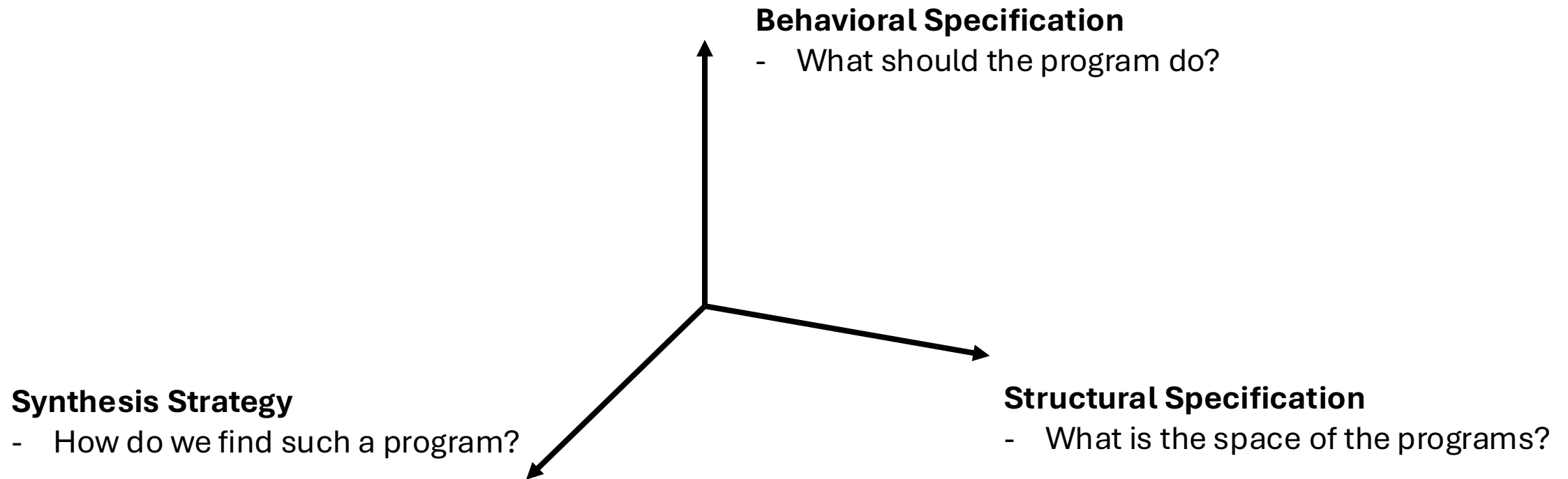


Machine Programming

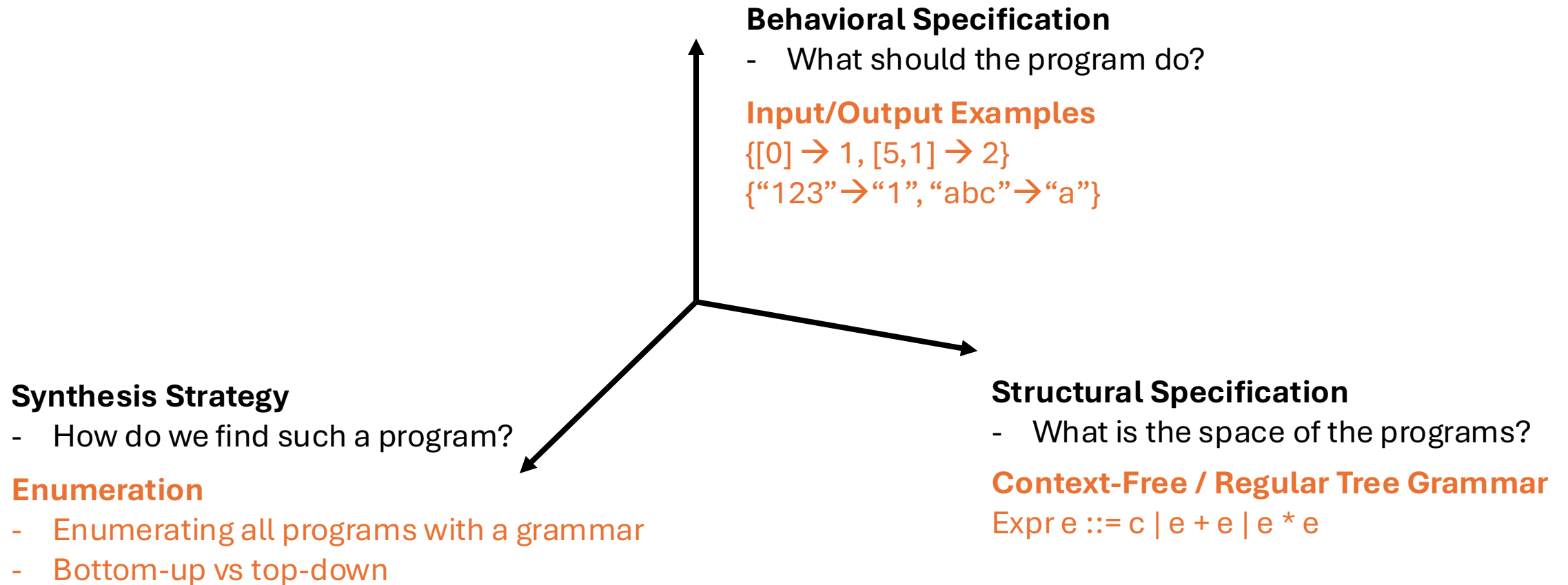
Lecture 2 – Inductive Synthesis

Ziyang Li

Dimensions in Program Synthesis



Today



Inductive Synthesis

=

Programming by Example

=

Inductive Program Synthesis

=

Inductive Programming

=

Inductive Learning

{“123”→“1”, “abc”→“a”} ➔ ???

{“123”→“1”, “abc”→“a”} ➔ input[0]

{“123”→“1”, “abc”→“a”}

{[0] → 1, [5,1] → 2}



input[0]



???

{“123”→“1”, “abc”→“a”}

{[0] → 1, [5,1] → 2}



input[0]



len(input)

{“123”→“1”, “abc”→“a”}
{[0] → 1, [5,1] → 2}



input[0], input[0:1], ...



len(input) , min(input) + 1, ...

High-level Picture

- Learning **abstraction** / **generalization** from a set of observations

Program Synthesis

$\{[0] \rightarrow 1, [5, 1] \rightarrow 2\}$ \rightarrow $\text{min}(\text{input}) + 1$
 $\{“123” \rightarrow “1”, “abc” \rightarrow “a”\}$ \rightarrow $\text{chars}(\text{input})[0]$

Deep Learning



MIT/LCS/TR-76

LEARNING STRUCTURAL DESCRIPTIONS FROM EXAMPLES

Patrick H. Winston

September 1970

LEARNING STRUCTURAL DESCRIPTIONS FROM EXAMPLES

Patrick H. Winston

September 1970

Abstract

The research here described centers on how a machine can recognize concepts and learn concepts to be recognized. Explanations are found in computer programs that build and manipulate abstract descriptions of scenes such as those children construct from toy blocks. One program uses sample scenes to create models of simple configurations like the three-brick arch. Another uses the resulting models in making identifications. Throughout emphasis is given to the importance of using good descriptions when exploring how machines can come to perceive and understand the visual environment.

High-level Picture

- Learning **abstraction** / **generalization** from a set of observations

Program Synthesis

$\{[0] \rightarrow 1, [5, 1] \rightarrow 2\}$ \rightarrow $\text{min}(\text{input}) + 1$
 $\{“123” \rightarrow “1”, “abc” \rightarrow “a”\}$ \rightarrow $\text{chars}(\text{input})[0]$

Deep Learning



Space of Programs

Program Synthesis

$\{[0] \rightarrow 1, [5, 1] \rightarrow 2\}$ \rightarrow `min(input) + 1`
 $\{"123" \rightarrow "1", "abc" \rightarrow "a"\}$ \rightarrow `chars(input)[0]`

Deep Learning



Space of Programs

Program Synthesis

$\{[0] \rightarrow 1, [5, 1] \rightarrow 2\}$ \rightarrow $\text{min}(\text{input}) + 1$
 $\{“123” \rightarrow “1”, “abc” \rightarrow “a”\}$ \rightarrow $\text{chars}(\text{input})[0]$

Deep Learning



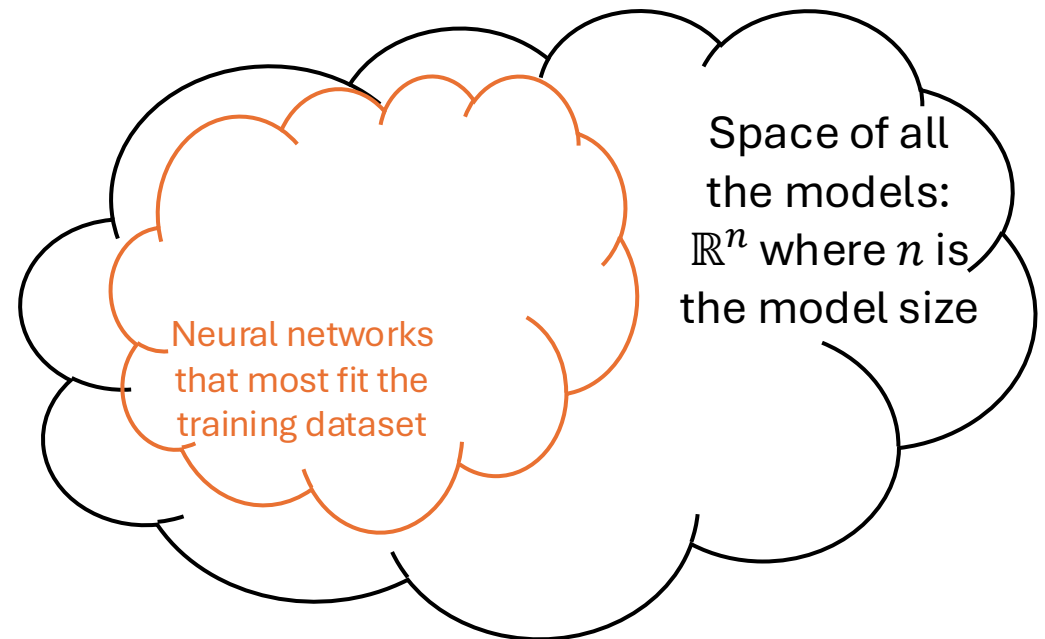
Space of all
the models:
 \mathbb{R}^n where n is
the model size

Space of Programs

Program Synthesis

$\{[0] \rightarrow 1, [5, 1] \rightarrow 2\}$ \rightarrow $\text{min}(\text{input}) + 1$
 $\{“123” \rightarrow “1”, “abc” \rightarrow “a”\}$ \rightarrow $\text{chars}(\text{input})[0]$

Deep Learning

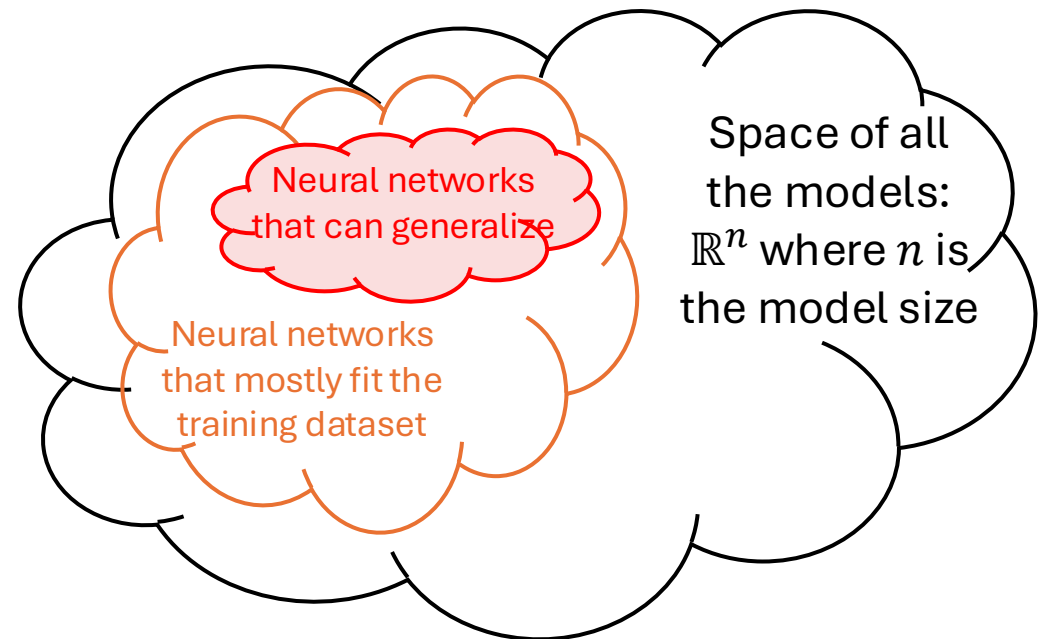


Space of Programs

Program Synthesis

$\{[0] \rightarrow 1, [5, 1] \rightarrow 2\} \rightarrow \text{min(input)} + 1$
 $\{"123" \rightarrow "1", "abc" \rightarrow "a"\} \rightarrow \text{chars(input)[0]}$

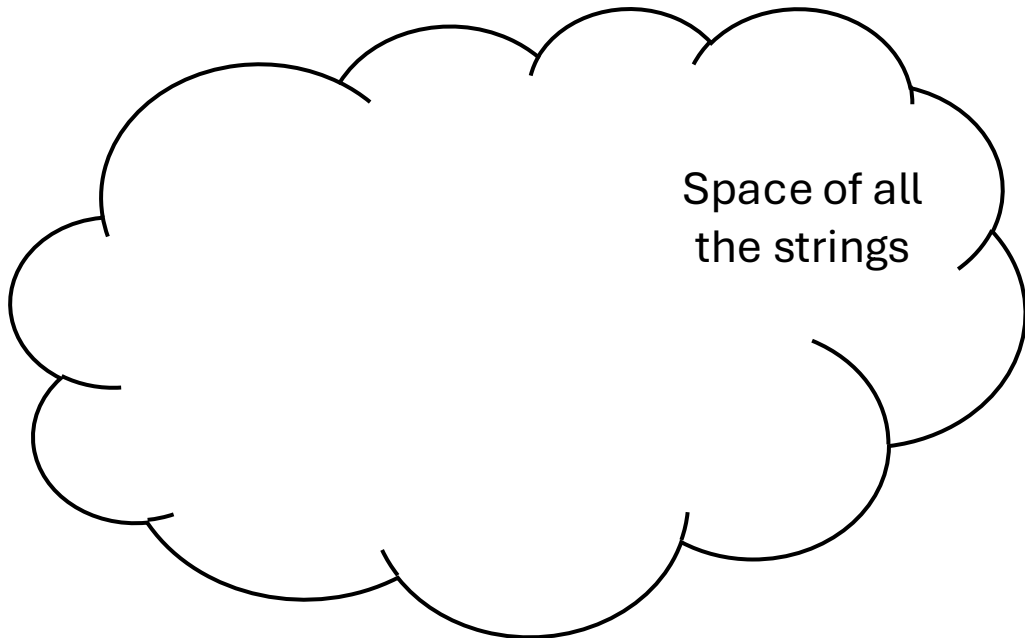
Deep Learning



Space of Programs

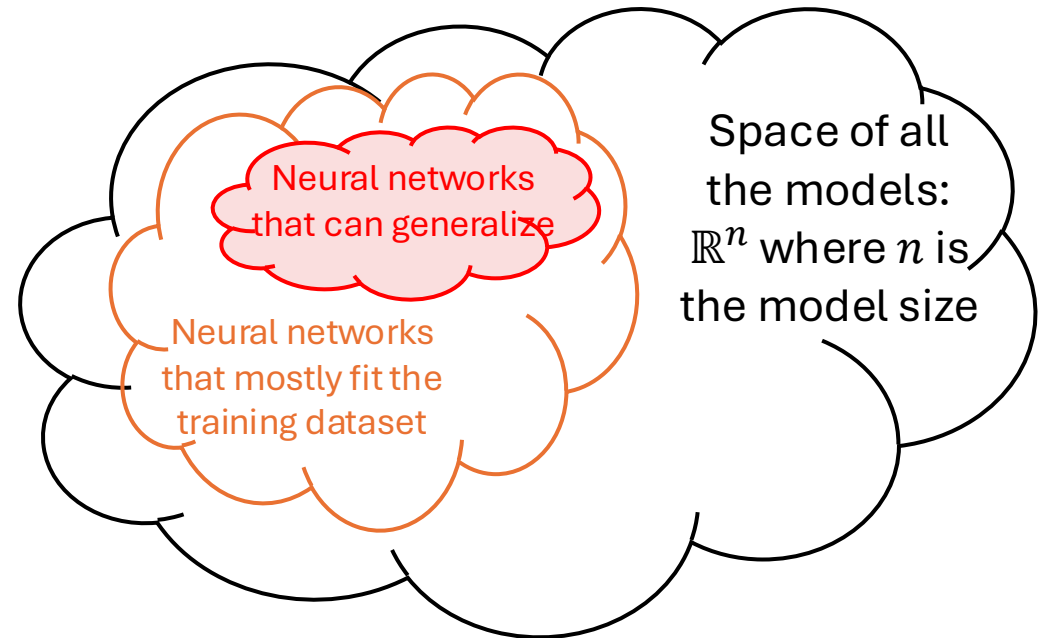
Program Synthesis

$\{[0] \rightarrow 1, [5, 1] \rightarrow 2\} \rightarrow \text{min(input)} + 1$
 $\{"123" \rightarrow "1", "abc" \rightarrow "a"\} \rightarrow \text{chars(input)[0]}$



Deep Learning

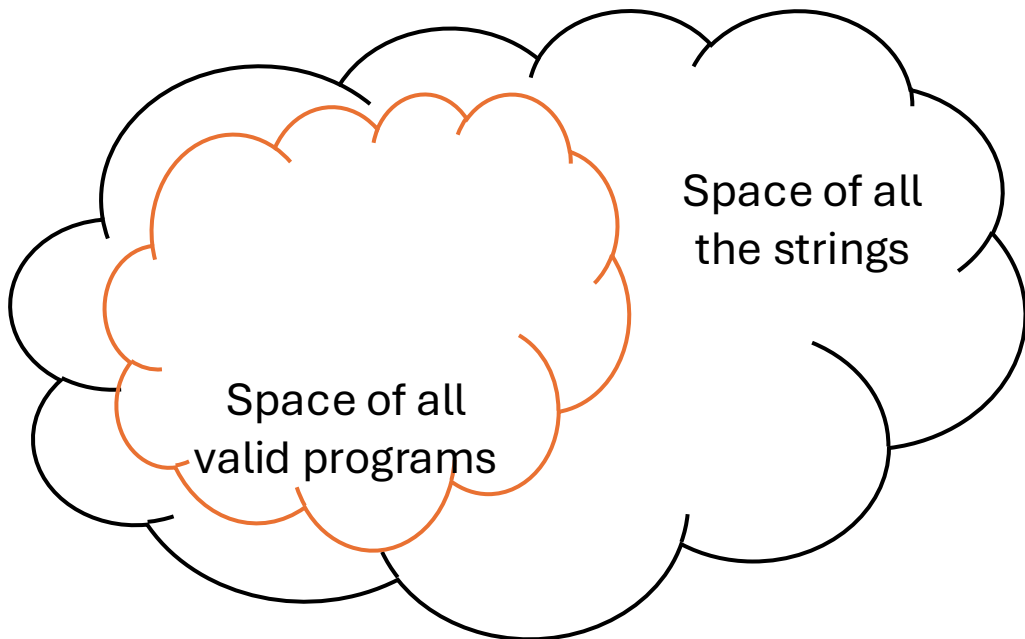
$\{ \text{dog image} \rightarrow \text{dog}, \text{cat image} \rightarrow \text{cat} \} \rightarrow$ 



Space of Programs

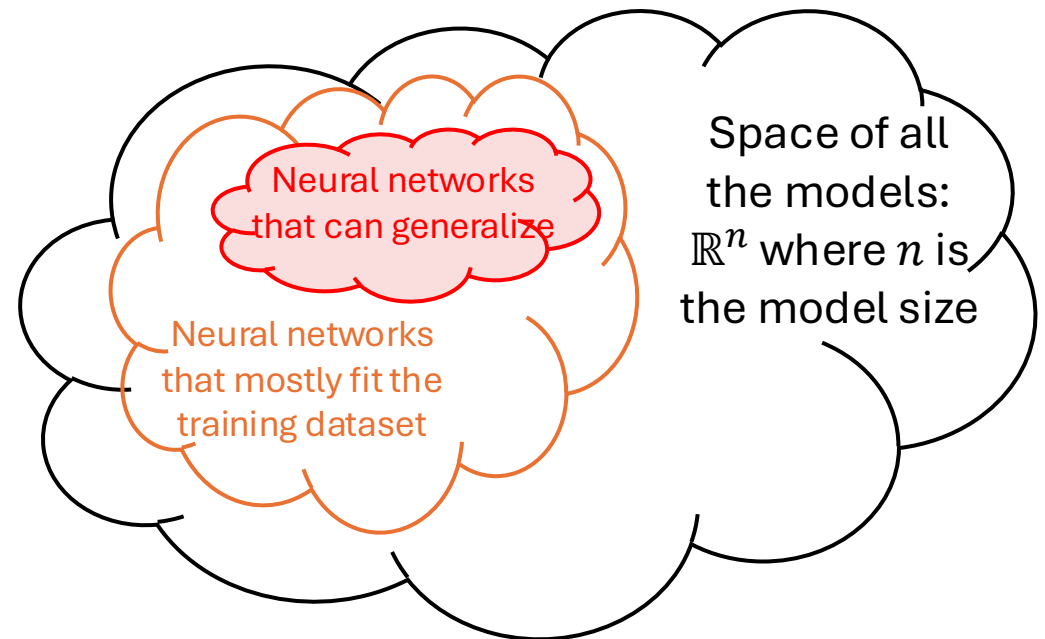
Program Synthesis

$\{[0] \rightarrow 1, [5, 1] \rightarrow 2\}$ \rightarrow $\text{min}(\text{input}) + 1$
 $\{“123” \rightarrow “1”, “abc” \rightarrow “a”\}$ \rightarrow $\text{chars}(\text{input})[0]$



Deep Learning

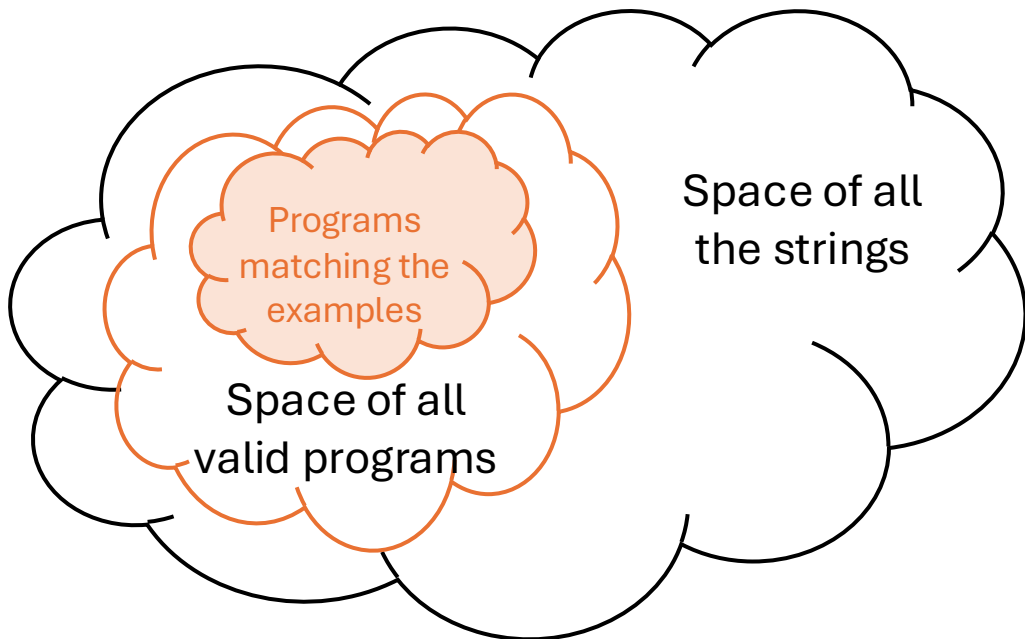
$\{ \text{dog image} \rightarrow \text{dog}, \text{cat image} \rightarrow \text{cat} \}$ \rightarrow 



Space of Programs

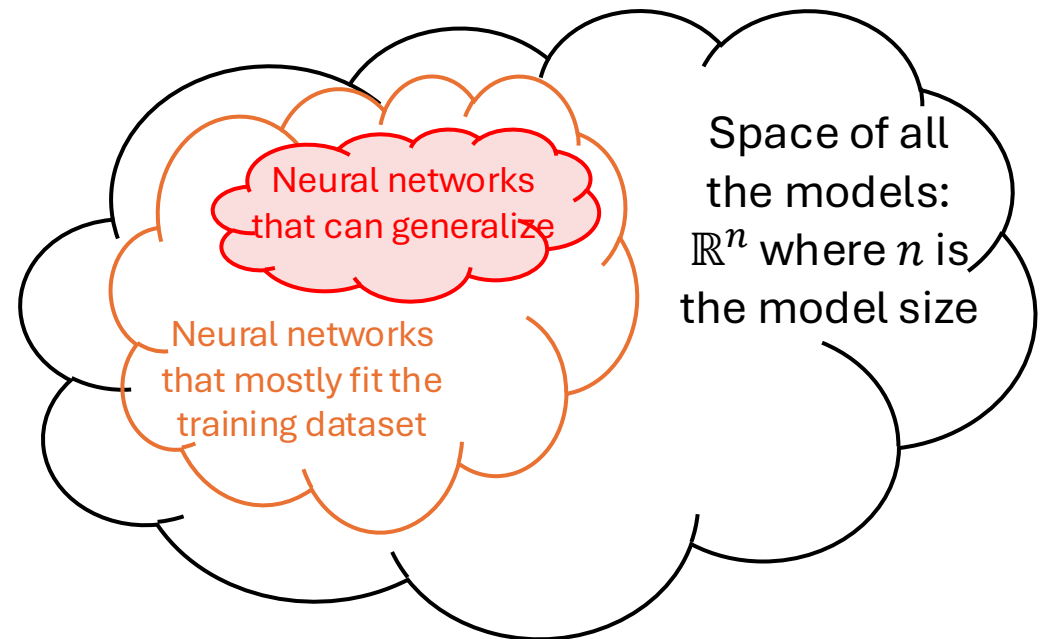
Program Synthesis

$\{[0] \rightarrow 1, [5, 1] \rightarrow 2\} \rightarrow \text{min(input)} + 1$
 $\{"123" \rightarrow "1", "abc" \rightarrow "a"\} \rightarrow \text{chars(input)[0]}$



Deep Learning

$\{ \text{dog image} \rightarrow \text{dog}, \text{cat image} \rightarrow \text{cat} \} \rightarrow$ 

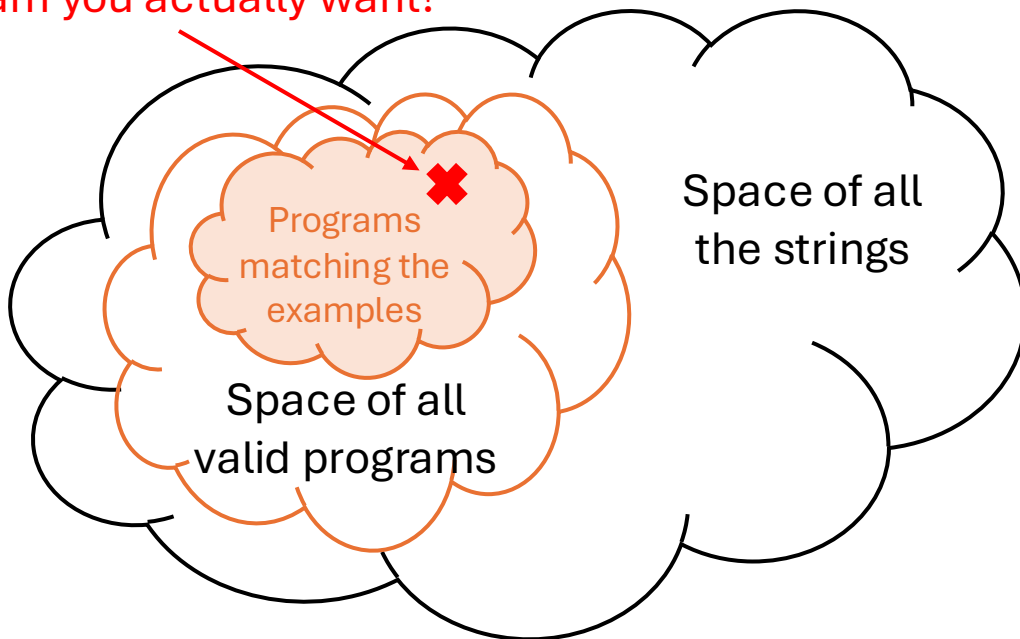


Space of Programs

Program Synthesis

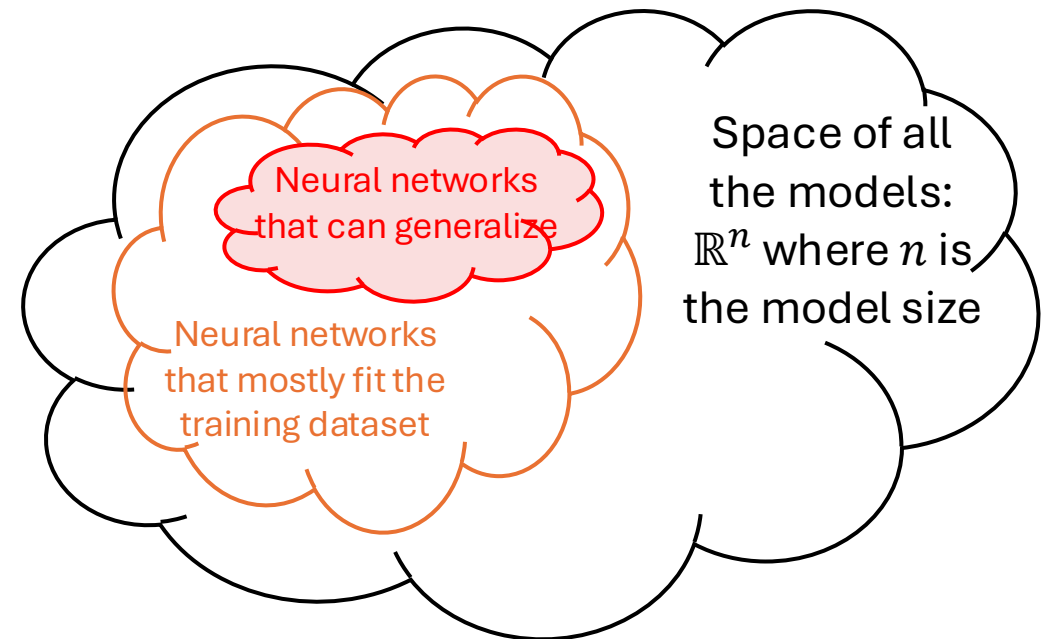
$\{[0] \rightarrow 1, [5, 1] \rightarrow 2\} \rightarrow \text{min(input)} + 1$
 $\{"123" \rightarrow "1", "abc" \rightarrow "a"\} \rightarrow \text{chars(input)[0]}$

Program you actually want!



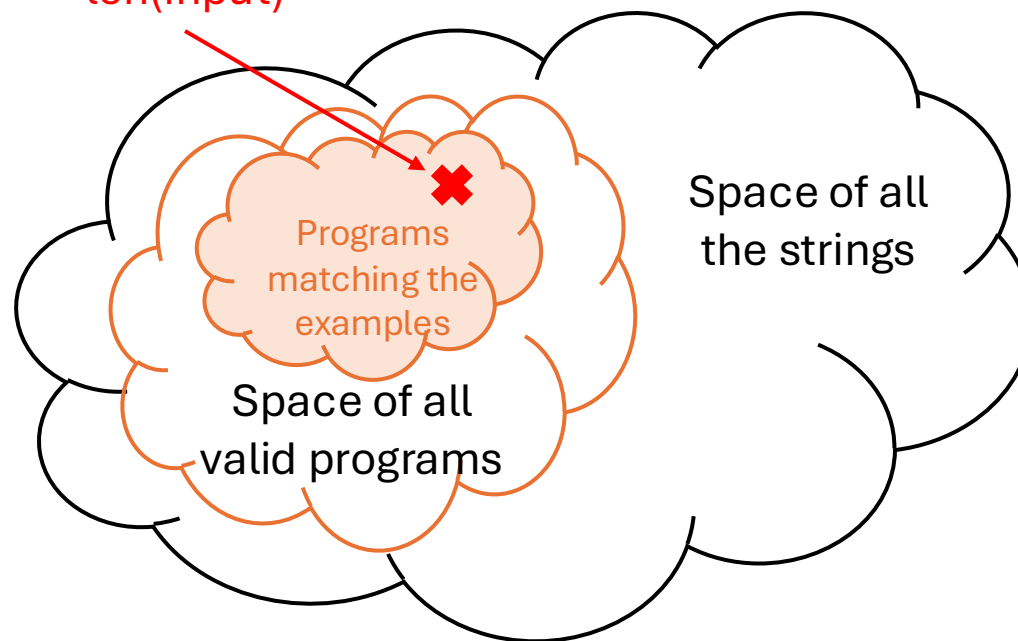
Deep Learning

$\{ \text{dog image} \rightarrow \text{dog}, \text{cat image} \rightarrow \text{cat} \} \rightarrow$ 

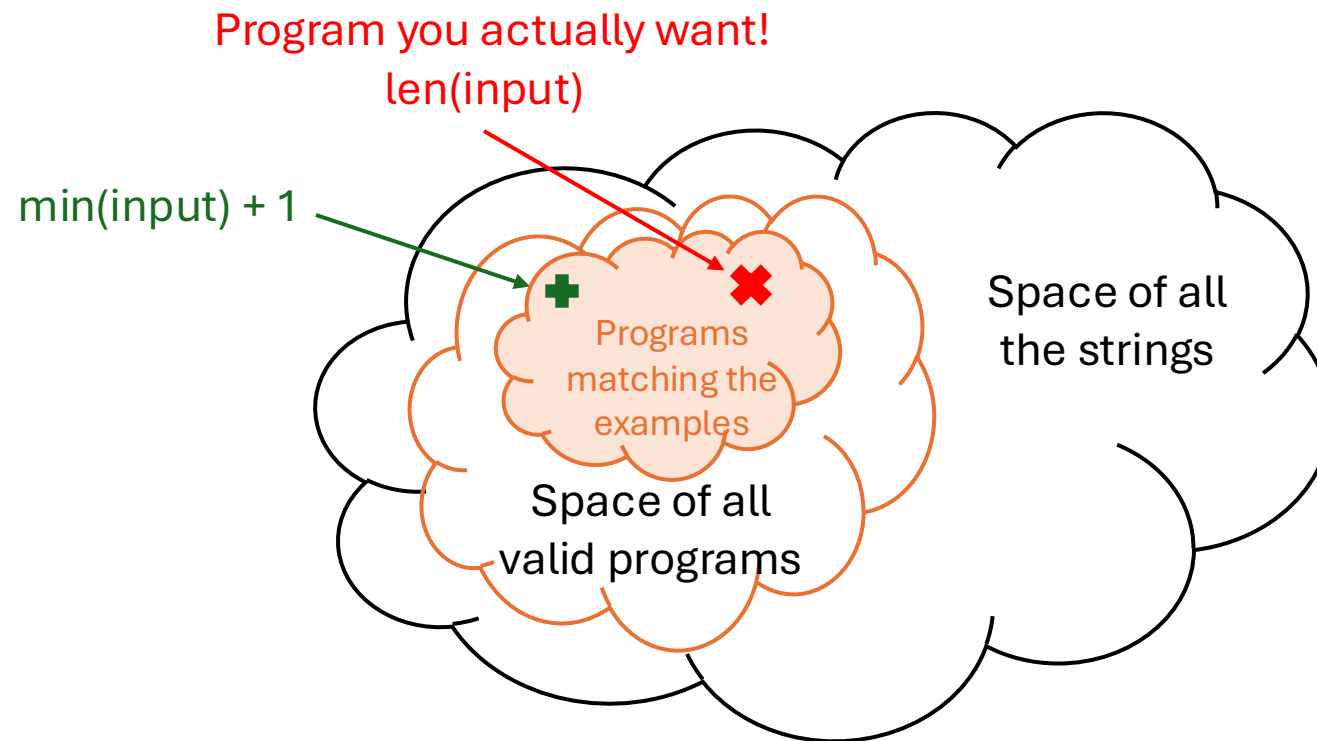


$\{[0] \rightarrow 1, [5,1] \rightarrow 2\} \rightarrow \text{len(input), min(input) + 1, ...}$

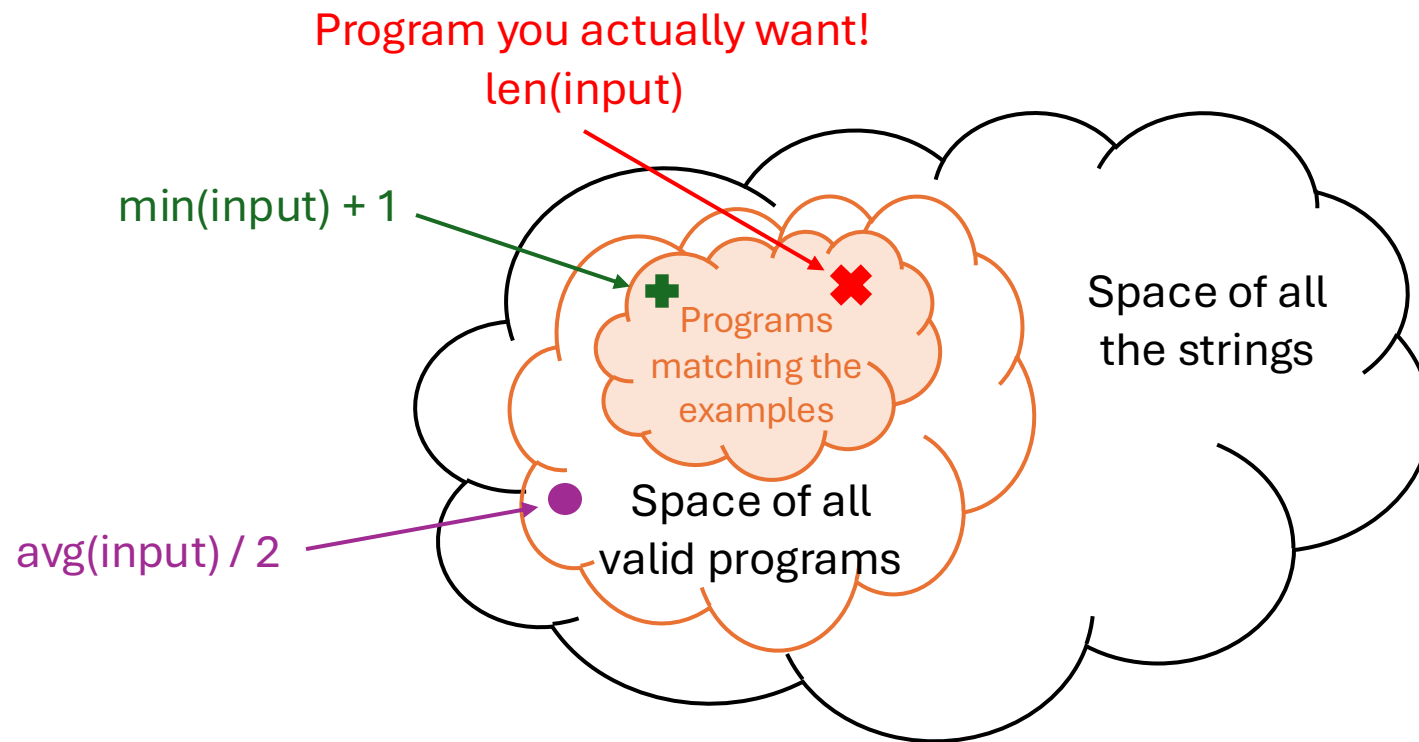
Program you actually want!
 len(input)



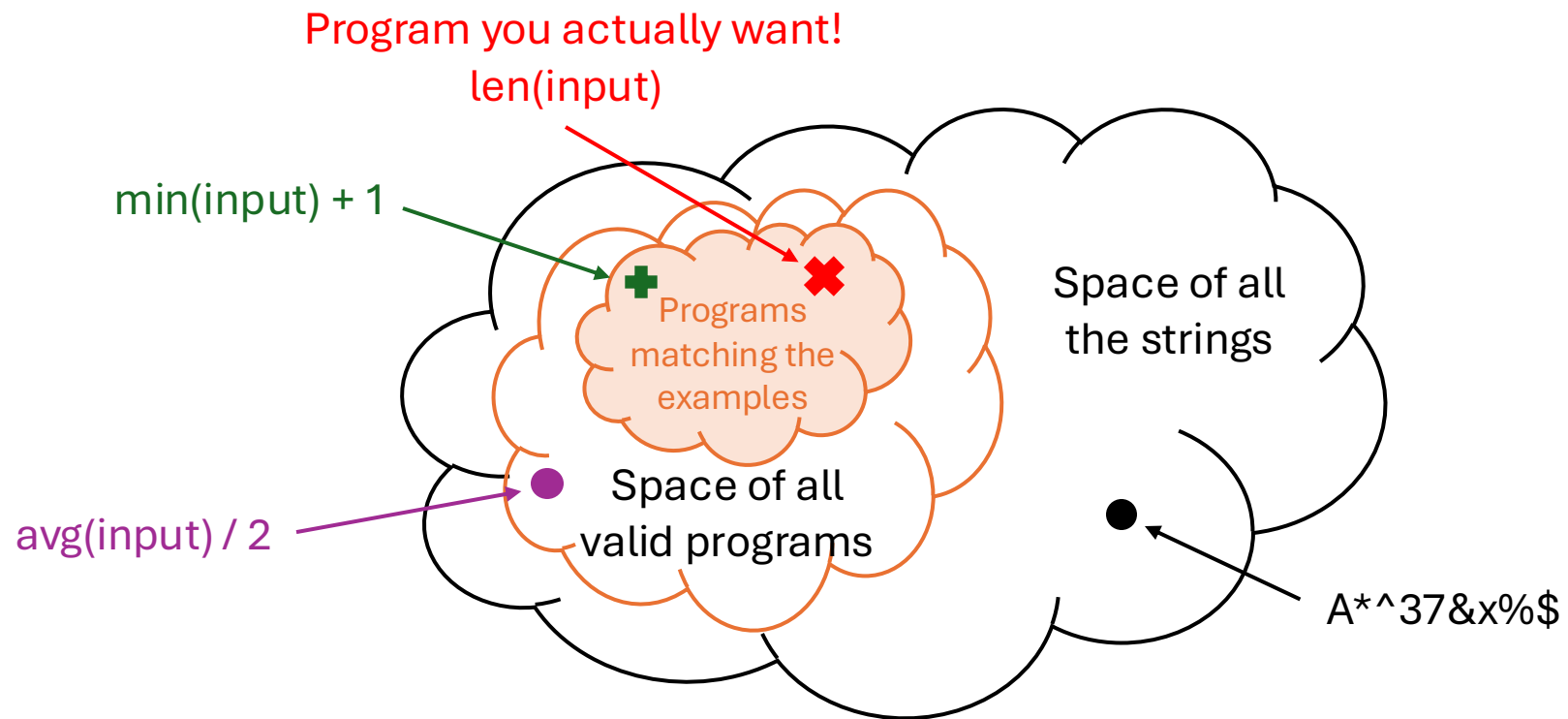
$\{[0] \rightarrow 1, [5,1] \rightarrow 2\} \rightarrow \text{len(input)}, \text{min(input)} + 1, \dots$



$\{[0] \rightarrow 1, [5,1] \rightarrow 2\} \rightarrow \text{len(input), min(input) + 1, ...}$



$\{[0] \rightarrow 1, [5,1] \rightarrow 2\} \rightarrow \text{len(input), min(input) + 1, ...}$

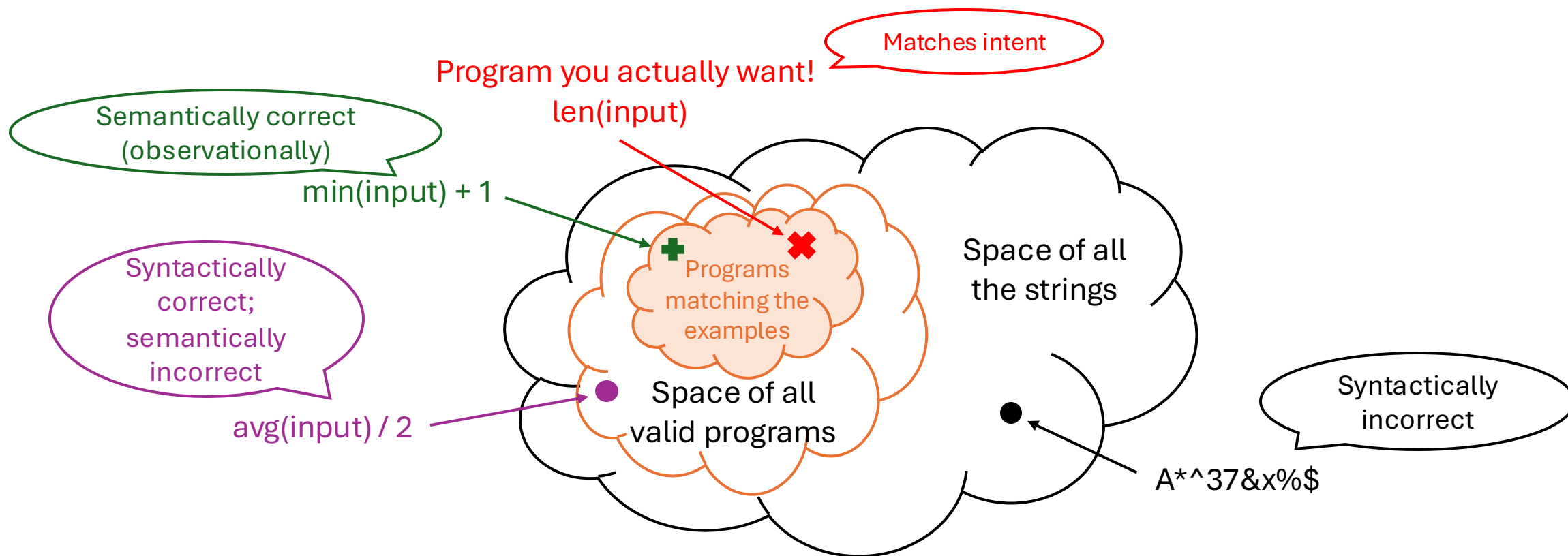


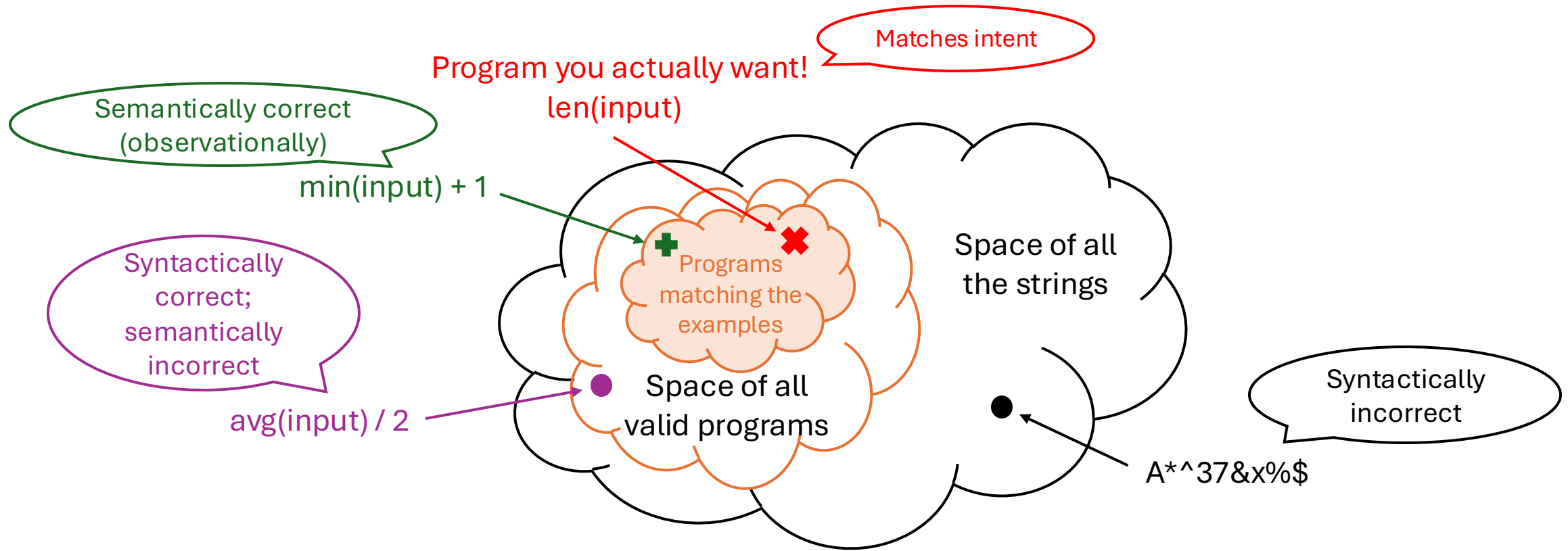
Behavioral Specification:

Examples

$\{[0] \rightarrow 1, [5, 1] \rightarrow 2\}$

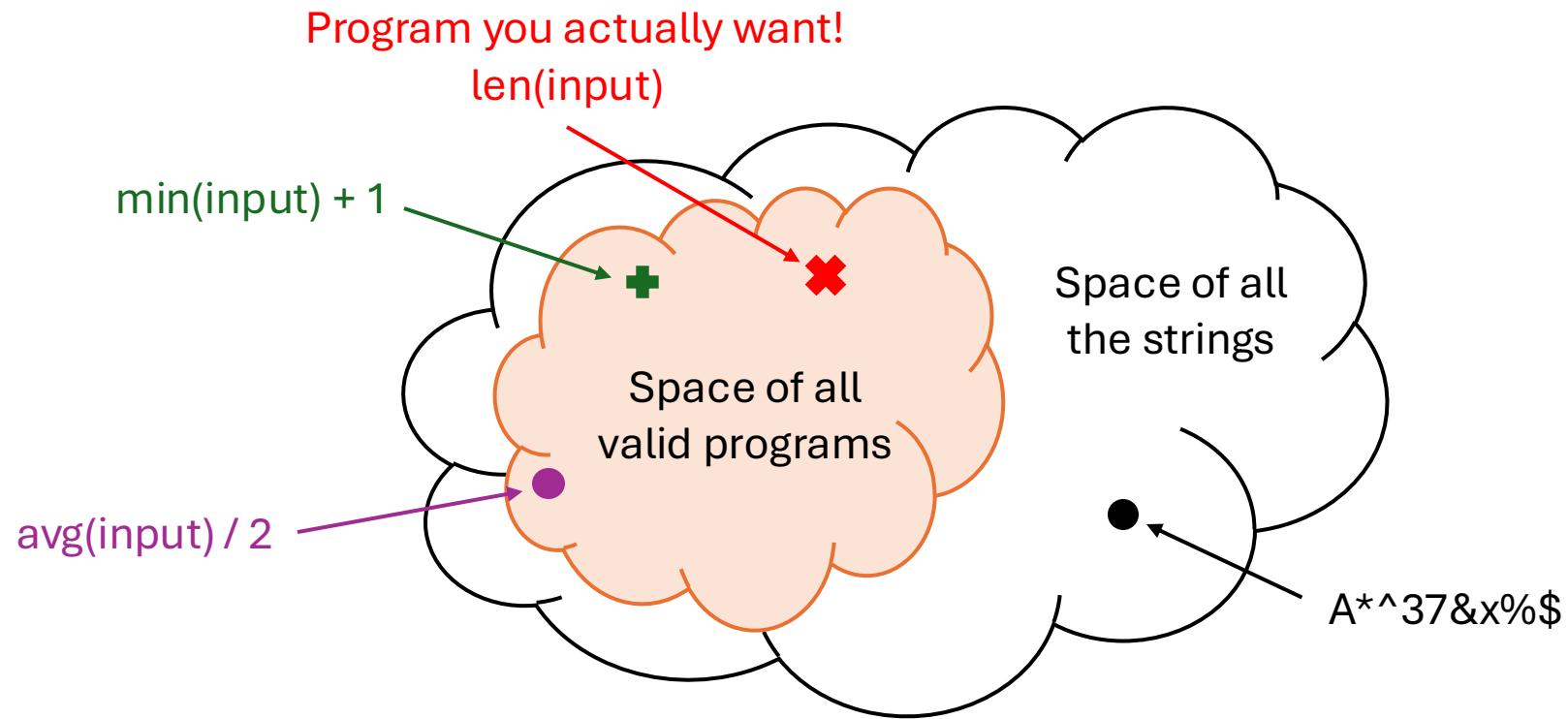
$\rightarrow \text{len}(\text{input}), \text{min}(\text{input}) + 1, \dots$





Syntax + Semantics

Syntax



Syntax: Example

$\{[0] \rightarrow 1, [5,1] \rightarrow 2\}$

➔ $\text{len}(\text{input}), \text{min}(\text{input}) + 1, \dots$

(Program) $P ::= L \mid N$

(List) $L ::= \text{input}$

| empty

| single(N)

| concat(L, L)

(Number) $N ::= \text{len}(L)$

| min(L)

| add(N, N)

| 0 | 1 | 2 | ...

// either a list expr or number expr

// the input list

// []

// [N]

// concat([1],[2,3]) = [1,2,3]

// len([0]) = 1

// min([1,2]) = 1

// add(2,1) = 3

// the constant numerical literals

Syntax: Example

```
(Program) P ::= L | N
(List) L ::= input
          | empty
          | single(N)
          | concat(L, L)
(Number) N ::= len(L)
            | min(L)
            | add(N, N)
            | 0 | 1 | 2 | ...
```

Syntax: Regular tree grammars (RTGs)

Name of nonterminals

(Program) $P ::= L \mid N$

(List) $L ::= \text{input}$

| empty

| $\text{single}(N)$

| $\text{concat}(L, L)$

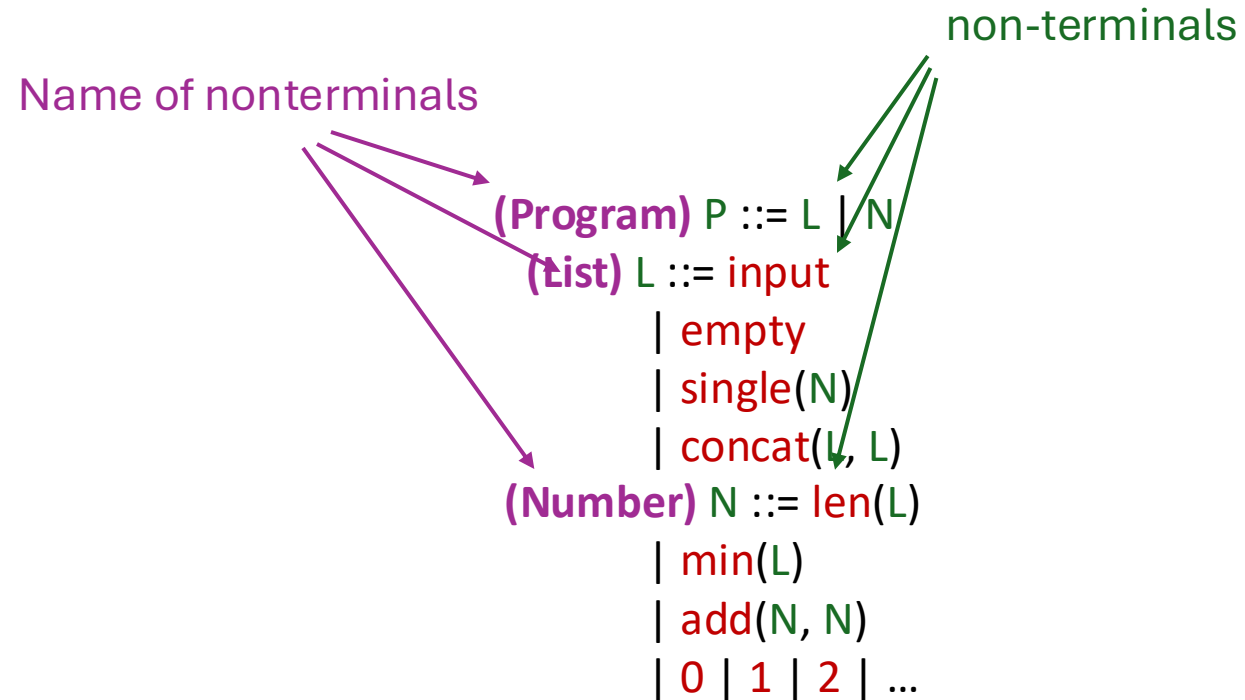
(Number) $N ::= \text{len}(L)$

| $\text{min}(L)$

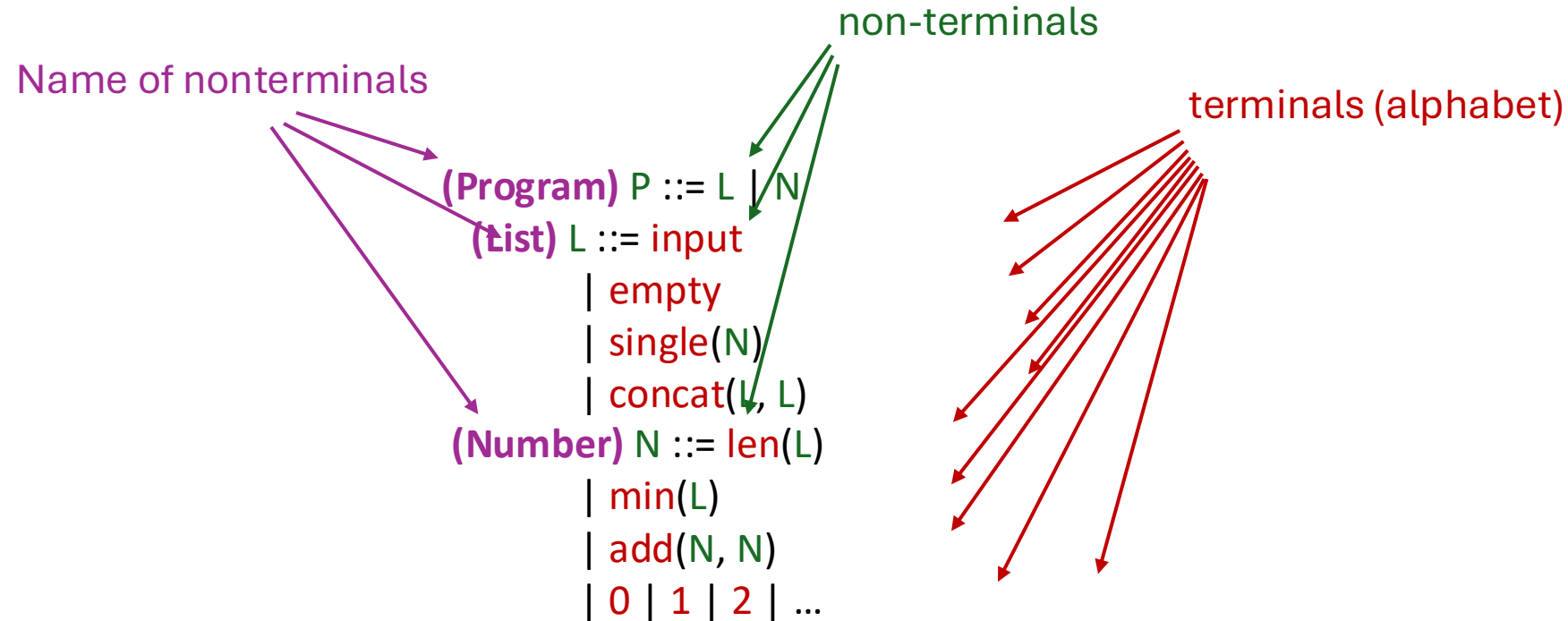
| $\text{add}(N, N)$

| $0 \mid 1 \mid 2 \mid \dots$

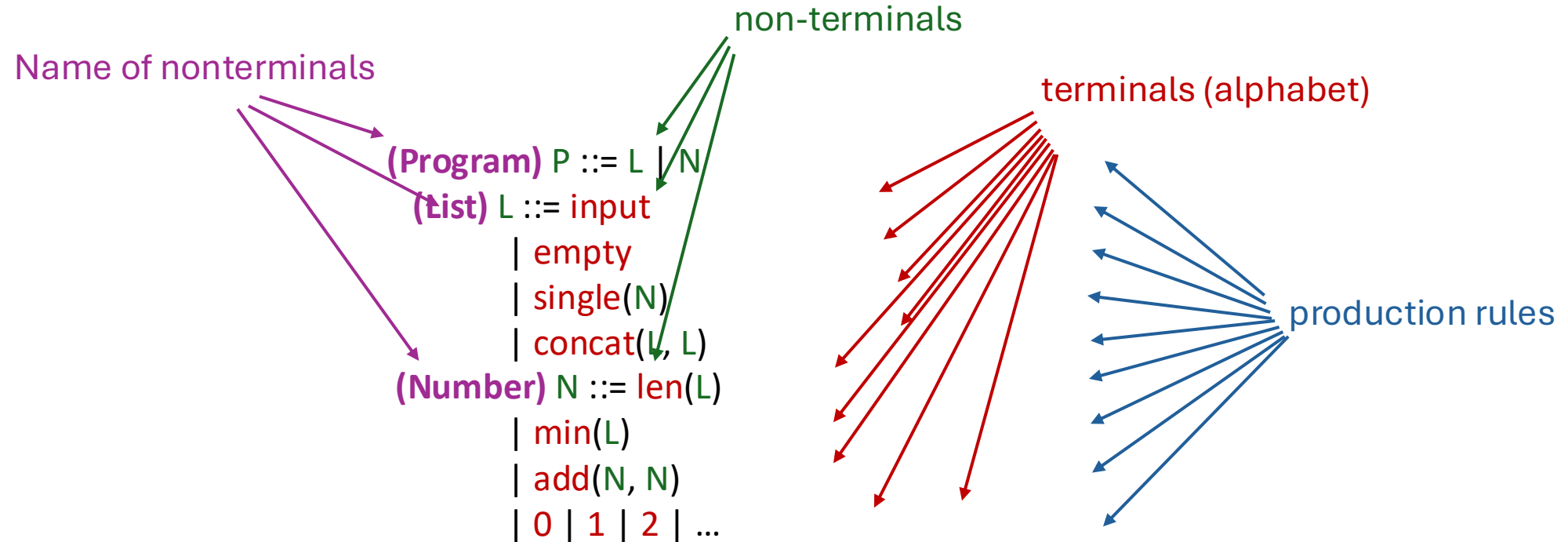
Syntax: Regular tree grammars (RTGs)



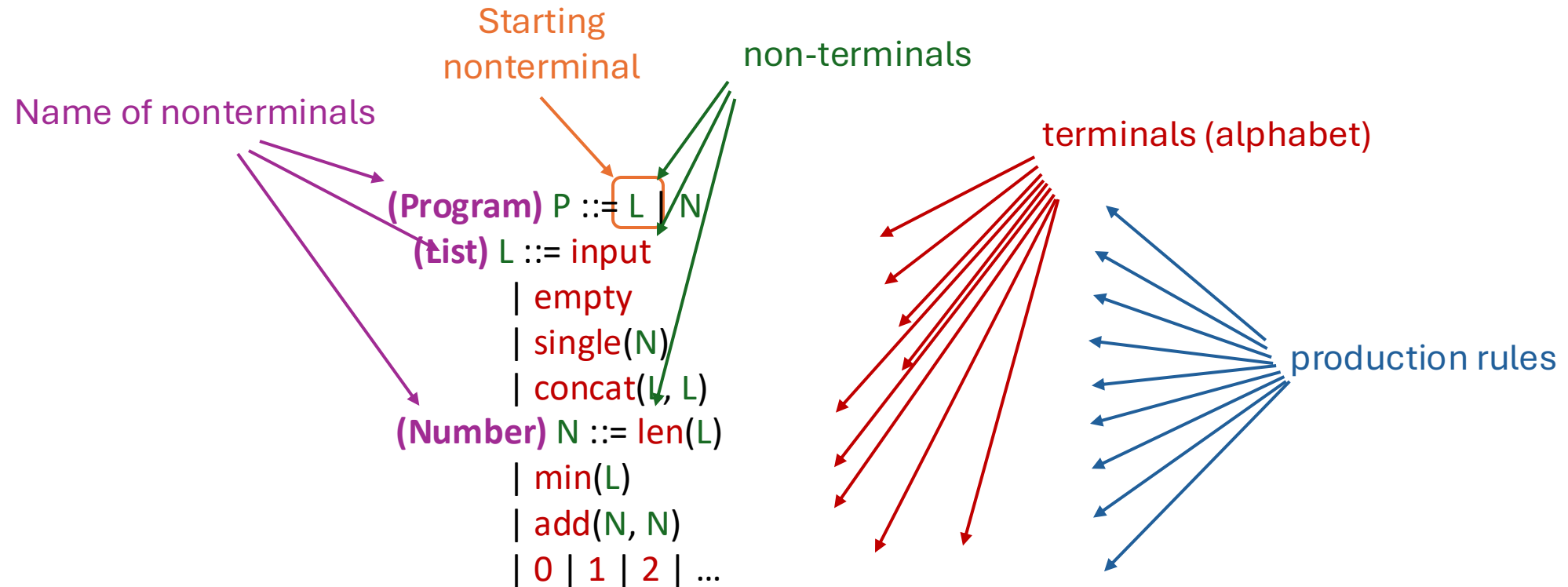
Syntax: Regular tree grammars (RTGs)



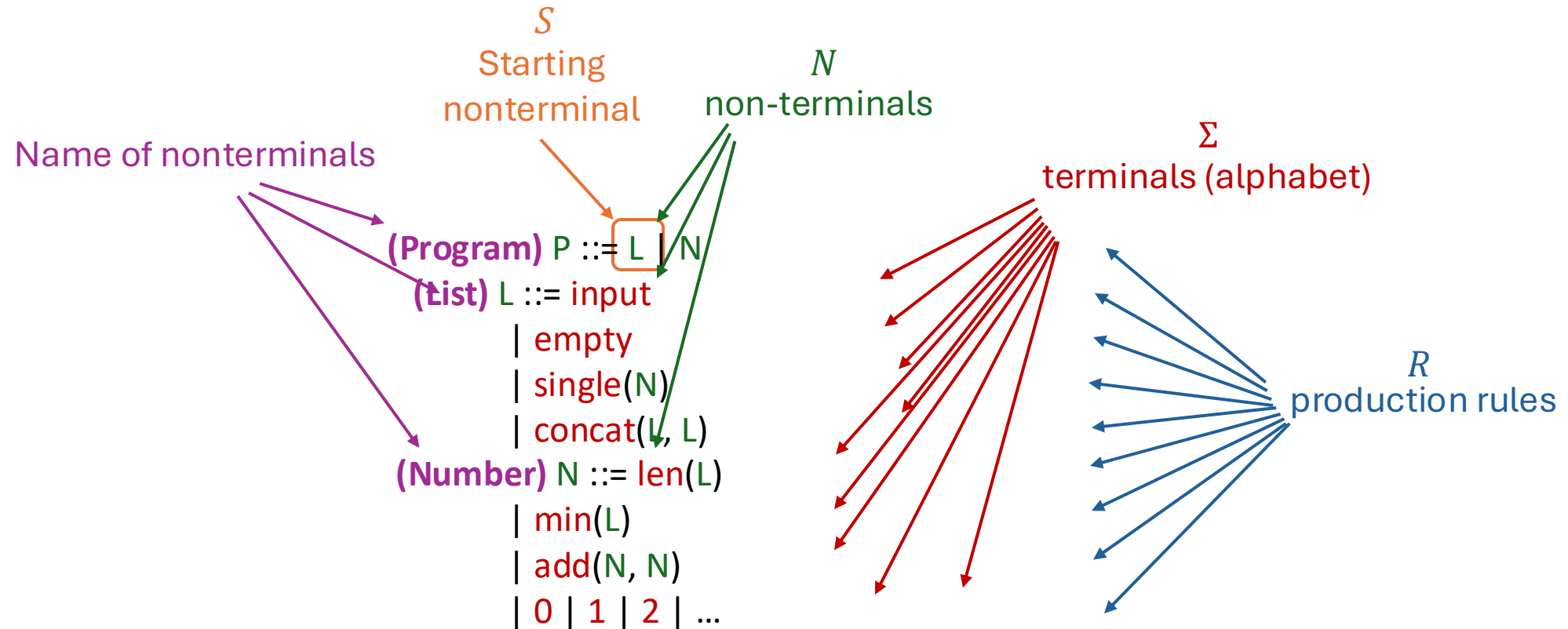
Syntax: Regular tree grammars (RTGs)



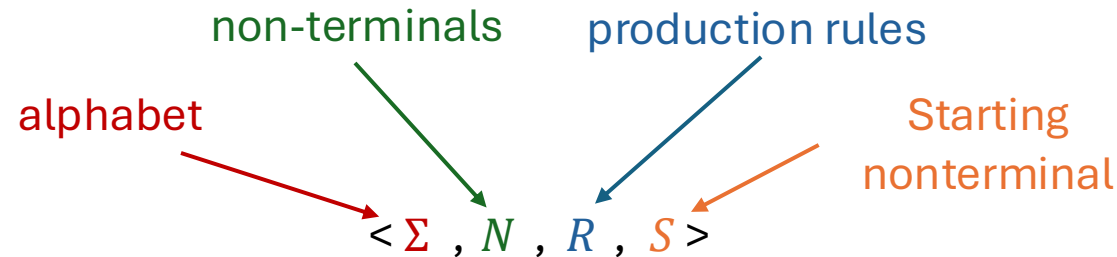
Syntax: Regular tree grammars (RTGs)



Syntax: Regular tree grammars (RTGs)



Syntax: Regular tree grammars (RTGs)



```
(Program) P ::= L | N
(List) L ::= input
| empty
| single(N)
| concat(L, L)
(Number) N ::= len(L)
| min(L)
| add(N, N)
| 0 | 1 | 2 | ...
```

Trees: $\tau \in T_{\Sigma}(N)$ = all trees made from $\Sigma \cup N$

Rules in R : $A \rightarrow \sigma(A_1, \dots, A_n)$ where $A \in N, A_i \in \Sigma \cup N$

Derivation in one step: \rightarrow

Derivations in multiple steps: \rightarrow^*

Incomplete Programs: a tree τ with non-terminals

- $\tau \in T_{\Sigma}(N)$ where $A \rightarrow^* \tau$

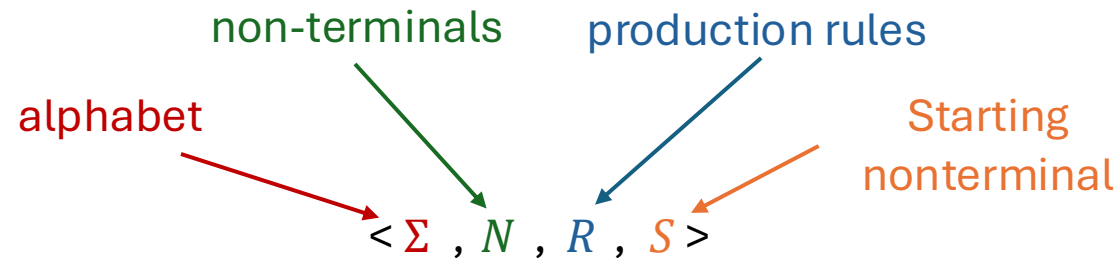
Complete Programs: a tree t without non-terminals

- $t \in T_{\Sigma}$ where $A \rightarrow^* t$

Whole Programs: a complete program t derivable by S

- $t \in T_{\Sigma}$ where $S \rightarrow^* t$

Syntax: Regular tree grammars (RTGs)



```

(Program) P ::= L | N
(List) L ::= input
        | empty
        | single(N)
        | concat(L, L)
(Number) N ::= len(L)
        | min(L)
        | add(N, N)
        | 0 | 1 | 2 | ...
    
```

Trees: $\tau \in T_{\Sigma}(N)$ = all trees made from $\Sigma \cup N$

Rules in R : $A \rightarrow \sigma(A_1, \dots, A_n)$ where $A \in N, A_i \in \Sigma \cup N$

Derivation in one step: \rightarrow

Derivations in multiple steps: \rightarrow^*

Incomplete Programs: a tree τ with non-terminals

- $\tau \in T_{\Sigma}(N)$ where $A \rightarrow^* \tau$

Complete Programs: a tree t without non-terminals

- $t \in T_{\Sigma}$ where $A \rightarrow^* t$

Whole Programs: a complete program t derivable by S

- $t \in T_{\Sigma}$ where $S \rightarrow^* t$

$\text{concat}(L, 0)$

$L \rightarrow \text{concat}(L, L)$

$\text{concat}(L, L) \rightarrow \text{concat}(\text{input}, L)$

$L \rightarrow^* \text{single}(\text{len}(L))$

$\text{len}(\text{concat}(L, L))$

$\text{len}(\text{concat}(\text{input}, \text{single}(1)))$

$\text{len}(\text{input})$

Syntax: Regular tree grammars (RTGs)

(Program) $\underline{P} ::= L \mid N$

(List) $L ::= \text{input}$

| empty

| $\text{single}(N)$

| $\text{concat}(L, L)$

(Number) $N ::= \text{len}(L)$

| $\text{min}(L)$

| $\text{add}(N, N)$

| $0 \mid 1 \mid 2 \mid \dots$

Space of programs

= the **language** of a Regular tree grammar

= all **complete** & **whole** programs

Syntax: How big is the space of programs?

$$E ::= x \mid f(E, E)$$

Syntax: How big is the space of programs?

$E ::= x \mid f(E, E)$

Depth ≤ 0



Size(0) = 1

Syntax: How big is the space of programs?

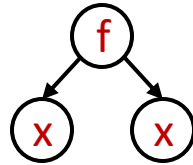
$E ::= x \mid f(E, E)$

Depth ≤ 0



Size(0) = 1

Depth ≤ 1



Size(1) = 2

Syntax: How big is the space of programs?

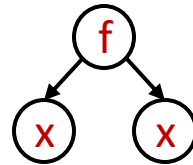
$E ::= x \mid f(E, E)$

Depth ≤ 0



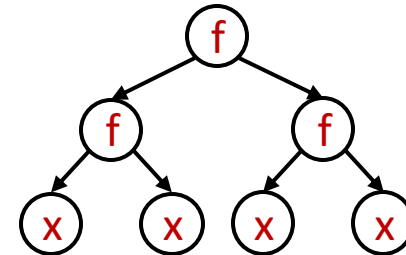
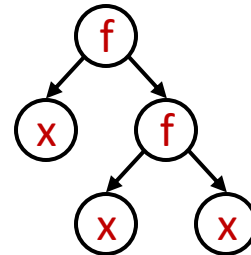
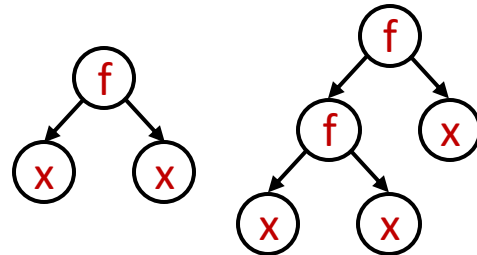
Size(0) = 1

Depth ≤ 1



Size(1) = 2

Depth ≤ 2



Size(2) = 5

Syntax: How big is the space of programs?

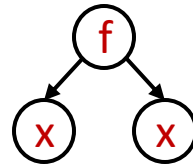
$E ::= x \mid f(E, E)$

Depth ≤ 0



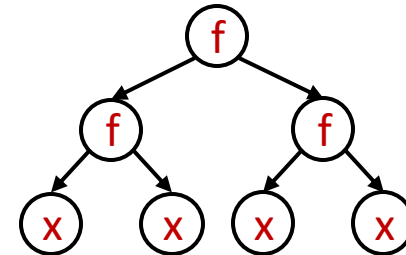
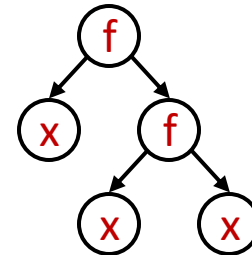
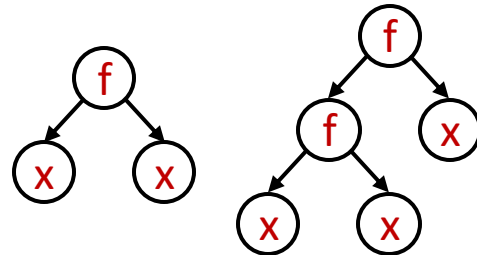
Size(0) = 1

Depth ≤ 1



Size(1) = 2

Depth ≤ 2



Size(2) = 5

Size(depth) = ???

Syntax: How big is the space of programs?

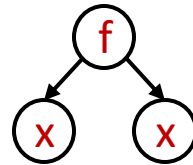
$E ::= x \mid f(E, E)$

Depth ≤ 0



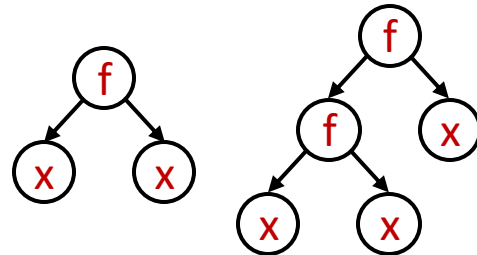
Size(0) = 1

Depth ≤ 1



Size(1) = 2

Depth ≤ 2



Size(2) = 5

$$\text{Size}(\text{depth}) = 1 + \text{Size}(\text{depth} - 1)^2$$

Syntax: How big is the space of programs?

$$E ::= x \mid f(E, E)$$
$$\text{Size}(\text{depth}) = 1 + \text{Size}(\text{depth} - 1)^2$$

size(1) = 1

size(2) = 2

size(3) = 5

size(4) = 26

size(5) = 677

size(6) = 458330

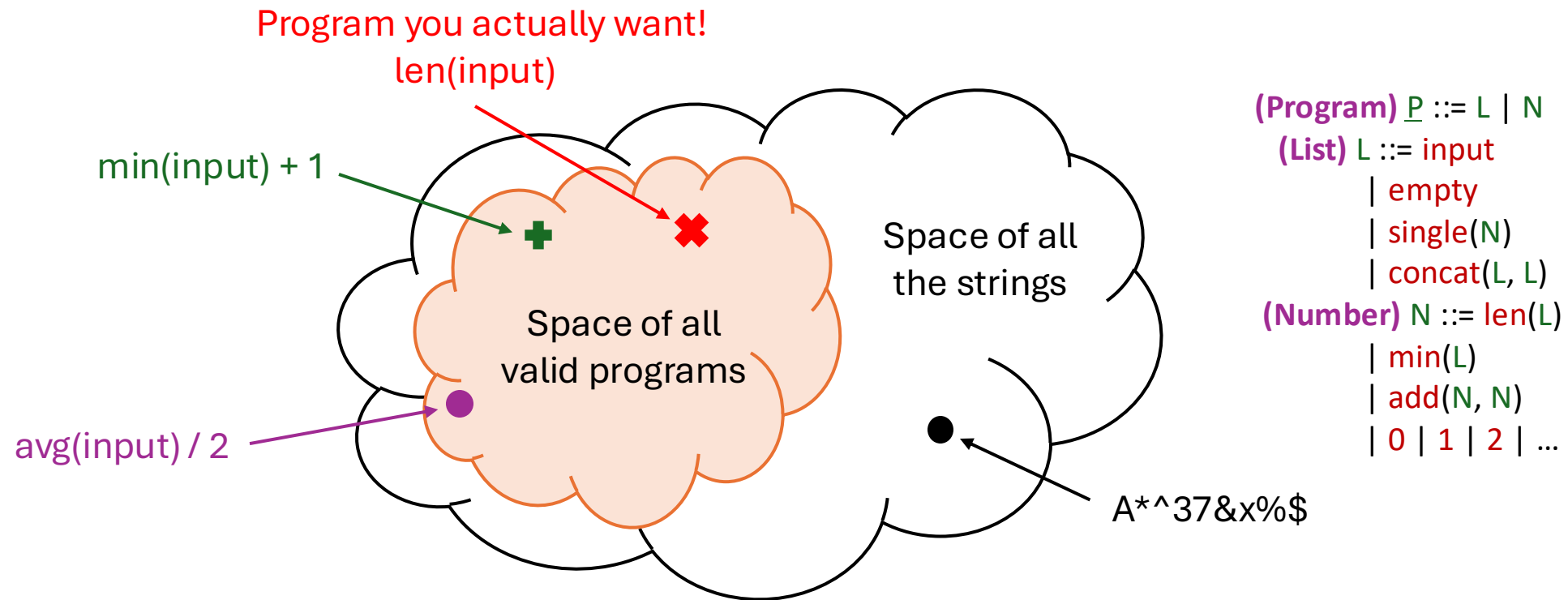
size(7) = 210066388901

size(8) = 44127887745906175987802

size(9) = 1947270476915296449559703445493848930452791205

size(10) = 3791862310265926082868235028027893277370233152247388584761734150717768254410341175325352026

Syntax: Sugars



Syntax: Sugars

min(input) + 1

(Program) $\underline{P} ::= L \mid N$

(List) $L ::= \text{input}$

| empty

| single(N)

| concat(L, L)

(Number) $N ::= \text{len}(L)$

| min(L)

| add(N, N)

| 0 | 1 | 2 | ...

(Program) $\underline{P} ::= L \mid N$

(List) $L ::= \text{input}$

| empty

| [N]

| L :: L

(Number) $N ::= \text{len}(L)$

| min(L)

| N + N

| 0 | 1 | 2 | ...

Syntax: Sugars

min(input) + 1

(Program) $\underline{P} ::= L \mid N$

(List) $L ::= \text{input}$

| empty

| single(N)

| concat(L, $_$)

(Number) $N ::= \text{len}(L)$

| min(L)

| add(N, N)

| 0 | 1 | 2 | ...

(Program) $\underline{P} ::= L \mid N$

(List) $L ::= \text{input}$

| []

| [N]

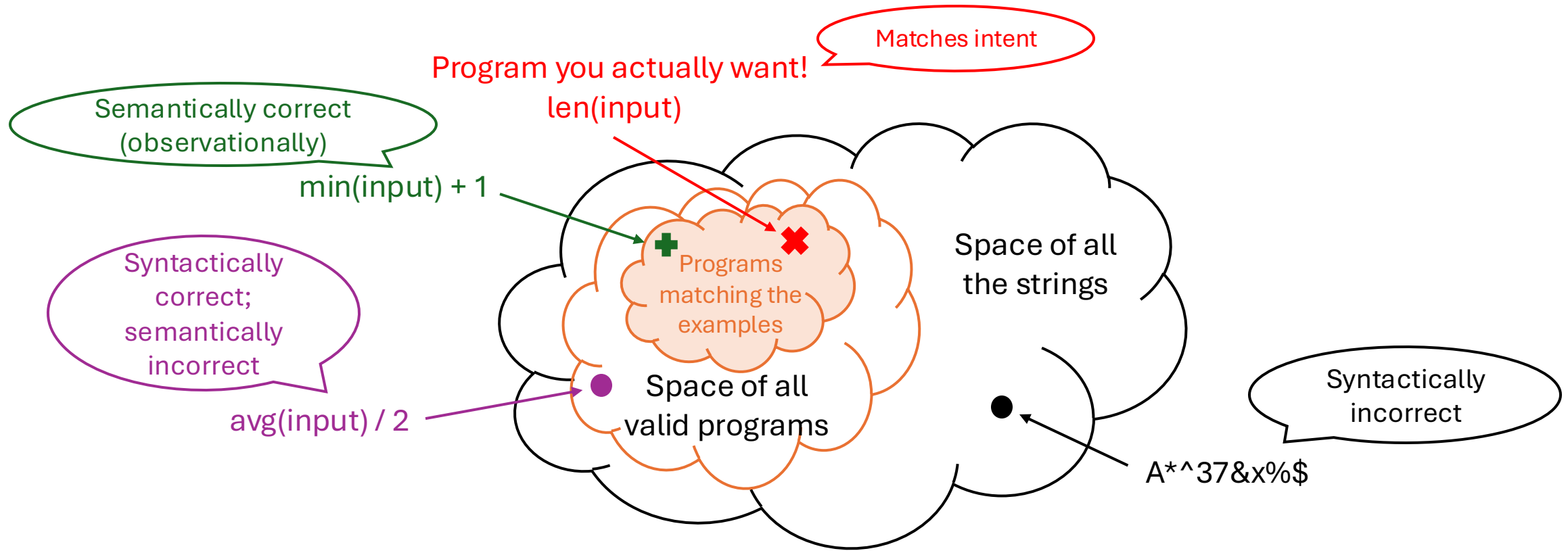
| $L :: L$

(Number) $N ::= \text{len}(L)$

| min(L)

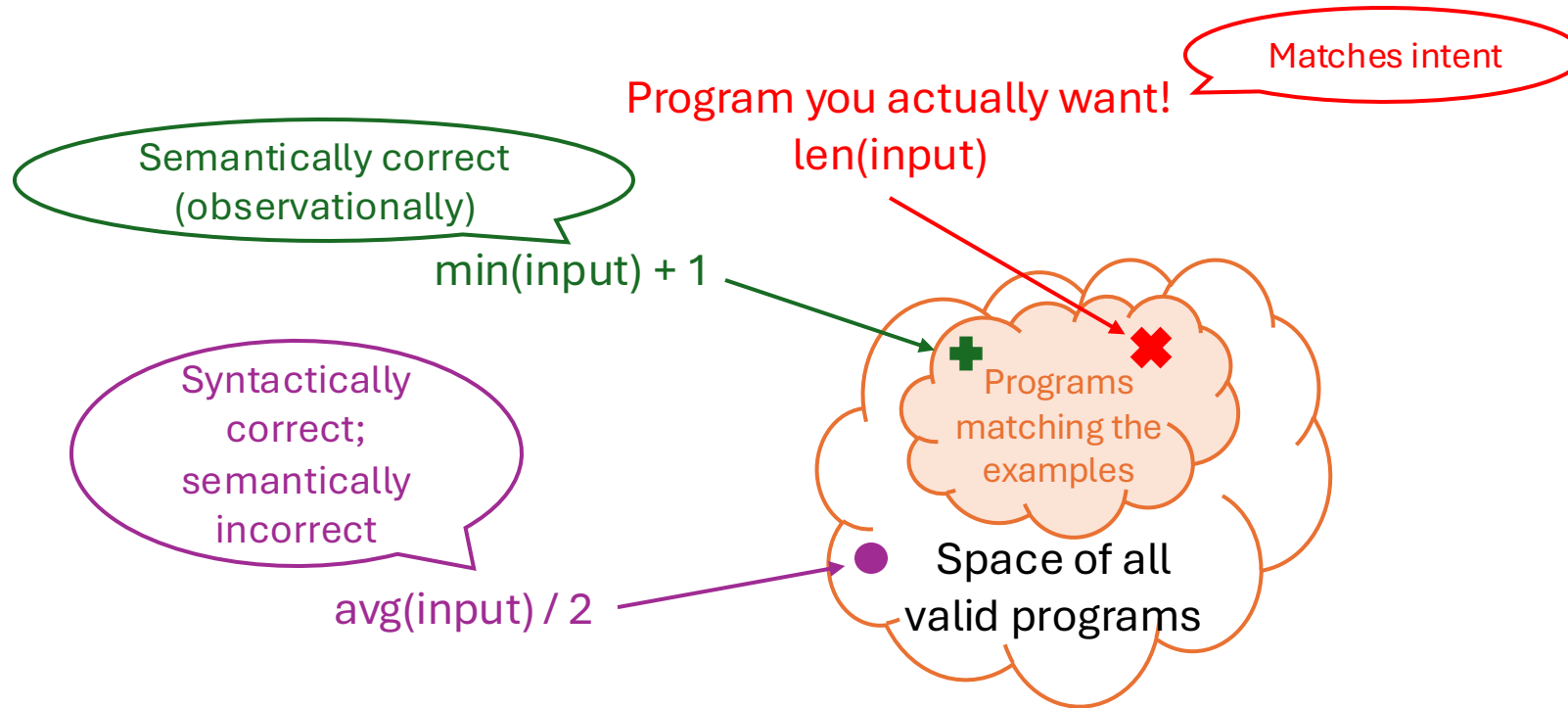
| $N + N$

| 0 | 1 | 2 | ...



Syntax + Semantics

Semantics



Semantics: Meaning of a Language

(Program) $\underline{P} ::= L \mid N$
(List) $L ::= \text{input}$
 $\mid \text{empty}$
 $\mid \text{single}(N)$
 $\mid \text{concat}(L, L)$
(Number) $N ::= \text{len}(L)$
 $\mid \text{min}(L)$
 $\mid \text{add}(N, N)$
 $\mid 0 \mid 1 \mid 2 \mid \dots$

$eval : T_{\Sigma} \times IntList \rightarrow List \mid Int$
 $eval(\text{'input'}, x) = x$
 $eval(\text{'empty'}, x) = []$
 $\forall \tau \in T_{\Sigma}, eval(\text{'single'}(\tau), x) = [eval(\tau, x)]$
 $\forall \tau_1, \tau_2 \in T_{\Sigma}, eval(\text{'concat'}(\tau_1, \tau_2), x) = eval(\tau_1, x) + eval(\tau_2, x)$
 $\forall \tau \in T_{\Sigma}, eval(\text{'len'}(\tau), x) = |eval(\tau, x)|$
 $\forall \tau \in T_{\Sigma}, eval(\text{'min'}(\tau), x) = \min_v (v \in eval(\tau, x))$
 $\forall \tau_1, \tau_2 \in T_{\Sigma}, eval(\text{'add'}(\tau_1, \tau_2), x) = eval(\tau_1, x) + eval(\tau_2, x)$
 $eval(\text{'0'}, x) = 0, \dots$

Denotational Semantics

Mathematical meaning to each program construct

Semantics: Meaning of a Language

(Program) $\underline{P} ::= L \mid N$
(List) $L ::= \text{input}$
 $\mid \text{empty}$
 $\mid \text{single}(N)$
 $\mid \text{concat}(L, L)$
(Number) $N ::= \text{len}(L)$
 $\mid \text{min}(L)$
 $\mid \text{add}(N, N)$
 $\mid 0 \mid 1 \mid 2 \mid \dots$

$[[\cdot]] : T_\Sigma \times \text{IntList} \rightarrow \text{List} \mid \text{Int}$
 $[[\text{input}]](x) = x$
 $[[\text{empty}]](x) = []$
 $\forall \tau \in T_\Sigma, [[\text{single}(\tau)]](x) = [[[\tau]](x)]$
 $\forall \tau_1, \tau_2 \in T_\Sigma, [[\text{concat}(\tau_1, \tau_2)]](x) = [[\tau_1]](x) + [[\tau_2]](x)$
 $\forall \tau \in T_\Sigma, [[\text{len}(\tau)]](x) = | [[\tau]](x) |$
 $\forall \tau \in T_\Sigma, [[\text{min}(\tau)]](x) = \min_v \left(v \in [[\tau]](x) \right)$
 $\forall \tau_1, \tau_2 \in T_\Sigma, [[\text{add}(\tau_1, \tau_2)]](x) = [[\tau_1]](x) + [[\tau_2]](x)$
 $[[0]](x) = 0, \dots$

Denotational Semantics

Mathematical meaning to each program construct

Semantics: Meaning of a Language

(Program) $\underline{P} ::= L \mid N$
(List) $L ::= \text{input}$
 | empty
 | $\text{single}(N)$
 | $\text{concat}(L, L)$
(Number) $N ::= \text{len}(L)$
 | $\text{min}(L)$
 | $\text{add}(N, N)$
 | $0 \mid 1 \mid 2 \mid \dots$

<hr/> $x \vdash \text{input} \Downarrow x$	<hr/> $x \vdash \text{empty} \Downarrow []$
$x \vdash N \Downarrow n$ <hr/>	$x \vdash L1 \Downarrow vL1 \quad x \vdash L2 \Downarrow vL2$ <hr/>
$x \vdash \text{single}(N) \Downarrow [n]$	$x \vdash \text{concat}(L1, L2) \Downarrow vL1 ++ vL2$
...	

Operational Semantics

Meaning in terms of computation steps

Semantics: Meaning of a Language

```
(Program)  $\underline{P} ::= L \mid N$   
(List)  $L ::= \text{input}$   
      |  $\text{empty}$   
      |  $\text{single}(N)$   
      |  $\text{concat}(L, L)$   
(Number)  $N ::= \text{len}(L)$   
        |  $\text{min}(L)$   
        |  $\text{add}(N, N)$   
        |  $0 \mid 1 \mid 2 \mid \dots$ 
```

```
def evaluate(program, input):  
    if isinstance(program, Empty):  
        return []  
    elif isinstance(program, Input):  
        return input  
    elif isinstance(program, Concat):  
        left = evaluate(program.left, input)  
        right = evaluate(program.right, input)  
        return left + right  
    elif ...
```

Semantics written in Python...

Meaning encoded as an evaluator / interpreter

Datafun

$\llbracket A \rrbracket \in \text{Poset}_0$
 $\llbracket 2 \rrbracket = 2$
 $\llbracket N \rrbracket = N_{\leq}$
 $\llbracket \text{str} \rrbracket = \text{Disc } S$
 $\llbracket A \times B \rrbracket = \llbracket A \rrbracket \times \llbracket B \rrbracket$
 $\llbracket A + B \rrbracket = \llbracket A \rrbracket + \llbracket B \rrbracket$
 $\llbracket A \xrightarrow{+} B \rrbracket = \llbracket A \rrbracket \Rightarrow \llbracket B \rrbracket$
 $\llbracket A \rightarrow B \rrbracket = \text{Disc } |\llbracket A \rrbracket| \Rightarrow \llbracket B \rrbracket$
 $\llbracket \{A\} \rrbracket = \mathcal{P}_{\text{fin}} |\llbracket A \rrbracket|$

$\llbracket \Delta \rrbracket, \llbracket \Gamma \rrbracket \in \text{Poset}_0$
 $\llbracket \cdot \rrbracket = 1$
 $\llbracket \Delta, x:A \rrbracket = \llbracket \Delta \rrbracket \times \llbracket A \rrbracket$
 $\llbracket \Gamma, x:A \rrbracket = \llbracket \Gamma \rrbracket \times \llbracket A \rrbracket$

Signature

$\text{alloc}(\tau_i)$
 $\overline{d_m} \leftarrow \text{eval}(\overline{s_n})$
 $\overline{d_n} \leftarrow \text{gather}(i, \overline{s_n})$
 $d \leftarrow \text{gather}(\alpha_{n,1})(\overline{i_n}, \overline{s_n})$
 $\text{store}(\rho)(\overline{s_n}, s_i)$
 $[\overline{s_n}, s_i] = \text{load}(\rho)(i)$
 $\overline{d_n} \leftarrow \text{build}(\overline{s_n})$
 $\overline{d_n} \leftarrow \text{count}(\overline{b_n}, h, \overline{a_n})$
 $\overline{d_n} \leftarrow \text{scan}(s)$
 $[d_i, d_r] \leftarrow \text{join}(W)(\overline{b_m}, \overline{a_n}, h, c, o)$
 $\overline{d_n} \leftarrow \text{copy}(\overline{s_n})$
 $\overline{d_n} \leftarrow \text{sort}(\overline{s_n})$
 $[\overline{d_n}, s] \leftarrow \text{unique}(\sigma)(\overline{s_n})$
 $\overline{d_n} \leftarrow \text{merge}(\overline{a_n}, \overline{b_n})$

Gather rows of $\overline{s_n}$
 Gather rows of $\overline{s_n}$
 Store registers $\overline{s_n}$
 Loads the columns and tags of relation ρ with arity n from the data
 Builds a hash index for register the table with columns $\overline{s_n}$.
 Count the number of occurrences of each tuple columns $\overline{a_n}$ via the hash index h .
 Computes the (exclusive) prefix sum of registers
 Produces the resulting indices from a W column the hash index h and count c and offset o .
 Copies from register $\overline{s_n}$, truncating if the destination
 Lexicographically sorts the table with columns
 Merges adjacent duplicate rows via σ from the unique elements s .
 Merges two sorted tables with columns $\overline{a_n}$ and

Lobster

SQL

Reduction

$$\frac{s; v^*; e^* \hookrightarrow_i s'; v'^*; e'^*}{s; v^*; L^k[e^*] \hookrightarrow_i s'; v'^*; L^k[e'^*]}$$

$L^0[\text{trap}] \hookrightarrow$

trap

if $L^0 \neq [-]$

$$(t.\text{const } c) t.\text{unop} \hookrightarrow t.\text{const } \text{unop}_t(c)$$

$$(t.\text{const } c_1) (t.\text{const } c_2) t.\text{binop} \hookrightarrow t.\text{const } c$$

if $c = \text{binop}_t(c_1, c_2)$

$$(t.\text{const } c_1) (t.\text{const } c_2) t.\text{binop} \hookrightarrow \text{trap}$$

otherwise

$$(t.\text{const } c) t.\text{testop} \hookrightarrow \text{i32.const } \text{testop}_t(c)$$

$$(t.\text{const } c_1) (t.\text{const } c_2) t.\text{relop} \hookrightarrow \text{i32.const } \text{relop}_t(c_1, c_2)$$

$$(t_1.\text{const } c) t_2.\text{convert } t_1 \text{--} s_x^? \hookrightarrow t_2.\text{const } c'$$

if $c' = \text{cvt}_{t_1, t_2}^{s_x^?}(c)$

$$(t_1.\text{const } c) t_2.\text{convert } t_1 \text{--} s_x^? \hookrightarrow \text{trap}$$

otherwise

$$(t_1.\text{const } c) t_2.\text{reinterpret } t_1 \hookrightarrow t_2.\text{const } \text{const}_{t_2}(\text{bits}_{t_1}(c))$$

unreachable \hookrightarrow **trap**

Expression semantics

$\alpha : \mathcal{U} \rightarrow \mathcal{U}, \quad \beta : \mathcal{U} \rightarrow \text{Bool}, \quad g : \mathcal{U} \rightarrow \mathcal{U}, \quad \llbracket e \rrbracket : \mathcal{F}_T \rightarrow \mathcal{U}_T$

$$\frac{t :: p(u) \in F_T}{t :: u \in \llbracket p \rrbracket(F_T)} \quad (\text{PREDICATE})$$

$$\frac{t :: u \in \llbracket e \rrbracket(F_T) \quad \beta(u) = \text{true}}{t :: u \in \llbracket \sigma_\beta(e) \rrbracket(F_T)} \quad (\text{SELECT})$$

$$\frac{t :: u \in \llbracket e \rrbracket(F_T) \quad u' = \alpha(u)}{t :: u' \in \llbracket \pi_\alpha(e) \rrbracket(F_T)} \quad (\text{PROJECT})$$

$$\llbracket R \rrbracket_{D, \eta, x} = R^D$$

$$\llbracket \tau : \beta \rrbracket_{D, \eta, x} = \llbracket T_1 \rrbracket_{D, \eta, 0} \times \cdots \times \llbracket T_k \rrbracket_{D, \eta, 0} \quad \text{for } \tau = (T_1, \dots, T_k)$$

$$\left\llbracket \begin{array}{l} \text{FROM } \tau : \beta \\ \text{WHERE } \theta \end{array} \right\rrbracket_{D, \eta, x} = \left\{ \left. \begin{array}{l} \bar{r}, \dots, \bar{r} \\ k \text{ times} \end{array} \right| \bar{r} \in k \llbracket \tau : \beta \rrbracket_{D, \eta, 0}, \llbracket \theta \rrbracket_{D, \eta'} = \text{t}, \eta' = \eta \oplus \ell(\tau : \beta) \right\}$$

$$\left\llbracket \begin{array}{l} \text{SELECT } \alpha : \beta' \\ \text{FROM } \tau : \beta \\ \text{WHERE } \theta \end{array} \right\rrbracket_{D, \eta, x} = \left\{ \left. \begin{array}{l} \llbracket \alpha \rrbracket_{\eta'}, \dots, \llbracket \alpha \rrbracket_{\eta'} \\ k \text{ times} \end{array} \right| \eta' = \eta \oplus \ell(\tau : \beta), \bar{r} \in k \left\llbracket \begin{array}{l} \text{FROM } \tau : \beta \\ \text{WHERE } \theta \end{array} \right\rrbracket_{D, \eta, x} \right\}$$

$$\left\llbracket \begin{array}{l} \text{SELECT } * \\ \text{FROM } \tau : \beta \\ \text{WHERE } \theta \end{array} \right\rrbracket_{D, \eta, 0} = \left\llbracket \begin{array}{l} \text{SELECT } \ell(\tau : \beta) : \ell(\tau) \\ \text{FROM } \tau : \beta \\ \text{WHERE } \theta \end{array} \right\rrbracket_{D, \eta, 0}$$

$$\left\llbracket \begin{array}{l} \text{SELECT } * \\ \text{FROM } \tau : \beta \\ \text{WHERE } \theta \end{array} \right\rrbracket_{D, \eta, 1} = \left\llbracket \begin{array}{l} \text{SELECT } c \text{ AS } N \\ \text{FROM } \tau : \beta \\ \text{WHERE } \theta \end{array} \right\rrbracket_{D, \eta, 1} \quad \text{for arbitrary } c \in \mathcal{C} \text{ and } N \in \mathbb{N}$$

$$\left\llbracket \begin{array}{l} \text{SELECT DISTINCT } \alpha : \beta' \mid * \\ \text{FROM } \tau : \beta \text{ WHERE } \theta \end{array} \right\rrbracket_{D, \eta, x} = \varepsilon \left(\left\llbracket \begin{array}{l} \text{SELECT } \alpha : \beta' \mid * \\ \text{FROM } \tau : \beta \text{ WHERE } \theta \end{array} \right\rrbracket_{D, \eta, x} \right)$$

WebAssembly (WASM)

$$s; v^*; e^* \hookrightarrow_i s; v^*; e^*$$

$$\frac{s; v^*; e^* \hookrightarrow_i s'; v'^*; e'^*}{s; v_0^*; \text{local}_n\{i; v^*\} e^* \text{ end} \hookrightarrow_j s'; v_0^*; \text{local}_n\{i; v'^*\} e'^* \text{ end}}$$

if $L^0 \neq [-]$

if $c = \text{binop}_t(c_1, c_2)$

otherwise

if $c' = \text{cvt}_{t_1, t_2}^{s_x^?}(c)$

otherwise

Scallop

Scallop

Grounded with concrete inputs...

Syntax

```
(Program)  $\underline{P} ::= L \mid N$   
(List)  $L ::= \text{input}$   
      | empty  
      | single( $N$ )  
      | concat( $L, L$ )  
(Number)  $N ::= \text{len}(L)$   
        | min( $L$ )  
        | add( $N, N$ )  
        | 0 | 1 | 2 | ...
```

Semantics

```
def evaluate(program, input):  
    if isinstance(program, Empty):  
        return []  
    elif isinstance(program, Input):  
        return input  
    elif isinstance(program, Concat):  
        left = evaluate(program.left, input)  
        right = evaluate(program.right, input)  
        return left + right  
    elif ...
```

Examples

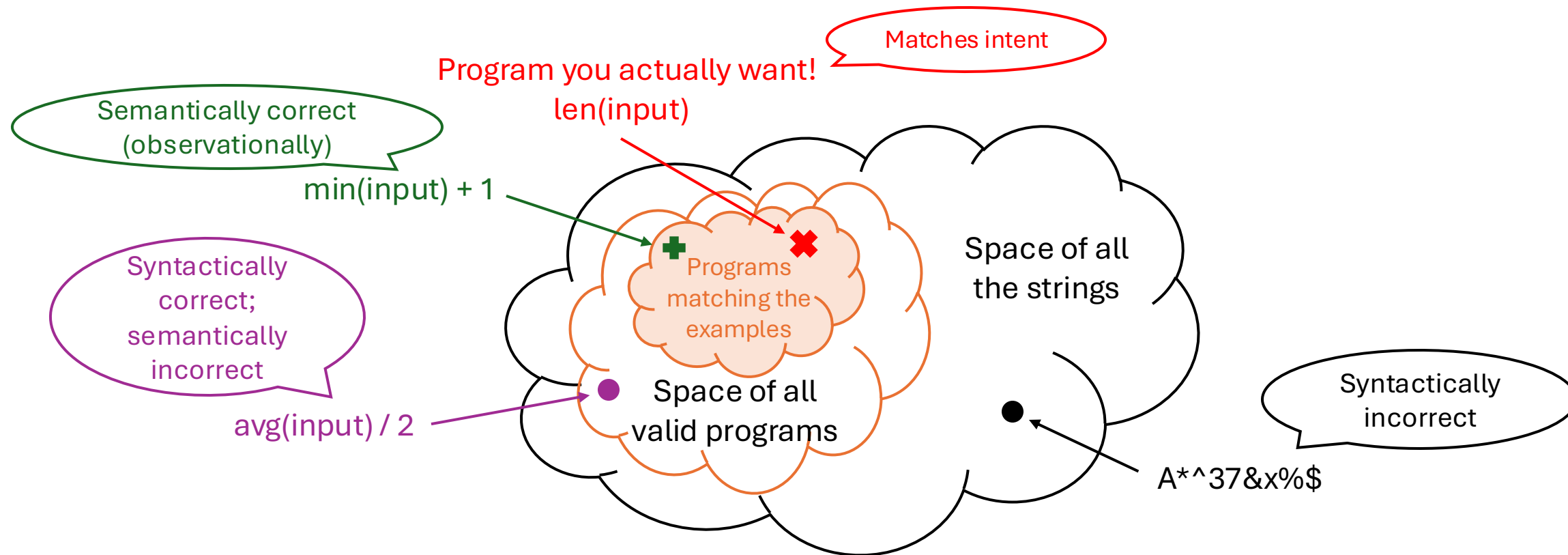
```
[0, 1]  -> 3  
[1]     -> 2  
[3, 5, 4] -> 4
```

`evaluate(`len(concat(single(3),input))`, [0, 1])` \rightarrow `len([3, 0, 1])` = 3

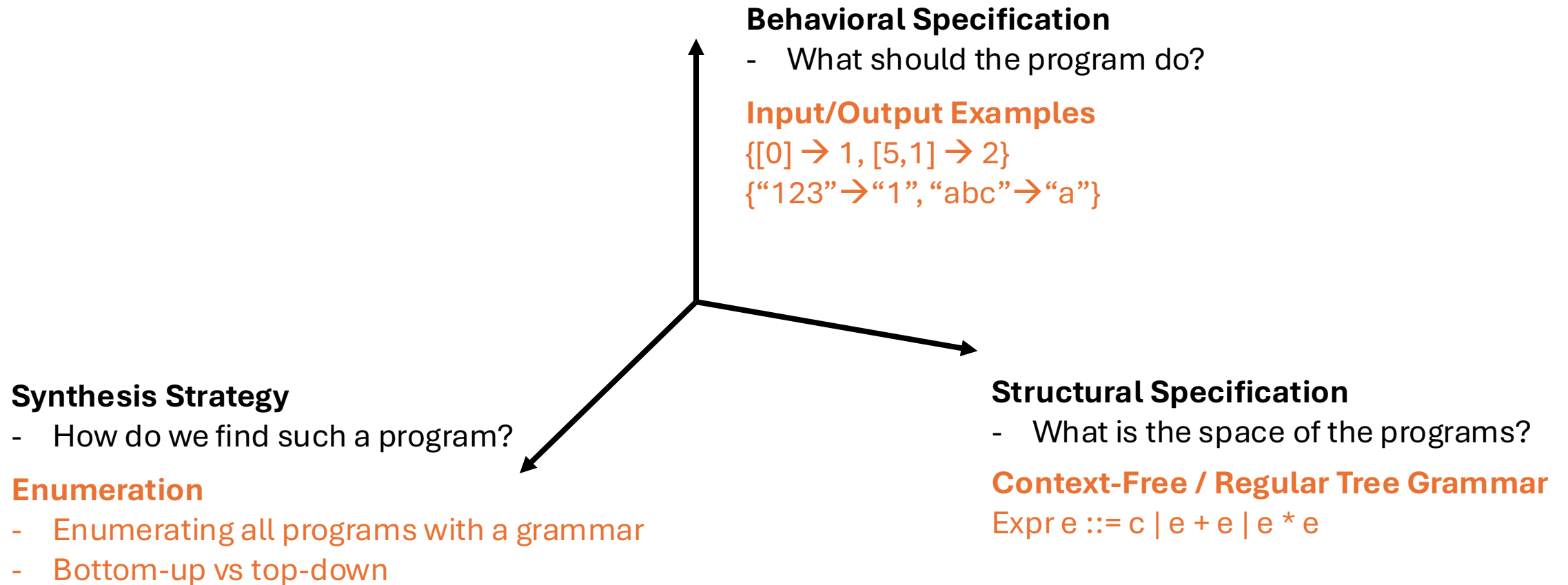
└─ `evaluate(`concat(single(3),input)`, [0, 1])` \rightarrow `[3] + [0, 1]` = `[3,0,1]`

└─ `evaluate(`single(3)`, [0, 1])` \rightarrow `[3]`

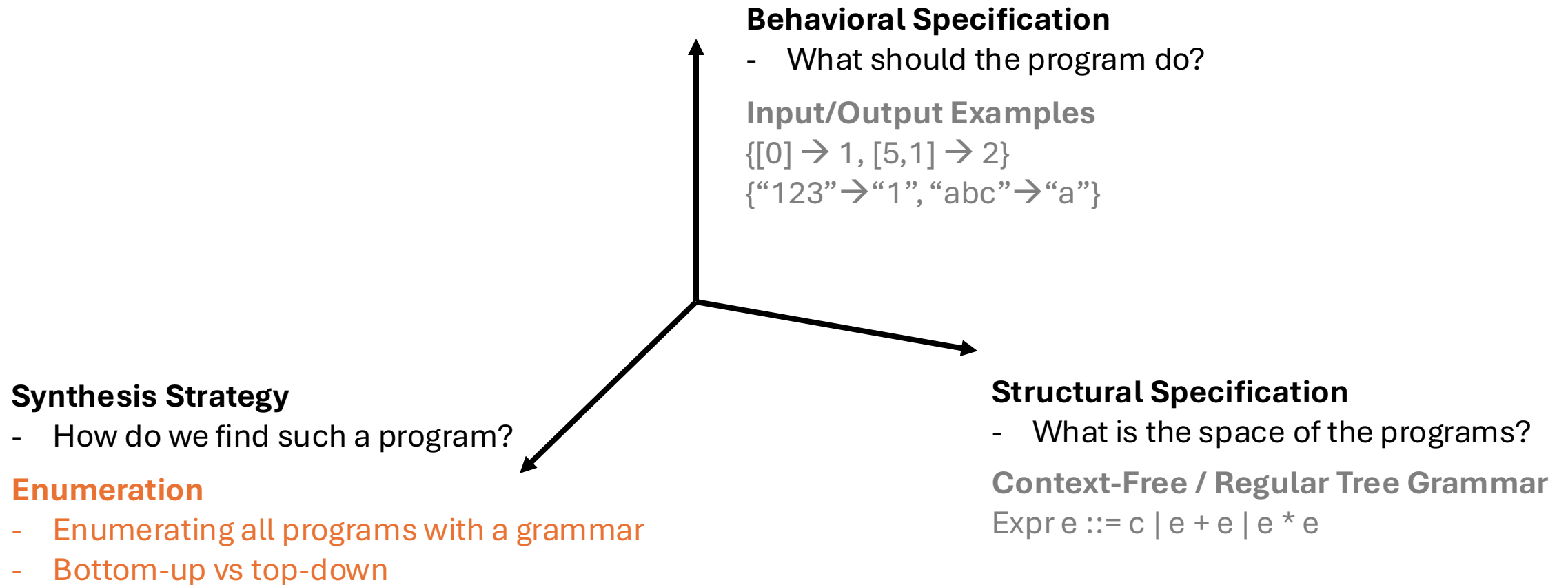
└─ `evaluate(`input`, [0, 1])` \rightarrow `[0, 1]`



Today



Today



Enumerative search

- **Idea**: enumerate programs from the grammar one by one and test them on the examples
- **Challenge**: How do we systematically enumerate all programs?
 - Bottom-up
 - Top-down

Bottom-up enumeration

- Maintain a **bank** of complete programs
 - Starting from all the terminal symbols
- Combine programs in the bank using **production rules**
 - Applying all possible production rules at each iteration

```
(Program)  $\underline{P} ::= L \mid N$   
(List)  $L ::= \text{input}$   
      | empty  
      | single(N)  
      | concat(L, L)  
(Number)  $N ::= \text{len}(L)$   
        | min(L)  
        | add(N, N)  
        | 0 | 1 | 2 | ...
```

Bottom-up enumeration: algorithm

```
bottom-up(< $\Sigma$ , N, R, S>, [i  $\rightarrow$  o], max_depth):  
  bank := {}  
  for depth in [0..max_depth]:  
    forall rule in R:  
      forall new_prog in grow(rule, depth, bank):  
        if (A = S  $\wedge$  new_prog([i]) = [o]):  
          return new_program  
        insert new_program to bank;  
  
grow(A  $\rightarrow$   $\sigma$ (A1...Ak), d, bank):  
  if (d = 0  $\wedge$  k = 0) yield  $\sigma$  // terminal  
  else forall <t1,...,tk> in bankk: // cartesian product  
    if Ai  $\rightarrow^*$  ti:  
      yield  $\sigma$ (t1,...,tk)
```

```
(Program)  $\underline{P}$  ::= L | N  
(List) L ::= input  
           | empty  
           | single(N)  
           | concat(L, L)  
(Number) N ::= len(L)  
            | min(L)  
            | add(N, N)  
            | 0 | 1 | 2 | ...
```

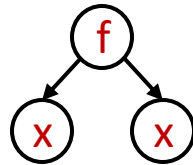
Bottom-up enumeration: example

$E ::= x \mid f(E, E)$

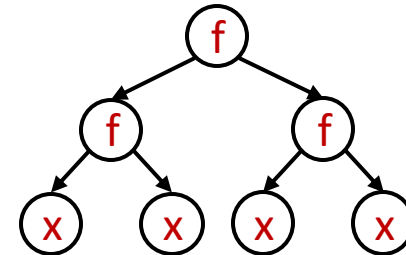
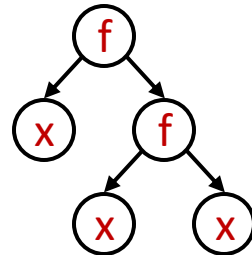
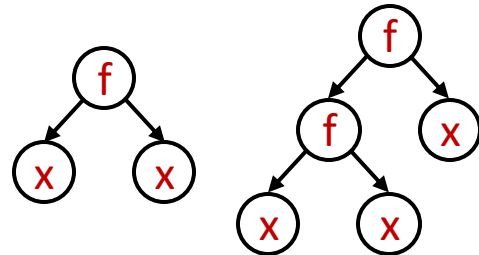
Depth ≤ 0



Depth ≤ 1



Depth ≤ 2



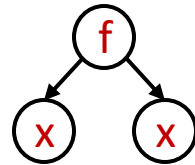
Bottom-up enumeration: example

$E ::= x \mid f(E, E)$

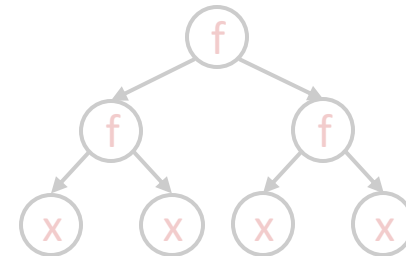
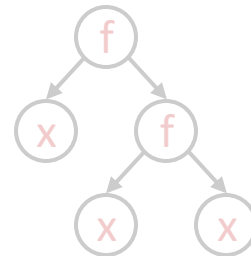
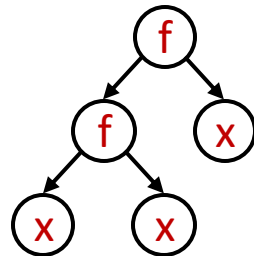
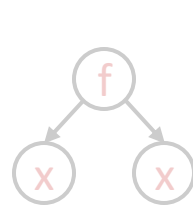
Depth ≤ 0



Depth ≤ 1



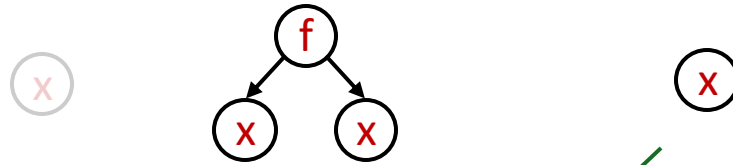
Depth ≤ 2



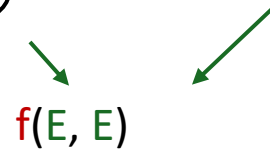
Bottom-up enumeration: example

$E ::= x \mid f(E, E)$

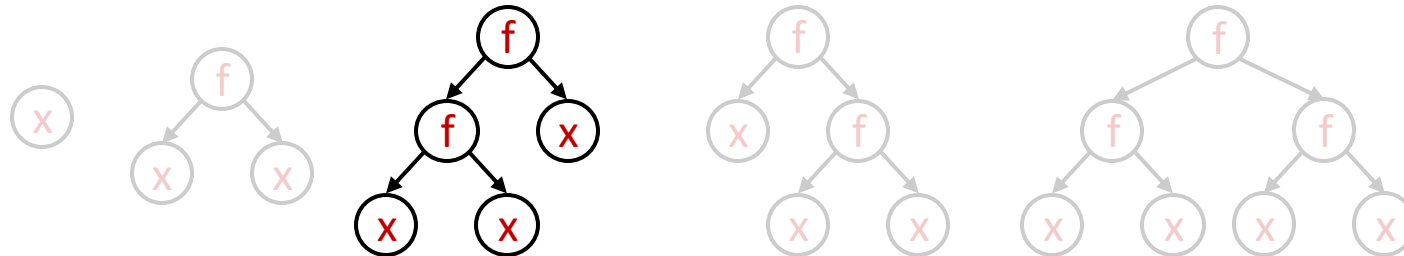
Depth ≤ 0



Depth ≤ 1



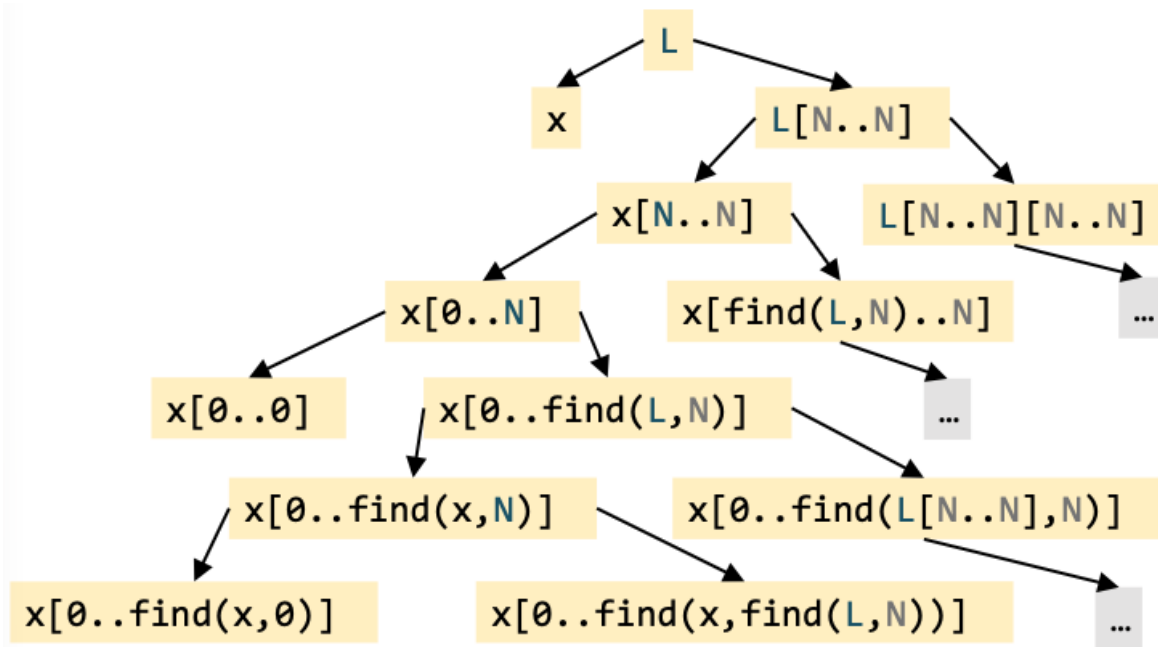
Depth ≤ 2



Top-down enumeration

- Search space is a tree, where:
 - Nodes are whole incomplete programs
 - Edges are derivations in a single step

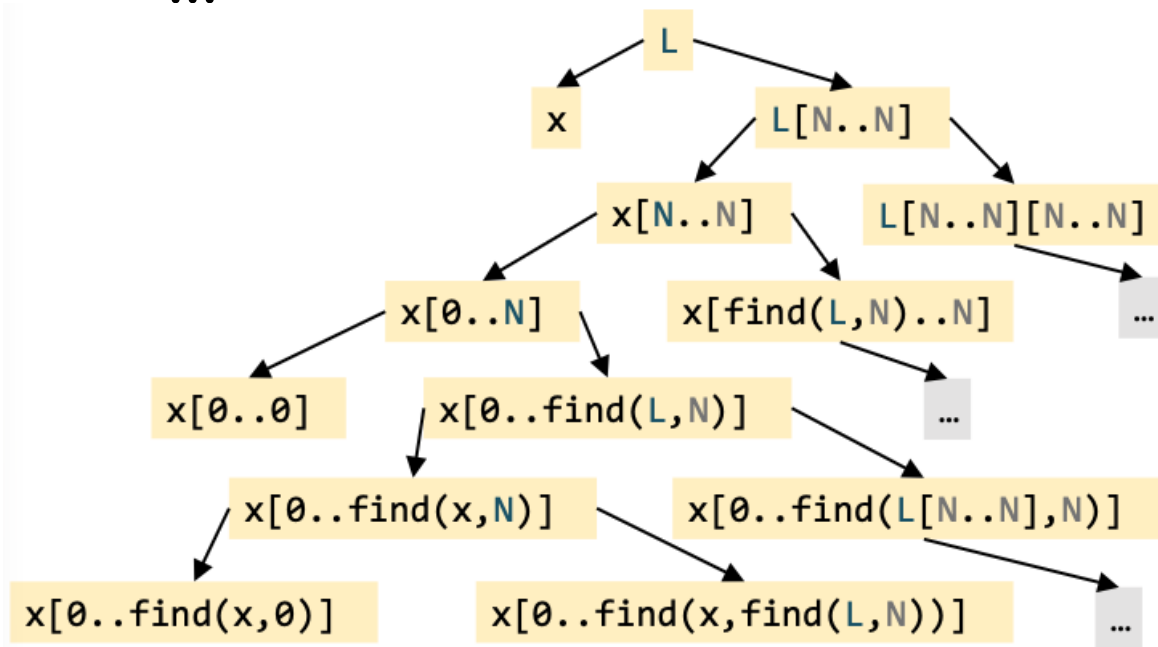
(List) $\underline{L} ::= L[N:N]$
| x // input
(Number) $N ::= \text{find}(L, N)$
| 0



Top-down enumeration

- Search tree can be traversed
 - Depth-first
 - Breadth-first
 - ...

(List) $\underline{L} ::= L[N:N]$
| x // input
(Number) $N ::= \text{find}(L, N)$
| 0



Bottom-up vs top-down

Top-down

Program candidates are **whole**
but might not be **complete**

- Cannot always run on inputs
- Can always relate to outputs

Bottom-up

Program candidates are **complete**
but might not be **whole**

- Can always run on inputs
- Cannot always relate to outputs

How to make it scale

Prune

- Discard useless subprograms

Prioritize

- Explore more promising candidates

Summary

- Syntax
- Semantics
- Enumerative algorithms
 - Bottom-up
 - Top-down

Week 1

- Assignment 1
 - Released: <https://github.com/machine-programming/assignment-1>
 - Autograder will be on GradeScope later today
 - API keys will be sent out later today
- Waitlisted students
 - Please contact me by sending emails; will add you to Courselore, GradeScope, and give you API keys
- Any questions?