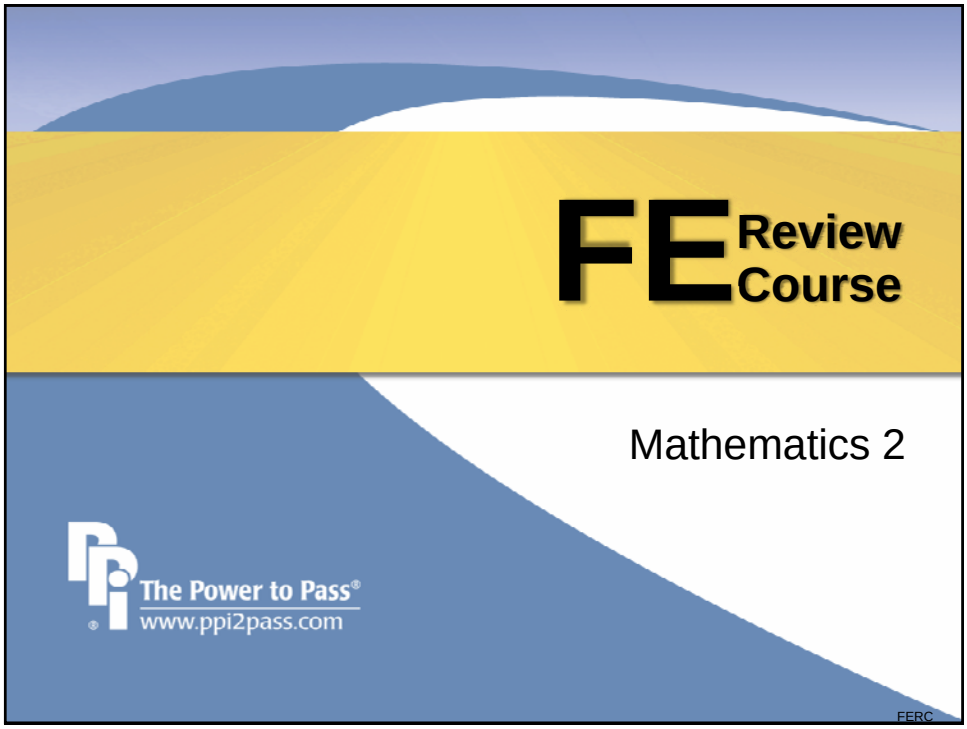

Mathematics 2 2-1a

Vectors and Matrices

Vector Addition and Subtraction

$$\mathbf{A} + \mathbf{B} = (a_x + b_x)\mathbf{i} + (a_y + b_y)\mathbf{j} + (a_z + b_z)\mathbf{k} \qquad 5.58$$

Example (FEIM):

What is the resultant of vectors $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$?

$\mathbf{F}_1 = 5\mathbf{i} + 6\mathbf{j} + 3\mathbf{k}$
$\mathbf{F}_2 = 11\mathbf{i} + 2\mathbf{j} + 9\mathbf{k}$
$\mathbf{F}_3 = 7\mathbf{i} - 6\mathbf{j} - 4\mathbf{k}$

$\mathbf{R} = (5 + 11 + 7)\mathbf{i} + (6 + 2 - 6)\mathbf{j} + (3 + 9 - 4)\mathbf{k}$
$= 23\mathbf{i} + 2\mathbf{j} + 8\mathbf{k}$

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Mathematics 2 2-1b

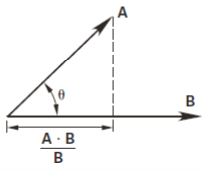
Vectors and Matrices

Vector Dot Product

$$\mathbf{A}\cdot\mathbf{B} = a_x b_x + a_y b_y + a_z b_z = |\mathbf{A}||\mathbf{B}|\cos\theta \qquad 5.60$$

Projection of a Vector

Figure 5.3 Vector Dot Product



Mathematics 2 2-1c

Vectors and Matrices

Example (FEIM):

What is the angle between the vectors $\mathbf{F}_1$ and $\mathbf{F}_2$?

$\mathbf{F}_1 = 5\mathbf{i} + 4\mathbf{j} + 6\mathbf{k}$

$\mathbf{F}_2 = 4\mathbf{i} + 10\mathbf{j} + 7\mathbf{k}$

$$\cos\theta = \frac{\mathbf{F}_1 \bullet \mathbf{F}_2}{|\mathbf{F}_1||\mathbf{F}_2|} = \frac{20+40+42}{\left(\sqrt{25+16+36}\right)\left(\sqrt{16+100+49}\right)} = 0.905$$

$$\theta = 25.2°$$

$\mathbf{F}_1 = 5\mathbf{i} + 4\mathbf{j} + 6\mathbf{k}$; $\mathbf{F}_2 = 4\mathbf{i} + 10\mathbf{j} + 7\mathbf{k}$

$$\text{Projection} = \frac{\mathbf{F}_1 \bullet \mathbf{F}_2}{|F_2|} = \frac{20+40+42}{\sqrt{16+100+49}} = 7.9$$

Mathematics 2 — 2-2a

Vectors and Matrices

Matrix Addition and Subtraction

$$\mathbf{C} \equiv c_{ij} \equiv a_{ij} + b_{ij} \qquad 5.38$$

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 6 & 8 \end{bmatrix}$$

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Mathematics 2 — 2-2b

Vectors and Matrices

Matrix Multiplication

$$\mathbf{C} \equiv c_{ij} \equiv \sum_{l=1}^{n} a_{il} b_{lj} \qquad 5.37$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 7 & 10 \\ 8 & 11 \\ 9 & 12 \end{bmatrix} = \begin{bmatrix} (1 \times 7) + (2 \times 8) + (3 \times 9) & 68 \\ 122 & 167 \end{bmatrix}$$

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Mathematics 2 2-2c

Vectors and Matrices

Identity Matrix: $a_{ij} = 1$ for $i = j; a_{ij} = 0$ for $i \neq j$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Transpose of a Matrix: $\mathbf{B} = \mathbf{A}^T$ if $b_{ij} = a_{ij}$

$$\begin{bmatrix} 1 & 6 & 9 \\ 5 & 4 & 2 \\ 7 & 3 & 8 \end{bmatrix}^T = \begin{bmatrix} 1 & 5 & 7 \\ 6 & 4 & 3 \\ 9 & 2 & 8 \end{bmatrix}$$

Mathematics 2 2-2d

Vectors and Matrices

Determinant of a Matrix

For a 2 x 2 matrix:

$$|\mathbf{A}| = \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} = a_1 b_2 - a_2 b_1 \qquad 5.40$$

For a 3 x 3 matrix:

$$\mathbf{A} = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix} \qquad 5.41(a)$$

$$\text{augmented } \mathbf{A} = \begin{bmatrix} a_1 & a_2 & a_3 & a_1 & a_2 \\ b_1 & b_2 & b_3 & b_1 & b_2 \\ c_1 & c_2 & c_3 & c_1 & c_2 \end{bmatrix} \qquad 5.41(b)$$

$$|\mathbf{A}| = a_1 b_2 c_3 + a_2 b_3 c_1 + a_3 b_1 c_2 - a_3 b_2 c_1 - a_2 b_1 c_3 - a_1 b_3 c_2 \qquad 5.42$$

$$\mathbf{A} = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix} \quad \begin{bmatrix} \text{first column chosen} \\ \text{as base column} \end{bmatrix}$$

$$\begin{aligned} |\mathbf{A}| &= a_1 \begin{vmatrix} b_2 & b_3 \\ c_2 & c_3 \end{vmatrix} - b_1 \begin{vmatrix} a_2 & a_3 \\ c_2 & c_3 \end{vmatrix} + c_1 \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} \\ &= a_1(b_2 c_3 - b_3 c_2) - b_1(a_2 c_3 - a_3 c_2) \\ &\quad + c_1(a_2 b_3 - a_3 b_2) \\ &= a_1 b_2 c_3 - a_1 b_3 c_2 - b_1 a_2 c_3 + b_1 a_3 c_2 \\ &\quad + c_1 a_2 b_3 - c_1 a_3 b_2 \end{aligned} \qquad 5.43$$

Mathematics 2 2-2e

Vectors and Matrices

The formula for 3 x 3 matrix in the NCEES Handbook is

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1b_2c_3 + a_2b_3c_1 + a_3b_1c_2 - a_3b_2c_1 - a_2b_1c_3 - a_1b_3c_2$$

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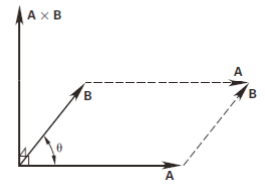
Mathematics 2 2-3a

Vectors and Matrices

Vector Cross Product

$$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix} = -\mathbf{B} \times \mathbf{A}$$
$$= |\mathbf{A}||\mathbf{B}|\mathbf{n} \sin\theta \qquad 5.61$$

Figure 5.4 Vector Cross Product



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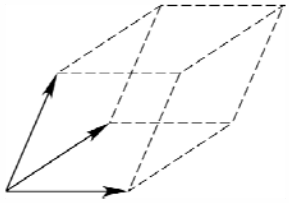
Mathematics 2 2-3b

Vectors and Matrices

Volume inside vectors $\mathbf{A}, \mathbf{B}, \mathbf{C} = \mathbf{A} \bullet (\mathbf{B} \times \mathbf{C})$



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Mathematics 2 2-3c

Vectors and Matrices

Example (FEIM):
What is the area of the parallelogram made by vectors $\mathbf{F}_1$ and $\mathbf{F}_2$?

$\mathbf{F}_1 = 5\mathbf{i} + 4\mathbf{j} + 6\mathbf{k}$

$\mathbf{F}_2 = 4\mathbf{i} + 10\mathbf{j} + 7\mathbf{k}$

$$\mathbf{F}_1 \times \mathbf{F}_2 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 5 & 4 & 6 \\ 4 & 10 & 7 \end{vmatrix} = (28-60)\mathbf{i} - (35-24)\mathbf{j} + (50-16)\mathbf{k} = -32\mathbf{i} - 11\mathbf{j} + 34\mathbf{k}$$

$$A = \sqrt{1024 + 121 + 1156} = 48$$

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Mathematics 2 2-3d

Vectors and Matrices

What is the volume inside the parallelepiped made by vectors $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$?

$\mathbf{F}_1 = -5\mathbf{i} - 4\mathbf{j} + 3\mathbf{k}$

$\mathbf{F}_2 = 5\mathbf{i} + 4\mathbf{j} + 6\mathbf{k}$

$\mathbf{F}_3 = 4\mathbf{i} + 10\mathbf{j} + 7\mathbf{k}$

$$V = \mathbf{F}_1 \bullet (\mathbf{F}_2 \times \mathbf{F}_3) = (-5\mathbf{i} - 4\mathbf{j} + 3\mathbf{k}) \bullet (-32\mathbf{i} - 11\mathbf{j} + 34\mathbf{k})$$
$$= 160 + 44 + 102 = 306$$

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Mathematics 2 2-4a

Vectors and Matrices

Cofactor Matrix

Cofactor matrix of $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 8 \end{bmatrix}$

The cofactor of 1 is $\begin{vmatrix} 5 & 6 \\ 8 & 8 \end{vmatrix}$ and so on.

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 8 \end{bmatrix} \xrightarrow{\text{Cofactor}} = \begin{bmatrix} -8 & 10 & -3 \\ 8 & -13 & 6 \\ -3 & 6 & -3 \end{bmatrix}$$

Classical Adjoint – transpose of the cofactor matrix

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 8 \end{bmatrix} \xrightarrow{\text{Adjoint}} = \begin{bmatrix} -8 & 8 & -3 \\ 10 & -13 & 6 \\ -3 & 6 & -3 \end{bmatrix}$$

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Mathematics 2 — 2-4b

Vectors and Matrices

Inverse Matrices

For 2 x 2 matrix **A**:

$$\mathbf{A}^{-1} = \frac{\begin{bmatrix} b_2 & -a_2 \\ -b_1 & a_1 \end{bmatrix}}{|\mathbf{A}|} \qquad 5.49$$

For 3 x 3 matrix **A**:

$$\mathbf{B} = \mathbf{A}^{-1} = \frac{\text{adj}(\mathbf{A})}{|\mathbf{A}|} \qquad 5.50$$

Example (FEIM):

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 8 \end{bmatrix}^{-1} = \frac{1}{3}\begin{bmatrix} -8 & 8 & -3 \\ 10 & -13 & 6 \\ -3 & 6 & -3 \end{bmatrix} = \begin{bmatrix} \frac{-8}{3} & \frac{8}{3} & -1 \\ \frac{10}{3} & \frac{-13}{3} & 2 \\ -1 & 2 & -1 \end{bmatrix}$$

Mathematics 2 — 2-4c

Vectors and Matrices

Matrices – Solve Simultaneous Equations

Gauss-Jordan Method

Example (FEIM):

$2x + 3y - 4z = 1$

$3x - y - 2z = 4$

$4x - 7y - 6z = -7$

$$\left[\begin{array}{ccc|c} 2 & 3 & -4 & 1 \\ 3 & -1 & -2 & 4 \\ 4 & -7 & -6 & -7 \end{array}\right] = \left[\begin{array}{ccc|c} 2 & 3 & -4 & 1 \\ 3 & -1 & -2 & 4 \\ 0 & -13 & 2 & -9 \end{array}\right] \text{ and so on until } = \left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

Mathematics 2 2-4d

Vectors and Matrices

Matrices – Solve Simultaneous Equations (cont)

Cramer's Rule:

$$x_1 = \frac{|\mathbf{A}_1|}{|\mathbf{A}|} \qquad 5.54$$

$$x_2 = \frac{|\mathbf{A}_2|}{|\mathbf{A}|} \qquad 5.55$$

$$x_3 = \frac{|\mathbf{A}_3|}{|\mathbf{A}|} \qquad 5.56$$

Example (FEIM):

$2x + 3y - 4z = 1$

$3x - y - 2z = 4$

$4x - 7y - 6z = -7$

$$x = \frac{\begin{vmatrix} 1 & 3 & -4 \\ 4 & -1 & -2 \\ -7 & -7 & -6 \end{vmatrix}}{\begin{vmatrix} 2 & 3 & -4 \\ 3 & -1 & -2 \\ 4 & -7 & -6 \end{vmatrix}} = \frac{246}{82} = 3$$

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Mathematics 2 2-5a

Progressions and Series

Arithmetic Progression

Subtract each number from the preceding (2^{nd} – 1^{st} etc.).

If the difference is a constant, the series is arithmetic.

or

Subtract the possible answers from the last number in the sequence.
If the difference is the same, then that is the correct answer.

Example (FEIM):
What is the next number in the sequence {14, 17, 20, 23,...}?

(A) 3
(B) 9
(C) 26
(D) 37

{14 + 3 =17 + 3 = 20 + 3 = 23 + 3 =...}
The series has a difference of +3 between each member, so the next number will be 26.
Therefore, (C) is correct.

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Mathematics 2 2-5b

Progressions and Series

Geometric Progression

Divide each number by the preceding (2nd / 1st etc.).
If the quotients are equal, the series is geometric.

or

If any of the possible answers are integer multiples of the last number, try that number on others in the series.

Example (FEIM):
What is the next number in the sequence {3, 21, 147, 1029,...}?

(A) 343
(B) 2000
(C) 3087
(D) 7203

{3 x 7 = 21 x 7 = 147 x 7 = 1029 x 7 = …}
Each number is seven times the previous number, so the next number in the series will be 7203.
Therefore, (D) is correct.

Mathematics 2 2-5c

Progressions and Series

Arithmetic Series

$$\ell = a + (n-1)d \qquad 5.77$$

$$S_n = \sum_{i=1}^{n}(a + (i-1)d) = \left(\frac{n}{2}\right)(a + \ell)$$

$$= \frac{n(2a + (n-1)d)}{2} \qquad 5.78$$

Example (FEIM):
What is the summation of the series 3 + (*n* – 1)7 for four terms?

(A) 7
(B) 24
(C) 45
(D) 54

$$S = \frac{4\left((2)(3) + (4-1)(7)\right)}{2} = 54$$

or

$$S = 3 + 10 + 17 + 24 = 54$$

Therefore, (D) is correct.

Mathematics 2 2-5d

Progressions and Series

Geometric Series

$$\ell = ar^{n-1} \qquad 5.79$$

$$S_n = \sum_{i=1}^{n} ar^{i-1} = \frac{a - r\ell}{1 - r}$$

$$= \frac{a(1 - r^n)}{1 - r} \qquad 5.80$$

$$S_n = \sum_{i=1}^{\infty} ar^{i-1}$$

$$= \frac{a}{1 - r} \qquad 5.81$$

Example (FEIM):
What is the summation of the series $3 \times 7^{n-1}$ for four terms?

(A) 54
(B) 149
(C) 1029
(D) 1200

$$S = 3\frac{(1 - 7^4)}{1 - 7} = 1200$$

or

$$S = 3 + 21 + 147 + 1029 = 1200$$

Therefore, (D) is correct.

Mathematics 2 2-5e

Progressions and Series

Power Series

$$\sum_{i=1}^{n} a_i x^{i-1} = a_1 + a_2 x + a_3 x^2 + \cdots + a_n x^{n-1} \qquad 5.82$$

Valid rules for power series:

$$\sum_{i=1}^{n} cx_i = c\sum_{i=1}^{n} x_i \qquad 5.84$$

$$\sum_{i=1}^{n} (x_i + y_i - z_i) = \sum_{i=1}^{n} x_i + \sum_{i=1}^{n} y_i - \sum_{i=1}^{n} z_i \qquad 5.85$$

$$\sum_{x=1}^{n} x = \frac{n + n^2}{2} \qquad 5.86$$

$$f'(x) = \sum_{i=1}^{n} \frac{d(a_i x^i)}{dx} \qquad 5.87$$

$$\int f(x)\,dx = \sum_{i=1}^{n} \int a_i x^i\,dx \qquad 5.88$$

Mathematics 2 2-5f

Progressions and Series

Taylor's Series

$$f(b) = f(a) + \left(\frac{f'(a)}{1!}\right)(b-a) + \left(\frac{f''(a)}{2!}\right)(b-a)^2 + \cdots + \left(\frac{f^n(a)}{n!}\right)(b-a)^n \qquad 5.89$$

Example (FEIM):

What is Taylor's series for sin *x* about *a* = 0 (or Maclaurin's series for sin *x*)?

$$\sin x = \frac{\sin 0 + \cos 0x - \sin 0x^2}{2!} - \frac{\cos 0x^3}{3!\ldots}$$

$$\approx x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots + (-1)^n \frac{x^{2n+1}}{(2n+1)!}$$

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Mathematics 2 2-6a

Probability and Statistics

Probability

- *a priori* knowledge about a phenomenon to predict the future

Statistics

- data taken about a phenomenon to predict the future

Sets – probability and statistics divide the universal set into what meets success or failure.

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Mathematics 2 2-6b

Probability and Statistics

Combinations

$$\binom{n}{r} = C(n,r) = \frac{P(n,r)}{r!} = \frac{n!}{r!(n-r)!} \quad [r \le n] \qquad 6.19$$

Example (FEIM):
A pizza restaurant offers 5 toppings. Given a one-topping minimum, how many combinations are possible?

(A) 5
(B) 10
(C) 31
(D) 36

$$C_{\text{total}} = \sum_{i=1}^{5} C_i = \frac{5!}{1!(5-1)!} + \frac{5!}{2!(5-2)!} + \frac{5!}{3!(5-3)!} + \frac{5!}{4!(5-4)!} + \frac{5!}{4!(5-4)!}$$

$$= 5 + 10 + 10 + 5 + 1$$

$$= 31$$

Therefore, (C) is correct.

Mathematics 2 2-6c

Probability and Statistics

Permutations

$$P(n,r) = \frac{n!}{(n-r)!} \quad [r \le n] \qquad 6.20$$

Examples (FEIM):
(a) A baseball coach has 9 players on a team. How many possible batting orders are there?

n permutations taken *n* at a time

$$P(n,n) = \frac{n!}{(n-n)} = \frac{n!}{0!} = n!$$

P(9,9) = 9! = 362,880

(b) A baseball coach has 11 players on the team. Any 9 can be in the batting order. How many possible batting orders are there?

$$P(11,9) = \frac{11!}{(11-9)} = 19,958,400$$

Mathematics 2 2-7a

Laws of Probability

1. General character of probability

$$P(E) = 1 - P(\text{not } E) \qquad 6.21$$

2. Law of total probability

$$P(A + B) = P(A) + P(B) - P(A, B) \qquad 6.22$$

3. Law of compound or joint probability

$$\begin{aligned} P(A, B) &= P(A)P(B/A) \\ &= P(B)P(A/B) \end{aligned} \qquad 6.23$$

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Mathematics 2 2-7b

Laws of Probability

Example 1 (FEIM):

One bowl contains eight white balls and two red balls. Another bowl contains four yellow balls and six black balls. What is the probability of getting a red ball from the first bowl and a yellow ball from the second bowl on one random draw from each bowl?

(A) 0.08
(B) 0.2
(C) 0.4
(D) 0.8

$$P(ry) = P(r)P(y) = \left(\frac{2}{10}\right)\left(\frac{4}{10}\right) = 0.08$$

Therefore, (A) is correct.

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Mathematics 2 2-7c

Laws of Probability

Example 2 (FEIM):

One bowl contains eight white balls, two red balls, four yellow balls, and six black balls. What is the probability of getting a red ball and then a yellow ball drawn at random without replacement?

There are 20 total balls and two are red, so for the first draw, $P(r) = 2/20$. Since we assume the first draw was successful, on the second draw there are only 19 balls left and four yellow balls, so $P(y|r) = 4/19$.

$$P(r,y) = P(r)P(y|r)$$

$$= \left(\frac{2}{20}\right)\left(\frac{4}{19}\right) = \frac{8}{380}$$

$$= 0.021$$

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Mathematics 2 2-7d

Laws of Probability

Probability Functions

- Discrete variables have distinct finite number of values.
- The sum total of all outcome probabilities is 1.

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Mathematics 2 2-7e

Laws of Probability

Binomial Distribution

$$F(x) = C(n,x)p^x q^{n-x} = \frac{n!}{x!(n-x)!}p^x q^{n-x} \qquad 6.39$$

Example (FEIM):
Five percent of students have red hair. If seven students are selected at random, what is the probability that exactly three will have red hair?

$$F(3) = \frac{7!}{3!(7-3)!}(0.05^3)(0.95^4) = 0.00356$$

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Mathematics 2 2-7f

Laws of Probability

Probability Cumulative Functions

- Continuous variables have infinite possible values.
- Define the probability that outcome is less than the value *x*.
- $F(-\infty)$ –0; $F(\infty) = 1$

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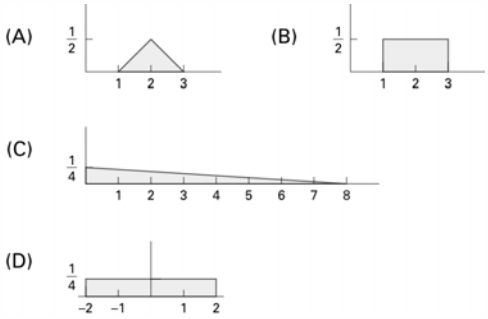
Mathematics 2 2-7g

Laws of Probability

Probability Density Functions

$$P(a \le X \le b) = \int_a^b f(x)\ dx \qquad 6.34$$

Example (FEIM):
Which of the following CANNOT be a probability density function?



The area under (A) is 1/2, so it cannot be a probability distribution.
Therefore, (A) is correct.

Mathematics 2 2-7h

Laws of Probability

Normal or Gaussian Distribution

Convert the distribution to a unit normal distribution:

$$x = \frac{X_0 - \mu}{\sigma} \qquad 6.42$$

Find the probability on the unit normal chart:

Column 1: $f(x)$ = probability density of one particular value
Column 2: $F(x)$ = probability values < x = 1 – $R(x)$
Column 3: $R(x)$ = probability values > x = 1 – $F(x)$
Column 4: $2R(x)$ = > x + < $-x$ = 1 – $W(x)$
Column 5: $W(x)$ = $-x$ < values < x = 1 – $2R(x)$

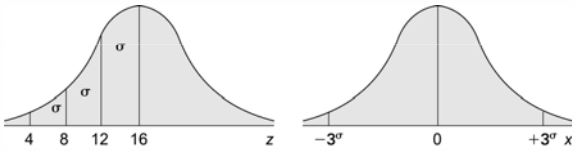
Mathematics 2 2-7i

Laws of Probability

Example (FEIM):
A normal distribution has a mean of 16 and a standard deviation of 4. What is the probability of values greater than 4?

(A) 0.1295
(B) 0.9987
(C) 0.0668
(D) 0.1336

$$z = \frac{x - \mu}{\sigma} = \frac{4 - 16}{4} = -3$$



Due to the symmetry of the distribution, $R(-z) = F(z)$, so from the NCEES Unit Normal Distribution Table, probability = 0.9987.
Therefore, (B) is correct.

Mathematics 2 2-7j

Laws of Probability

t-Distribution

Convert the distribution to unit normal distribution:

$$t = \frac{x - \mu}{\sigma}$$

For n degrees of freedom, find $t_{\alpha,n}$ that leads to probability α.
Calculate probability like the normal distribution columns.

$\text{Values} > t_{\alpha,n} : \alpha$

$\text{Values} > t_{\alpha,n} : 1 - \alpha$

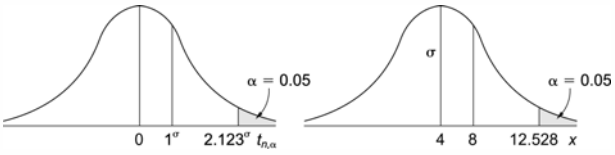
$\text{Values} > t_{\alpha,n} + \text{Values} < -t_{\alpha,n} : 2\alpha$

$t_{\alpha,n} < \text{Values} < t_{\alpha,n} : 1 - 2\alpha$

Mathematics 2 2-7k

Laws of Probability

Example (FEIM):
A *t*-distribution with 4 degrees of freedom has a mean of 4 and a standard deviation of 4. If 5% of the population is greater than a value, what is that value?



From the NCEES *t*-Distribution Table for $\alpha = 0.05$ and $n = 4, t_{n,\alpha} = 2.132$

$x = t_{n,\alpha}\sigma + \mu = (2.132)(4) + 4 = 12.528$

Mathematics 2 2-8a

Statistical Calculations

Arithmetic Mean

$$\overline{X} = \left(\frac{1}{n}\right)(X_1 + X_2 + \cdots + X_n) = \frac{1}{n}\sum_{i=1}^{n} X_i \qquad 6.25$$

Example (FEIM):
What is the mean of the following data?

61, 62, 63, 63, 64, 64, 66, 66, 67, 68, 68, 68, 68, 69, 69, 69, 69, 70, 70, 70, 70, 71, 71, 72, 73, 74, 74, 75, 76, 79

$$\overline{X} = \frac{61 + 62 + 2 \times 63 + 2 \times 64 + \ldots + 79}{30} = \frac{2069}{30} = 68.97$$

Mathematics 2 2-8b

Statistical Calculations

Weighted Arithmetic Mean

$$\overline{X}_w = \frac{\sum_{i=1}^{n} w_i X_i}{\sum_{i=1}^{n} w_i} \qquad 6.26$$

Example (FEIM):
What is the weighted arithmetic mean of the following data?

data	weighting factor
62	1
62	2
72	3

$$\overline{X}_w = \frac{\sum w_i X_i}{\sum w_i} = \frac{(1)(62)+(2)(62)+(3)(72)}{1+2+3} = 67$$

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Mathematics 2 2-8c

Statistical Calculations

Median

- Half of the data points are less than the median, half of the data points are greater than the median.

Example (FEIM):
What is the median of the following data?
61, 62, 63, 63, 64, 64, 66, 66, 67, 68, 68, 68, 68, 69, 69, 69, 69, 70, 70, 70, 70, 71, 71, 72, 73, 74, 74, 75, 76, 79

There are 30 data points, so we start counting at the lowest value and count 15 data points. We see that both the 15th and 16th data points are 69, so the median is 69. If there is an even number of data points and the points on either side of the median are not the same, then the median is halfway in between the two middle points.

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Mathematics 2 2-8d

Statistical Calculations

Mode

- The data value that occurs most frequently.

Example (FEIM):
What is the mode of the following data?
61, 62, 63, 63, 64, 64, 66, 66, 67, 68, 68, 68, 69, 69, 69, 69, 69, 70, 70, 70, 70, 71, 71, 72, 73, 74, 74, 75, 76, 79

69 is the most frequently represented number, so it is the mode.

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Mathematics 2 2-8e

Statistical Calculations

Variance:

$$\sigma^2 = \frac{1}{N}\left((X_1 - \mu)^2 + (X_2 - \mu)^2 + \cdots + (X_N - \mu)^2\right)$$

$$= \frac{1}{N}\sum_{i=1}^{N}(X_i - \mu)^2 \qquad 6.31$$

Standard Deviation:

$$\sigma = \sqrt{\left(\frac{1}{N}\right)\sum_{i=1}^{N}(X_i - \mu)^2} \qquad 6.29$$

Example (FEIM):
What is the variance and standard deviation of the following data?
61, 62, 63, 63, 64, 64, 66, 66, 67, 68, 68, 68, 68, 69, 69, 69, 69, 70, 70, 70, 70, 71, 71, 72, 73, 74, 74, 75, 76, 79

$$\sigma^2 = \frac{\sum X_i^2}{N} - \mu^2 = \frac{143,225}{30} - \left(\frac{2069}{30}\right)^2 = 17.77$$

$$\sigma = 4.214$$

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Mathematics 2 2-8f

Statistical Calculations

Sample Variance:

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \overline{X})^2 \qquad 6.32$$

Sample Standard Deviation:

$$\sigma = \sqrt{\left(\frac{1}{N}\right)\sum_{i=1}^{N}(X_i - \mu)^2} \qquad 6.29$$

Coefficient of Variation:

$$\mathrm{CV} = \frac{s}{\overline{X}} \qquad 6.33$$

$$\text{geometric mean} = \sqrt[n]{X_1 X_2 X_3 \cdots X_n} \quad [X_i > 0] \qquad 6.27$$

Root-Mean-Square:

$$X_{\text{rms}} = \sqrt{\left(\frac{1}{n}\right)\sum_{i=1}^{n}X_i^2} \qquad 6.28$$

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