FE Review Course

Mechanics/Strength of Materials

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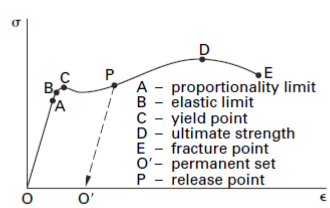
Mechanics/Strength of Materials 13-1

Stress-Strain Curve for Mild Steel

Figure 41.3 Typical Stress-Strain Curve for Steel



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Definitions

- Hooke's Law $\sigma = E\epsilon$ 18.5
- Shear Modulus: $\tau = G\gamma$ 18.7

  $G = \dfrac{E}{2(1+\nu)}$ 18.8
- Stress: $\sigma = \dfrac{P_{\text{normal to area}}}{A}$ 18.1
- Strain: $\nu = -\dfrac{\epsilon_{\text{lateral}}}{\epsilon_{\text{axial}}}$ 18.6
- Poisson's Ratio:
- Normal stress or strain = ⊥ to the surface
- Shear stress = || to the surface

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Mechanics/Strength of Materials 13-2b

Definitions

Uniaxial Load and Deformation

$$\sigma = \frac{P_{\text{normal to area}}}{A} \quad 18.1$$

$$\epsilon = \frac{\delta}{L} \quad 18.3$$

$$\delta = L\epsilon = L\left(\frac{\sigma}{E}\right) = \frac{PL}{AE} \quad 18.10$$

Thermal Deformation

$$\delta_{\text{th}} = \alpha L(T - T_0) \quad 19.1$$

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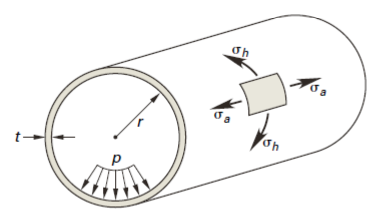
Stress and Strain

Thin-Walled Tanks

Figure 19.2 Stresses in a Thin-Walled Tank



Hoop Stress:

$$\sigma_t = \frac{pD}{2t} = \frac{pr}{t} \qquad 19.5$$

Axial Stress:

$$\sigma_a = \frac{pD}{4t} = \frac{pr}{2t} = \frac{\sigma_t}{2} \qquad 19.6$$

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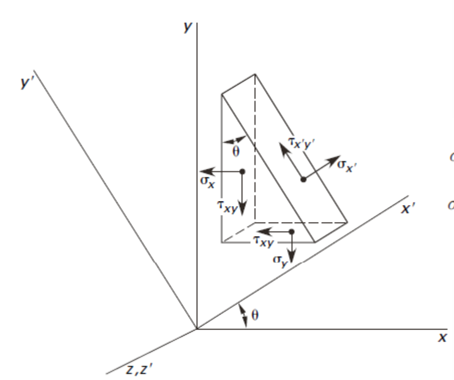
Stress and Strain

Transformation of Axes

Figure 18.3 Transformation of Axes



$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta + \tau_{xy}\sin 2\theta \qquad 18.21$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta - \tau_{xy}\sin 2\theta \qquad 18.22$$

$$\tau_{x'y'} = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta \qquad 18.23$$

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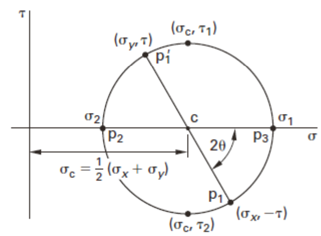
Stress and Strain

Figure 18.4 Mohr's Circle for Stress



Five simplified steps to construct Mohr's circle

1. Determine the applied stresses (σ_x, σ_y, τ_{xy}).
2. Draw a set of σ - τ axes.
3. Locate the center: $\sigma_c = \frac{1}{2}(\sigma_x + \sigma_y)$.
4. Find the radius (or τ_{max}): $r = \sqrt{\frac{1}{4}(\sigma_x - \sigma_y)^2 + \tau_{xy}^2}$ 18.28
5. Draw Mohr's circle.

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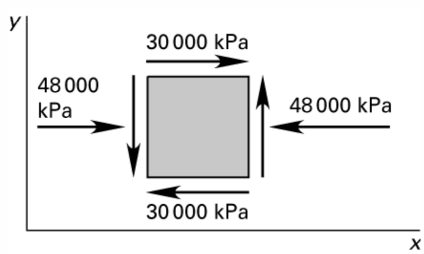
Stress and Strain

For examples 1 and 2, use the following illustration.



Example 1 (FEIM)

The principal stresses (σ_2, σ_1) are most nearly

(A) –62 400 kPa and 14 400 kPa
(B) 84 000 kPa and 28 000 kPa
(C) 70 000 kPa and 14 000 kPa
(D) 112 000 kPa and –28 000 kPa

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Stress and Strain

The center of Mohr's circle is at

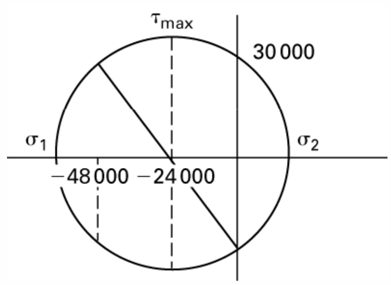
$$\sigma_c = \tfrac{1}{2}(\sigma_x + \sigma_y) = \tfrac{1}{2}(-48000 \text{ kPa} + 0) = -24000 \text{ kPa}$$

Using the Pythagorean theorem, the radius of Mohr's circle (τ_{max}) is:

$$\tau_{max} = \sqrt{(30000 \text{ kPa})^2 + (24000 \text{ kPa})^2} = 38419 \text{ kPa}$$

$$\sigma_1 = \sigma_c - \tau_{max} = (-24000 \text{ kPa} - 38419 \text{ kPa}) = -62419 \text{ kPa}$$

$$\sigma_2 = \sigma_c + \tau_{max} = (-24000 \text{ kPa} + 38419 \text{ kPa}) = 14418 \text{ kPa}$$



Therefore, (D) is correct.

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Stress and Strain

Example 2 (FEIM):

The maximum shear stress is most nearly

(A) 24 000 kPa
(B) 33 500 kPa
(C) 38 400 kPa
(D) 218 000 kPa

In the previous example problem, the radius of Mohr's circle (τ_{max}) was

$$\tau_{max} = \sqrt{(30000 \text{ kPa})^2 + (24000 \text{ kPa})^2}$$

$$= 38419 \text{ kPa} \quad (38400 \text{ kPa})$$

Therefore, (C) is correct.

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Stress and Strain

General Strain

$$\epsilon_x = \frac{1}{E}\left(\sigma_x - \nu(\sigma_y + \sigma_z)\right) \qquad 18.29$$

$$\epsilon_y = \frac{1}{E}\left(\sigma_y - \nu(\sigma_z + \sigma_x)\right) \qquad 18.30$$

$$\epsilon_z = \frac{1}{E}\left(\sigma_z - \nu(\sigma_x + \sigma_y)\right) \qquad 18.31$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G} \qquad 18.32$$

$$\gamma_{yz} = \frac{\tau_{yz}}{G} \qquad 18.33$$

$$\gamma_{zx} = \frac{\tau_{zx}}{G} \qquad 18.34$$

Note that σ_x is no longer proportional to ε_x.

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Mechanics/Strength of Materials 13-3h

Stress and Strain

Static Loading Failure Theory

Maximum Normal Stress: A material fails if

- $\sigma \geq S_t$

Or

- $\sigma \leq S_c$

This is true of brittle materials.
For ductile materials:

Maximum Shear

$$T_{\max} = \max\left(\frac{|\sigma_1 - \sigma_2|}{2}, \frac{|\sigma_1 - \sigma_3|}{2}, \frac{|\sigma_2 - \sigma_3|}{2}\right) > \frac{S_{yt}}{2}$$

Distortion Energy (von Mises Stress)

$$\sigma' = \sqrt{\tfrac{1}{2}\left((\sigma_1 - \sigma_2)^2 + (\sigma_1 - \sigma_3)^2 + (\sigma_2 - \sigma_3)^2\right)} > S_{yt}$$

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Stress and Strain

Torsion

- For a body with radius *r* being strained to an angle), the shear strain and stress are:

$$\gamma = r\frac{d\phi}{dz} \qquad \tau = G\gamma = Gr\frac{d\phi}{dz}$$

- For a body with polar moment of inertia (*J*), the torque (*T*) is:

$$T = G\frac{d\phi}{dz}\int_A r^2 dA = GJ\frac{d\phi}{dz}$$

The shear stress is:

$$\tau_{\phi z} = Gr\frac{T}{GJ} = \frac{Tr}{J}$$

- For a body, the general angular displacement (ϕ) is:

$$\phi = \int_0^L \frac{T}{GJ}dz$$

- For a shaft of length (*L*), the total angular displacement (ϕ) is:

$$\phi = \frac{TL}{GJ} \quad \text{[radians]} \qquad 19.16$$

Torsional stiffness:

$$k = \frac{T}{\phi} = \frac{GJ}{L} \qquad 19.17$$

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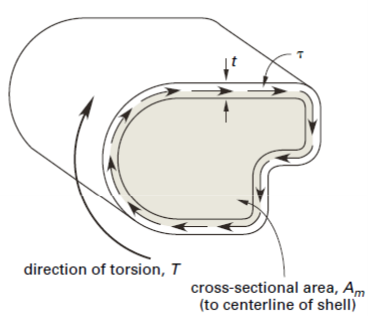
Mechanics/Strength of Materials 13-3j

Stress and Strain

Hollow, Thin-Walled Shafts

$$J = \frac{\pi}{2}(r_o^4 - r_i^4) = \frac{\pi}{32}(D_o^4 - D_i^4) \qquad 19.15$$

Figure 19.3 *Torsion in Thin-Walled Shells*



$$\tau = \frac{T}{2A_m t} \qquad 19.18$$

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Mechanics/Strength of Materials 13-4a

Beams

Figure 20.2 Bending Moment Sign Conventions

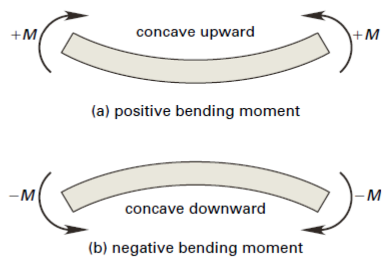
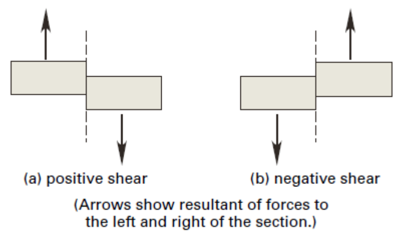


Figure 20.1 Shear Sign Conventions



Mechanics/Strength of Materials 13-4b

Beams

Load, Shear, and Moment Relations

Load: $$w(x) = -\frac{dV(x)}{dx} \quad \text{20.4}$$

Shear: $$V(x) = \frac{dM(x)}{dx} \quad \text{20.6}$$

$$V_2 - V_1 = \int_{x_1}^{x_2} w(x)\,dx \quad \text{20.3}$$

$$M_2 - M_1 = \int_{x_1}^{x_2} V(x)\,dx \quad \text{20.5}$$

For a beam deflected to a radius of curvature (ρ), the axial strain at a distance (y) from the neutral axis is $\varepsilon_x = -y/\rho$.

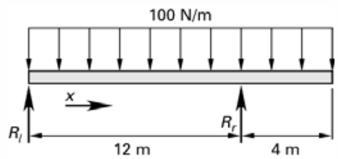
Mechanics/Strength of Materials 13-4c

Beams

Shear and Bending Moment Diagrams

Example 1 (FEIM):

Draw the shear and bending moment diagrams for the following beam.



Mechanics/Strength of Materials 13-4d

Beams

$$R_l + R_r = \left(100\,\frac{\text{N}}{\text{m}}\right)(16\ \text{m}) = 1600\ \text{N}$$

$$R_l = (8) - R_r(4) = 0$$

Therefore, R_l = 533.3 N and R_r = 1066.7 N

From 0 m to 12 m, $V = R_l - \left(100\,\frac{\text{N}}{\text{m}}\right)x = 533.3\ \text{N} - \left(100\,\frac{\text{N}}{\text{m}}\right)x;\ 0\ \text{m} < x < 12\ \text{m}$

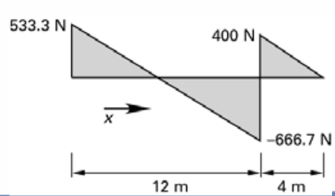
Shear is undefined at concentrated force points, but just short of x = 12 m

$$V(12^-) = 533.3\ \text{N} - \left(100\,\frac{\text{N}}{\text{m}}\right)(12\ \text{m}) = -666.7\ \text{N}$$

From 12 m to 16 m, $V = V(12^-) + R_r - (100\ \text{N})(x - 12)$

$$V = 1600\ \text{N} - \left(100\,\frac{\text{N}}{\text{m}}\right)x;\ 12 < x \le 16\ \text{m}$$

So the shear diagram is:



Mechanics/Strength of Materials 13-4e

Beams

The bending moment is the integral of the shear.

$M = 533.3x - 50x^2$; $0\text{ m} < x < 12\text{ m}$

$$M = \int_0^{12} S dx + \int_{12}^{x} S dx = -800 + \int_0^{12}\left(1600\text{ N} - 100\frac{\text{N}}{\text{m}}x\right)dx$$

$$= -800\text{ N}\cdot\text{m} + \left((1600\text{ N})x - \left(50\frac{\text{N}}{\text{m}}\right)x^2\right)_{12}^{x}$$

$$M = -800\text{ N}\cdot\text{m} + (1600\text{ N})x - \left(50\frac{\text{N}}{\text{m}}\right)x^2 - (1600\text{ N})(12\text{ m}) + \left(50\frac{\text{N}}{\text{m}}\right)(12\text{ m})^2$$

$$M = -12800\text{ N}\cdot\text{m} + (1600\text{ N})x - \left(50\frac{\text{N}}{\text{m}}\right)x^2$$

$12\text{ m} < x \le 16\text{ m}$

Or, let the right end of the beam be $x = 0$ m

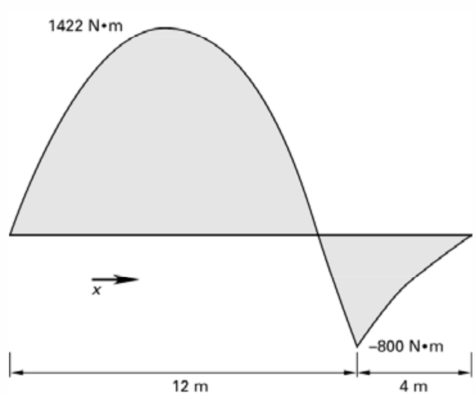
Then, $S = -\left(100\frac{\text{N}}{\text{m}}\right)x;\quad -4\text{ m} < x \le 0\text{ m}$

$$M = \int_x^0 S dx = \int_x^0 -\left(100\frac{\text{N}}{\text{m}}\right)x = -\left(50\frac{\text{N}}{\text{m}}\right)x^2$$

Mechanics/Strength of Materials 13-4f

Beams

The bending moment diagram is:



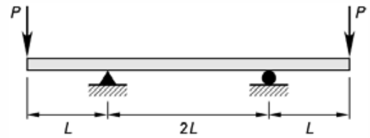
Mechanics/Strength of Materials 13-4g

Beams

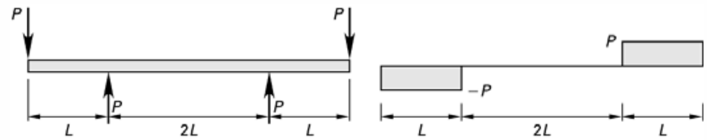
Example 2 (FEIM):

The vertical shear for the section at the midpoint of the beam shown is

(A) 0
(B) $\frac{1}{2}P$
(C) P
(D) none of these



Drawing the force diagram and the shear diagram,



Therefore, (A) is correct.

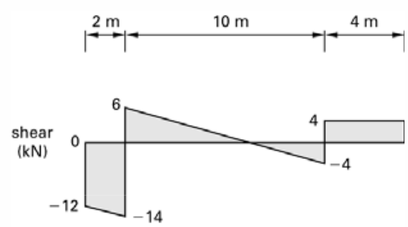
Mechanics/Strength of Materials 13-4h

Beams

Example 3 (FEIM):

For the shear diagram shown, what is the maximum bending moment? The bending moment at the ends is zero, and there are no concentrated couples.

(A) 8 kN·m
(B) 16 kN·m
(C) 18 kN·m
(D) 26 kN·m



Starting from the left end of the beam, areas begin to cancel after 2 m. Starting from the right end of the beam, areas begin to cancel after 4 m. The rectangle on the right has an area of 16 kN·m. The trapezoid on the left has an area of (1/2)(12 kN + 14 kN) (2 m) = 26 kN·m. The trapezoid has the largest bending moment.

Therefore, (A) is correct.

Mechanics/Strength of Materials 13-4i

Beams

Bending Stress

$$\sigma_b = -\frac{My}{I} \qquad 20.7$$

$$\sigma_{b,\max} = \frac{Mc}{I} \qquad 20.8$$

Shear Stress

$$\tau_{xy} = \frac{VQ}{Ib} \qquad 20.13$$

$$Q = A'y' \qquad 20.15$$

Deflection

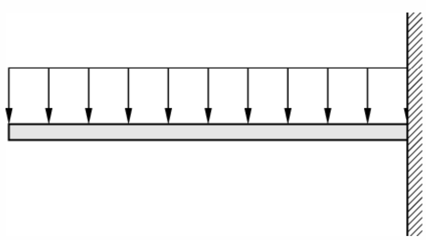
$$EIy = \iint M(x)dx \qquad 20.26$$

Note: Beam deflection formulas are given in the NCEES Handbook for any situation that might be on the exam.

Mechanics/Strength of Materials 13-4j

Beams

Example (FEIM):



Find the tip deflection of the beam shown. *EI* is 3.47 × 106 N·m^2, the load is 11 379 N/m, and the beam is 3.7 m long.

From the NCEES Handbook:

$$\delta = \frac{w_o L^4}{8EI} = \frac{\left(11379\,\frac{\text{N}}{\text{m}}\right)(3.7\text{ m})^4}{(8)\left(3.47\times10^6\text{ N}\cdot\text{m}^2\right)} = 0.077\text{ m}$$

Mechanics/Strength of Materials 13-5a

Columns

Beam-Columns (Axially Loaded Beams)

Maximum and minimum stresses in an eccentrically loaded column:

$$\sigma_{\text{max,min}} = \frac{F}{A} \pm \frac{Mc}{I} = \frac{F}{A} \pm \frac{Fec}{I} \qquad 21.1$$

Mechanics/Strength of Materials 13-5b

Columns

Euler's Formula

Critical load that causes a long column to buckle:

$$P_{\text{cr}} = \frac{\pi^2 EI}{(Kl)^2} \qquad 21.5$$

r = the radius of gyration
k = the end-resistant coefficient
kl = the effective length
$\frac{l}{r}$ = slenderness ratio

Mechanics/Strength of Materials 13-5c

Columns

Elastic Strain Energy:

$$U = \tfrac{1}{2}P\delta = \frac{P^2 L}{2AE} \qquad 18.16$$

Strain energy per unit volume for tension:

$$u = \frac{U}{AL} = \frac{\sigma^2}{2E} \qquad 18.17$$

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