysis
15C	Heading 4.0	Deformation and Stiffness *(repeat from previous Section 9.0 Mechanics of Materials)*
15D	Heading 5.0	Springs
15E	Heading 6.0	Pressure Vessels
15F	Heading 7.0	Beams
15G	Heading 8.0	Piping *(repeat from hydraulic components below)*
15H	Heading 9.0	Bearings
15I	Heading 10.0	Power Screws
15J	Heading 11.0	Power Transmission
15K	Heading 12.0	Joining Methods
15L	Heading 13.0	Manufacturability
15M	Heading 14.0	Quality and Reliability
15N	Heading 15.0	Hydraulic Components
15O	Heading 16.0	Pneumatic Components
15P	Heading 17.0	Electromechanical Components
	Heading 18.0	Practice Exam Problems

2.0 STRESS ANALYSIS OF MACHINE ELEMENTS

Stress analysis of machine elements is discussed in *Section 9.0 Mechanics of Materials*, there are many topics within that section that discuss the stress for all possible types of loads.

3.0 Failure Theories and Analysis

Failure can mean a component has been completely fractured; permanently distorted or its function has been compromised. In *Section 10.0 Material Properties and Processing*, various strengths of material properties have been presented. Unfortunately, these strengths which can be used to determine the stress levels at which failure will occur apply only to simple loadings like tension, compression or shear that occurs in one axis. In real life situations and for most of the FE exam, the simple loadings can be assumed. However, there may be a couple of questions on the FE exam where there are complex loadings. For these questions you need to use one of the following failure theories, (1) Maximum Normal Stress Theory for Brittle Materials, (2) Mohr's Theory for Brittle Materials, (3) Maximum Shear Stress Theory for Ductile Materials and (4) Distortion Energy Theory for Ductile Materials also known as Von Mises Theory. Before you learn how to use each of the theories, you need to first understand factor of safety.

In failure theories, the term factor of safety is used to describe the ratio of the load at which the object will fail to the load at which the object will be allowed. The allowed loading can also be described as the design loading. For example, if you design a beam for a loading of 1,000 N then you could choose a beam that fails at 2,000 N. This will result in a factor of safety of 2.

$$Factor\ of\ safety\ (F.S.) = \frac{Failure\ loading}{Allowed\ or\ Design\ loading}$$

$$FS = 2 = \frac{2{,}000\ N\ beam\ failure}{1{,}000\ N\ beam\ design}$$

3.1 Maximum Normal Stress Theory - Brittle

The maximum normal stress theory is similar to the methods presented in *Section 10.0 Material Properties and Processing.* The theory is that if the normal stress in tension or compression is greater than the ultimate strength of the material, then failure will occur.

$$Failure\ Occurs \rightarrow \sigma_{tensile} \geq S_{ut}; S_{ut} = ultimate\ tensile\ strength$$

$$Failure\ Occurs \rightarrow \sigma_{compression} \leq -S_{uc}; S_{uc} = ultimate\ compressive\ strength$$

3.2 Mohr Theory - Brittle

Mohr's theory is another theory used to assess when a material will fail. This theory is used when a material does not have similar compressive and tensile yield strengths. Mohr's theory can be best understood through the use of Mohr's Circle. This circle is a representation of 2-d stresses in the tensile/compressive direction and in shear. The circle is used to analyze complex stresses to determine the principle stresses that are applied to a component. The principle stresses can then be compared to the tensile, compressive and shear strengths.

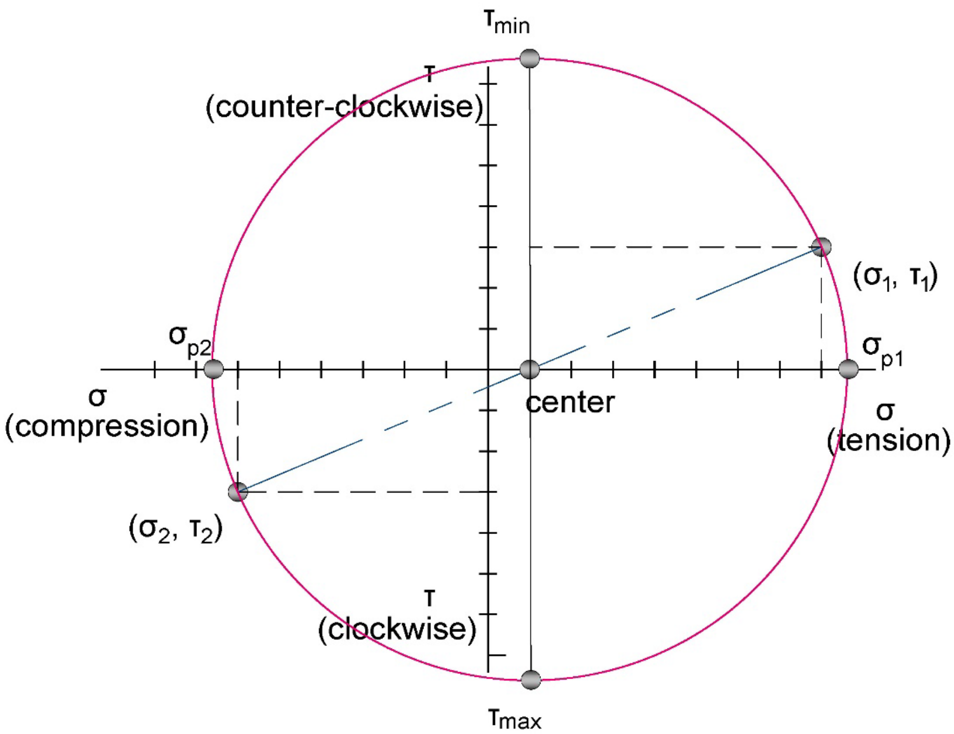


Figure 1: Mohr's circle is used to find the primary stresses in tension, compression and shear.

The complex stresses of (σ_1, τ_1) and (σ_2, τ_2) are typically given in a FE problem. Then you must use Mohr's circle to find the principal forces that act in tension, compression and shear. These principal forces are shown as the extreme points on the circle.

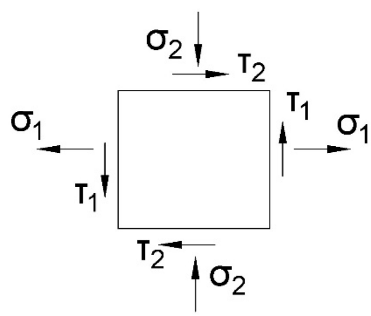


Figure 2: The given stresses that act upon an element within a component are shown in this figure. Notice how the clockwise and counter-clockwise shears correspond to the points shown on the previous figure. Also notice how the tension and compression stresses are shown in this figure correspond to the correct location in the previous figure.

Mohr's circle is created by first determining the center point of the circle from the complex stresses given in the problem. The center is found through this equation.

Control Chart Factors

n	A2	A3	~~c4~~	B3	B4	~~d2~~	~~1/d2~~	~~d3~~	D3	D4
2	1.880	2.659	~~0.7979~~	0.000	3.267	~~1.128~~	~~0.8862~~	~~0.853~~	0.000	3.267
3	1.023	1.954	~~0.8862~~	0.000	2.568	~~1.693~~	~~0.5908~~	~~0.888~~	0.000	2.575
4	0.729	1.628	~~0.9213~~	0.000	2.266	~~2.059~~	~~0.4857~~	~~0.880~~	0.000	2.282
5	0.577	1.427	~~0.9400~~	0.000	2.089	~~2.326~~	~~0.4299~~	~~0.864~~	0.000	2.114
6	0.483	1.287	~~0.9515~~	0.030	1.970	~~2.534~~	~~0.3946~~	~~0.848~~	0.000	2.004
7	0.419	1.182	~~0.9594~~	0.118	1.882	~~2.704~~	~~0.3698~~	~~0.833~~	0.076	1.924
8	0.373	1.099	~~0.9650~~	0.185	1.815	~~2.847~~	~~0.3512~~	~~0.820~~	0.136	1.864
9	0.337	1.032	~~0.9693~~	0.239	1.761	~~2.970~~	~~0.3367~~	~~0.808~~	0.184	1.816
10	0.308	0.975	~~0.9727~~	0.284	1.716	~~3.078~~	~~0.3249~~	~~0.797~~	0.223	1.777
11	0.285	0.927	~~0.9754~~	0.321	1.679	~~3.173~~	~~0.3152~~	~~0.787~~	0.256	1.744
12	0.266	0.886	~~0.9776~~	0.354	1.646	~~3.258~~	~~0.3069~~	~~0.778~~	0.283	1.717
13	0.249	0.850	~~0.9794~~	0.382	1.618	~~3.336~~	~~0.2998~~	~~0.770~~	0.307	1.693
14	0.235	0.817	~~0.9810~~	0.406	1.594	~~3.407~~	~~0.2935~~	~~0.763~~	0.328	1.672
15	0.223	0.789	~~0.9823~~	0.428	1.572	~~3.472~~	~~0.2880~~	~~0.756~~	0.347	1.653
16	0.212	0.763	~~0.9835~~	0.448	1.552	~~3.532~~	~~0.2831~~	~~0.750~~	0.363	1.637
17	0.203	0.739	~~0.9845~~	0.466	1.534	~~3.588~~	~~0.2787~~	~~0.744~~	0.378	1.622
18	0.194	0.718	~~0.9854~~	0.482	1.518	~~3.640~~	~~0.2747~~	~~0.739~~	0.391	1.609
19	0.187	0.698	~~0.9862~~	0.497	1.503	~~3.689~~	~~0.2711~~	~~0.733~~	0.404	1.596
20	0.180	0.680	~~0.9869~~	0.510	1.490	~~3.735~~	~~0.2677~~	~~0.729~~	0.415	1.585
21	0.173	0.663	~~0.9876~~	0.523	1.477	~~3.778~~	~~0.2647~~	~~0.724~~	0.425	1.575
22	0.167	0.647	~~0.9882~~	0.534	1.466	~~3.819~~	~~0.2618~~	~~0.720~~	0.435	1.565
23	0.162	0.633	~~0.9887~~	0.545	1.455	~~3.858~~	~~0.2592~~	~~0.716~~	0.443	1.557
24	0.157	0.619	~~0.9892~~	0.555	1.445	~~3.895~~	~~0.2567~~	~~0.712~~	0.452	1.548
25	0.153	0.606	~~0.9896~~	0.565	1.435	~~3.931~~	~~0.2544~~	~~0.708~~	0.459	1.541

d_2 and $1/d_2$: These two terms are crossed out because these values are only used to determine A_2.

$$A_2 = \frac{3}{d_2\sqrt{n}};\ where\ n = sample\ size$$

d_3: This term is crossed out because it is only used to determine D_3 and D_4.

$$D_3 = 1 - \frac{3d_3}{d_2};\ D_4 = 1 + \frac{3d_3}{d_2};$$

B_3, B_4: These terms are used in s-charts to find UCL and LCL.

$$UCL_s = B_4\bar{s};\ LCL_s = B_3\bar{s}, where\ \bar{s}\ is\ the\ average\ standard\ deviation\ (centerline)$$

18.2 Practice Problem 2 – Quality and Reliability

A machined product is measured and the following shows six samples, each with 15 observations per sample.

Sample	Mean	Range
1	78.1	14.1
2	72.1	8.5
3	63.7	18.1
4	89.4	18.9
5	80.8	9.2
6	78.8	14.7

n	A2	A3	~~c4~~	B3	B4	~~d2~~	~~1/d2~~	~~d3~~	D3	D4
15	0.223	0.789	~~0.9823~~	0.428	1.572	~~3.472~~	~~0.2880~~	~~0.756~~	0.347	1.653
16	0.212	0.763	~~0.9835~~	0.448	1.552	~~3.532~~	~~0.2831~~	~~0.750~~	0.363	1.637
17	0.203	0.739	~~0.9845~~	0.466	1.534	~~3.588~~	~~0.2787~~	~~0.744~~	0.378	1.622
18	0.194	0.718	~~0.9854~~	0.482	1.518	~~3.640~~	~~0.2747~~	~~0.739~~	0.391	1.609
19	0.187	0.698	~~0.9862~~	0.497	1.503	~~3.689~~	~~0.2711~~	~~0.733~~	0.404	1.596
20	0.180	0.680	~~0.9869~~	0.510	1.490	~~3.735~~	~~0.2677~~	~~0.729~~	0.415	1.585

What are the LCL and UCL for the range chart?

(A) $\boldsymbol{UCL = 23.0; LCL = 4.8}$

(B) $UCL = 13.9; LCL = 13.9$

(C) $UCL = 18.9; LCL = 8.5$

(D) $UCL = 13.9; LCL = 8.5$

18.14.1 Solution 14 – Hydraulic Components

A pump is sized for 200 GPM at 150 ft of head. If the impeller diameter is decreased in size by 25%, what is the new flow of the pump? Assume the speed remains the same.

Use the affinity laws.

$$\frac{Q_1}{Q_2} = \frac{D_1}{D_2}; if\ speed\ is\ held\ constant$$

$$\frac{200}{Q_2} = \frac{x}{0.75x}$$

$$200 * 0.75 * x = Q_2 * x$$

$$x = 150\ gpm$$

18.15 PROBLEM 15 – JOINING METHODS

A bolt is initially used to secure two components with a force of 500 lbf. The bolt has a stiffness coefficient of 1.8 lbf/in and the clamping stiffness coefficient is 13 lbf/in. If an external load of 1,000 lbf is applied to the components, then what is the total amount of force acting upon the bolt? The bolt is SAE grade no. 1, 1" with a proof strength of 55 kips and a tensile strength of 75 kips.

(a) 622 lbf

(b) 999 lbf

(c) 1,381 lbf

(d) 1,500 lbf

18.15.1 Solution 15 – Joining Methods

A bolt is initially used to secure two components with a force of 500 lbf. The bolt has a stiffness coefficient of 1.8 lbf/in and the clamping stiffness coefficient is 13 lbf/in. If an external load of 1,000 lbf is applied to the components, then what is the total amount of force acting upon the bolt? The bolt is SAE grade no. 1, 1" with a proof strength of 55 kips and a tensile strength of 75 kips.

For this problem, use the following equation from the NCEES FE handbook. The other information about the bolt is unnecessary.

$$F_b = F_i + \frac{k_b}{k_b + k_c} F_e$$

$$F_b = 500\ lbf + \frac{1.8\frac{lbf}{in}}{1.8\frac{lbf}{in} + \frac{13 lbf}{in}} 1{,}000\ lbf$$

$$F_b = 621.6\ lbf$$

The correct answer is most nearly, **(a) 622 lbf**.