FE Review Course

Fluid Mechanics

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Fluid Mechanics 9-1a

Definitions

Fluids

- Substances in either the liquid or gas phase
- Cannot support shear

Density

- Mass per unit volume

Specific Volume

$$v = \frac{1}{\rho} \qquad 22.2$$

Specific Weight

$$\gamma = \lim_{\Delta V \to 0} \left(\frac{g \Delta m}{\Delta V} \right) = \rho g$$

Specific Gravity

$$\mathrm{SG} = \frac{\gamma}{\gamma_{\mathrm{water}}} = \frac{\rho}{\rho_{\mathrm{water}}} \qquad 22.4$$

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Fluid Mechanics 9-1b

Definitions

Example (FEIM):

Determine the specific gravity of carbon dioxide gas (molecular weight = 44) at 66°C and 138 kPa compared to STP air.

$$R_{\text{carbon dioxide}} = \frac{8314\ \dfrac{\text{J}}{\text{kmol}\cdot\text{K}}}{44\ \dfrac{\text{kg}}{\text{kmol}}} = 189\ \text{J/kg}\cdot\text{K}$$

$$R_{\text{air}} = \frac{8314\ \dfrac{\text{J}}{\text{kmol}\cdot\text{K}}}{29\ \dfrac{\text{kg}}{\text{kmol}}} = 287\ \text{J/kg}\cdot\text{K}$$

$$\text{SG} = \frac{\rho}{\rho_{\text{STP}}} = \frac{PR_{\text{air}}T_{\text{STP}}}{R_{\text{CO}_2}Tp_{\text{STP}}} = \left(\frac{1.38\times10^5\ \text{Pa}}{\left(189\ \dfrac{\text{J}}{\text{kg}\cdot\text{K}}\right)(66°\text{C}+273.16)}\right)\left(\frac{\left(287\ \dfrac{\text{J}}{\text{kg}\cdot\text{K}}\right)(273.16)}{1.013\times10^5\ \text{Pa}}\right) = 1.67$$

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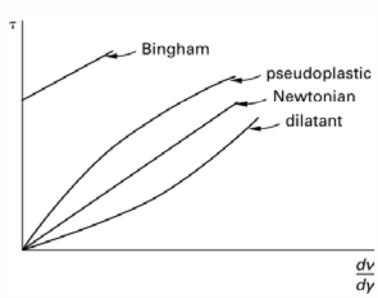
Fluid Mechanics 9-1c

Definitions

Shear Stress

- Normal Component: $\tau_n = -p$ 22.8
- Tangential Component
  - For a Newtonian fluid: $\tau_t = \mu \dfrac{dv}{dy}$ 22.10
  - For a pseudoplastic or dilatant fluid: $\tau_t = K\left(\dfrac{dv}{dy}\right)^n$ 22.11



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Fluid Mechanics 9-1d

Definitions

Absolute Viscosity

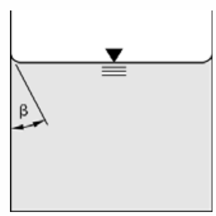
- Ratio of shear stress to rate of shear deformation

Surface Tension

$$\sigma = \frac{F}{L} \qquad 22.13$$

Capillary Rise

$$h = \frac{4\sigma \cos\beta}{\rho g d_{\text{tube}}} = \frac{4\sigma \cos\beta}{\gamma d_{\text{tube}}} \qquad \text{[SI]} \qquad 22.16(a)$$



Fluid Mechanics 9-1e

Definitions

Example (FEIM):
Find the height to which ethyl alcohol will rise in a glass capillary tube 0.127 mm in diameter.

Density is 790 kg/m^3, $\sigma = 0.0227$ N/m, and $\beta = 0°$.

$$h = \frac{4\sigma\cos\beta}{\rho g d} = \frac{(4)\left(0.0227\ \frac{\text{kg}}{\text{s}^2}\right)(1.0)}{\left(790\ \frac{\text{kg}}{\text{m}^3}\right)\left(9.8\ \frac{\text{m}}{\text{s}^2}\right)(0.127\times 10^{-3}\text{m})} = 0.00923\ \text{m}$$

Fluid Mechanics 9-2a

Fluid Statics

Gage and Absolute Pressure

$$p_{\text{absolute}} = p_{\text{gage}} + p_{\text{atmospheric}}$$

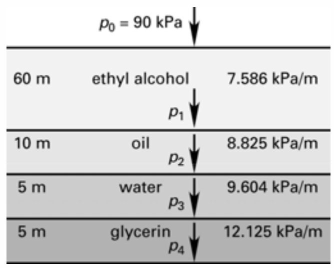
Hydrostatic Pressure

$$p = \gamma h + \rho g h$$

$$p_2 - p_1 = -\gamma(z_2 - z_1)$$

Example (FEIM):

In which fluid is 700 kPa first achieved?



(A) ethyl alcohol
(B) oil
(C) water
(D) glycerin

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Fluid Mechanics 9-2b

Fluid Statics

$$p_0 = 90 \text{ kPa}$$

$$p_1 = p_0 + \gamma_1 h_1 = 90 \text{ kPa} + \left(7.586 \frac{\text{kPa}}{\text{m}}\right)(60 \text{ m}) = 545.16 \text{ kPa}$$

$$p_2 = p_1 + \gamma_2 h_2 = 545.16 \text{ kPa} + \left(8.825 \frac{\text{kPa}}{\text{m}}\right)(10 \text{ m}) = 633.41 \text{ kPa}$$

$$p_3 = p_2 + \gamma_3 h_3 = 633.41 \text{ kPa} + \left(9.604 \frac{\text{kPa}}{\text{m}}\right)(5 \text{ m}) = 681.43 \text{ kPa}$$

$$p_4 = p_3 + \gamma_4 h_4 = 681.43 \text{ kPa} + \left(12.125 \frac{\text{kPa}}{\text{m}}\right)(5 \text{ m}) = 742 \text{ kPa}$$

Therefore, (D) is correct.

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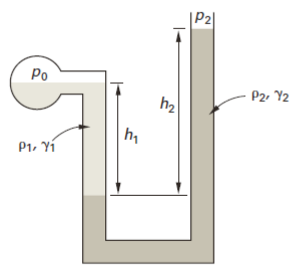
Fluid Mechanics 9-2c

Fluid Statics

Manometers

Figure 23.3 Open Manometer



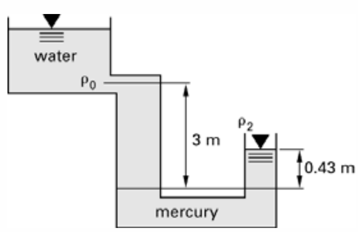
$$p_0 = p_2 + \frac{\rho_2 g}{g_c} h_2 - \frac{\rho_1 g}{g_c} h_1 = p_2 + \gamma_2 h_2 - \gamma_1 h_1 \qquad \text{[U.S.]} \quad 23.4(b)$$

Fluid Mechanics 9-2d

Fluid Statics

Example (FEIM):

The pressure at the bottom of a tank of water is measured with a mercury manometer. The height of the water is 3.0 m and the height of the mercury is 0.43 m. What is the gage pressure at the bottom of the tank?



From the table in the NCEES Handbook,

$$\rho_{mercury} = 13\,560 \ \frac{\text{kg}}{\text{m}^3} \quad \rho_{water} = 997 \text{ kg/m}^3$$

$$\Delta p = g(\rho_2 h_2 - \rho_1 h_1)$$

$$= \left(9.81 \frac{\text{m}}{\text{s}^2}\right)\left(\left(13\,560 \frac{\text{kg}}{\text{m}^3}\right)(0.43 \text{ m}) - \left(997 \frac{\text{kg}}{\text{m}^3}\right)(3.0 \text{ m})\right)$$

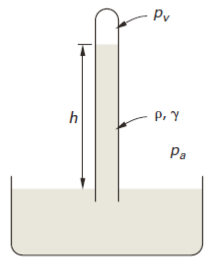
$$= 27\,858 \text{ Pa}$$

Fluid Mechanics 9-2e

Fluid Statics

Barometer

Figure 23.4 Barometer



Atmospheric Pressure

$$p_a = p_v + \rho g h = p_v + \gamma h \qquad \text{[SI]} \qquad 23.7(a)$$

Fluid Mechanics 9-2f

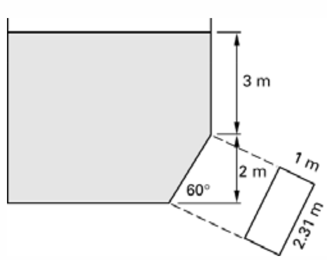
Fluid Statics

Forces on Submerged Surfaces

$$R = pA \qquad 23.8$$

$$\overline{p} = \tfrac{1}{2}\rho g(h_1 + h_2) = \tfrac{1}{2}\gamma(h_1 + h_2) \qquad \text{[SI]} \qquad 23.10(a)$$

Example (FEIM):
The tank shown is filled with water. Find the force on 1 m width of the inclined portion.



The average pressure on the inclined section is:

$$p_{ave} = \left(\tfrac{1}{2}\right)\left(997\,\frac{\text{kg}}{\text{m}^3}\right)\left(9.81\,\frac{\text{m}}{\text{s}^2}\right)(3\text{ m} + 5\text{ m})$$
$$= 39122\text{ Pa}$$

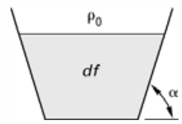
The resultant force is

$$R = p_{ave}A = (39122\text{ Pa})(2.31\text{ m})(1\text{ m})$$
$$= 90\,372\text{ N}$$

Fluid Mechanics 9-2g

Fluid Statics

Center of Pressure



$$y^* = \frac{\rho g I_{yz} \sin\alpha}{p_c A} = \frac{\gamma I_{yz} \sin\alpha}{p_c A} \quad \text{[SI]} \quad 23.17(a)$$

$$z^* = \frac{\rho g I_y \sin\alpha}{p_c A} = \frac{\gamma I_y \sin\alpha}{p_c A} \quad \text{[SI]} \quad 23.18(a)$$

If the surface is open to the atmosphere, then p_0 = 0 and

$$p_c = \overline{p} = \rho g z_c \sin\alpha = \gamma z_c \sin\alpha \quad \text{[SI]} \quad 23.19(a)$$

$$y_{cp} - y_c = y^* = \frac{I_{yz}}{A z_c} \quad 23.20$$

$$z_{cp} - z_c = z^* = \frac{I_y}{A z_c} \quad 23.21$$

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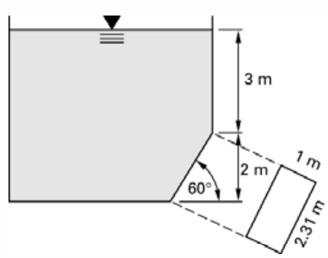
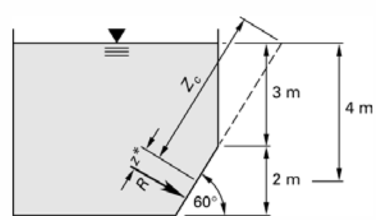
Fluid Mechanics 9-2h

Fluid Statics

Example 1 (FEIM):
The tank shown is filled with water. At what depth does the resultant force act?





The surface under pressure is a rectangle 1 m at the base and 2.31 m tall.

$$A = bh$$

$$I_{y_c} = \frac{b^3 h}{12}$$

$$Z_c = \frac{4 \text{ m}}{\sin 60^\circ} = 4.618 \text{ m}$$

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Fluid Mechanics 9-2i

Fluid Statics

Using the moment of inertia for a rectangle given in the NCEES Handbook,

$$z^* = \frac{I_y}{AZ_c} = \frac{b^3h}{12bhZ_c} = \frac{b^2}{12Z_c}$$

$$= \frac{(2.31\ \text{m})^2}{(12)(4.618\ \text{m})} = 0.0963\ \text{m}$$

$R_{\text{depth}} = (Z_c + z^*) \sin 60° = (4.618\ \text{m} + 0.0963\ \text{m}) \sin 60° = 4.08\ \text{m}$

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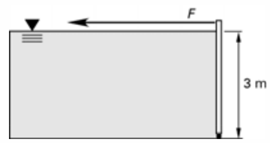
Fluid Mechanics 9-2j

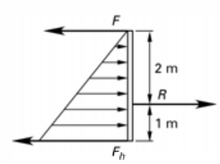
Fluid Statics

Example 2 (FEIM):

The rectangular gate shown is 3 m high and has a frictionless hinge at the bottom. The fluid has a density of 1600 kg/m^3. The magnitude of the force F per meter of width to keep the gate closed is most nearly



(A) 0 kN/m
(B) 24 kN/m
(C) 71 kN/m
(D) 370 kN/m



$$p_{\text{ave}} = \rho g z_{\text{ave}} (1600 \frac{\text{kg}}{\text{m}^3})(9.81 \frac{\text{m}}{\text{s}^2})(\tfrac{1}{2})(3\ \text{m})$$

$$= 23544\ \text{Pa}$$

$$\frac{R}{w} = p_{\text{ave}}h = (23544\ \text{Pa})(3\ \text{m}) = 70662\ \text{N/m}$$

$$F + F_h = R$$

R is one-third from the bottom (centroid of a triangle from the NCEES Handbook). Taking the moments about R,

$2F = F_h$

$$\frac{F}{w} = \left(\frac{1}{3}\right)\left(\frac{R}{w}\right) = \frac{70{,}667\ \frac{\text{N}}{\text{m}}}{3} = 23.6\ \text{kN/m}$$

Therefore, (B) is correct.

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Fluid Mechanics 9-2k

Fluid Statics

Archimedes' Principle and Buoyancy

- The buoyant force on a submerged or floating object is equal to the weight of the displaced fluid.
- A body floating at the interface between two fluids will have buoyant force equal to the weights of both fluids displaced.

$$F_{\text{buoyant}} = \gamma_{\text{water}} V_{\text{displaced}}$$

Fluid Mechanics 9-3a

Fluid Dynamics

Hydraulic Radius for Pipes

$$R_H = \frac{\text{area in flow}}{\text{wetted perimeter}} \qquad 24.26$$

Example (FEIM):
A pipe has diameter of 6 m and carries water to a depth of 2 m. What is the hydraulic radius?

$r = 3\text{ m}$

$d = 2\text{ m}$

$\phi = (2\text{ m})(\arccos((r-d)/r)) = (2\text{ m})(\arccos\tfrac{1}{3}) = 2.46\text{ radians}$

(Careful! Degrees are very wrong here.)

$s = r\phi = (3\text{ m})(2.46\text{ radians}) = 7.38\text{ m}$

$A = \tfrac{1}{2}(r^2(\phi - \sin\phi)) = (\tfrac{1}{2})((3\text{ m})^2(2.46\text{ radians} - \sin 2.46)) = 8.235\text{ m}^2$

$$R_H = \frac{A}{s} = \frac{8.235\text{ m}^2}{7.38\text{ m}} = 1.12\text{ m}$$

Fluid Mechanics 9-3b

Fluid Dynamics

Continuity Equation

$$\dot{m} = \rho A \mathrm{v} = \rho Q \qquad 24.2$$

$$\rho_1 A_1 \mathrm{v}_1 = \rho_2 A_2 \mathrm{v}_2 \qquad 24.3$$

If the fluid is incompressible, then $\rho_1 = \rho_2$.

$$Q = A_1 \mathrm{v}_1 = A_2 \mathrm{v}_2 \qquad 24.4$$

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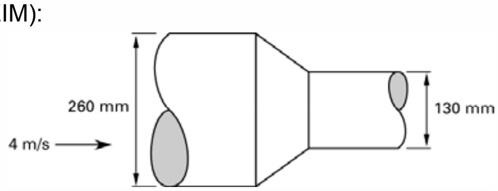
Fluid Mechanics 9-3c

Fluid Dynamics

Example (FEIM):



The speed of an incompressible fluid is 4 m/s entering the 260 mm pipe. The speed in the 130 mm pipe is most nearly

(A) 1 m/s
(B) 2 m/s
(C) 4 m/s
(D) 16 m/s

$$A_1 v_1 = A_2 v_2$$

$$A_1 = 4A_2$$

$$\text{so } v_2 = 4v_1 = (4)\left(4\,\frac{\text{m}}{\text{s}}\right) = 16 \text{ m/s}$$

Therefore, (D) is correct.

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Fluid Mechanics 9-3d

Fluid Dynamics

Bernoulli Equation

$$\frac{p_1 g_c}{\rho_1 g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2 g_c}{\rho_2 g} + \frac{v_2^2}{2g} + z_2$$

$$\frac{p_1}{\gamma_1} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\gamma_2} + \frac{v_2^2}{2g} + z_2 \qquad \text{[U.S.]} \qquad 24.11(b)$$

- In the form of energy per unit mass:

$$\frac{p_1}{\rho_1} + \frac{v_1^2}{2} + gz_1 = \frac{p_2}{\rho_2} + \frac{v_2^2}{2} + gz_2$$

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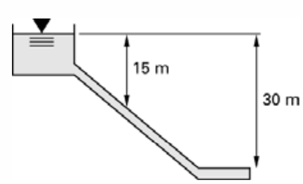
Fluid Mechanics 9-3e

Fluid Dynamics

Example (FEIM):
A pipe draws water from a reservoir and discharges it freely 30 m below the surface. The flow is frictionless. What is the total specific energy at an elevation of 15 m below the surface? What is the velocity at the discharge?



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Fluid Mechanics 9-3f

Fluid Dynamics

Let the discharge level be defined as z = 0, so the energy is entirely potential energy at the surface.

$$E_{\text{surface}} = z_{\text{surface}}\, g = (30\text{ m})\left(9.81\frac{\text{m}}{\text{s}^2}\right) = 294.3\text{ J/kg}$$

(Note that m^2/s^2 is equivalent to J/kg.)

The specific energy must be the same 15 m below the surface as at the surface.

$E_{15\text{ m}} = E_{\text{surface}} = 294.3\text{ J/kg}$

The energy at discharge is entirely kinetic, so

$$E_{\text{discharge}} = 0 + 0 + \tfrac{1}{2}\text{v}^2$$

$$\text{v} = \sqrt{(2)\left(294.3\frac{\text{J}}{\text{kg}}\right)} = 24.3\text{ m/s}$$

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Fluid Mechanics 9-3g

Fluid Dynamics

Flow of a Real Fluid

- Bernoulli equation + head loss due to friction

$$\frac{p_1 g_c}{\rho g} + \frac{\text{v}_1^2}{2g} + z_1 = \frac{p_2 g_c}{\rho g} + \frac{\text{v}_2^2}{2g} + z_2 + h_f$$

$$\frac{p_1}{\gamma} + \frac{\text{v}_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{\text{v}_2^2}{2g} + z_2 + h_f \qquad \text{[U.S.]} \quad 24.12(b)$$

$$h_f = \frac{(p_1 - p_2)g_c}{\rho g}$$

$$= \frac{p_1 - p_2}{\gamma} \qquad \text{[U.S.]} \quad 24.13(b)$$

(h_f is the head loss due to friction)

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Fluid Mechanics 9-3h

Fluid Dynamics

Fluid Flow Distribution

If the flow is laminar (no turbulence) and the pipe is circular, then the velocity distribution is:

$$v_r = v_{max}\left(1 - \left(\frac{r}{R}\right)^2\right) \qquad 24.20$$

r = the distance from the center of the pipe
v = the velocity at r
R = the radius of the pipe
v_{max} = the velocity at the center of the pipe

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Fluid Mechanics 9-3i

Fluid Dynamics

Reynolds Number
For a Newtonian fluid:

$$\mathrm{Re} = \frac{vD\rho}{\mu} \qquad \text{[SI]} \quad 24.14(a)$$

$$\mathrm{Re} = \frac{vD}{\nu} \qquad 24.15$$

D = hydraulic diameter = $4R_H$
ν = kinematic viscosity
μ = dynamic viscosity

For a pseudoplastic or dilatant fluid:

$$\mathrm{Re}' = \frac{v^{2-n}D^n\rho}{K\left(\frac{3n+1}{4n}\right)^n 8^{n-1}} \qquad 24.16$$

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Fluid Mechanics 9-3j

Fluid Dynamics

Example (FEIM):
What is the Reynolds number for water flowing through an open channel 2 m wide when the flow is 1 m deep? The flow rate is 800 L/s. The kinematic viscosity is 1.23×10^{-6} m²/s.

$$D = 4R_H = 4\frac{A}{p} = \frac{(4)(1\text{ m})(2\text{ m})}{2\text{ m} + 1\text{ m} + 1\text{ m}} = 2\text{ m}$$

$$v = \frac{Q}{A} = \frac{800\,\frac{\text{L}}{\text{s}}}{2\text{ m}^2} = 0.4\text{ m/s}$$

$$\text{Re} = \frac{vD}{\nu} = \frac{\left(0.4\,\frac{\text{m}}{\text{s}}\right)(2\text{ m})}{1.23 \times 10^{-6}\,\frac{\text{m}^2}{\text{s}}} = 6.5 \times 10^5$$

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Fluid Mechanics 9-3k

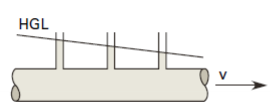
Fluid Dynamics

Hydraulic Gradient

- The decrease in pressure head per unit length of pipe

$$\dot{m} = \rho A v = \rho Q \qquad 24.2$$

Figure 24.2 Hydraulic Grade Line in a Horizontal Pipe



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Fluid Mechanics 9-4a

Head Loss in Conduits and Pipes

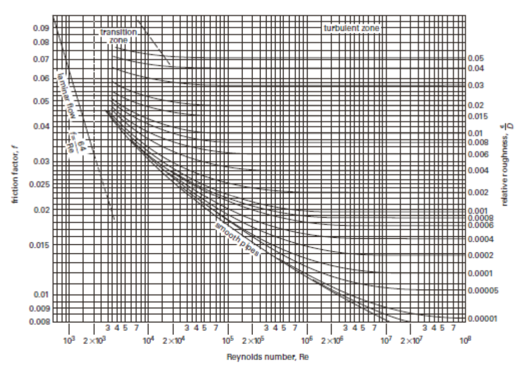
Darcy Equation

- calculates friction head loss

$$h_f = \frac{fLv^2}{2Dg} \qquad 24.24$$

Moody (Stanton) Diagram:

Figure 24.6 Moody Friction Factor Chart



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Fluid Mechanics 9-4b

Head Loss in Conduits and Pipes

Minor Losses in Fittings, Contractions, and Expansions

- Bernoulli equation + loss due to fittings in the line and contractions or expansions in the flow area

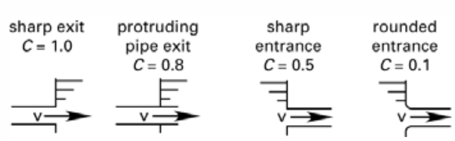
$$\frac{p_1 g_c}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2 g_c}{\rho g} + \frac{v_2^2}{2g} + z_2 + h_f + h_{L,\text{fitting}}$$

$$\frac{p_1}{\gamma} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{v_2^2}{2g} + z_2 + h_f + h_{L,\text{fitting}} \qquad \text{[U.S.]} \quad 24.30(b)$$

$$h_{L,\text{fitting}} = C\left(\frac{v^2}{2g}\right) \qquad 24.31$$

Entrance and Exit Losses

- When entering or exiting a pipe, there will be pressure head loss described by the following loss coefficients:



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Fluid Mechanics 9-5

Pump Power Equation

$$P = \dot{W} = \frac{Q\gamma h}{\eta}$$

$$= \frac{Q\rho g h}{\eta} \qquad 25.1$$

$$\dot{W} = \frac{\dot{m} g h}{\eta} \qquad 25.2$$

Fluid Mechanics 9-6a

Impulse-Momentum Principle

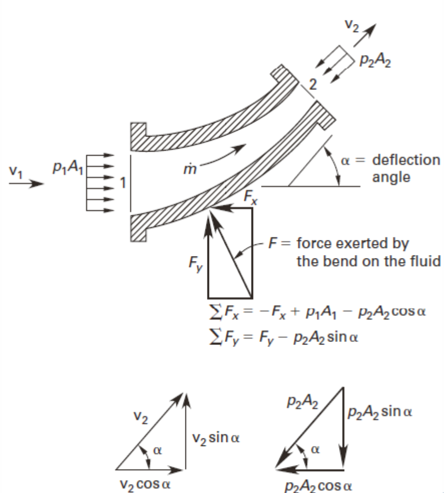
$$\sum \mathbf{F} = Q_2\rho_2\mathbf{v}_2 - Q_1\rho_1\mathbf{v}_1 \qquad \text{[SI]} \quad 24.45(a)$$

Pipe Bends, Enlargements, and Contractions

$$-F_x = p_2A_2\cos\alpha - p_1A_1 + Q\rho(\mathrm{v}_2\cos\alpha - \mathrm{v}_1) \qquad \text{[SI]} \quad 24.46(a)$$

$$F_y = (p_2A_2 + Q\rho\mathrm{v}_2)\sin\alpha + m_{\text{fluid}}g \qquad \text{[SI]} \quad 24.47(a)$$

Figure 24.8 *Forces on a Pipe Bend*



Fluid Mechanics 9-6b

Impulse-Momentum Principle

Example (FEIM):

Water at 15.5˚C, 275 kPa, and 997 kg/m³ enters a 0.3 m × 0.2 m reducing elbow at 3 m/s and is turned through 30˚. The elevation of the water is increased by 1 m. What is the resultant force exerted on the water by the elbow? Ignore the weight of the water.

$$r_1 = \frac{0.3\text{ m}}{2} = 0.15\text{ m}$$

$$r_2 = \frac{0.2\text{ m}}{2} = 0.10\text{ m}$$

$$A_1 = \pi r_1^2 = \pi(0.15\text{ m})^2 = 0.0707\text{ m}^2$$

$$A_2 = \pi r_2^2 = \pi(0.10\text{ m})^2 = 0.0314\text{ m}^2$$

By the continuity equation:

$$\text{v}_2 = \frac{\text{v}_1 A_1}{A_2} = \frac{\left(3\frac{\text{m}}{\text{s}}\right)(0.0707\text{ m}^2)}{0.0314\text{ m}^2} = 6.75\text{ m/s}$$

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Fluid Mechanics 9-6c

Impulse-Momentum Principle

Use the Bernoulli equation to calculate ρ_2:

$$p_2 = \rho\left(-\frac{\text{v}_2^2}{2} + \frac{p_1}{\rho} + \frac{\text{v}_1^2}{2} + g(z_1 - z_2)\right)$$

$$= \left(997\frac{\text{kg}}{\text{m}^3}\right)\left(-\frac{\left(6.75\frac{\text{m}}{\text{s}}\right)^2}{2} + \frac{275000\text{ Pa}}{997\frac{\text{kg}}{\text{m}^3}} + \frac{\left(3\frac{\text{m}}{\text{s}}\right)^2}{2} + \left(9.8\frac{\text{m}}{\text{s}^2}\right)(0\text{ m} - 1\text{ m})\right)$$

$$= 247000\text{ Pa} \quad (247\text{ kPa})$$

$$Q = \text{v}A$$

$$F_x = -Q\rho(\text{v}_2\cos\alpha - \text{v}_1) + P_1A_1 + P_2A_2\cos\alpha$$

$$= -(3)(0.0707)\left(997\frac{\text{kg}}{\text{m}^3}\right)\left(\left(6.75\frac{\text{m}}{\text{s}}\right)\cos 30° - 3\frac{\text{m}}{\text{s}}\right) + (275\times10^3\text{ Pa})(0.0707)$$

$$+ (247\times10^3\text{ Pa})(0.0314\text{ m}^2)\cos 30°$$

$$= 256\times10^4\text{ N}$$

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Fluid Mechanics 9-6d

Impulse-Momentum Principle

$$F_y = Qp(v_2 \sin\alpha - 0) + P_2A_2 \sin\alpha$$
$$= (3)(0.0707)\left(997\,\frac{\text{kg}}{\text{m}^3}\right)\left(\left(6.75\,\frac{\text{m}}{\text{s}}\right)\sin 30°\right)$$
$$+(247\times10^3\ \text{Pa})(0.0314\ \text{m}^2)\sin 30°$$
$$= 4592\times10^4\ \text{N}$$

$$R = \sqrt{F_x^2+F_y^2} = \sqrt{(25\,600\ \text{kN})^2+(4592\ \text{kN})^2} = 26\,008\ \text{kN}$$

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Fluid Mechanics 9-7a

Impulse-Momentum Principle

Initial Jet Velocity:

$$v = \sqrt{2gh} \qquad 24.48$$

Jet Propulsion:

$$\begin{aligned} F &= \dot{m}(v_2 - v_1) \\ &= \dot{m}(v_2 - 0) \\ &= Q\rho v_2 \\ &= v_2A_2\rho v_2 \\ &= A_2\rho v_2^2 \\ &= A_2\rho(\sqrt{2gh})^2 \\ &= 2g\rho hA_2 \\ &= 2\gamma hA_2 \qquad 24.49 \end{aligned}$$

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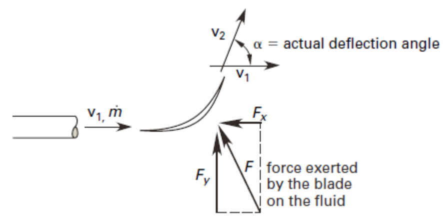
Fluid Mechanics 9-7b

Impulse-Momentum Principle

Fixed Blades

Figure 24.10 Open Jet on a Stationary Blade



$$-F_x = Q\rho(v_2 \cos\alpha - v_1) \quad \text{[SI]} \quad 24.50(a)$$

$$F_y = Q\rho(v_1 - v)\sin\alpha \quad \text{[SI]} \quad 24.53(a)$$

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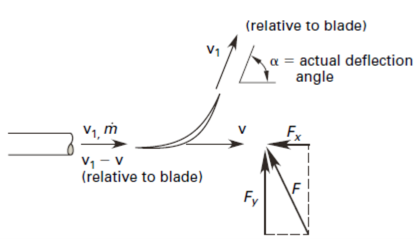
Fluid Mechanics 9-7c

Impulse-Momentum Principle

Moving Blades

Figure 24.11 Open Jet on a Moving Blade



$$-F_x = -Q\rho(v_1 - v)(1 - \cos\alpha) \quad \text{[SI]} \quad 24.52(a)$$

$$F_y = Q\rho(v_1 - v)\sin\alpha \quad \text{[SI]} \quad 24.53(a)$$

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Fluid Mechanics 9-7d

Impulse-Momentum Principle

Impulse Turbine

Figure 24.12 *Impulse Turbine*

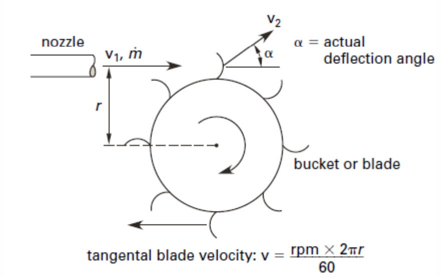
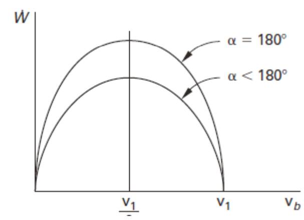


Figure 24.13 *Turbine Power*



$$W = Q\rho(v_1 - v)(1 - \cos\alpha)v \qquad \text{[SI]} \qquad 24.54(a)$$

The maximum power possible is the kinetic energy in the flow.

$$\dot{W}_{max} = \frac{Q\rho v_1^2}{2} = \frac{Q\gamma v_1^2}{2g} \qquad \text{[SI]} \qquad 24.56(a)$$

$$\dot{W}_{max} = \frac{Q\gamma v_1^2}{2g} \qquad \text{[U.S.]} \qquad 24.56(b)$$

The maximum power transferred to the turbine is the component in the direction of the flow.

$$\dot{W}_{max} = Q\rho\left(\frac{v_1^2}{4}\right)(1 - \cos\alpha) \qquad \text{[SI]} \qquad 24.55(a)$$

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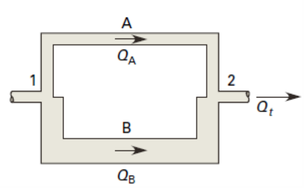
Fluid Mechanics 9-8

Multipath Pipelines

Figure 24.7 *Parallel Pipe Loop System*



- Mass must be conserved.

$$D^2 v = D_A^2 v_A + D_B^2 v_B$$

1) The flow divides as to make the head loss in each branch the same.

$$h_{f,A} = h_{f,B} \qquad 24.32$$

$$\frac{f_A L_A v_A^2}{2D_A g} = \frac{f_B L_B v_B^2}{2D_B g} \qquad 24.33$$

2) The head loss between the two junctions is the same as the head loss in each branch.

$$h_{f,1-2} = h_{f,A} = h_{f,B} \qquad 24.34$$

3) The total flow rate is the sum of the flow rate in the two branches.

$$\frac{\pi}{4}D_1^2 v_1 = \frac{\pi}{4}D_A^2 v_A + \frac{\pi}{4}D_B^2 v_B = \frac{\pi}{4}D_2^2 v_2 \qquad 24.36$$

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Fluid Mechanics 9-9

Speed of Sound

In an ideal gas: $c = \sqrt{kRT}$ [SI] *26.48(a)*

Mach number: $\mathrm{M} = \frac{\mathrm{v}}{c}$ *26.49*

Example (FEIM):
What is the speed of sound in air at a temperature of 339K?
The heat capacity ratio is k = 1.4.

$$c = \sqrt{kRT} = \sqrt{(1.4)\left(286.7\,\frac{\mathrm{m}^2}{\mathrm{s}^2\cdot\mathrm{K}}\right)(339\mathrm{K})} = 369\ \mathrm{m/s}$$

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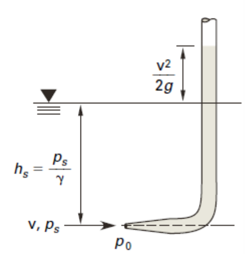
Fluid Mechanics 9-10a

Fluid Measurements

Pitot Tube – measures flow velocity

Figure 25.1 *Pitot Tube*



- The static pressure of the fluid at the depth of the pitot tube (p_0) must be known. For incompressible fluids and compressible fluids with M ≤ 0.3,

$$\mathrm{v} = \sqrt{\frac{2(p_0 - p_s)}{\rho}} = \sqrt{\frac{2g(p_0 - p_s)}{\gamma}}$$ [SI] *25.12(a)*

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Fluid Mechanics 9-10b

Fluid Measurements

Example (FEIM):
Air has a static pressure of 68.95 kPa and a density 1.2 kg/m^3. A pitot tube indicates 0.52 m of mercury. Losses are insignificant. What is the velocity of the flow?

$$p_0 = \rho_{\text{mercury}} gh = \left(13560\ \frac{\text{kg}}{\text{m}^3}\right)\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(0.52\ \text{m}) = 69380\ \text{Pa}$$

$$\text{v} = \sqrt{\frac{2(p_0 - p_s)}{\rho}} = \sqrt{\frac{(2)(69380\ \text{Pa} - 68950\ \text{Pa})}{1.2\ \frac{\text{kg}}{\text{m}^3}}} = 26.8\ \text{m/s}$$

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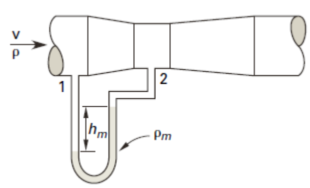
Fluid Mechanics 9-10c

Fluid Measurements

Venturi Meters – measures the flow rate in a pipe system

- The changes in pressure and elevation determine the flow rate. In this diagram, $z_1 = z_2$, so there is no change in height.

Figure 25.2 *Venturi Meter with Differential Manometer*



$$Q = \left(\frac{C_\text{v} A_2}{\sqrt{1 - \left(\frac{A_2}{A_1}\right)^2}}\right)\sqrt{2\left(\frac{p_1 g_c}{\rho} + g z_1 - \frac{p_2 g_c}{\rho} - g z_2\right)}$$

$$= \left(\frac{C_\text{v} A_2}{\sqrt{1 - \left(\frac{A_2}{A_1}\right)^2}}\right)\sqrt{2g\left(\frac{p_1}{\gamma} + z_1 - \frac{p_2}{\gamma} - z_2\right)}$$

[U.S.] *25.15(b)*

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Fluid Mechanics 9-10d

Fluid Measurements

Example (FEIM):
Pressure gauges in a venturi meter read 200 kPa at a 0.3 m diameter and 150 kPa at a 0.1 m diameter. What is the mass flow rate? There is no change in elevation through the venturi meter.
Assume $C_v = 1$ and $\rho = 1000\ \text{kg/m}^3$.

(A) 52 kg/s

(B) 61 kg/s

(C) 65 kg/s

(D) 79 kg/s

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Fluid Mechanics 9-10e

Fluid Measurements

$$Q = \left(\frac{C_v A_2}{\sqrt{1 - \left(\frac{A_2}{A_1} \right)^2}} \right) \sqrt{2g\left(\frac{p_1}{\gamma} + z_1 - \frac{p_2}{\gamma} - z_2 \right)}$$

$$= \left(\frac{\pi (0.05\ \text{m}^2)^2}{\sqrt{1 - \left(\frac{0.05}{0.15} \right)^2}} \right) \sqrt{2\left(\frac{200000\ \text{Pa} - 150000\ \text{Pa}}{1000\ \frac{\text{kg}}{\text{m}^3}} \right)} = 0.079\ \text{m}^3/\text{s}$$

$$\dot{m} = \rho Q = \left(1000\ \frac{\text{kg}}{\text{m}^3}\right)\left(0.079\ \frac{\text{m}^3}{\text{s}}\right) = 79\ \text{kg/s}$$

Therefore, (D) is correct.

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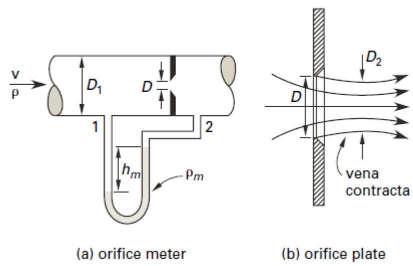
Fluid Mechanics 9-10f

Fluid Measurements

Orifices

Figure 25.3 Orifice Meter with Differential Manometer



$$Q = CA\sqrt{2\left(\frac{p_1 g_c}{\rho} + gz_1 - \frac{p_2 g_c}{\rho} - gz_2\right)}$$

$$= CA\sqrt{2g\left(\frac{p_1}{\gamma} + z_1 - \frac{p_2}{\gamma} - z_2\right)} \quad \text{[U.S.]} \quad 25.18(b)$$

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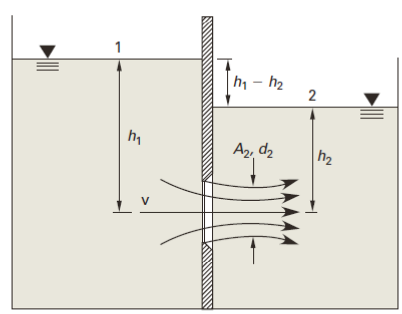
Fluid Mechanics 9-10g

Fluid Measurements

Submerged Orifice

Figure 25.4 Submerged Orifice



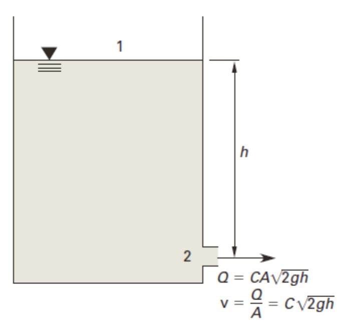
$$Q = A_2 v_2 = C_c C_v A\sqrt{2g(h_1 - h_2)} \quad 25.19$$

$$C = C_c C_v \quad 25.20$$

and C_c = coefficient of contraction

Orifice Discharging Freely into Atmosphere

Figure 25.5 Orifice Discharging Freely into the Atmosphere



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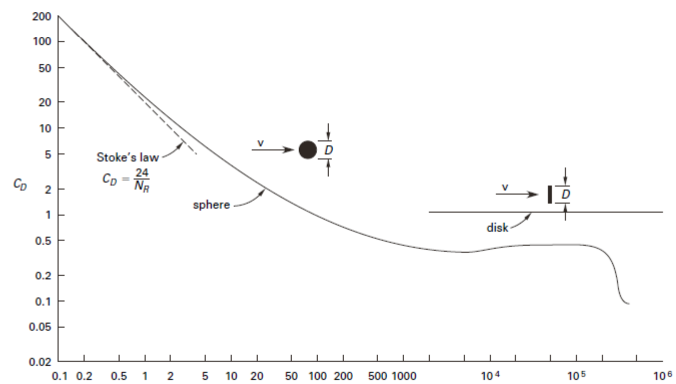
Fluid Mechanics 9-10h

Fluid Measurements

Drag Coefficients for Spheres and Circular Flat Disks

Figure 24.14 *Drag Coefficients for Spheres and Circular Flat Disks*



$$F_D = \frac{C_D A \rho v^2}{2} \qquad \text{[SI]} \qquad 24.57(a)$$

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