Topic C: Powers

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2024/2025

Key points

- laws for manipulating indices
- simplifying expressions involving indices
- the use of negative powers
- square roots, cube roots and fractional powers

Where will I ever use this?!?

- Operating Systems & Internetworking (CS, CSFC, CMP)
- Security and Cryptography (CSFC, CS, CN, SE) for Diffie-Hellman Key Exchange and RSA
- THEOCS (CS degree)

Introduction

Examples:

2^4 is read as '2 *to the power of* 4' or '2 *raised to the power of* 4' and it means

$$2^4 = 2 \times 2 \times 2 \times 2 = 16$$

2 is the **base** and 4 is the **index** or **power**

Now try:

(a) $5^2 = 5 \times 5 = 25$
(b) $3^3 = 3 \times 3 \times 3 = 27$

a^0, a^1

Generalising:
If a is any number and n a positive whole number, then:

$$a^n = \underbrace{a \times a \times \cdots \times a}_{n}$$

You need to remember:
$a^0 = 1$ anything (except 0) raised to the power of zero is 1, 0^0 is undefined (has no meaning)

$a^1 = a$

Exercises

Now try:

(a) $5^0 = 1$
(b) $3^1 = 3$
(c) $6^3 = 6 \times 6 \times 6 = 216$
(d) $3^0 = 1$

Laws of indices

The following rules apply to powers:

(I) $a^n \times a^m = a^{n+m}$ (when multiplying, add the indices)
Example: $2^4 \times 2^2 = 2^{4+2} = 2^6 = 64$

(II) $\dfrac{a^n}{a^m} = a^{n-m}$ (when dividing, substract the indices)
Example: $\dfrac{3^4}{3^3} = 3^{4-3} = 3^1 = 3$

(III) $(a^n)^m = a^{n \times m}$ (when raising one power to another, multiply the indices)
Example: $(2^4)^2 = 2^{4\times 2} = 2^8 = 256$

Exercises

Now try:

(a) $3^{-5} \times 3^7 = 3^2 = 9$

(b) $2^2 \times 2^3 = 2^5 = 32$

(c) $\dfrac{6^6}{6^4} = 6^2 = 36$

(d) $\dfrac{5^2}{5^{-1}} = 5^3 = 125$

(e) $4x^2 \times 3x^5 = 12x^7$
(the numbers are multiplied, but the powers of the same variable are added)

(f) $(2x^5)^2 = 4x^{10}$

(g) $\dfrac{12x^4}{3x} = 4x^3$

Negative Powers

What is $\dfrac{a^3}{a^5}$?

- Using the rule for division $\dfrac{a^3}{a^5} = a^{-2}$.
- Simplifying the expression we get

$$\frac{a^3}{a^5} = \frac{a \times a \times a}{a \times a \times a \times a \times a} = \frac{1}{a^2}$$

Thus $\dfrac{1}{a^2} = a^{-2}$.

It is important to remember the following general rule

$$a^{-n} = \frac{1}{a^n}$$

Exercises

Now try:

(a) $\dfrac{3^5}{3^7} = 3^5 \div 3^7 = 3^{-2} = \dfrac{1}{3^2} = \dfrac{1}{9}$

(b) $2^{-2} \div 2^3 = 2^{-5} = \dfrac{1}{2^5} = \dfrac{1}{32}$

(c) $\dfrac{6^6}{6^4} = 6^2 = 36$

(d) $\dfrac{5^2}{5^{-1}} = 5^3 = 125$

(e) $12x^4 \div 3x^5 = 4x^{-1}$
(the numbers are divided and the powers of the same variable are subtracted)

Fractional Powers

What is the square root of 36? The anwer is 6 since $6^2 = 36$.
We can rewrite it as $\sqrt{36} = 36^{\frac{1}{2}} = 6$
The general rule is: if a, n are positive numbers, then

$$a^{\frac{1}{n}} = \sqrt[n]{a}$$

Now practise:

(a) $100^{\frac{1}{2}} = \sqrt{100} = 10$

(b) $27^{\frac{1}{3}} = \sqrt[3]{27} = 3$ (since $3 \times 3 \times 3 = 27$)

Fractional Powers

Important for counting is the following:

$$(a^{\frac{1}{n}})^m = a^{\frac{m}{n}} = (a^m)^{\frac{1}{n}}$$

Example:

$$81^{\frac{3}{4}} = (81^{\frac{1}{4}})^3 = 3^3 = 27$$

Practise:

(a) $(8^5)^{\frac{1}{3}} = \left(8^{\frac{1}{3}}\right)^5 = 2^5 = 32$

(b) $[(a^{-\frac{1}{4}})^8]^{\frac{2}{3}} = (a^{-2})^{\frac{2}{3}} = a^{-\frac{4}{3}}$

(c) $(a^{\frac{1}{3}})^2 \times (a^{-\frac{1}{3}})^2 = a^{\frac{2}{3}} \times a^{-\frac{2}{3}} = a^0 = 1$

Exercises

Now try to simplify the following expressions:

(a)

$$\sqrt{\frac{72a^{12}b^7c^2}{2a^2b^3c^{-10}}} = \sqrt{36a^{10}b^4c^{12}} = (36a^{10}b^4c^{12})^{\frac{1}{2}} = 6a^5b^2c^6$$

(b)

$$\frac{a^4b^{-7}c^2}{a^2b^4c^{-8}} \div \frac{abc}{a^2b^3c^{-1}} = a^2b^{-11}c^{10} \div a^{-1}b^{-2}c^2 = a^3b^{-9}c^8$$