FE Review Course

Mathematics 1

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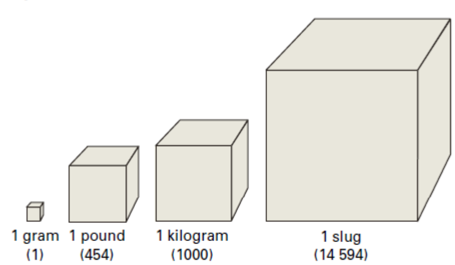
Mathematics 1 — 1-1

Units

FERM Fig. 1.1 Common Units of Mass

Figure 1.1 Common Units of Mass



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Mathematics 1 1-2

Units

Review the Units section of the

NCEES Handbook

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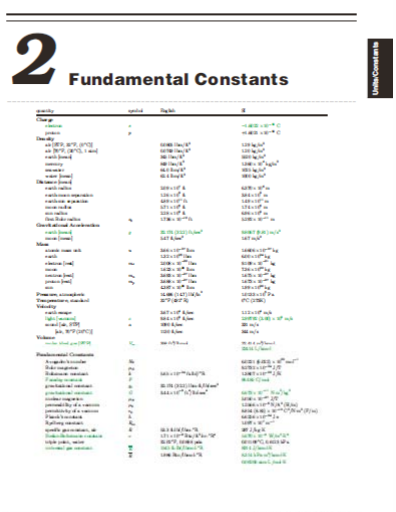
Mathematics 1 1-3

Fundamental Constants



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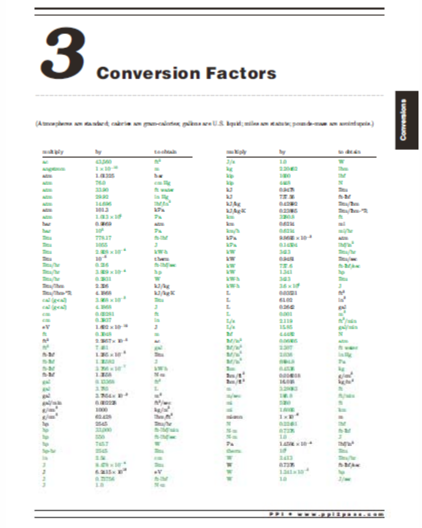
Mathematics 1 — 1-4

Conversion Factors



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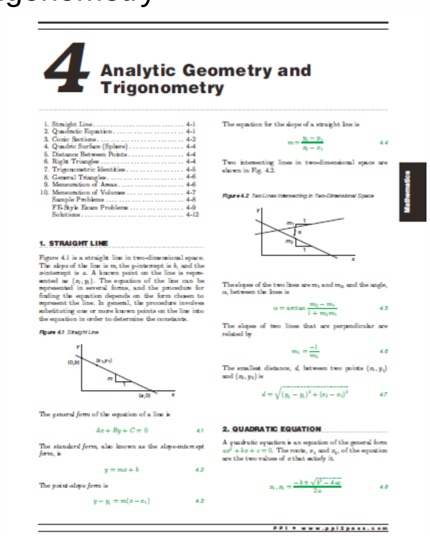
Mathematics 1 — 1-5

Geometry and Trigonometry



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Mathematics 1 1-6a

Straight Line

Figure 4.1 *Straight Line*

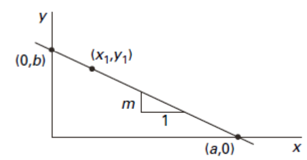
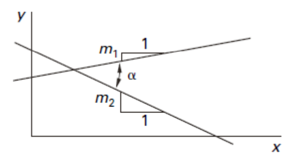


Figure 4.2 *Two Lines Intersecting in Two-Dimensional Space*



$$Ax + By + C = 0 \qquad 4.1$$

$$y = mx + b \qquad 4.2$$

$$y - y_1 = m(x - x_1) \qquad 4.3$$

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad 4.4$$

$$m_1 = \frac{-1}{m_2} \qquad 4.6$$

$$d = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2} \qquad 4.7$$

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Mathematics 1 1-6b

Straight Line

Example (EFPRB):

MATHEMATICS–30

The equation $y = a_1 + a_2x$ is an algebraic expression for which of the following?

(A) a cosine expansion series
(B) projectile motion
(C) a circle in polar form
(D) a straight line

$y = mx + b$ is the slope-intercept form of the equation of a straight line. Thus, $y = a_1 + a_2x$ describes a straight line.

The answer is (D).

MATHEMATICS–31

Find the slope of the line defined by $y - x = 5$.

(A) $5 + x$ (B) $-1/2$ (C) $1/4$ (D) 1

The slope-intercept form of the equation of a straight line is $y = mx + b$, where m is the slope and b is the y-intercept.

$$y - x = 5$$
$$y = x + 5$$

The coefficient of x, m, is

$$m = 1$$

The answer is (D).

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Mathematics 1 — 1-6c

Straight Line

Example (EFPRB):

MATHEMATICS–32

Find the equation of a line with slope = 2 and y-intercept = -3.

(A) $y = -3x + 2$
(B) $y = 2x - 3$
(C) $y = \frac{2}{3}x + 1$
(D) $y = 2x + 3$

The slope-intercept form of the given equation is

$$y = 2x - 3$$

The answer is (B).

Mathematics 1 — 1-6d

Straight Line

Example (EFPRB):

MATHEMATICS–33

Find the equation of the line that passes through the points $(0, 0)$ and $(2, -2)$.

(A) $y = x$ (B) $y = -2x + 2$ (C) $y = -2x$ (D) $y = -x$

Since the line passes through the origin, the y-intercept is 0. Thus, the equation simplifies to $y = mx$. Substituting for the known points,

$$\begin{aligned} y &= \left(\frac{-2-0}{2-0}\right)x \\ &= -x \end{aligned}$$

The answer is (D).

Mathematics 1 1-6e

Straight Line

Example (EFPRB):

MATHEMATICS–29

In finding the distance, d, between two points, which equation is the appropriate one to use?

(A) $d=\sqrt{(x_1-x_2)^2-(y_2-y_1)^2}$

(B) $d=\sqrt{(x_1-y_1)^2+(x_2-y_2)^2}$

(C) $d=\sqrt{(x_1^2-x_2^2)+(y_1^2-y_2^2)}$

(D) $d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

The distance formula is defined as follows.

$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$\boxed{\text{The answer is (D).}}$

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Mathematics 1 1-7a

Quadratic Equations

$$x_1, x_2=\frac{-b\pm\sqrt{b^2-4ac}}{2a} \qquad 4.8$$

Example (FERM):

Problem 5

What are the solutions to the following equation?

$$x^2-x-12=0$$

(A) $x_1=1;\ x_2=12$

(B) $x_1=4;\ x_2=-3$

(C) $x_1=-1;\ x_2=4$

(D) $x_1=6;\ x_2=-2$

Solution

There are two ways to solve the equation. The first method is to factor the equation.

$$x^2-x-12=(x+3)(x-4)=0$$
$$x=-3 \text{ or } x=4$$

The second method is to use the quadratic equation.

$$\begin{aligned} x_1, x_2 &= \frac{-b\pm\sqrt{b^2-4ac}}{2a} \\ &= \frac{-(-1)\pm\sqrt{(-1)^2-(4)(1)(-12)}}{(2)(1)} \\ &= \frac{1\pm 7}{2} \\ x_1 &= 4 \\ x_2 &= -3 \end{aligned}$$

Answer is B.

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Mathematics 1 — 1-7b

Quadratic Equations

Example (EFPRB):

MATHEMATICS–6

What is the solution of the equation $50x^2 + 5(x-2)^2 = -1$, where x is a real-valued variable?

(A) -6.12 and -3.88 (B) -0.52 and 0.700
(C) 7.55 (D) no solution

For real-valued x, the left-hand side of the equation must always be greater than or equal to zero, since all terms containing x are squared. There is no solution to this equation for real values of x.

The answer is (D).

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Mathematics 1 — 1-7c

Quadratic Equations

Example (EFPRB):

MATHEMATICS–7

What are the roots of the cubic equation $x^3 - 8x - 3 = 0$?

(A) $x = -7.90, -3, -0.38$
(B) $x = -3, -2, 2$
(C) $x = -3, -0.38, 2$
(D) $x = -2.62, -0.38, 3$

By inspection, $+3$ is a root, and $(x-3)$ is a factor. Factor out $(x-3)$.

$$\frac{x^3 - 8x - 3}{x - 3} = x^2 + 3x + 1$$

Use the quadratic equation to solve $x^2 + 3x + 1 = 0$.

$$\begin{aligned} x &= 3, \frac{-3 \pm \sqrt{9-4}}{2} \\ &= -2.62,\ -0.38,\ 3 \end{aligned}$$

The answer is (D).

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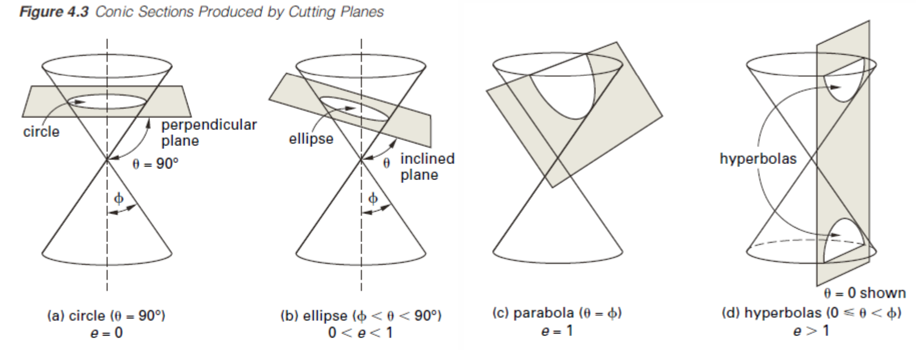
Mathematics 1 1-8a

Conic Sections

Figure 4.3 Conic Sections Produced by Cutting Planes



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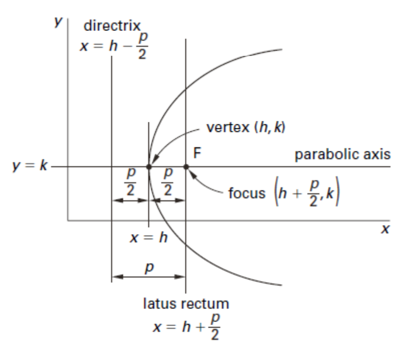
Mathematics 1 1-8b

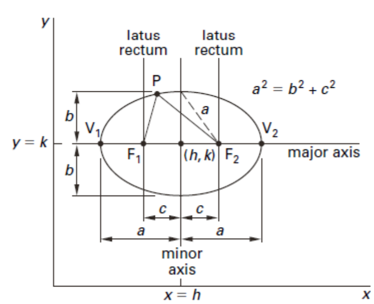
Conic Sections

Figure 4.4 Parabola



$$(y-k)^2 = 2p(x-h) \qquad 4.12$$

Figure 4.5 Ellipse



$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1 \qquad 4.14$$

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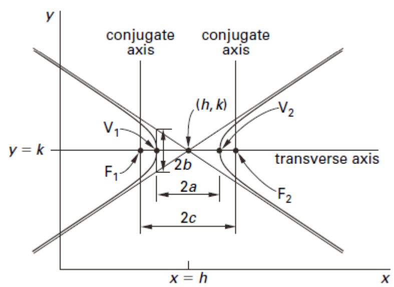
Mathematics 1 1-8c

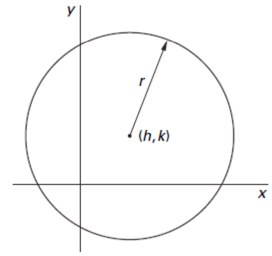
Conic Sections

Figure 4.6 Hyperbola



$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1 \qquad 4.18$$

Figure 4.7 Circle



$$(x-h)^2 + (y-k)^2 = r^2 \qquad 4.22$$

Mathematics 1 1-8d

Conic Sections

Example (EFPRB):

MATHEMATICS–9

What is the radius of the circle defined by $x^2 + y^2 - 4x + 8y = 7$?

(A) $\sqrt{3}$ (B) $2\sqrt{5}$ (C) $3\sqrt{3}$ (D) $4\sqrt{3}$

Since the general equation for a circle is $(x-a)^2 + (y-b)^2 = r^2$, rearrange the equation given to fit the general equation.

$x^2 - 4x + y^2 + 8y = 7$

Complete the binomial forms.

$x^2 - 4x + 4 + y^2 + 8y + 16 = 7 + 4 + 16$

$(x-2)^2 + (y+4)^2 = 27$

$r^2 = 27$

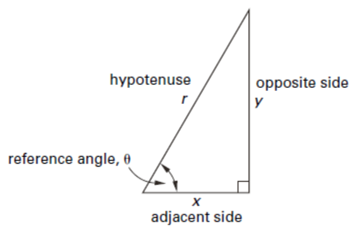
$r = 3\sqrt{3}$

The answer is (C).

Mathematics 1 — 1-9

Right Triangles

Figure 4.9 *Right Triangle*



$$\sin\theta = \frac{y}{r} \quad 4.30$$

$$\cos\theta = \frac{x}{r} \quad 4.31$$

$$\tan\theta = \frac{y}{x} \quad 4.32$$

$$\cot\theta = \frac{x}{y} \quad 4.33$$

$$\csc\theta = \frac{r}{y} \quad 4.34$$

$$\sec\theta = \frac{r}{x} \quad 4.35$$

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Mathematics 1 — 1-10a

Trigonometric Identities

$$\csc\theta = \frac{1}{\sin\theta} \quad 4.36$$

$$\sec\theta = \frac{1}{\cos\theta} \quad 4.37$$

$$\cot\theta = \frac{1}{\tan\theta} \quad 4.38$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \quad 4.39$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{\cos\theta}{\sin\theta} \quad 4.40$$

$$\sin^2\theta + \cos^2\theta = 1 \quad 4.41$$

$$\tan^2\theta + 1 = \sec^2\theta \quad 4.42$$

$$\cot^2\theta + 1 = \csc^2\theta \quad 4.43$$

$$\sin 2\alpha = 2\sin\alpha\cos\alpha \quad 4.44$$

$$\begin{aligned}\cos 2\alpha &= \cos^2\alpha - \sin^2\alpha \\ &= 1 - 2\sin^2\alpha \\ &= 2\cos^2\alpha - 1 \quad 4.45\end{aligned}$$

$$\tan 2\alpha = \frac{2\tan\alpha}{1-\tan^2\alpha} \quad 4.46$$

$$\cot 2\alpha = \frac{\cot^2\alpha - 1}{2\cot\alpha} \quad 4.47$$

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Mathematics 1 1-10b

Trigonometric Identities

- *Two-angle formulas*

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta \qquad 4.48$$

$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta \qquad 4.49$$

$$\tan(\alpha+\beta) = \frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta} \qquad 4.50$$

$$\cot(\alpha+\beta) = \frac{\cot\alpha\cot\beta-1}{\cot\alpha+\cot\beta} \qquad 4.51$$

$$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta \qquad 4.52$$

$$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta \qquad 4.53$$

$$\tan(\alpha-\beta) = \frac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta} \qquad 4.54$$

$$\cot(\alpha-\beta) = \frac{\cot\alpha\cot\beta+1}{\cot\beta-\cot\alpha} \qquad 4.55$$

- *Half-angle formulas*

$$\sin\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{2}} \qquad 4.56$$

$$\cos\frac{\alpha}{2} = \pm\sqrt{\frac{1+\cos\alpha}{2}} \qquad 4.57$$

$$\tan\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}} \qquad 4.58$$

$$\cot\frac{\alpha}{2} = \pm\sqrt{\frac{1+\cos\alpha}{1-\cos\alpha}} \qquad 4.59$$

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Mathematics 1 1-10c

Trigonometric Identities

- *Miscellaneous formulas*

$$\sin\alpha\sin\beta = \tfrac{1}{2}\big(\cos(\alpha-\beta)-\cos(\alpha+\beta)\big) \qquad 4.60$$

$$\cos\alpha\cos\beta = \tfrac{1}{2}\big(\cos(\alpha-\beta)+\cos(\alpha+\beta)\big) \qquad 4.61$$

$$\sin\alpha\cos\beta = \tfrac{1}{2}\big(\sin(\alpha+\beta)+\sin(\alpha-\beta)\big) \qquad 4.62$$

$$\sin\alpha+\sin\beta = 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right) \qquad 4.63$$

$$\sin\alpha-\sin\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right) \qquad 4.64$$

$$\cos\alpha+\cos\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right) \qquad 4.65$$

$$\cos\alpha-\cos\beta = -2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right) \qquad 4.66$$

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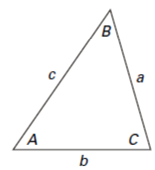
Mathematics 1 — 1-11a

General Triangles

Figure 4.11 General Triangle



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \qquad 4.67$$

$$a^2 = b^2 + c^2 - 2bc\cos A \qquad 4.68$$

$$b^2 = a^2 + c^2 - 2ac\cos B \qquad 4.69$$

$$c^2 = a^2 + b^2 - 2ab\cos C \qquad 4.70$$

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Mathematics 1 — 1-11b

General Triangles

Example (EFPRB):

MATHEMATICS–19

If $\sin\alpha = x$, what is $\sec\alpha$?

(A) $\sqrt{1-x^2}$ (B) $\frac{x}{\sqrt{1-x^2}}$ (C) $\frac{1}{\sqrt{1-x^2}}$ (D) $\frac{x}{\sqrt{1+x^2}}$

Since $\sin\alpha$ is the side facing angle α divided by the hypotenuse, the hypotenuse = 1. Therefore,

$$\text{side adjacent to angle } \alpha = \sqrt{1-x^2}$$

$$\sec\alpha = \frac{\text{hypotenuse}}{\text{side adjacent to angle } \alpha} = \frac{1}{\sqrt{1-x^2}}$$

The answer is (C).

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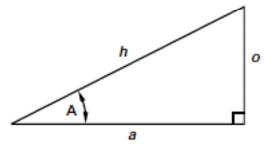
Mathematics 1 — 1-11c

General Triangles

Example (EFPRB):

MATHEMATICS–21

If the sine of angle A is given as K, what is the tangent of angle A?



(A) $\frac{hK}{o}$ (B) $\frac{aK}{h}$ (C) $\frac{ha}{K}$ (D) $\frac{hK}{a}$

$$\begin{aligned}\sin A &= \frac{o}{h} \\ &= K \\ \tan A &= \frac{o}{a} \\ &= \left(\frac{h}{a}\right)\left(\frac{o}{h}\right) \\ &= \frac{hK}{a}\end{aligned}$$

The answer is (D).

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Mathematics 1 — 1-11d

General Triangles

Example (EFPRB):

MATHEMATICS–22

Which is true regarding the signs of the natural functions for angles between 90° and 180°?

(A) The tangent is positive.
(B) The cotangent is positive.
(C) The cosine is negative.
(D) The sine is negative.

In the second quadrant, the natural functions and their signs are as follows.

sin	positive
cos	negative
tan	negative
cot	negative
sec	negative
csc	positive

The answer is (C).

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Mathematics 1 1-11e

General Triangles

Example (EFPRB):

MATHEMATICS–23

What is the inverse natural function of the cosecant?

(A) secant (B) sine (C) cosine (D) tangent

In a right triangle, the cosecant is the hypotenuse divided by the opposite side. The sine is the opposite side divided by the hypotenuse.

$$\sin\theta = \frac{1}{\csc\theta}$$

$\boxed{\text{The answer is (B).}}$

Mathematics 1 1-11f

General Triangles

Example (EFPRB):

MATHEMATICS–24

What is the sum of the squares of the sine and cosine of an angle?

(A) 0 (B) 1 (C) $\sqrt{3}$ (D) 2

For any angle,

$$\cos^2 x + \sin^2 x = 1$$

$\boxed{\text{The answer is (B).}}$

Mathematics 1 — 1-11g

General Triangles

Example (EFPRB):

MATHEMATICS–25

What is an equivalent expression for $\sin 2x$?

(A) $\frac{1}{2}\sin x\cos x$ (B) $2\sin x\cos\frac{1}{2}x$ (C) $-2\sin x\cos x$ (D) $\frac{2\sin x}{\sec x}$

The double angle formula for the sine function is

$$\begin{aligned}\sin 2x &= 2\sin x\cos x\\ &= \frac{2\sin x}{\sec x}\end{aligned}$$

The answer is (D).

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Mathematics 1 — 1-12a

Mensuration of Areas and Volumes

Figure 4.12 *Parabola*

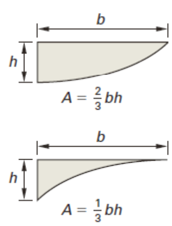
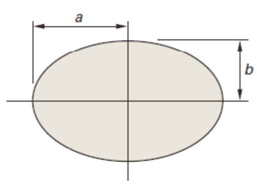


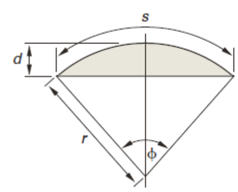
Figure 4.13 *Ellipse*



$A = \pi ab$

$P \approx 2\pi\sqrt{\frac{1}{2}(a^2 + b^2)}$

Figure 4.14 *Circular Segment*



$A = \frac{1}{2}r^2(\phi - \sin\phi)$ [ϕ must be in radians]

$\phi = \frac{s}{r} = 2\arccos\left(\frac{r-d}{r}\right)$

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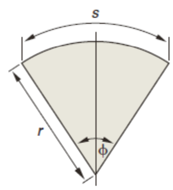
Mathematics 1 — 1-12b

Mensuration of Areas and Volumes

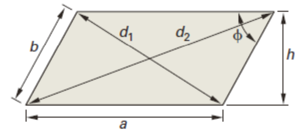
Figure 4.15 *Circular Sector*



$$A = \tfrac{1}{2}\phi r^2 = \tfrac{1}{2}sr$$

$$\phi = \frac{s}{r}$$

Figure 4.16 *Parallelogram*



$$P = 2(a + b)$$

$$d_1 = \sqrt{a^2 + b^2 - 2ab\cos\phi}$$

$$d_2 = \sqrt{a^2 + b^2 + 2ab\cos\phi}$$

$$d_1^2 + d_2^2 = 2(a^2 + b^2)$$

$$A = ah = ab\sin\phi$$

If $a = b$, the parallelogram is a rhombus.

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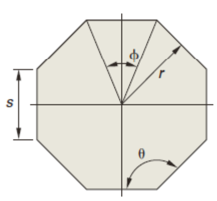
Mathematics 1 — 1-12c

Mensuration of Areas and Volumes

Figure 4.17 *Regular Polygon (n equal sides)*



$$\phi = \frac{2\pi}{n}$$

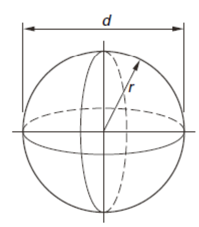
$$\theta = \frac{\pi(n-2)}{n} = \pi\left(1 - \frac{2}{n}\right) = \pi - \phi$$

$$P = ns$$

$$s = 2r\tan\frac{\phi}{2}$$

$$A = \frac{nsr}{2}$$

Figure 4.18 *Sphere*



$$V = \frac{4\pi r^3}{3} = \frac{\pi d^3}{6}$$

$$A = 4\pi r^2 = \pi d^2$$

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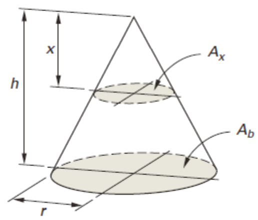
Mathematics 1 — 1-12d

Mensuration of Areas and Volumes

Figure 4.19 *Right Circular Cone*

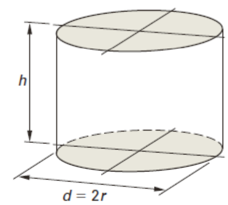


$$V = \frac{1}{3}\pi r^2 h$$

$$A = \text{side area} + \text{base area}$$
$$= \pi r\left(r + \sqrt{r^2 + h^2}\right)$$

$$A_x : A_b = x^2 : h^2$$

Figure 4.20 *Right Circular Cylinder*



$$V = \pi r^2 h = \frac{1}{4}\pi d^2 h$$
$$A = \text{side area} + \text{end areas}$$
$$= 2\pi r(h + r)$$

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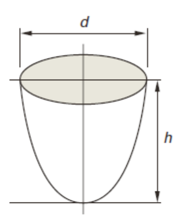
Mathematics 1 — 1-12e

Mensuration of Areas and Volumes

Figure 4.21 *Paraboloid of Revolution*



$$V = \frac{1}{8}\pi d^2 h$$

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