



Electricity & Magnetism Basic Complex Algebra Review FERM Ch. 43 and Mathematics 1.

14-2

Electricity & Magnetism

14-3a

Electrostatics

Charges

1 Coulomb (C) = charge on 6.24×10^{18} electrons Charge on 1 e⁻ = 1.6022×10^{-19} C (the inverse of 1 Coulomb)

Force on Charged Object

· General case:

$$\mathbf{F} = Q\mathbf{E}$$

43.17

- Specific for 2 point charges: $\mathbf{F}_2 = \mathit{Q}_2\mathbf{E}_1 = \frac{\mathit{Q}_1\mathit{Q}_2}{4\pi\epsilon\mathit{r}^2}\mathbf{a}$

 - a is a unit vector pointing from point charge 1 to point charge 2.

14-3b

Electrostatics

Permittivity

$$\varepsilon_{r} = \frac{\varepsilon}{\varepsilon_{0}} = \frac{\phi_{\text{actual}}}{\phi_{\text{vacuum}}}$$
$$\varepsilon = \varepsilon_{r} \varepsilon_{0}$$

NOTE: On the FE exam, assume the permittivity is Σ_0 = 8.85 × 10^-12 F/m unless another value is provided.

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14-3c

Electrostatics

Electric Field Intensity

• Due to a point charge Q₁: ${\bf E} = \frac{Q_1}{4\pi\epsilon r^2} {\bf a}$

$$\mathbf{E} = \frac{Q_1}{4\pi\epsilon r^2} \mathbf{a}$$

43.20

For a line charge ρ_L:

$$\mathbf{E}_L = \frac{\rho L}{2\pi \epsilon \mathbf{r}} \mathbf{a}$$

43.21

- a is a unit vector normal to the line.
- For a sheet charge ρ_s :

$$\mathbf{E}_S = \frac{\rho_S}{2\epsilon}$$
a

43.22

- a is a unit vector normal to the sheet charge.

14-3d

Electrostatics

Example (FEIM):

A point charge of 0.001 C is placed 10 m from a sheet charge of -0.001 C/m², and a 10 m diameter sphere of charge 0.001 C is placed half-way in between on a straight line, all in a vacuum. What is the force on the point charge?

$$\begin{aligned} \mathbf{F} &= \mathbf{F}_{\text{sheet}} + \mathbf{F}_{\text{sphere}} = \mathbf{Q}_{\text{point}} \left(\mathbf{E}_{\text{sheet}} + \mathbf{E}_{\text{sphere}} \right) \\ F &= Q_{\text{point}} \left(\frac{\rho_{\text{sheet}}}{2\epsilon} + \frac{Q_{\text{sphere}}}{4\pi\epsilon r^2} \right) = \left(\frac{0.001 \text{ C}}{8.85 \times 10^{-12} \frac{\text{F}}{\text{m}}} \right) \left(-\frac{0.001 \frac{\text{C}}{\text{m}^2}}{2} + \frac{0.001 \text{C}}{4\pi (5 \text{ m})^2} \right) \\ &= -5.61 \times 10^4 \, \text{N} \end{aligned}$$

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Electrostatics

Electric Flux - Gauss' Law

$$Q_{\text{encl}} = \bigoplus_{S} \epsilon \mathbf{E} \cdot d\mathbf{S}$$
 43.24

If **E** is constant and parallel to d**S**, then

$$\mathbf{Q}_{\mathsf{encl}} = \int_{\mathbf{S}} \boldsymbol{\epsilon} \mathbf{E} \cdot d\mathbf{S} = \boldsymbol{\epsilon} \boldsymbol{E} \int d\mathbf{S}$$

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Electrostatics

Work (W) done by moving a charge Q_1 radially from distance r_1 to r_2 in an electric field:

$$W = -Q_1 \int_{r_1}^{r_2} \mathbf{E} \cdot d\mathbf{L}$$
 43.25

 For a uniform field, the work done by moving a charge Q a distance d parallel to the uniform field:

$$\begin{split} W &= -\mathbf{F} \cdot \mathbf{d} = -EQd \\ &= \frac{-V_{\text{plates}} \, Qd}{r} \\ &= -Q\Delta \, V \end{split} \tag{43.27}$$

Note that Eq. 43.27 is always true, for all fields. (V may not be easy to compute.)

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Electrostatics

Voltage

- · A scalar quantity that describes the electrical field.
- The field **E** is the gradient of the voltage, *V*.
- The voltage differential between two points is the work to bring a unit charge from one point to the other.
- · The choice of zero potential is arbitrary.
- · Electric field strength between two parallel plates:

$$E = \frac{V}{d}$$
 43.28

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14-3h

Electrostatics

Example (FEIM):

A source at zero potential emits electrons at negligible velocity. An open grid at 18 V is located 0.003 m from the source. At what velocity will the electrons pass through the grid?

- (A) 490 m/s
- (B) 16000 m/s
- (C) 8.3×10^5 m/s
- (D) 2.5×10^6 m/s

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Electrostatics

The mass of an electron is 9.11 x 10-31 kg.

The work done by the grid on an electron is equal to the change in kinetic energy of the electron and is equal to the charge on the electron times the change in voltage potential.

$$W = \frac{1}{2} m v^2 = q \Delta V$$

$$V = \sqrt{\frac{2q\Delta V}{m}} = \sqrt{\frac{(2)(1.6022 \times 10^{-19} \text{ C})(18 \text{ V})}{9.11 \times 10^{-31} \text{ kg}}}$$

$$= 2.516 \times 10^6 \text{ m/s} \quad (2.5 \times 10^6 \text{ m/s})$$

Therefore, the answer is (D).

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Current

Change in charge per unit time

$$i(t) = \frac{dq(t)}{dt} 43.29$$

Current Density (ρ)

• The density of charge moving per unit time through a volume

Volume Current Density (J)

· The vector current density

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14-5a

Magnetism

Magnetic field around a current-carrying wire

$$H = \frac{B}{\mu} = \frac{I}{2\pi r}a$$
 [straight wire] 43.35

Force on current-carrying conductor

$$\mathbf{F} = I\mathbf{L} \times \mathbf{B}$$
 43.36

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