FE Review Course

Alternating Current Electricity

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Alternating Current Electricity 15-1

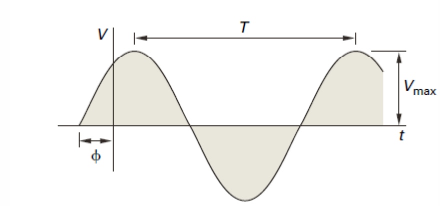
Frequency and Period

Frequency (linear and angular)

$$f = \frac{1}{T} = \frac{\omega}{2\pi} \qquad 45.4$$

Period = *T*

Figure 45.1 *Sinusoidal Waveform with Phase Angle*



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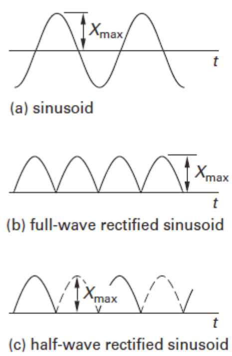
Average Value

$$X_{ave} = \frac{1}{T}\int_0^T x(t)\,dt \qquad 45.9$$

Figure 45.2 Average and Effective Values



(a) $X_{ave} = 0$, $X_{eff} = \sqrt{\frac{1}{T}\int_0^T x^2(t)\,dt}$

(b) $X_{ave} = \frac{2X_{max}}{\pi}$, $X_{eff} = \frac{X_{max}}{2}$

(c) $X_{ave} = \frac{X_{max}}{\pi}$, $X_{eff} = \frac{X_{max}}{2}$

Square wave,
positive pulse = negative pulse: $X_{ave} = 0$

Pulse pattern, positive, all the same:

$$X_{ave} = \frac{tX_{max}}{T}$$

where t is the duration of the pulse and T is the period.

Sawtooth: $X_{ave} = \frac{1}{2}X_{max}$

Symmetrical triangular wave: $X_{ave} = 0$

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Alternating Current Electricity 15-2b

Average Value

Example (FEIM):

A plating tank with an effective resistance of 100 Ω is connected to the output of a full-wave rectifier. The applied voltage is sinusoidal with a maximum of 170 V. How long does it take to transfer 0.005 faradays?

$$V_{ave} = \frac{2V_{max}}{\pi} = \frac{(2)(170\text{ V})}{\pi} = 108.2\text{ V}$$

$$I_{ave} = \frac{V_{ave}}{R} = \frac{108.2\text{ V}}{100\,\Omega} = 1.082\text{ A}$$

$$t = \frac{q}{I_{ave}} = \frac{(0.005\text{ F})\left(96{,}487\,\frac{\text{A}\cdot\text{s}}{\text{F}}\right)}{1.082\text{ A}}$$

$$= 446\text{ s}$$

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Effective or Root-Mean-Squared (RMS) Value

Effective value of an alternating waveform:

$$X_{\text{eff}} = X_{\text{rms}} = \sqrt{\frac{1}{T}\int_0^T x^2(t)\,dt} \quad \textit{45.12}$$

For a sinusoidal waveform,

$$X_{\text{eff}} = X_{\text{rms}} = \frac{X_{\text{max}}}{\sqrt{2}} \quad \textit{45.13}$$

For a half-wave sinusoidal waveform,

$$X_{\text{eff}} = X_{\text{rms}} = \frac{X_{\text{max}}}{2} \quad \textit{45.14}$$

Pulse pattern, positive, all the same:

$$X_{\text{rms}} = \sqrt{\frac{t}{T}}\,X_{\text{max}}$$

where t is the duration of the pulse and T is the period.

Symmetrical triangular: $X_{\text{rms}} = \frac{1}{\sqrt{3}} X_{\text{max}}$

Sawtooth: $X_{\text{rms}} = \frac{1}{\sqrt{3}} X_{\text{max}}$

Alternating Current Electricity 15-3b

Effective or Root-Mean-Squared (RMS) Value

Example 1 (FEIM):

A 170 V (maximum value) sinusoidal voltage is connected across a 4 Ω resistor. What is the power dissipated by the resistor?

$$V_{\text{rms}} = \frac{V_{\text{max}}}{\sqrt{2}} = \frac{170\ \text{V}}{\sqrt{2}} = 120.2\ \text{V}$$

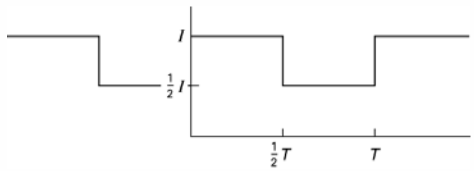
$$P = \frac{V_{\text{rms}}^2}{R} = \frac{(120.2\ \text{V})^2}{4\ \Omega} = 3.61\times 10^3\ \text{W}$$

Alternating Current Electricity 15-3c

Effective or Root-Mean-Squared (RMS) Value

Example 2 (FEIM):

What is the I_{rms} value for the following waveform?



(A) $\dfrac{\sqrt{2}}{4}I$

(B) $\dfrac{\sqrt{3}}{4}I$

(C) $\dfrac{\sqrt{10}}{4}I$

(D) $\dfrac{\sqrt{3}}{2}I$

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Alternating Current Electricity 15-3d

Effective or Root-Mean-Squared (RMS) Value

$$I_{\text{rms}} = \sqrt{\frac{1}{T}\int_0^T (I(t))^2} = \sqrt{\frac{1}{T}\int_0^{\frac{T}{2}} I^2 dt + \frac{1}{T}\int_{\frac{T}{2}}^{T}\left(\frac{I}{2}\right)^2 dt}$$

$$= \sqrt{\frac{I^2}{T}\left(\frac{T}{2}\right) + \left(\frac{1}{T}\right)\left(\frac{I^2}{4}\right)(T)\Big|_{\frac{T}{2}}^{T}} = \sqrt{\frac{I^2}{2} + \left(\frac{I^2}{4T}\right)\left(\frac{T}{2}\right)}$$

$$= \sqrt{\frac{I^2}{2} + \frac{I^2}{8}} = \sqrt{\frac{5}{8}I^2} = \frac{\sqrt{10}}{4}I$$

Therefore, the answer is (C).

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Alternating Current Electricity 15-3e

Effective or Root-Mean-Squared (RMS) Value

Example 3 (FEIM):

A sinusoidal AC voltage with a value of V_{rms} = 60 V is applied to a purely resistive circuit. What steady-state voltage would dissipate the same power as the AC voltage?

(A) 30 V
(B) 42 V
(C) 60 V
(D) 85 V

The answer is right in the problem statement. Since the voltage is given as an rms value, the equivalent DC voltage is simply 60 V.

Therefore, the answer is (C).

Alternating Current Electricity 15-4

Phasor Transforms

Phase angle = $\theta - \phi$

ϕ as the angle between the reference and voltage

θ as the angle between the reference and current

If the phase angle is positive, the signal is called “leading” or “capacitive.”

If the phase angle is negative, the signal is called “lagging” or “inductive.”

If the phase angle is zero, the signal is called “in phase.”

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Alternating Current Electricity 15-5a

Impedance

Definitions

Impedance: $\mathbf{Z} \equiv R \pm jX$ 45.20

Resistor: $\mathbf{Z}_R = R\angle 0^\circ = R + j0 = R$ 45.23

Capacitor: $\mathbf{Z}_C = X_C\angle{-90^\circ} = -jX_C = \frac{1}{j\omega C}$ 45.24

Inductor: $\mathbf{Z}_L = X_L\angle 90^\circ = jX_L = j\omega L$ 45.26

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Alternating Current Electricity 15-5b

Impedance

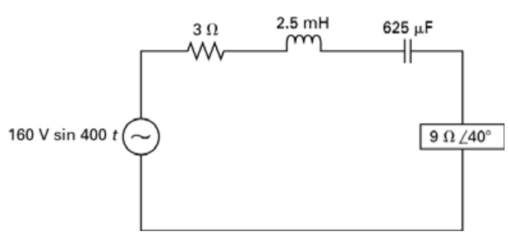
Example (FEIM):

For the following circuit:

(a) What is the impedance in polar form?

(b) Is the current leading or lagging?

(c) What is the voltage across the inductor?



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Alternating Current Electricity 15-5c

Impedance

(a) $$Z = 3\,\Omega + j\left(400\frac{1}{\text{s}}\right)(2.5\times10^{-3}\text{ H}) - j\frac{1}{\left(400\frac{1}{\text{s}}\right)(625\times10^{-3}\text{ F})} + 9\,\Omega\angle 40°$$

$$= 3\,\Omega + j1\Omega - j4\,\Omega + (9\,\Omega)\cos 40° + (j9\,\Omega)\sin 40°$$

$$= 3\,\Omega + 6.894\,\Omega + j(1\Omega - 4\,\Omega + 5.785\,\Omega)$$

$$= 9.894\,\Omega + j2.785\,\Omega$$

$$|Z| = \sqrt{(9.894\Omega)^2 + (2.785\Omega)^2} = 10.28\Omega$$

$$\text{Phase angle} = \tan^{-1}\left(\frac{\text{imaginary}}{\text{real}}\right)$$

$$= \tan^{-1}\left(\frac{2.785\Omega}{9.894\Omega}\right) = 15.72°$$

$$Z = 10.28\Omega\angle 15.72°$$

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Alternating Current Electricity 15-5d

Impedance

(b) $$I = \frac{V}{Z} = \left(\frac{160\text{ V}\angle 0°}{10.28\Omega\angle 15.72°}\right) = 16.56\text{ A}\angle -15.72°$$

Therefore, the current is lagging (ELI).

(c) $$V_L = IZ_L = (15.56\text{ A}\angle -15.72°)(1\Omega\angle 90°)$$

$$= 15.56\text{ V}\angle 74.28°$$

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Alternating Current Electricity 15-6

Admittance

$$\mathbf{Y} = \frac{1}{\mathbf{Z}} = \frac{1}{Z}\angle -\theta \qquad 45.28$$

$$G = \frac{1}{R} \qquad 45.29$$

$$B = \frac{1}{X} \qquad 45.30$$

Multiplying by the complex conjugate,

$$\begin{aligned}\mathbf{Y} &= G + jB \\ &= \left(\frac{1}{R+jX}\right)\left(\frac{R-jX}{R-jX}\right) \\ &= \frac{R}{R^2+X^2} - j\left(\frac{X}{R^2+X^2}\right) \qquad 45.31\end{aligned}$$

$$\begin{aligned}\mathbf{Z} &= R + jX \\ &= \left(\frac{1}{G+jB}\right)\left(\frac{G-jB}{G-jB}\right) \\ &= \frac{G}{G^2+B^2} - j\left(\frac{B}{G^2+B^2}\right) \qquad 45.32\end{aligned}$$

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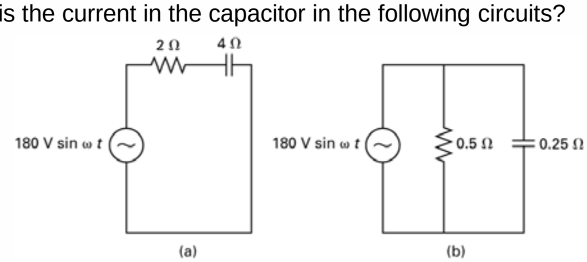
Ohm's Law for AC Circuits

$$\mathbf{V} = \mathbf{IZ} \qquad 45.35$$

Example (FEIM):
What is the current in the capacitor in the following circuits?



(a) $Z = 2\,\Omega - j4\,\Omega = 4.47\,\Omega\angle -63.4°$

$$I = \frac{V}{Z} = \frac{180\text{ V}\angle 0°}{4.47\,\Omega\angle -63.4°} = 40.3\text{ A }\angle 63.4°$$

(b) $Z_c = 0.25\,\Omega\angle -90°$

$$I = \frac{V}{Z_c} = \frac{180\text{ V}\angle 0°}{0.25\,\Omega\angle -90°} = 720\text{ A }\angle 90°$$

Alternating Current Electricity 15-8a

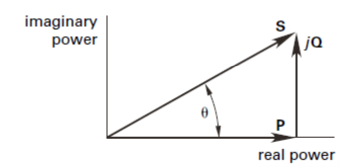
Complex Power

P: real power (W)

Q: reactive power (VAR)

S: apparent power (VA)

Figure 45.5 *Lagging Complex Power Triangle*



- Power Factor:

$$\text{pf} = \cos\theta \qquad 45.40$$

- Real Power:

$$P = \tfrac{1}{2}V_{\max}I_{\max}\cos\theta = V_{\text{rms}}I_{\text{rms}}\cos\theta \qquad 45.38$$

- Reactive Power:

$$Q = \tfrac{1}{2}V_{\max}I_{\max}\sin\theta = V_{\text{rms}}I_{\text{rms}}\sin\theta = \frac{V_{\text{rms}}^2}{X} \qquad 45.39$$

$$\mathbf{S} = \mathbf{VI}^* \qquad 45.37$$

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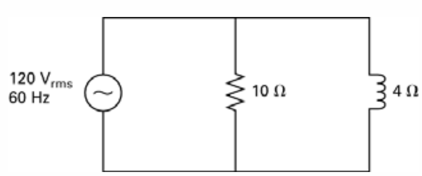
Alternating Current Electricity 15-8b

Complex Power

Example (FEIM):

For the following circuit, find the (a) real power, and (b) reactive power.
(c) Draw the power triangle.



(a) The real power is: $P = \dfrac{V_R^2}{R} = \dfrac{(120\text{ V})^2}{10\ \Omega} = 1440\text{ W}$

(b) The reactive power is: $Q = \dfrac{V_L^2}{X_L} = \dfrac{(120\text{ V})^2}{4\ \Omega} = 3600\text{ VAR}$

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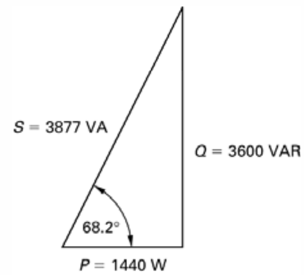
Alternating Current Electricity 15-8c

Complex Power

(c) The power triangle is:



In this example, the impedance of the inductor has a lagging current, so the current has a negative phase angle. The complex conjugate of the current has a positive phase angle, so the reactive power, Q, is positive and the power triangle is in the first quadrant.

For a leading current (which has a positive phase angle compared to the voltage) the power triangle has a negative imaginary part and a negative power angle, so it is in the fourth quadrant.

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Alternating Current Electricity 15-9a

Resonance

$$\omega_0 = \frac{1}{\sqrt{LC}} = 2\pi f_0 \qquad 45.43$$

$$\omega_0 L = \frac{1}{\omega_0 C} \qquad 45.47$$

Series Resonance

Quality Factor: $$Q = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 CR} \qquad 45.52$$

Bandwidth: $$\mathrm{BW} = f_2 - f_1 = \frac{f_0}{Q} \quad [\text{in Hz}]$$

$$= \omega_2 - \omega_1 = \frac{\omega_0}{Q} \quad [\text{in rad/s}] \qquad 45.48$$

Half-power point is when $R = X$: $\omega_{1/2\text{power}} = \omega_0 \pm \frac{1}{2}\mathrm{BW}$

Parallel Resonance

Quality Factor: $$Q = \omega_0 RC = \frac{R}{\omega_0 L} \qquad 45.53$$

Bandwidth: $$\mathrm{BW} = f_2 - f_1 = \frac{f_0}{Q} \quad [\text{in Hz}]$$

$$= \omega_2 - \omega_1 = \frac{\omega_0}{Q} \quad [\text{in rad/s}] \qquad 45.48$$

Half-power point is when $R = X$: $\omega_{1/2\text{power}} = \omega_0 \pm \frac{1}{2}Q$

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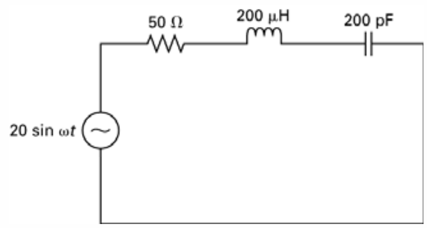
Alternating Current Electricity 15-9b

Resonance

Example (FEIM):

For the following circuit, find
(a) the resonance frequency (rad/s)
(b) the half-power points (rad/s)
(c) the peak current (at resonance)
(d) the peak voltage across each component at resonance



(a) $$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(200\times10^{-6}\text{ H})(200\times10^{-12}\text{ F})}} = 5\times10^{6}\ \frac{\text{rad}}{\text{s}}$$

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Resonance

(b) $$\omega_{\frac{1}{2}\text{power}} = \omega_0 \pm \frac{1}{2}\text{BW}$$
$$= \omega_0 \pm \frac{R}{2L} = 5\times10^{6}\ \frac{\text{rad}}{\text{s}} \pm \frac{50\ \Omega}{(2)(200\times10^{-6}\text{ H})}$$
$$= 5.125\times10^{6},\ 4.875\times10^{6}\ \frac{\text{rad}}{\text{s}}$$

(c) At resonance, $Z = R$, $I(\text{resonance}) = \frac{V}{R} = \frac{20\text{ V}\angle 0°}{50\ \Omega} = 0.4\text{ A}\angle 0°$

(d) $$V_R = I_0 R = (0.4\text{ A}\angle 0°)(50\ \Omega) = 20\text{ V}\angle 0°$$
$$V_L = I_0 X_L = I_0 j\omega L = (0.4\text{ A}\angle 0°)\left(5\times10^{6}\ \frac{\text{rad}}{\text{s}}\right)(200\times10^{-6}\text{ H})\angle 90°$$
$$= 400\angle 90°$$
$$V_C = -V_L = 400\text{ V}\angle -90°$$

Alternating Current Electricity 15-9d

Resonance

Example (FEIM):

A parallel resonance circuit with a 10 Ω resistor has a resonance frequency of 1 MHz and bandwidth of 10 kHz. What resistor value will increase the BW to 20 kHz?

$$\text{BW} = \frac{\omega_0}{Q} = \frac{\omega_0}{\omega_0 RC} = \frac{1}{RC}$$

C remains the same, so to double the BW.

$$R_{\text{new}} = \frac{1}{2} R_{\text{old}} = 5\ \Omega$$

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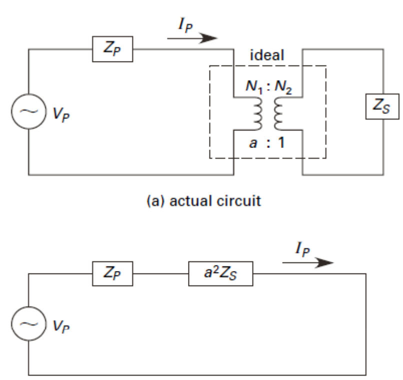
Alternating Current Electricity 15-10a

Transformers

Figure 45.9 *Equivalent Circuit with Secondary Impedance*



(a) actual circuit

(b) equivalent circuit

$$a = \frac{N_1}{N_2} = \frac{V_P}{V_S} = \frac{I_S}{I_P} \qquad 45.56$$

$$Z_{\text{eq}} = \frac{V_P}{I_P} = Z_P + a^2 Z_S \qquad 45.57$$

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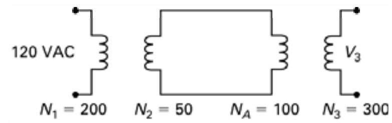
Alternating Current Electricity 15-10b

Transformers

Example 1 (FEIM):

What is the voltage V_3? Disregard losses.



(A) 45 V
(B) 65 V
(C) 75 V
(D) 90 V

$$V_2 = \frac{N_2}{N_1} 120 \text{ V} \qquad V_3 = \frac{N_3}{N_A} V_2$$

$$V_3 = \left(\frac{N_3}{N_A}\right)\left(\frac{N_2}{N_1}\right)(120 \text{ V}) = \left(\frac{300}{100}\right)\left(\frac{50}{200}\right)(120 \text{ V}) = 90 \text{ V}$$

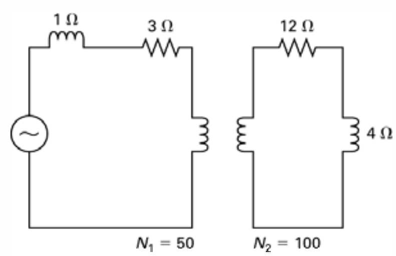
Therefore, the answer is (D).

Alternating Current Electricity 15-10c

Transformers

Example 2 (FEIM):

What is the total impedance in the primary circuit? Assume an ideal transformer.



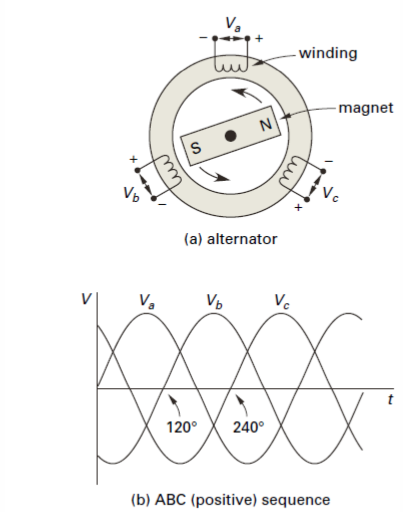
$$Z_{\text{total}} = Z_p + Z = \left(\frac{50}{100}\right)^2 (12\ \Omega + j4\ \Omega) + 3\ \Omega + j\Omega$$

$$= 4\Omega + 3\Omega + j(1\Omega + 1\Omega) = 7\Omega + j2\Omega$$

Alternating Current Electricity 15-11a

Rotating Machines

Figure 46.1 *Rotating Magnetic Field*



$$n_s = \frac{120f}{p} = \frac{60\Omega}{2\pi} = \frac{60\omega}{\pi p} \quad \text{[synchronous speed]} \qquad 46.1$$

Synchronous Machines

1. Synchronous motors
2. Synchronous generators

Alternating Current Electricity 15-11b

Rotating Machines

Example (FEIM):

A six-pole, three-phase synchronous generator supplies current with a frequency of 60 Hz. What is the angular velocity of the rotor in the generator?

(A) $10\pi \frac{\text{rad}}{\text{s}}$

(B) $40\pi \frac{\text{rad}}{\text{s}}$

(C) $60\pi \frac{\text{rad}}{\text{s}}$

(D) $80\pi \frac{\text{rad}}{\text{s}}$

$$n_s = \frac{120f}{p} = \frac{60\Omega}{2\pi}$$

(Note: Here, Ω does not mean ohms. It is the angular velocity.)

$$\Omega = \frac{4\pi f}{p}$$

$$= \frac{\left(4\pi \frac{\text{rad}}{\text{cycle}}\right)\left(60 \frac{\text{cycle}}{\text{s}}\right)}{6}$$

$$= 40\pi \text{ rad/s}$$

Therefore, the answer is (B).

Alternating Current Electricity 15-11c

Rotating Machines

Induction Motors

- A constant-speed device that receives power though induction without using brushes or slip rings

Slip:

$$s = \frac{n_s - n}{n_s} = \frac{\Omega_s - \Omega}{\Omega_s} \qquad 46.2$$

$$\text{percent slip} = s \times 100\% \qquad 46.3$$

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Alternating Current Electricity 15-11d

Rotating Machines

Example (FEIM):

A four-pole induction motor operates on a three-phase, 240 V_{rms} line-to-line supply. The slip is 5%. The operating speed is 1600 rpm. What is most nearly the operating frequency?

(A) 56 Hz
(B) 60 Hz
(C) 64 Hz
(D) 102 Hz

$$\begin{aligned} n_s &= n(1+s) \\ &= (1600\ \text{rpm})(1+0.05) \\ &= 1680\ \text{rpm} \end{aligned}$$

Solving the synchronous speed equation for the operating frequency yields

$$\begin{aligned} f &= \frac{pn_s}{120} \\ &= \frac{(4)(1680\ \text{rpm})}{120} \\ &= 56\ \text{Hz} \end{aligned}$$

Therefore, the answer is (A).

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Alternating Current Electricity 15-11e

Rotating Machines

DC Machines

- Have a constant magnetic field in the stator; the magnetic field of rotor responds to the stator field
- The magnetic flux produced is described by the equation

$$\phi = K_f I_f \qquad 46.4$$

Alternating Current Electricity 15-11f

Rotating Machines

DC Generators

- A device that produces DC potential

Figure 46.2 *Commutator Action*

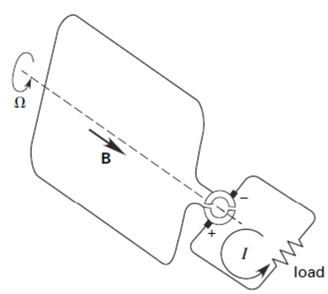
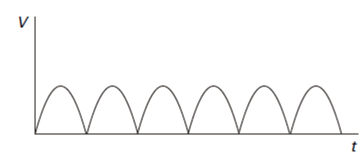


Figure 46.3 *Rectified DC Voltage Induced in a Single Coil*



Alternating Current Electricity 15-11g

Rotating Machines

DC Generators

Figure 46.6 *DC Machine Equivalent Circuit*

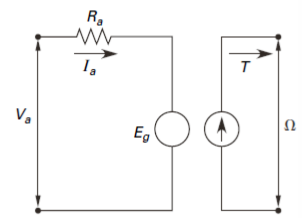
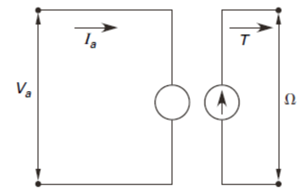


Figure 46.7 *Simplified DC Machine Equivalent Circuit*



For the DC machine equivalent circuit

$$E_g = K_a n\phi \quad \textit{46.5}$$

Terminal Voltage:

$$V_a = E_g + I_a R_a = K_a n\phi + I_a R_a \quad \textit{46.6}$$

For the simplified DC machine equivalent circuit

Terminal Voltage:

$$V_a = K_a n\phi \quad \textit{46.7}$$

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Alternating Current Electricity 15-11h

Rotating Machines

Example (FEIM):

A DC generator provides a current of 12 A for a resistive load when operating at 1800 rpm. The speed of the armature is reduced, and the new steady-state current is 10 A. What is most nearly the operating speed when the generator reaches steady-state conditions? Ignore armature resistance.

(A) 1500 rpm
(B) 1700 rpm
(C) 1800 rpm
(D) 2200 rpm

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Alternating Current Electricity 15-11i

Rotating Machines

The current is proportional to the voltage by Ohm's law. From the equation for the armature voltage, ignoring the armature resistance, the armature voltage is proportional to the speed.

$$V_a = K_a n\phi$$

Therefore, the armature current is proportional to the speed.

$$\frac{I_2}{I_1} = \frac{V_2}{V_1} = \frac{n_2}{n_1}$$

Rearranging yields the new operating speed.

$$\begin{aligned} n_2 &= \frac{I_2}{I_1} n_1 \\ &= \left(\frac{10\text{ A}}{12\text{ A}}\right)(1800\text{ rpm}) \\ &= 1500\text{ rpm} \end{aligned}$$

Therefore, the answer is (A).

Alternating Current Electricity 15-11j

Rotating Machines

DC Motors

- Very similar to a DC generator, only the direction of current changes

Power (for the motor model in the DC machine equivalent circuit):

$$P_e = P_h + P_m = I_a^2 R_a + I_a E_g \qquad 46.8$$

- Ignoring the power dissipated as heat for the DC machine equivalent circuit,

$$P_m = V_a I_a \qquad 46.9$$

Torque:

$$T_m = \frac{60}{2\pi} K_a \phi I_a \qquad 46.10$$

Mechanical Power:

$$P_m = T\Omega \qquad 46.11$$

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Rotating Machines

Example (FEIM):

A DC motor is operating on 100 V with a magnetic flux of 0.02 Wb produces an output torque of 20 N·m. The magnetic flux is changed to 0.03 Wb. What is most nearly the new steady-state output torque? Ignore armature resistance.

(A) = 19 N·m
(B) = 30 N·m
(C) = 32 N·m
(D) = 51 N·m

The motor voltage is not important to the solution because the torque equation is proportional to the magnetic flux.

$$\frac{T_1}{T_2} = \frac{\phi_1}{\phi_2}$$

$$T_2 = \frac{\phi_2}{\phi_1} T_1$$

$$= \left(\frac{0.03 \text{ Wb}}{0.02 \text{ Wb}}\right)(20 \text{ N}\cdot\text{m})$$

$$= 30 \text{ N}\cdot\text{m}$$

The answer is (B).

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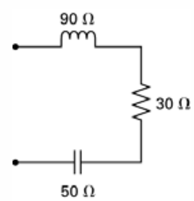
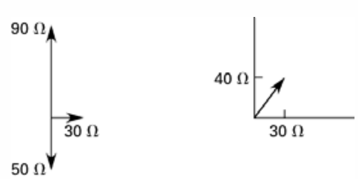
Additional Examples

Example 1 (FEIM):

For the circuit elements shown, draw the conventional impedance diagram.





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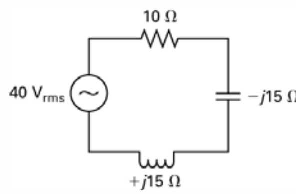
Additional Examples

Example 2 (FEIM):

What is the steady-state magnitude of the rms voltage across the capacitor?

(A) 15 V
(B) 30 V
(C) 45 V
(D) 60 V



$$Z = 10\ \Omega + j(15\ \Omega - 15\ \Omega) = 10\ \Omega$$

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{R} = \frac{40\ \text{V}}{10\ \Omega} = 4\ \text{A}$$

$$V_{c\ \text{rms}} = I_{\text{rms}} X_c = (4\ \text{A})(15\ \Omega) = 60\ \text{V}$$

Therefore, the answer is (D).

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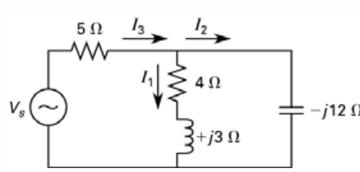
Additional Examples

Example 3 (FEIM):

For the following circuit, the rms steady-state currents are $I_1 = 14.4\angle{-36.9^\circ}$ and $I_2 = 6\angle 90^\circ$.



The impedance seen by the voltage source is most nearly

(A) $9.5\ \Omega\angle 18.4^\circ$
(B) $10.0\ \Omega\angle 36.9^\circ$
(C) $10.7\ \Omega\angle 16.0^\circ$
(D) $11.0\ \Omega\angle 7.1^\circ$

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Additional Examples

$$Z = R + \frac{1}{\frac{1}{Z_1} + \frac{1}{Z_2}} = R + \frac{Z_1 Z_2}{Z_1 + Z_2}$$

$$= 5\,\Omega + \frac{(4 + j3)(-j12)}{4 + j3 - j12}\Omega = 5\,\Omega + \left(\frac{36 - j48}{4 - j9}\right)\left(\frac{4 + j9}{4 + j9}\right)\Omega$$

Note: The preceding calculation is an example of rationalizing the denominator of a complex number without converting it to polar form.

$$Z = 5\,\Omega + \frac{144\,\Omega + 432\,\Omega + j(324\,\Omega - 192\,\Omega)}{16\,\Omega + 81\,\Omega} = 5\,\Omega + \frac{576\,\Omega}{97\,\Omega} + j\left(\frac{132\,\Omega}{97\,\Omega}\right)$$

$$= 5\,\Omega + 5.938\,\Omega + j1.361\,\Omega = 10.938\,\Omega + j1.361\,\Omega$$

$$|Z| = \sqrt{(10.938\,\Omega)^2 + (1.361\,\Omega)^2} = 11.0\,\Omega$$

$$\tan^{-1}\frac{1.361\,\Omega}{10.938\,\Omega} = 7.1°$$

$$Z = 11\,\Omega\angle 7.1°$$

Therefore, the answer is (D).

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Additional Examples

Example 4 (FEIM):

The phasor form of I_3 is most nearly

(A) 10.3 A $\angle$–32.5°

(B) 11.8 A $\angle$–12.9°

(C) 18.6 A $\angle$51.9°

(D) 18.6 A $\angle$–51.9°

$$I_3 = I_1 + I_2$$

$$= 14.4\text{ A}\angle -36.9° + 6\text{ A}\angle 90°$$

$$= (14.4\text{ A})\cos(-36.9°) + j(14.4\text{ A})\sin -36.9° + j6\text{ A}$$

$$= 11.515\text{ A} - j8.646\text{ A} + j6\text{ A}$$

$$= 11.515\text{ A} - j2.646\text{ A}$$

$$|I_3| = \sqrt{(11.515\text{ A})^2 + (2.646\text{ A})^2} = 11.8\text{ A}$$

$$\arctan\frac{-2.646\text{ A}}{11.515\text{ A}} = -12.9°$$

$$I_3 = 11.8\text{ A}\angle -12.9°$$

Therefore, the answer is (B).

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