Powers

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2,40,0,505

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### Topic C: Powers

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## Key points

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- laws for manipulating indices
- simplifying expressions involving indices
- the use of negative powers
- square roots, cube roots and fractional powers

#### Where will I ever use this?!?

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- Operating Systems & Internetworking (CS, CSFC, CMP)
- Security and Cryptography (CSFC, CS, CN, SE) for Diffie-Hellman Key Exchange and RSA
- THEOCS (CS degree)

#### Introduction

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#### **Examples:**

2<sup>4</sup> is read as '2 to the power of 4' or '2 raised to the power of 4' and it means

$$2^4 = 2 \times 2 \times 2 \times 2 = 16$$

2 is the base and 4 is the index or power

Now try:

(a) 
$$5^2 = 5 \times 5 = 25$$

(b) 
$$3^3 = 3 \times 3 \times 3 = 27$$

$$a^0, a^1$$

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#### **Generalising:**

If a is any number and n a positive whole number, then:

$$a^n = \underbrace{a \times a \times \cdots \times a}_n$$

You need to remember:

 $a^0 = 1$  anything (except 0) raised to the power of zero is 1,  $0^0$  is undefined (has no meaning)

$$a^1 = a$$

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Now try:

(a) 
$$5^0 = 1$$

(b) 
$$3^1 = 3$$

(c) 
$$6^3 = 6 \times 6 \times 6 = 216$$

(d) 
$$3^0 = 1$$

### Laws of indices

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The following rules apply to powers:

(I) 
$$a^n \times a^m = a^{n+m}$$
 (when multiplying, add the indices)

Example:  $2^4 \times 2^2 = 2^{4+2} = 2^6 = 64$ 

(II) 
$$\frac{a^n}{a^m} = a^{n-m}$$
 (when dividing, substract the indices)

Example: 
$$\frac{3^4}{3^3} = 3^{4-3} = 3^1 = 3$$

(III)  $(a^n)^m = a^{n \times m}$  (when raising one power to another, multiply the indices)

Example: 
$$(2^4)^2 = 2^{4 \times 2} = 2^8 = 256$$

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Now try:

(a)  $3^{-5} \times 3^7 = 3^2 = 9$ 

(b)  $2^2 \times 2^3 = 2^5 = 32$ 

(c)  $\frac{6^6}{6^4} = 6^2 = 36$ 

(d)  $\frac{5^2}{5^{-1}} = 5^3 = 125$ 

(e)  $4x^2 \times 3x^5 = 12x^7$ 

(the numbers are multiplied, but the powers of the same

variable are added) (f)  $(2x^5)^2 = 4x^{10}$ 

(g)  $\frac{12x^4}{3x} = 4x^3$ 

### **Negative Powers**

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What is  $\frac{a^3}{a^5}$ ?

- Using the rule for division  $\frac{a^3}{a^5} = a^{-2}$ .
- Simplifying the expression we get

$$\frac{a^3}{a^5} = \frac{a \times a \times a}{a \times a \times a \times a \times a} = \frac{1}{a^2}$$

Thus 
$$\frac{1}{a^2} = a^{-2}$$
.

It is important to remember the following general rule

$$a^{-n} = \frac{1}{a^n}$$

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Now try:

(a)  $\frac{3^5}{3^7} = 3^5 \div 3^7 = 3^{-2} = \frac{1}{3^2} = \frac{1}{9}$ 

(b) 
$$2^{-2} \div 2^3 = 2^{-5} = \frac{1}{2^5} = \frac{1}{32}$$

(c) 
$$\frac{6^6}{6^4} = 6^2 = 36$$

(d) 
$$\frac{5^2}{5^{-1}} = 5^3 = 125$$

(e)  $12x^4 \div 3x^5 = 4x^{-1}$  (the numbers are divided and the powers of the same

variable are subtracted)

#### **Fractional Powers**

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What is the square root of 36? The anwer is 6 since  $6^2 = 36$ .

We can rewrite it as  $\sqrt{36} = 36^{\frac{1}{2}} = 6$ 

The general rule is: if a, n are positive numbers, then

$$a^{\frac{1}{n}} = \sqrt[n]{a}$$

Now practise:

(a) 
$$100^{\frac{1}{2}} = \sqrt{100} = 10$$

(b) 
$$27^{\frac{1}{3}} = \sqrt[3]{27} = 3$$
 (since  $3 \times 3 \times 3 = 27$ )

### Fractional Powers

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Important for counting is the following:

$$(a^{\frac{1}{n}})^m = a^{\frac{m}{n}} = (a^m)^{\frac{1}{n}}$$

Example:

$$81^{\frac{3}{4}} = (81^{\frac{1}{4}})^3 = 3^3 = 27$$

Practise:

(a) 
$$(8^5)^{\frac{1}{3}} = (8^{\frac{1}{3}})^5 = 2^5 = 32$$

(b) 
$$[(a^{-\frac{1}{4}})^8]^{\frac{2}{3}} = (a^{-2})^{\frac{2}{3}} = a^{-\frac{4}{3}}$$

(c) 
$$(a^{\frac{1}{3}})^2 \times (a^{-\frac{1}{3}})^2 = a^{\frac{2}{3}} \times a^{-\frac{2}{3}} = a^0 = 1$$

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Now try to simplify the following expressions: (a)

$$\sqrt{\frac{72a^{12}b^7c^2}{2a^2b^3c^{-10}}} = \sqrt{36a^{10}b^4c^{12}} = (36a^{10}b^4c^{12})^{\frac{1}{2}} = 6a^5b^2c^6$$

$$\frac{a^4b^{-7}c^2}{a^2b^4c^{-8}} \div \frac{abc}{a^2b^3c^{-1}} = a^2b^{-11}c^{10} \div a^{-1}b^{-2}c^2 = a^3b^{-9}c^8$$