3.5.1

Given: $\frac{dY}{dt} = \begin{pmatrix} -3 & 0 \\ 1 & -3 \end{pmatrix} Y$

w/ initial condition $Y_0 = (1, 0)$

ⓐ Find the eigenvalue

$$\begin{pmatrix} -3 & 0 \\ 1 & -3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \lambda \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\begin{cases} -3x + \cancel{0y} = \lambda x \rightarrow -\lambda x - 3x = 0 \\ 1x + -3y = \lambda y \rightarrow x - 3y - \lambda y = 0 \end{cases}$$

$$\begin{cases} x(-\lambda - 3) = 0 \\ x - y(-3 - \lambda) = 0 \end{cases}$$

$$\det \begin{pmatrix} (-\lambda - 3) & 0 \\ 1 & (-3 - \lambda) \end{pmatrix} = 0$$

$$(-\lambda - 3)(-3 - \lambda) - 1(0) = 0$$

$$\lambda = -3,\ \lambda = -3$$

Repeat eigenen value!

(b) Find an eigenvector

set $\lambda = -3$

$$\frac{dY}{dt} = AV = \lambda V$$

$$= \begin{pmatrix} -3 & 0 \\ 1 & -3 \end{pmatrix} \begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = -3 \begin{pmatrix} x_1 \\ y_1 \end{pmatrix}$$

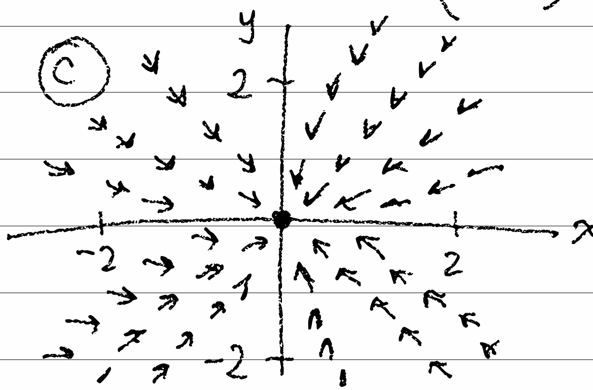
$$\begin{cases} -3x_1 + \cancel{0y_1} = -3x_1 \rightarrow 0x_1 = 0 \; ; \; x_1 : \\ x_1 + -3y_1 = -3y_1 \rightarrow x_1 + 0y_1 = 0 \end{cases}$$

any such ~~that~~: $0y_1 = x_1$

$x_1 = 0$ & $y_1 = 0, +, -$

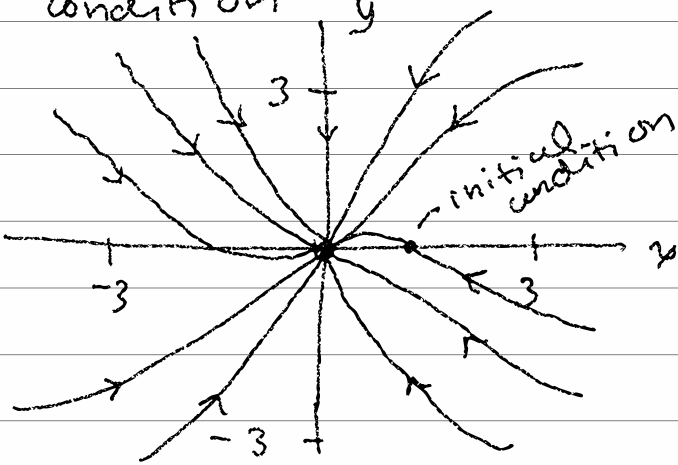
ex: $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

(c)

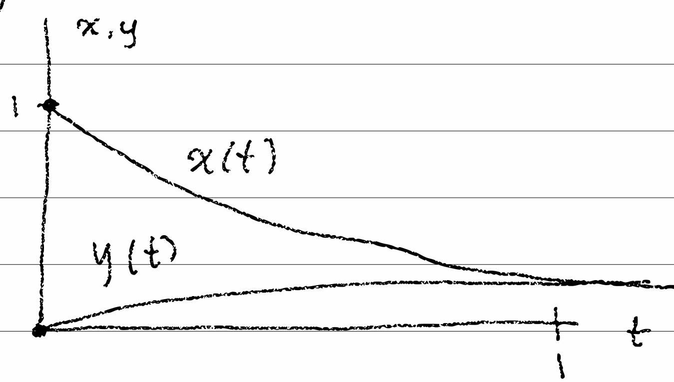


Sketch ~~the~~ direction field!

(d) sketch phase plane, including ~~the~~ soln $\bar{w}$ ~~the~~ given initial condition



(e) sketch ~~the~~ $x(t)$ & $y(t)$ graphs of ~~the~~ soln $\bar{w}$ ~~the~~ given initial condition!



3.5.5

for: $\frac{dY}{dt} = \begin{pmatrix} -3 & 0 \\ 1 & -3 \end{pmatrix} Y$ w $Y_0 = (1, 0)$

(a) find the general soln!

assume

$$\frac{dx}{dt} = -3x + 0y$$

$$\frac{dy}{dt} = x - 3y$$

We know for this system of, there are 2 repeat eigenvalues $(\lambda = -3)$ and the general soln has the form: $Y(t) = K_1 e^{\lambda t} V_1 + K_2 e^{\lambda t} (t V_1 + V_2)$

where:

$$Y_2(t) = e^{\lambda t} (t V_1 + V_2)$$

where V_2:

$$A V_2 - \lambda V_2 = V_1$$

$$K_1 e^{-3t} \begin{pmatrix} 0 \\ 1 \end{pmatrix} + K_2 e^{-3t} \left(t \begin{pmatrix} 0 \\ 1 \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right)$$ ✓

computation (incorrect?)

$$V_2 (A - \lambda) = V_1$$

$$V_2 = \frac{V_1}{A - \lambda} \quad \text{NOT CORRECT!}$$

$$\underbrace{V_2 (A - \lambda I)} = V_1 \; ; \; I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{pmatrix} -3 & 0 \\ 1 & -3 \end{pmatrix} + 3 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} -3 & 0 \\ 1 & -3 \end{pmatrix} + \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

$$V_2 \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$V_2 \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}^{-1} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}^{-1}$$

$$V_2 I = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \begin{pmatrix} 1/0 & 1/0 \\ 1/1 & 1/0 \end{pmatrix} \rightarrow \text{can't be!}$$

matrix can be inverted

if $\det \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \neq 0$

(b) Find the particular soln for the given initial condition;

Set $t=0$ to the general soln and solve for K_1 & K_2

$$K_1 e^{-3\theta}\begin{pmatrix}0\\1\end{pmatrix} + K_2 e^{-3\theta}\left(\theta\begin{pmatrix}0\\1\end{pmatrix}+\begin{pmatrix}1\\0\end{pmatrix}\right)=\begin{pmatrix}1\\0\end{pmatrix}$$

$$K_1 \cancel{e}^{0}\begin{pmatrix}0\\1\end{pmatrix} + K_2 \cancel{e}^{0}\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}1\\0\end{pmatrix}$$

$$K_1\begin{pmatrix}0\\1\end{pmatrix} + K_2\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}1\\0\end{pmatrix}$$

$$\cancel{K_1 \cdot 0} + K_2 \cdot 1 = 1 \qquad K_2 = 1$$

$$K_1 \cdot 1 + \cancel{K_2 \cdot 0} = 0 \qquad K_1 = 0$$

$$K_1 \cdot 1 = 0$$

$$Y(t) = 1 e^{-3t}\left(t\begin{pmatrix}0\\1\end{pmatrix}+\begin{pmatrix}1\\0\end{pmatrix}\right)$$

Recall, once you have the general soln & all vectors, you can solve for the initial value problem!

(c) Sketch the $x(t)$ & $y(t)$ graphs of the solution.