3.6.1

$m = 1, k = 7, b = 8, y(0) = -1,$
$v(0) = 5$

(a) write the second-order diff. equation and the corresponding 1st order system

$$\frac{d^2y}{dt^2} + 8\frac{dy}{dt} + 7y = 0 \rightarrow 2^{nd} \text{ order}$$

$dy/dt = v$; $dy/dt = v$; $\frac{dv}{dt} = \frac{d^2y}{dt^2}$

$$\frac{dv}{dt} = -7y - 8v$$

(b) Find the eigenvalues & eigenvectors of the linear system

$dy/dt = v$
$dv/dt = -7y - 8v$

$$\frac{dY}{dt} = \begin{pmatrix} 0 & 1 \\ -7 & -8 \end{pmatrix} Y$$

$$\begin{pmatrix} 0 & 1 \\ -7 & -8 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \lambda \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\begin{cases} 0 \cdot x + 1 \cdot y = \lambda x \rightarrow -\lambda x + y = 0 \\ -7 \cdot x + 8 \cdot y = \lambda y \rightarrow -7x + y(-8 - \lambda) = 0 \end{cases}$$

$$\begin{cases} -\lambda x + y = 0 \\ -7x + -8y - \lambda y = 0 \end{cases}$$

$$\begin{cases} -\lambda x + y = 0 \\ -7x + y(-8-\lambda) = 0 \end{cases}$$

$$\det\begin{pmatrix} -\lambda & 1 \\ -7 & (-8-\lambda) \end{pmatrix} = 0$$

$$(-\lambda)(-8-\lambda) - (1)(-7) = 0$$

$$8\lambda + \lambda^2 + 7 = 0$$

$$\lambda^2 + 8\lambda + 7 = 0$$

$$(\lambda+1)(\lambda+7) = 0$$

$$\left.\begin{array}{l} \lambda = -1 \\ \lambda = -7 \end{array}\right\} \text{ eigenvalues}$$

eigenvectors:

$$AV = \lambda V$$

$$\lambda_1 = -1 \quad \begin{pmatrix} 0 & 1 \\ -7 & -8 \end{pmatrix}\begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = -1\begin{pmatrix} x_1 \\ y_1 \end{pmatrix}$$

$y_1 = -1$

$$\begin{cases} \cancel{0 \cdot x_1} + 1 \cdot y_1 = -1x_1 \longrightarrow y_1 + x_1 = 0 \longrightarrow y_1 = -x_1 \\ -7 \cdot x_1 + -8 \cdot y_1 = -1y_1 \longrightarrow -7x_1 + -7y_1 = 0 \quad -7x_1 = 7y_1 \end{cases}$$

$$-7y_1 = 7x_1$$

$$x_1 = -1$$

$$V_1 = (y_1, v_1) \,;\, v_1 = -y_1$$

$\lambda_2 = -7$

similar approach to above!

$V_2 = (y_2, v_2)$

$v_2 = -7y_2$

(c) Classify the oscillator: overdamped

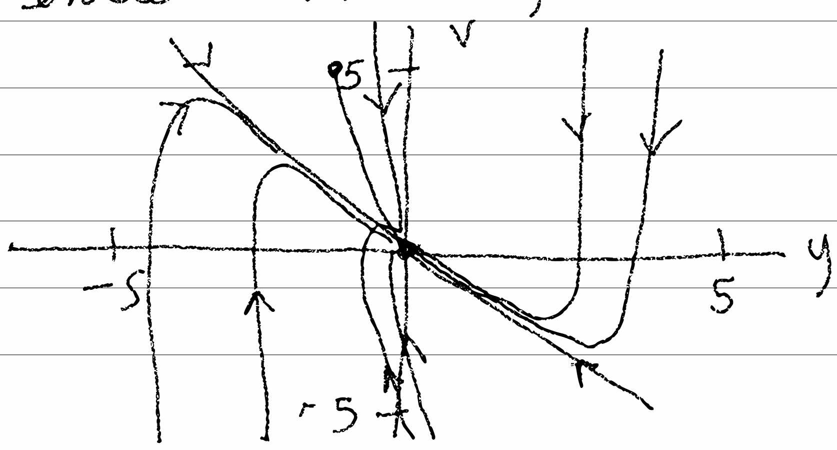
compute: $b^2 - 4km$

$$8^2 - 4(7)(1)$$

$$64 - 28 = 36 = +$$

(d)

sketch the phase portrait of the associated linear system and include the solution curve for the given initial condition;



(e) Sketch the y(t) & v(t) graphs of the soln w̄ the given initial condition.