FE Review Course

Dynamics

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Dynamics 8-1

Overview

Dynamics—the study of moving objects.

Kinematics—the study of a body's motion independent of the forces on the body.

Kinetics—the study of motion and the forces that cause motion.

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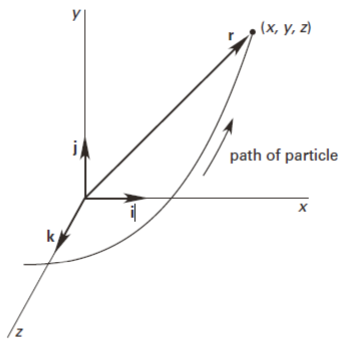
Dynamics 8-2

Kinematics—Rectangular Coordinates

Figure 14.1 Rectangular Coordinates



$$\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k} \qquad 14.7$$

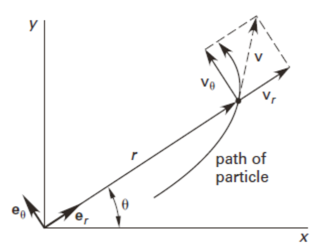
$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = \dot{x}\mathbf{i} + \dot{y}\mathbf{j} + \dot{z}\mathbf{k} \qquad 14.8$$

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = \frac{d^2\mathbf{r}}{dt^2} = \ddot{x}\mathbf{i} + \ddot{y}\mathbf{j} + \ddot{z}\mathbf{k} \qquad 14.9$$

Dynamics 8-3a

Kinematics—Polar Coordinates

Figure 14.2 Radial and Transverse Coordinates



$$\mathbf{r} = r\mathbf{e}_r \quad \text{[position]} \qquad 14.14$$

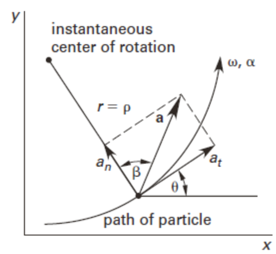
$$\mathbf{v} = \mathrm{v}_r\mathbf{e}_r + \mathrm{v}_\theta\mathbf{e}_\theta = \dot{r}\mathbf{e}_r + r\dot{\theta}\mathbf{e}_\theta \quad \text{[velocity]} \qquad 14.15$$

$$\mathbf{a} = a_r\mathbf{e}_r + a_\theta\mathbf{e}_\theta = (\ddot{r} - r\dot{\theta}^2)\mathbf{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\mathbf{e}_\theta \quad \text{[acceleration]} \qquad 14.16$$

Dynamics 8-3b

Kinematics—Polar Coordinates

Figure 14.3 *Tangential and Normal Coordinates*



$$\mathbf{v} = \mathrm{v}_t \mathbf{e}_t \qquad 14.17$$

$$\mathbf{a} = \frac{d\mathrm{v}_t}{dt}\mathbf{e}_t + \frac{\mathrm{v}_t^2}{\rho}\mathbf{e}_n = a_t\mathbf{e}_t + \frac{\mathrm{v}_t^2}{\rho}\mathbf{e}_n \qquad 14.18$$

Dynamics 8-4a

Kinematics—Circular Motion

$$\theta \quad \text{[angular position]} \qquad 14.19$$

$$\omega = \frac{d\theta}{dt} \quad \text{[angular velocity]} \qquad 14.20$$

$$\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2} \quad \text{[angular acceleration]} \qquad 14.21$$

Dynamics 8-4b

Kinematics—Circular Motion

Angular velocity = $\omega = \dot{\theta} = \frac{v_t}{r}$

Angular acceleration = $\alpha = \dot{\omega} = \ddot{\theta} = \frac{a_t}{r}$

Tangential acceleration = $a_t = r\alpha = \frac{dv_t}{dt}$ 14.24

Normal acceleration = $a_n = \frac{v_t^2}{r} = r\omega^2$ 14.25

Dynamics 8-4c

Kinematics—Circular Motion

Example (FEIM):

A turntable starts from rest and accelerates uniformly at 1.5 rad/s^2. How many revolutions does it take for the rotational frequency to reach 33.33 rpm?

$$\omega = 2\pi f = \left(2\pi \frac{\text{rad}}{\text{rev}}\right)\left(33.33 \frac{\text{rev}}{\text{min}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right) = 3.49 \text{ rad/s}$$

$$\theta = \int_0^{t_f} \omega dt = \int_0^{t_f} \alpha t dt = \frac{\alpha t_f^2}{2}$$

$$t = \frac{\omega}{\alpha}$$

$$\theta = \frac{\omega^2}{2\alpha} = \frac{\left(3.49 \frac{\text{rad}}{\text{s}}\right)^2}{(2)\left(1.5 \frac{\text{rad}}{\text{s}^2}\right)} = 4.06 \text{ rad}$$

$$n = \frac{4.06 \text{ rad}}{2\pi \frac{\text{rad}}{\text{rev}}} = 0.65 \text{ revolution}$$

Dynamics 8-5a

Kinematics—Projectile Motion

Constant acceleration formulas:

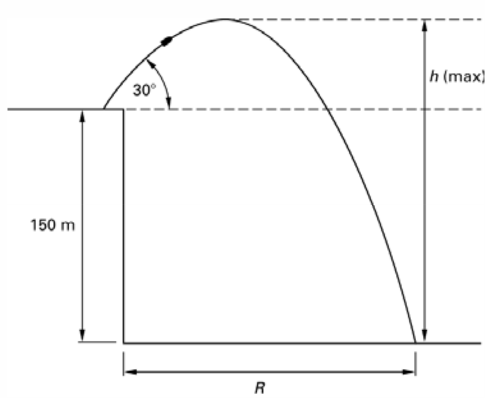
$$s = s_0 + v_0 t + \tfrac{1}{2} a t^2$$
$$v = v_0 + a_0 t$$
$$v^2 = v_0^2 + 2a_0 (s - s_0)$$

Dynamics 8-5b

Kinematics—Projectile Motion

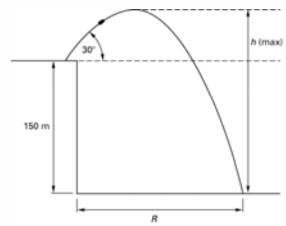
Example 1 (FEIM):

A projectile is launched at 180 m/s at a 30° incline. The launch point is 150 m above the impact plane. Find the maximum height, flight time, and range.



Dynamics 8-5c

Kinematics—Projectile Motion



$$h'_{max} = v_0 \sin\theta + at = 0$$

$$t_{max} = -v_0 \frac{\sin\theta}{a} = -\frac{\left(180 \frac{m}{s}\right)(0.5)}{\left(-9.8 \frac{m}{s}\right)} = 9.18 \text{ s}$$

$$h = h_0 + v_0 t \sin\theta + \tfrac{1}{2}at^2$$

$$h_{max} = 150 \text{ m} + \left(180 \frac{m}{s^2}\right)(9.18 \text{ s})(0.5) - \left(\tfrac{1}{2}\right)\left(9.8 \frac{m}{s^2}\right)(9.18 \text{ s})^2 = 563 \text{ m}$$

$$h_{impact} = 0 = 150 \text{ m} + 90t - 4.9t^2$$

By the quadratic equation, $t_{impact} = 19.9$ s

$\cos 30° = 0.866$

$$R_{impact} = R_0 + v_0 t_{impact} \cos\theta$$

$$= 0 + \left(180 \frac{m}{s}\right)(19.9 \text{ s})(0.866) = 3100 \text{ m}$$

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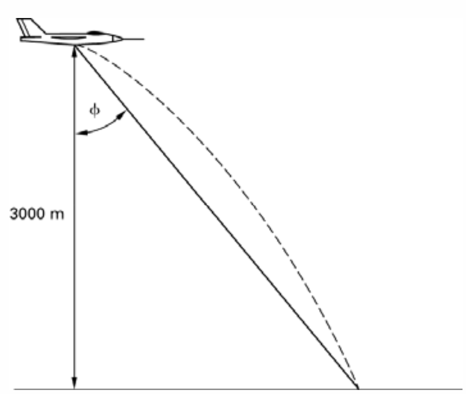
Dynamics 8-5d

Kinematics—Projectile Motion

Example 2 (FEIM):

A bomber flies horizontally at 275 km/h and an altitude of 3000 m. At what viewing angle from the bomber to the target should the bomb be dropped?



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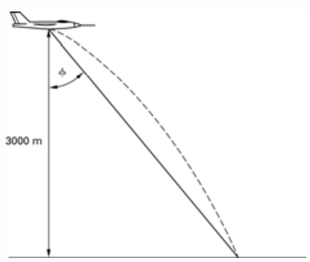
Dynamics 8-5e

Kinematics—Projectile Motion



$$h_{\text{impact}} = \tfrac{1}{2}at^2 = \left(\tfrac{1}{2}\right)\left(-9.8\,\frac{\text{m}}{\text{s}^2}\right)t^2 = -3000\text{ m}$$

$$t = \sqrt{\frac{2h_{\text{impact}}}{g}} = 24.7\text{ s}$$

$$\left(275\,\frac{\text{km}}{\text{hr}}\right)\left(1000\,\frac{\text{m}}{\text{km}}\right)\left(\frac{1\text{ hr}}{3600\text{ s}}\right) = 76.39\text{ m/s}$$

$$R_{\text{impact}} = R_0 + \text{v}_0 t = 0 + \left(76.39\,\frac{\text{m}}{\text{s}}\right)(24.7\text{ s}) = 1887\text{ m}$$

$$\phi = \arctan\frac{R_{\text{impact}}}{h_{\text{impact}}} = \arctan\frac{1887\text{ m}}{3000\text{ m}} = 32.2°$$

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Dynamics 8-6a

Kinetics—Newton's 2nd Law of Motion

$$\mathbf{F} = \frac{d\mathbf{p}}{dt} \qquad 15.2$$

For a constant mass,

$$\begin{aligned} \mathbf{F} &= \frac{d\mathbf{p}}{dt} = \frac{d(m\mathbf{v})}{dt} \\ &= m\frac{d\mathbf{v}}{dt} \\ &= m\mathbf{a} \qquad \text{[SI]} \qquad 15.3(a) \end{aligned}$$

One-dimension motion

$$s(t) = \int \text{v}(t)\,dt = \iint a(t)\,dt^2 \qquad 14.4$$

$$\text{v}(t) = \frac{ds(t)}{dt} = \int a(t)\,dt \qquad 14.5$$

$$a(t) = \frac{d\text{v}(t)}{dt} = \frac{d^2 s(t)}{dt^2} \qquad 14.6$$

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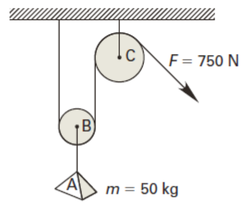
Dynamics 8-6b

Kinetics—Newton's 2nd Law of Motion

Example (FERM prob. 6, p. 15-5):

Problem 6

A constant force of 750 N is applied through a pulley system to lift a mass of 50 kg, as shown. Neglecting the mass and friction of the pulley system, what is the acceleration of the 50 kg mass?



(A) 5.20 m/s^2
(B) 8.72 m/s^2
(C) 16.2 m/s^2
(D) 20.2 m/s^2

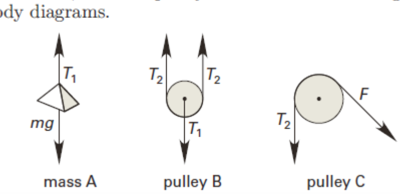
Dynamics 8-6d

Kinetics—Newton's 2nd Law of Motion

Example (FERM prob. 6, p. 15-5):

Solution

Apply Newton's second law to the mass and to the two frictionless, massless pulleys. Refer to the following free-body diagrams.



mass A: $T_1 - mg = ma$

pulley B: $2T_2 - T_1 = 0$

pulley C: $T_2 = F = 750$ N

$$a = \frac{T_1 - mg}{m} = \frac{2T_2 - mg}{m} = \frac{2F - mg}{m}$$

$$= \frac{(2)(750\ \text{N}) - (50\ \text{kg})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)}{50\ \text{kg}}$$

$$= 20.2\ \text{m/s}^2$$

Answer is D.

Dynamics 8-7a

Kinetics—Impulse and Momentum

Impulse (constant mass in one dimension)

$$\mathbf{Imp} = \int_{t_1}^{t_2} \mathbf{F}\,dt \qquad 17.14$$

Momentum

$$\mathbf{p} = m\mathbf{v} \qquad \text{[SI]} \qquad 15.1(a)$$

Impulse-Momentum Principle

$$\mathbf{Imp} = \Delta\mathbf{p} \qquad 17.16$$

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Dynamics 8-7b

Kinetics—Impulse and Momentum

Example 1 (FEIM):

A 0.046 kg marble attains a velocity of 76 m/s in a slingshot. Contact with the slingshot is 1/25 of a second. What is the average force on the marble during the launch?

$$F_{\text{ave}} = \frac{m\Delta \text{v}}{\Delta t} = \frac{(0.046\ \text{kg})\left(76\,\frac{\text{m}}{\text{s}}\right)}{0.04\ \text{s}} = 87.4\ \text{N}$$

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Dynamics 8-7c

Kinetics—Impulse and Momentum

Example 2 (FEIM):

A 2000 kg cannon fires a 10 kg projectile horizontally at 600 m/s. It takes 0.007 s for the projectile to pass through the barrel. What is the recoil velocity if the cannon is not restrained? What average force must be exerted on the cannon to keep it from moving?

$$m_{\text{projectile}}\Delta \text{v}_{\text{projectile}} = m_{\text{cannon}}\Delta \text{v}_{\text{cannon}}$$

$$(10\text{ kg})\left(600\,\frac{\text{m}}{\text{s}}\right) = (2000\text{ kg})(\text{v}_{\text{cannon}})$$

$$\text{v}_{\text{cannon}} = 3\text{ m/s} = \text{initial recoil velocity}$$

$$F = \frac{m\Delta \text{v}}{\Delta t} = \frac{(10\text{ kg})\left(600\,\frac{\text{m}}{\text{s}}\right)}{0.007\text{ s}} = 8.57\times10^{5}\text{ N}$$

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Dynamics 8-8a

Work & Energy

Work

$$W = \int \mathbf{F}\cdot d\mathbf{r} \qquad 17.1$$

$$W = E_2 - E_1 \qquad 17.11$$

Kinetic Energy of a Mass

$$T = \tfrac{1}{2}m\text{v}^2 \qquad \text{[SI]} \qquad 17.2(a)$$

$$W = T_2 - T_1 = \tfrac{1}{2}m(\text{v}_2^2 - \text{v}_1^2)$$

Kinetic Energy of a Rotating Body

$$T = \tfrac{1}{2}I\omega^2 \qquad \text{[SI]} \qquad 17.3(a)$$

$$W = T_2 - T_1 = \tfrac{1}{2}I(\omega_2^2 - \omega_1^2)$$

Potential Energy

- Gravity

$$U = mgh \qquad \text{[SI]} \qquad 17.6(a)$$

$$W = U_2 - U_1 = mg(h_2 - h_1)$$

- Spring (linear)
  $F_s = kx$ where the spring is compressed a distance x

$$U = \tfrac{1}{2}kx^2 \qquad 17.7$$

$$W = U_2 - U_1 = \tfrac{1}{2}k(x_2^2 - x_1^2)$$

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Dynamics 8-8b

Work & Energy

Conservation of Energy

- For a closed system (no external work), the change in potential energy equals the change in kinetic energy.

$$U_1 - U_2 = T_2 - T_1$$
$$U_1 + T_1 = U_2 + T_2$$

- For a system with external work, *W* equals $\Delta U + \Delta T$.

$$W_{1\to 2} = (U_1 - U_2) + (T_2 - T_1)$$
$$U_1 + T_1 + W_{1\to 2} = U_2 + T_2$$

Dynamics 8-8c

Work & Energy

Impacts: Momentum is always conserved.
Elastic Impacts: Kinetic energy is conserved.

$$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2' \quad \text{[always true]} \quad 17.20$$

Example 1 (FEIM):

Two identical balls collide along their centerlines in an elastic collision. The initial velocity of ball 1 is 0.85 m/s. The initial velocity of ball 2 is –0.53 m/s. What is the relative velocity of each ball after the collision?

(A) –0.53 m/s and 0.85 m/s
(B) –0.72 m/s and 1.2 m/s
(C) –5.1 m/s and 1.2 m/s
(D) 0.98 m/s and 1.8 m/s

$$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2'$$
$$m_1 = m_2 \text{ so } v_1 + v_2 = v_1' + v_2' = 0.85 - 0.53 = 0.32$$
$$\tfrac{1}{2} m v_1^2 + \tfrac{1}{2} m v_2^2 = \tfrac{1}{2} m {v_1'}^2 + \tfrac{1}{2} m {v_2'}^2$$
$$v_1^2 + v_2^2 = {v_1'}^2 + {v_2'}^2$$

Solving two equations and two unkowns:

$$v_1' = -0.53 \text{ m/s}$$
$$v_2' = 0.85 \text{ m/s}$$

Therefore, (A) is correct.

Dynamics 8-8d

Work & Energy

Example 2 (FEIM):

Ball A of 200 kg is traveling at 16.7 m/s. It strikes stationary ball B of 200 kg along the centerline. What is the velocity of ball A after the collision? Assume the collision is elastic.

(A) –16.7 m/s
(B) –8.35 m/s
(C) 0
(D) 8.35 m/s

$$m_A = m_B \text{so} \, \mathrm{v}_A + \mathrm{v}_B = \mathrm{v}'_A + \mathrm{v}'_B = 16.7 \text{ m/s}$$

$$\mathrm{v}_A^2 + \mathrm{v}_B^2 = \mathrm{v}'^{\,2}_A + \mathrm{v}'^{\,2}_B$$

There are two possible solutions for these equations.

$$\mathrm{v}'_A = 0, \ \mathrm{v}'_B = 16.7 \text{ m/s}$$

or

$$\mathrm{v}'_A = 16.7 \text{ m/s}, \ \mathrm{v}'_B = 0$$

Since there must be a change in the collision, ball A's velocity must be 0. Therefore, (C) is correct.

Dynamics 8-8e

Work & Energy

Inelastic Impacts:
Kinetic energy does not have to be conserved if some energy is converted to another form.

$$\mathrm{v}'_1 - \mathrm{v}'_2 = -e(\mathrm{v}_1 - \mathrm{v}_2)$$

where e = coefficient of restitution

$$\mathrm{v}'_1 = \frac{m_2\mathrm{v}_2(1+e) + (m_1 - em_2)\mathrm{v}_1}{m_1 + m_2}$$

$$\mathrm{v}'_2 = \frac{m_1\mathrm{v}_1(1+e) + (em_1 - m_2)\mathrm{v}_2}{m_1 + m_2}$$

Example 1 (FEIM):

A ball is dropped from an initial height h_o. If the coefficient of restitution is 0.90, how high will the ball rebound?

(A) $0.45h_o$
(B) $0.81h_o$
(C) $0.85h_o$
(D) $0.9h_o$

$$mgh = \tfrac{1}{2}m\mathrm{v}^2$$

$$\mathrm{v} = \sqrt{2gh_o}$$

$$\mathrm{v}' = -e\mathrm{v} = -e\sqrt{2gh_o} = \sqrt{2gh'}$$

$$h' = e^2 h_o = (0.9)^2 h_o = 0.81h_o$$

Since there must be a change in the collision, ball A's velocity must be 0. Therefore, (B) is correct.

Dynamics 8-8f

Work & Energy

Example 2 (FEIM):

Two masses collide in a perfectly inelastic collision. What is the velocity of the combined mass after collision?

$m_1 = 4m_2 \quad v_1 = 10 \text{ m/s} \quad v_2 = -20 \text{ m/s}$

(A) 0

(B) 4 m/s

(C) –5m/s

(D) 10 m/s

"Perfectly inelastic" means the masses collide and stick together.

$$m_1 v_1 + m_2 v_2 = m_3 v_3$$

$$m_3 = m_1 + m_2 = 5m_2$$

$$4m_2\left(10\frac{\text{m}}{\text{s}}\right) + m_2\left(-20\frac{\text{m}}{\text{s}}\right) = 5m_2 v_3$$

$$5m_2 v_3 = \left(40\frac{\text{m}}{\text{s}}\right)m_2 - \left(20\frac{\text{m}}{\text{s}}\right)m_2$$

$$v_3 = 4 \text{ m/s}$$

Therefore, (B) is correct.

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Dynamics 8-9a

Kinetics

Friction

$F_f = \mu N$ 15.5

Example (FEIM):

A snowmobile tows a sled with a weight of 3000 N. It accelerates up a 15° slope at 0.9 m/s². The coefficient of friction between the sled and the snow is 0.1. What is the tension in the tow rope?

$$F_{\text{slope}} = F_{\text{rope}} - (F_{\text{friction}} + F_{\text{gravity}}) = ma_{\text{slope}}$$

$$F_{\text{rope}} = F_{\text{friction}} + F_{\text{gravity}} + ma_{\text{slope}}$$

$$= mg\sin 15° + mg\mu\cos 15° + ma_{\text{slope}}$$

$$= (3000)(0.2588) + (3000 \text{ N})(0.1)(0.9659)$$

$$+ \left(\frac{3000 \text{ N}}{9.8\frac{\text{m}}{\text{s}^2}}\right)\left(0.9\frac{\text{m}}{\text{s}^2}\right)$$

$$= 1342 \text{ N}$$

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Dynamics 8-9b

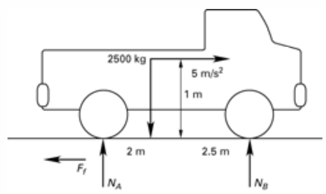
Kinetics

Plane Motion of a Rigid Body

Similar equations can be written for the *y*-direction or any other coordinate direction.

$$F_x = ma_x \qquad \text{[SI]} \quad 15.8$$

Example (FEIM):
A 2500 kg truck skids with a deceleration of 5 m/s². What is the coefficient of sliding friction? What are the frictional forces and normal reactions (per axle) at the tires?



Dynamics 8-9c

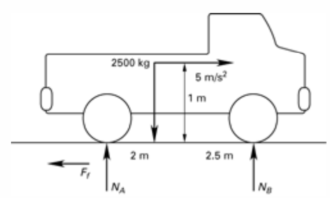
Kinetics

The force of deceleration is equal to the friction force.

$$F_{\text{deceleration}} = (2500\ \text{kg})\left(5\ \frac{\text{m}}{\text{s}^2}\right) = F_{\text{friction}} = \mu m g$$

$$= \mu(2500\ \text{kg})\left(9.8\ \frac{\text{m}}{\text{s}^2}\right)$$

$$\mu = \left(5\ \frac{\text{m}}{\text{s}^2}\right)\left(9.8\ \frac{\text{m}}{\text{s}^2}\right) = 0.51$$



The moment about the center of gravity, M_A, must be equal to zero.

$$\sum M_A = (4.5\ \text{m})N_B - (2\ \text{m})mg - (1\ \text{m})(F_{\text{deceleration}}) = 0$$

$$= (4.5\ \text{m})N_B - (2\ \text{m})(2500\ \text{kg})\left(9.8\ \frac{\text{m}}{\text{s}^2}\right) - (2500\ \text{kg})\left(5\ \frac{\text{m}}{\text{s}^2}\right) = 0$$

$$N_B = 13{,}667\ \text{N}$$

$$N_A = mg - N_B = (2500\ \text{kg})\left(9.8\ \frac{\text{m}}{\text{s}^2}\right) - 13667\ \text{N} = 10833\ \text{N}$$

Dynamics 8-10a

Rotation

Rotation about Fixed Axis

$$\alpha = \frac{M}{I} \qquad 16.12$$

$$\omega = \int \alpha dt = \omega_0 + \left(\frac{M}{I}\right)t$$
$$= \omega_0 + \alpha t \qquad 16.13$$

$$\theta = \iint \alpha dt^2 = \theta_0 + \omega_0 t + \left(\frac{M}{2I}\right)t^2$$
$$= \theta_0 + \omega_0 t + \frac{\alpha t^2}{2} \qquad 16.14$$

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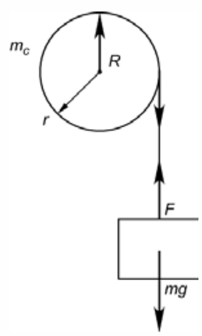
Dynamics 8-10b

Rotation

Example (FEIM):

A mass m is attached to a rope wound around a cylinder of mass m_C and radius r. What is the acceleration of the falling mass? What is the rope tension?



There are three simultaneous equations for the movement of the mass, the cylinder, and the relationship between the two:

$mg - F = ma$

$Fr = I\alpha$

$\alpha r = a$

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Dynamics 8-10c

Rotation

Rearranging,

$$Fr = I\frac{a}{r}$$

$$F = \frac{m_c r^2 a}{2r^2} = \frac{m_c}{2}a$$

$$mg - \frac{m_c}{2}a = ma$$

$$a = g\frac{m}{m + \frac{m_c}{2}}$$

Solving for *F*,

$$F = \frac{m_c}{2}a = g\frac{m_c m}{2\left(m + \frac{m_c}{2}\right)} = \frac{m m_c}{2m + m_c}g$$

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Dynamics 8-10d

Rotation

Centripetal Force

- The force required to keep a body rotating about an axis.

$$F_c = ma_n = \frac{mv_t^2}{r} = mr\omega^2 \qquad \text{[SI]} \quad 16.16(a)$$

(r is the distance from the center of mass to the center of rotation.)

Centrifugal Force

- The "reaction" to centripetal force.
- The centrifugal force, like any inertia force, should not be used in free-body diagrams.

Example (FEIM):

A 2000 kg car travels 65 km/hr around a curve of radius 60 m. What is the centripetal force?

$$65 \text{ km/hr} = 18.06 \text{ m/s}$$

$$F_c = \frac{m\text{v}^2}{r} = \frac{(2000 \text{ kg})\left(18.06\,\frac{\text{m}}{\text{s}}\right)^2}{60 \text{ m}} = 10900 \text{ N}$$

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Dynamics 8-10e

Rotation

Banking Curves

$$\tan\theta = \frac{v_t^2}{gr} \qquad 16.18$$

Example (FEIM):
A 2000 kg car travels at 64 km/hr around a banked curve with a radius of 150 m. What should the angle between the roadway and the horizontal be so tire friction is not needed to prevent sliding?

$$\left(64\,\frac{\text{km}}{\text{hr}}\right)\left(1000\,\frac{\text{m}}{\text{km}}\right)\left(\frac{1\text{hr}}{3600\ \text{s}}\right) = 17.8\,\frac{\text{m}}{\text{s}}$$

$$\theta = \arctan\left(\frac{v^2}{gr}\right) = \arctan\left(\frac{\left(17.8\,\frac{\text{m}}{\text{s}}\right)}{\left(9.8\,\frac{\text{m}}{\text{s}}\right)(150\ \text{m})}\right) = 12.1°$$

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Dynamics 8-10d

Rotation

Free Vibration

Spring and Mass: $mg - k(x + \delta_{st}) = m\ddot{x}$ [SI] 15.20

Natural Frequency: $\omega = \sqrt{\frac{k}{m}}$ [SI] 15.24(a)

Solution: $x(t) = C_1 \cos\omega t + C_2 \sin\omega t$ 15.23

For initial conditions $x(0) = x_0$ and $x'(0) = v_0$,

$$x(t) = x_0 \cos\omega t + \left(\frac{v_0}{\omega}\right)\sin\omega t \qquad 15.27$$

For initial conditions $x(0) = x_0$ and $x'(0) = 0$,

$$x(t) = x_0 \cos\omega t \qquad 15.28$$

Torsional Free Vibration:

$$\ddot{\theta} + \left(\frac{k_t}{I}\right)\theta = \ddot{\theta} + \omega_n^2\theta = 0 \qquad 16.19$$

Natural frequency: $\omega_n = \sqrt{\frac{k_t}{I}}$

Solution: $\theta(t) = \theta_0 \cos\omega_n t + \left(\frac{\omega_0}{\omega_n}\right)\sin\omega_n t$ 16.21

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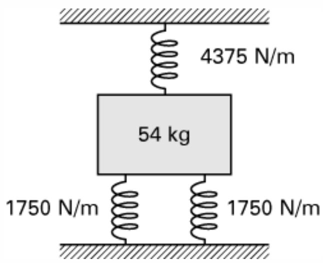
Dynamics 8-10e

Rotation

Example (FEIM):
A 54 kg mass is supported by three springs, as shown. The starting position is 5.0 cm down from the equilibrium position. No external forces act on the mass after it is released. What are the maximum velocity and acceleration?



Dynamics 8-10f

Rotation

From the solution to the differential equation,

$$x(t) = x_0 \cos \omega_n t + \left(\frac{\mathrm{v}_0}{\omega_n}\right) \sin \omega_n t$$

$$\mathrm{v}(t) = x'(t) = -x_0 \omega_n \sin \omega_n t + \mathrm{v}_0 \cos \omega_n t$$

$$\text{But, } \mathrm{v}_0 = 0; \text{ so } \mathrm{v}(t) = -x_0 \omega_n \sin \omega_n t$$

$$x_0 = 0.05 \text{ m}$$

$$k = k_1 + k_2 + k_3 = 1750 \ \frac{\text{N}}{\text{m}} + 1750 \ \frac{\text{N}}{\text{m}} + 4375 \ \frac{\text{N}}{\text{m}} = 7875 \text{ N/m}$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{7875 \ \frac{\text{N}}{\text{m}}}{54 \text{ kg}}} = 12.08 \text{ rad/s}$$

$$v = x' = (-0.05 \text{ m})\left(12.08 \ \frac{\text{rad}}{\text{s}}\right) \sin 12.08t$$

$$= -0.604 \text{ m/s } \sin 12.08t$$

Dynamics 8-10g

Rotation

The maximum velocity is when sin 12.08t = 1. Because the motion is oscillatory, the maximum velocity occurs in both directions.

$$v_{max} = \pm 0.604 \text{ m/s}$$

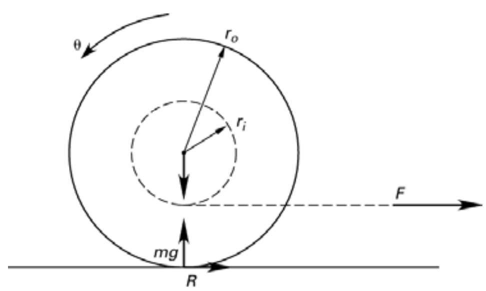
$$a = v' = (-0.05 \text{ m})\left(12.08 \ \frac{\text{rad}}{\text{s}}\right)^2 \cos 12.08t$$

$$a_{max} = \pm 7.30 \text{ m/s}^2$$

Dynamics 8-11a

Review

Example:



A yo-yo (mass m, inertia I about center of gravity) is placed on a horizontal surface with a coefficient of friction μ. The string, wrapped underneath, is pulled with a force F. Determine if the yo-yo will slip on the floor and which direction it will rotate as a function of F.

The two equations for the movement of the center of gravity and the rotation are

$$F + R = ma$$

$$Fr_i + Rr_o = I\alpha$$

Dynamics 8-11b

Review

There are two possibilities for the third equation: slip or no slip.

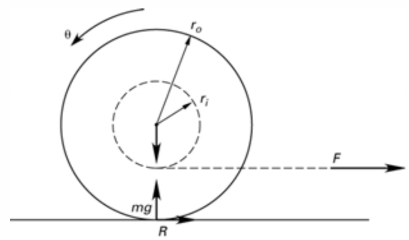
No slip: $|R| < \mu mg \Leftrightarrow a = -r_o \alpha$

Solving these 3 equations with 3 unknowns (α, a, and R) leads to

$$\alpha = -F \frac{r_o - r_i}{I + mr_o^2} \therefore \alpha < 0$$

$$a = \frac{F r_o (r_o - r_i)}{I + mr_o^2} \therefore a > 0$$

$$R = -F \frac{I + mr_i^2}{I + mr_o^2}$$

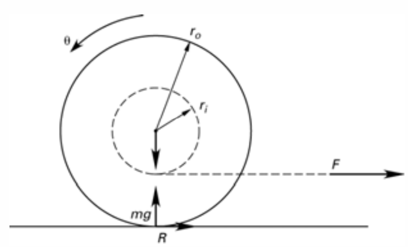


The yo-yo is moving to the right while wrapping itself on the string. The solution for R gives the condition for no slip.

$$F < \mu mg \frac{I + mr_o^2}{I + mr_i^2}$$

Dynamics 8-11c

Review



Slip: $R = -\mu mg$

Solving these 3 equations lead to

$$a = \frac{F - \mu mg}{m}$$

$$= \frac{F r_i - \mu mg r_o}{I}$$

The yo-yo always moves to the right, but can rotate forward or backard depending on the value for F.