Topic A: Basic Numeracy and Basic Algebra

Dr Paolo Serafino

School of Computing
University of Portsmouth

September 30, 2024

Where will I ever use this stuff...

At least in the following modules:

- MATHFUN/Discrete Mathematics (CS)
- THEOCS (CS)
- Programming
- Operating Systems & Internetworking (CS, CSFC, CMP)
- Security Management

Key points

- negative numbers
- fractions
- order of operations, expanding brackets
- algebraic expressions, simplifying algebraic expressions
- solving simple linear equations

Negative numbers

- subtracting a negative number is equivalent to adding a positive number, for example

$$2-(-5)=2+5=7$$

Negative numbers

- the result of multiplying/dividing two numbers of the same sign (i.e. if either both are positive or both are negative) is always positive, for example

$$2 \times 3 = 6, \quad (-2) \times (-4) = 8, \quad \frac{-4}{-2} = 2$$

- the result of multiplying/dividing two numbers of opposite signs (i.e. one is positive and the other one is negative) is always negative, for example

$$2 \times (-3) = -6, \quad (-2) \times 4 = -8, \quad \frac{4}{-2} = -2$$

BIDMAS (BEDMAS, BIMDAS, . . .)

When evaluating numerical expressions the order in which operations are carried out is important, e.g. $2 + 3 \times 5 = 17$ (the multiplication is carried out first!). The order of carrying out arithemtical operations is:

First	Brackets ()
Second	Indices (powers, exponentials) a^b
Third	Division $\div$
Third	Multiplication $\times$
Fourth	Addition $+$
Fourth	Subtraction $-$

BIDMAS (BEDMAS, BIMDAS, ...)

Evaluate and check the results by clicking. One result is not correct, do you know which one?

$$\begin{aligned}
(2^3 - 3^2) \times 4 - 2 &= -6 \\
(2^3 - 3^2) \times (4 - 2) &= -2 \\
2^3 - 3^2 \times 4 - 2 &= -30 \\
2^3 - 3^2 \times (4 - 2) &= -10 \\
(2 - 3) \times (2 - 4) \times (-3) &= -6 \\
9 + 6 \div 2 \times (-3) &= 0 \\
\left(\frac{3^3}{3^2} + 6 \div 2 \times (-3)\right) \times 2 &= 2525
\end{aligned}$$

Equivalent fractions

$$\text{fraction} = \frac{\text{numerator}}{\text{denominator}} = \frac{p}{q}$$

Equivalent fractions: can be expressed in a different form, but always have the same value, for example the fractions $\frac{1}{2}, \frac{2}{4}, \frac{4}{8}, \frac{8}{16}, \ldots$ are equivalent.

Multiplying or dividing both numerator and denominator by the same number produces an equivalent fraction.

$$\frac{3}{6} = \frac{3 \times 2}{6 \times 2} = \frac{6}{12} = \frac{6 \div 6}{12 \div 6} = \frac{1}{2}$$

Addition/subtraction of fractions

- rewrite each fraction so that they will have the same denominator (compute the Least Common Multiple)

$$\frac{3}{10} + \frac{1}{4} = \frac{6}{20} + \frac{5}{20}$$

- calculate the sum/difference of the numerators

$$\frac{6}{20} + \frac{5}{20} = \frac{11}{20}$$

- simplify, if necessary $\frac{3}{10} + \frac{1}{4} = \frac{11}{20}$

Examples

Evaluate and check the results by clicking.

$$\frac{3}{4} + \frac{5}{6} - \frac{1}{2} = \frac{13}{12}$$

$$\frac{7}{8} - \frac{3}{4} + \frac{5}{12} = \frac{13}{24}$$

Multiplication of fractions

- numerator of the result: multiply the numerators, denominator of the result: multiply the denominators

$$\frac{3}{10} \times \frac{5}{8} = \frac{3 \times 5}{10 \times 8} = \frac{15}{80}$$

- find the simplest form of the result, if possible

Evaluate and check the results by clicking:

$$\frac{3}{4} \times \frac{5}{6} = \frac{5}{8}, \qquad \frac{3}{7} \times \frac{14}{6} = 1, \qquad \frac{2}{9} \times \frac{6}{20} = \frac{1}{15}$$

Division of fractions

- multiply the first fraction by the reciprocal of the second fraction (invert the fraction)

$$\frac{3}{10} \div \frac{5}{8} = \frac{3}{10} \times \frac{8}{5} = \frac{24}{50}$$

- simplify, if necessary $\frac{24}{50} = \frac{12}{25}$

Evaluate and check the results by clicking:

$$\frac{3}{4} \div \frac{5}{6} = \frac{9}{10}, \qquad \frac{3}{7} \div \frac{3}{7} = 1, \qquad \frac{2}{9} \div \frac{1}{3} = \frac{2}{3}$$

Simplest form of fractions

A fraction is in its simplest form when there are no factors other than 1 common to both numerator and denominator, for example $\frac{5}{17}, \frac{3}{7}, \frac{1}{8}$ are in their simplest form.

Express $\frac{24}{36}$ in its simplest form.

$$\frac{24}{36} = \frac{12}{18} = \frac{4}{6} = \frac{2}{3}$$

(or we could find the highest common factor of 24, 36 and use it directly).

Express the following fractions in their simplest form:

$$\frac{18}{24} = \frac{3}{4}, \quad \frac{15}{25} = \frac{3}{5}, \quad \frac{24}{48} = \frac{1}{2}, \quad \frac{16}{36} = \frac{4}{9}$$

What is algebra?

- So far we've talked only about **arithmetic**: whole numbers, fractions, ...
- We need to generalize and work with letters/symbols (variables or constants) instead of numbers – it is called algebra. (Nothing new to you!)

Algebra defines the exact rules how to manipulate expressions containing symbols.

Why algebra is so important?

Example 1. How can we convert Celsius temperatures from Fahrenheit? By a formula:

$$F = \frac{9}{5}C + 32,$$

where C is Celsius temperature, F is Fahrenheit.

- if $C = 10$ then $F = 50$,
- if $C = 5$ then $F = 41$,
- ...

Example 2. To find the area A of a rectangle, multiply the length L by the width W. The formula is $A = L \times W$. For a rectangle with a length of 8 cm and a witdh of 3 cm the area is: $A = L \times W = 8 \times 3 = 24$ cm^2

Addition/subtraction of like terms

- a term is either a single number or variable, or the product of several numbers and/or variables, for example 2, $3y$ ($3y$ means $3 \times y = y + y + y$), $4yz$, $2y^2$ ($2 \times y \times y$), $-4, x^3, y^4, \dots$
- a constant – a term without symbol, e.g. 2, -5, 3, …
- like terms are multiples of the same quantity (the same variables), for example $3y, 18y, -y, 25y$
- like terms can be added/subtracted in order to simplify the expression, for example $x + 3x + 6x = 10x$

Evaluate and check the results by clicking:

$$24y^2 + 7x + 12xy - 4x - 5y^2 + 3xy = 19y^2 + 15xy + 3x$$

Multiplying algebraic expressions

- multiplication of terms: multiply the numbers and multiply the variables (use rules for multiplication of indices if possible), for example $3a \times 4a = 12a^2$
- don't forget about the sign rules when multiplying terms with different signs, for example $2(-3b) = -6b$

Evaluate and check the results by clicking:

$(2b)(-4b^2) = -8b^3, \quad (2a)(6ab^2) = 12a^2b^2,$
$(4xy)(-2xy) = -8x^2y^2, \quad 4z(3yx) = 12xyz,$

Is there a difference between ab^2 and $(ab)^2$? Yes!

Removing brackets

In an expression such as $a(b+c)$, a is to be multiplied by all the bracketed terms:

$$a(b+c) = ab + ac$$
$$a(b-c) = ab - ac$$

In the expression $(a+b)(c+d)$, $(a+b)$ is to be multiplied by both the c and the d in the second pair of brackets:

$$(a+b)(c+d) = (a+b)c + (a+b)d = ac + bc + ad + bd$$

Removing brackets

Example. Expand $(x+6)(x-3)$.

$$\begin{aligned}(x+6)(x-3) &= (x+6)x + (x+6)(-3)\\ &= x^2 + 6x - 3x - 18\\ &= x^2 + 3x - 18\end{aligned}$$

Example. Expand $3(x+1)(x-1)$.

$$\begin{aligned}3(x+1)(x-1) &= 3((x+1)x + (x+1)(-1))\\ &= 3(x^2 + x - x - 1)\\ &= 3(x^2 - 1)\\ &= 3x^2 - 3\end{aligned}$$

Substitution/evaluation

Substitution means replacing letters by actual numerical values.

Example. Evaluate mk when $m = 5$, $k = -3$.

$$mk = 5 \times (-3) = -15$$

Example. Find the value of a^3 when $a = 3$.

$$a^3 = 3^3 = 27$$

Solving simple linear equations

An equation states that two quantities are equal – will always contains an unknown quantity that needs to be found!

To solve an equation means to find all value of the unknown quantity that can be substituted into the equation so that the left side equals the right side.

Each such values is called a solution.

A linear equation is one of the form $ax + b = 0$, a, b given numbers, x unknown quantity.

Example. $4x + 13 = -7$ is a linear equation ...

Solving simple linear equations

Example. Solve the equation $4x + 8 = 0$.
Subtracting 8 from both sides we find:

$$4x + 8 - 8 = 0 - 8 = -8$$

That is

$$4x = -8$$

Then dividing both sides by 4 gives:

$$\frac{4x}{4} = \frac{-8}{4}$$

so that $x = -2$ is the solution.