Topic B: Everyday Maths

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Key points

- Percentage, Profit
- Ratio
- Average
- Basic Probability
- Miscellaneous (exchange ratios, areas, . . .)

Always think of a program/function which would solve our examples or their generalisation!

Percentage

A percentage is a fraction whose denominator is 100.

"Percent" corresponds to "per 100"

So,

100% means all,

50% means half (of all)

25% means a quarter (of all)

5% means 5/100ths (of all) . . .

Percentage – examples

Example 1. Peter is required to pay a 25% deposit on his monthly rent of £300 in advance. How much is Peter's deposit in pounds?

Solution.
1st approach: 25% is $\frac{1}{4}$ ($why\ ? = \frac{25}{100}$) of the total , hence $\frac{1}{4}$ of £300 is £75.
To summarise: $\frac{25}{100} \times 300 = 75$

2nd approach: 100% is 300. How much is 1%?
1% is $\frac{300}{100} = 3$.
And now how much is 25%? $\ldots 25 \times 3 = 75$
To summarise: $\frac{300}{100} \times 25 = 75$

Now think of a program with deposit in % and rent as the inputs and deposit in £ as an output!

Percentage – examples

Example 2. (a) If only 10 of 100 apples were bad, what percentage is that?

Solution. 10% ... *Why?*

$\frac{10}{100} = \frac{1}{10}$th of all were bad, hence $\frac{1}{10}$ of 100% which means 10%.

Another approach:
100 apples is 100%, hence 1 apple is 1%

Hence, 10 apples is 10%

Percentage – examples

Example 2(cont).
(b) If only 10 of 200 apples were bad, what percentage is that?

Solution. 5% ... *Why?*

$\frac{10}{200} = \frac{1}{20}$th of all were bad, hence $\frac{1}{20}$ of 100% which means 5% .

Another approach:
200 apples is 100%, hence 1 apple is $\frac{100}{200} = 0.5\%$

Hence, 10 apples is 5%

Percentage – examples

Example 2(cont). (c) If only 10 of 50 apples were bad, what percentage is that?

Solution. 20% ... *Why?*

$\frac{10}{50} = \frac{1}{5}$th of all were bad, hence $\frac{1}{5}$ of 100% which means 20%

Now think of a program with number of apples/bad apples as inputs and % as output!

$$\frac{\text{number of bad apples}}{\text{total number of apples}} \times 100\%$$

Percentage – examples

Example 3. The price of a television is reduced by 20% in the sales. It now costs £250. What was the original price?

Solution. The sale price is 100% - 20% = 80% of the pre-sale price.
Hence 1% is $\frac{250}{80} = 3.125$
and all 100% equals $3.125 \times 100 = 312.50$.
The original price was £312.50.

Now think of a program with given % reduction/reduced price as inputs and the original price as output!

$$\frac{\text{reduced price}}{\text{100 - \% reduction}} \times 100$$

Percentage – examples

Example 4 (from programming). `Write a` **futureValue** `function that uses a loop to calculate the future value of an investment amount, assuming an annual interest rate of 5.5%. The function should ask the user for the initial amount and the number of years that it is to be invested, and should output the final value of the investment using compound interest with the interest compounded every year.`

OK, first we need to translate the problem to our maths language. . .

Solution.
Let an initial amount be £200.

After the 1st year: 105.5% of 200, means
$1.055 \times 200 = 211$. *Why?*
100% is 200, 105.5% is $\frac{105.5}{100} \times 200$

After the 2nd year: 105.5% of 211, or 105.5% of 105.5% of 200, hence:

$$1.055 \times 1.055 \times 200$$

After k years: $\underbrace{1.055 \times \cdots \times 1.055}_{k} \times 200$

`Back to programming`: in the loop "number of years" multiply 1.055 by itself, then multiply the result by the initial amount.

Percentage – examples

Example 5. Mike is taking our Programming module. He has received the following marks:

I/1: Java tests (20%): 36

I/2: Python tests (20%): 56

I/3: maths part (20%): 95

II: Python coursework (20%): 47

III: Java coursework (20%): 25

Calculate Mike's final mark for the unit.

Percentage – examples

Solution: Mark for part I/1 is 36 (out of 100%) and that is 20% of the overall mark.Hence the overall contribution is:

$$\frac{36}{100} \times 20 = 7.2$$

Similarly for the other parts, and summing together:

$$\frac{36}{100}\times 20+\frac{56}{100}\times 20+\frac{95}{100}\times 20+\frac{47}{100}\times 20+\frac{25}{100}\times 20 = 51.8$$

So Mike's final mark would be 52 (not bad :-), but mainly due his performance in the maths part!)

Now think of a program with given marks as inputs and the final mark as output!

Ratio

A ratio is used to compare two or more quantities. The symbol for '**Compared to**' is a **colon** (:). To simplify ratios, divide both parts of the ratio by the highest common factor.

Ratio – examples

Example 6. A forest has 25 000 trees. Oak and ash trees are present in the forest in the ratio 2 : 3. How many ash trees are there? What percentage of the trees are ash trees?

Solution.

- $2 + 3 = 5$ parts
- 5 parts is 25 000
- 1 part is $\frac{25000}{5} = 5\,000$

Ash has 3 parts, so $3 \times 5000 = 15\,000$.

Percentage: $\frac{15000}{25000} \times 100 = 60\%$.
Or 1 part corresponds to 20%, so 3 parts corresponds to 60%.

Ratio – examples

Example 7. The same brand of breakfast cereal is sold in two different-sized packets. Which packet is better value for money?

A: 125 g cost £1.06

B: 750 g cost £2.81

Solution.

- Cost of 1 g of A is $\frac{106}{125} = 0.848$ [pence per gram]
- Cost of 1 g of B is $\frac{281}{750} = 0.3746$ [pence per gram]

Since the 750 g packet costs less per gram, it is better value for money.

Ratio – examples

Example 8. It took 8 people 6 days to build a house. At the same rate how long would it take 3 people?

Solution.

- Time for 8 people: 6 days
- Time for 1 person: $8 \times 6 = 48$ days
- Time for 3 people: $\frac{48}{3} = 16$ days

Think of a program with the number of days and the number of people (in both cases) as inputs and the number of days in the second case as output!

Average

Average sometimes known as the mean can be calculated as:

$$\text{average} = \frac{\text{sum of a set of values}}{\text{number of values}}$$

Example 9. A football team scored the following number of goals in the first ten matches:
2, 4, 0, 1, 2, 2, 3, 6, 2, 4
Find the average number of goals scored.

Solution. Average $= \frac{2+4+0+1+2+2+3+6+2+4}{10} = 2.6$ goals

Think of a program with certain (?) numbers as inputs and the average as the output!

Average – examples

Example 10. The average of four numbers is 7. Three of the numbers are 10, 4 and 8. Find the value of the fourth number.

Solution.
The sum of the 4 numbers is 28 (*why?*).
If x is the missing number, then

$$10 + 4 + 8 + x = 28.$$

The fourth number is 6.

Basic probability

- Probability expresses how likely it is for some event to happen.
- Probability is expressed as a (decimal) number, all probabilities lie between 0 and 1. No event has a probability less than 0 or greater than 1.

Some probabilities can be calculated using the fact that each outcome is equally likely, if this is the case.

$$P(\text{event}) = \frac{\text{number of outcomes that constitute the event}}{\text{total number of possible outcomes}}$$

Basic probability – examples

Example 11. A bag has 3 red, 4 green and 7 blue balls in it. If John takes out a ball at random, what is the probability that it is:

- a red ball
  Answer. $P(\text{red}) = \frac{3}{14}$
- a red or green ball
  Answer. $P(\text{red or green}) = \frac{3}{14} + \frac{4}{14} = \frac{7}{14} = \frac{1}{2}$
- a orange ball
  Answer. $P(\text{orange}) = \frac{0}{14} = 0$
- a red, green or blue ball
  Answer. $P(\text{red, green or blue}) = \frac{14}{14} = 1$

Miscellaneous

Example 12 (from programming). Write a function **costOfStone** that asks the user for the diameter (not the radius) of a circular garden (in m), and then outputs the cost of the ornamental stones. Suppose the user wants to use the ornamental stones for only 65% of the garden area. Assume that the cost of the ornamental stones is £8.20 per square metre.

Miscellaneous

Solution.

- the diameter is d, the radius is $\frac{d}{2}$
- the total area of the garden: $\pi\frac{d^2}{4}$,
- the area of the garden with the ornamental stones: $\pi\frac{d^2}{4} \times 0.65$
- the cost of the ornamental stones: $\pi\frac{d^2}{4} \times 0.65 \times 8.20$

Now think about a program . . . !

Miscellaneous

Example 13 (from programming). Write a function **euros2pounds** which converts an amount in Euros entered by the user to a corresponding amount in Pounds. Assume that the exchange rate is 1.26 Euros to the Pound.

Miscellaneous

Solution.

- let d be an amount in Euros entered by the user
- 1.26 Euros corresponds to 1 Pound, hence 1 Euro corresponds to $\frac{1}{1.26} = 0.79365 \cdots \approx 0.794$ Pounds
- hence d Euros: $d \times 0.794$

Now it is easy to think about a program ...!