BC Unit 6: Integration and Accumulation of Change

6.1 Exploring Accumulations of Change

6.1: Daily Video 1

In this video, we will explore the connection between area and the accumulation of a rate of change in context.

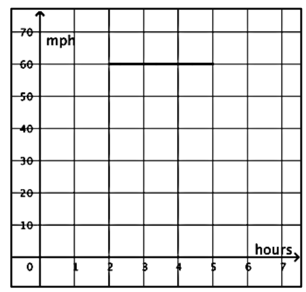
4.B *Communication and Notation* — Use appropriate units of measure.

WHAT WILL WE LEARN?

Accumulation of a Rate of Change
Up to this point, we have worked with functions and found their rate of change using the derivative.
In this video, we will reverse this process and start with a rate of change to find information about the function whose rate of change is known.

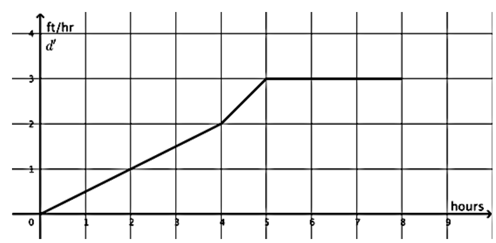
Velocity

During a road trip, a car traveled at a constant velocity of 60 miles per hour during hours 2 through 5.



Filling a Pool

The function, d'(t) is the rate that the depth of water in feet changes per hour. If it takes 8 hours to fill the pool, what is the depth of the water?



WHAT SHOULD WE TAKE AWAY?

Area and Accumulation

- Using the units for the rate of change function and the unit of the independent value will give you the units for the area.
- The units will help you describe the accumulation of change over the interval.
- The area of a region bounded by velocity and time is represents the change in position.

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BC Unit 6: Integration and Accumulation of Change

6.1 Exploring Accumulations of Change

6.1: Daily Video 2

In this video, we will explore positive and negative accumulation and practice solving problems involving accumulation of change over an interval.

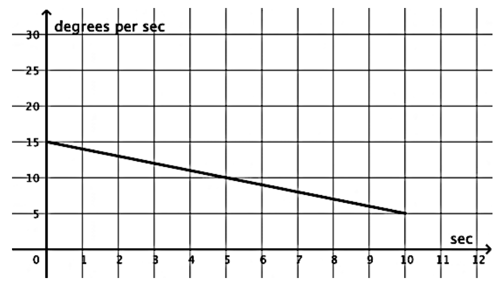
4.B *Communication and Notation* — Use appropriate units of measure.

WHAT WILL WE LEARN?

Accumulation of Change
- If a rate of change is positive over an interval, then the accumulated change is positive.
- If a rate of change is negative over an interval, then the accumulated change is negative.

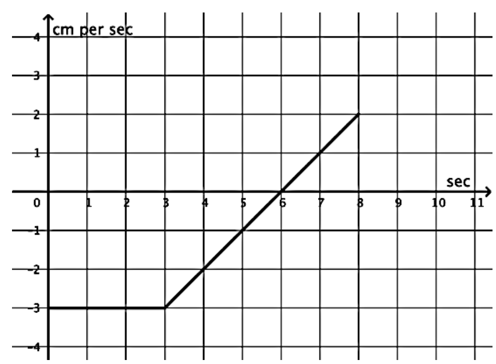
Temperature – Cooling

A cup of hot water is set on the counter to cool. The rate of change of the temperature in degrees per second is graphed. What do we know about the water after 10 seconds?



Rectilinear Motion

An object starts at the origin and moves horizontally as time passes. The velocity of the object in cm per second is graphed. After 8 seconds have passed, what do we know about the object?



BC Unit 6: Integration and Accumulation of Change

6.2 Approximating Area with Riemann Sums

6.2: Daily Video 1

In this video, we will use left and right Riemann sums a to approximate the area under the curve.

1.F *Implementing Mathematical Processes*

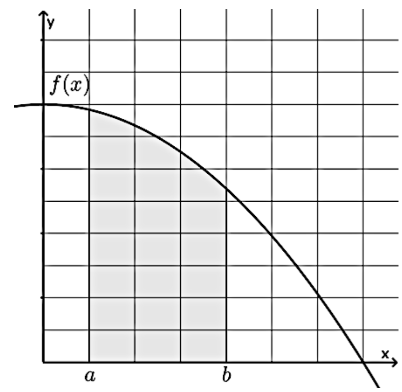
Explain how an approximated value relates to the actual value.

WHAT WILL WE LEARN?

Approximating Area
We will learn how to approximate area bounded by a function, the lines *x*=*a*, *x*=*b*, and the *x*-axis.

Left Riemann Sum

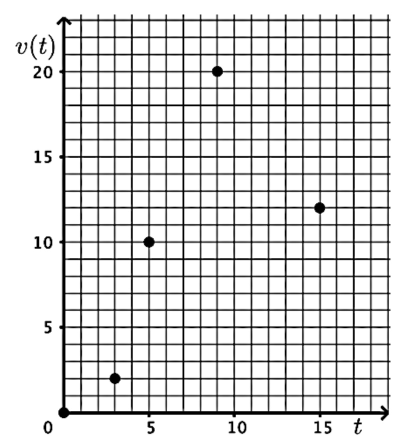
Given a continuous function *f*, using rectangles, approximate the area under the curve bounded by the *x*-axis and *x*=*a* and *x*=*b*.



Tabular Data

v(*t*) is the velocity of an object moving horizontally measured in feet per second. *v*(*t*) is continuous, and values of *v*(*t*) are given in the table. Approximate the distance traveled from *t*=0 to *t*=15 seconds using a left Riemann sum.

t seconds	0	3	5	9	15
v(t) ft/sec	0	2	10	20	12



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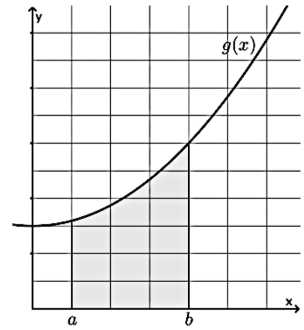
Right Riemann Sum

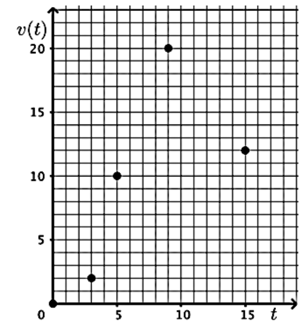
Given a continuous function *g*, using rectangles, approximate the area under the curve bounded by the *x*-axis and *x*=*a* and *x*=*b*.



Tabular Data

v(*t*) is the velocity of an object moving horizontally measured in feet per second. *v*(*t*) is continuous, and values of *v*(*t*) are given in the table. Approximate the distance traveled from *t*=0 to *t*=15 seconds using a right Riemann sum.

t seconds	0	3	5	9	15
v(t) ft/sec	0	2	10	20	12

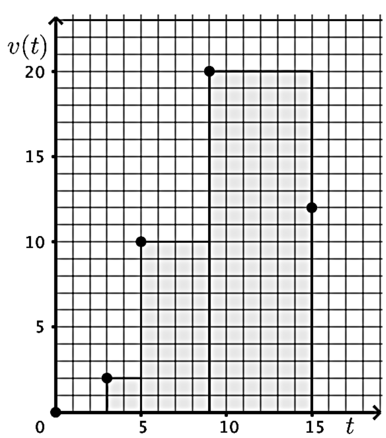


WHAT SHOULD WE TAKE AWAY?

Estimated Areas

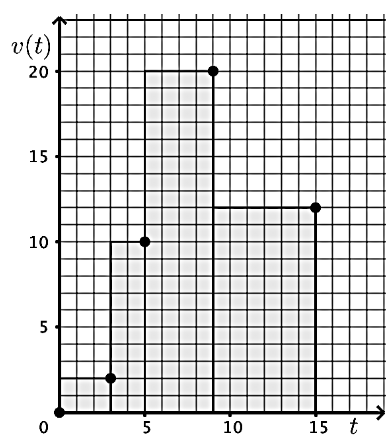
Left Riemann Sum

164 feet



Right Riemann Sum

178 feet



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BC Unit 6: Integration and Accumulation of Change

6.2 Approximating Area with Riemann Sums

6.2: Daily Video 2

In this video, we will use midpoint Riemann sums and trapezoidal sums to approximate the area under the curve.

1.F *Implementing Mathematical Processes*

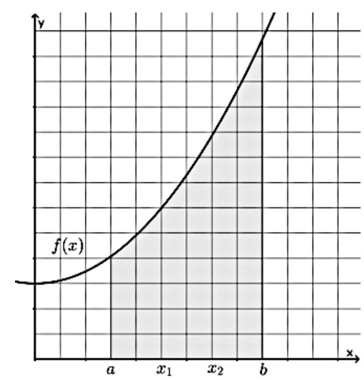
Explain how an approximated value relates to the actual value.

WHAT WILL WE LEARN?

Approximating Area
In this video will learn how to approximate area bounded by a function, the lines $x = a$, $x = b$, and the x-axis using midpoint Riemann sums and trapezoidal sums.

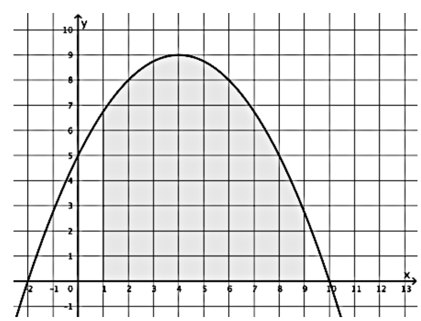
Midpoint Riemann Sum

Given a continuous function *f*, using rectangles, approximate the area under the curve bounded by the *x*-axis and $x=a$ and $x=b$.



Midpoint Riemann Sum Example

Find the midpoint Riemann sum approximation of the area bounded by $x = 1$, $x = 9$, the curve $f(x) = -\frac{1}{4}(x-4)^2 + 9$ and the x-axis using 4 subintervals of equal length.



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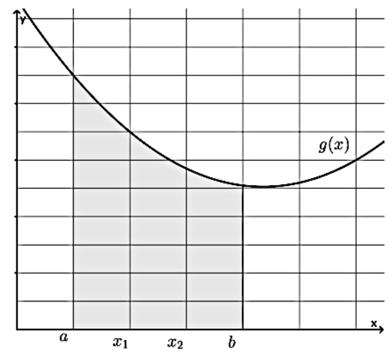
Trapezoidal Sum

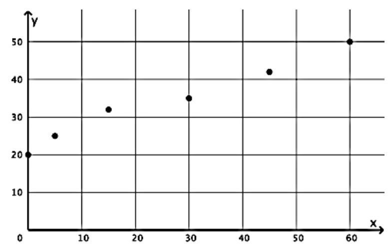
Given a continuous function *g*, using right trapezoids, approximate the area under the curve bounded by the *x*-axis and *x*=*a* and *x*=*b*.



Tabular Data

Water is dripping from a faucet measured in drops per minute. Values for the rate of drops are recorded at the different times in the table. Use a trapezoidal sum to estimate the number of drops of water that dripped from *t* = 15 min to *t* = 60 min.

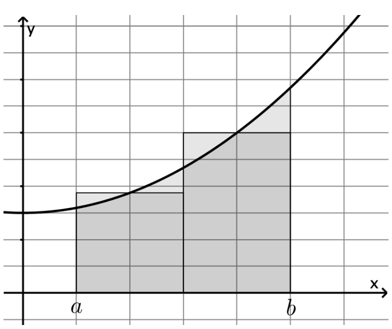
t min	0	5	15	30	45	60
d(t) drops/min	20	25	32	35	42	50



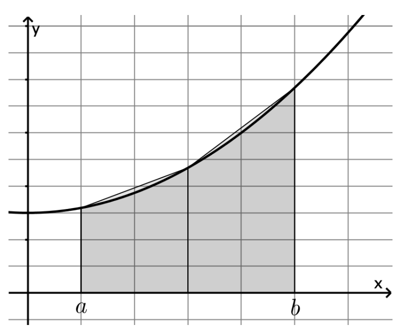
WHAT SHOULD WE TAKE AWAY?

Estimated Areas

Midpoint Riemann Sum



Trapezoidal Riemann Sum



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BC Unit 6: Integration and Accumulation of Change

6.2 Approximating Area with Riemann Sums

6.2: Daily Video 3

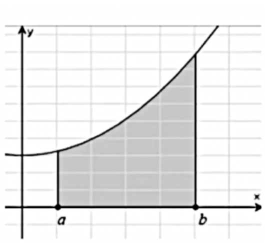
In this video, we will make the connection between the behavior of the function and whether an approximation for a definite integral is an underestimate or overestimate when possible.

1.F *Implementing Mathematical Processes* — Explain how an approximated value relates to the actual value.

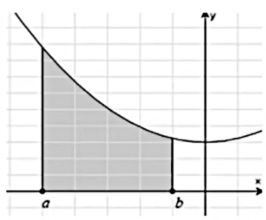
WHAT WILL WE LEARN? In this video, we will connect the behavior of the function to whether the estimate is an overestimate or underestimate where possible.

Behaviors of Functions

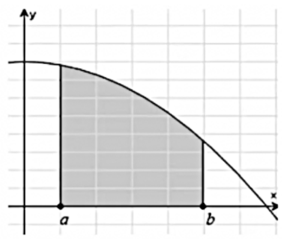
Think about all the behaviors of functions that we have described in the past.



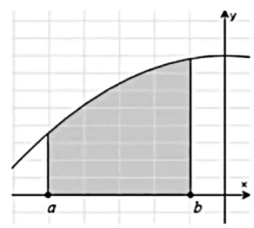
Increasing Concave Up



Decreasing Concave Up

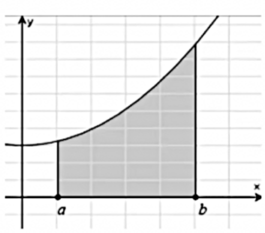


Decreasing Concave Down

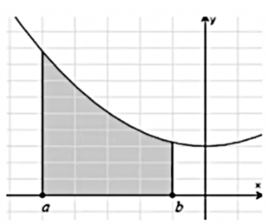


Increasing Concave Down

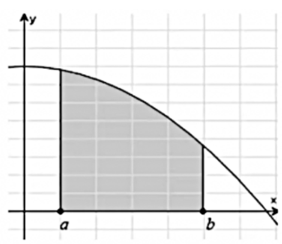
Left Riemann Sum



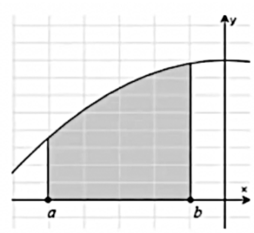
Increasing Concave Up



Decreasing Concave Up



Decreasing Concave Down



Increasing Concave Down

For a strictly increasing function, the left Riemann sum is an ____________________

For a strictly decreasing function, the left Riemann sum is an ____________________

Concavity is not considered.

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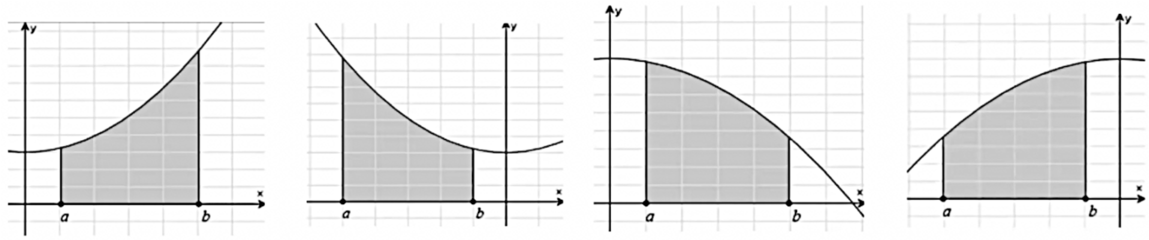
Right Riemann Sum

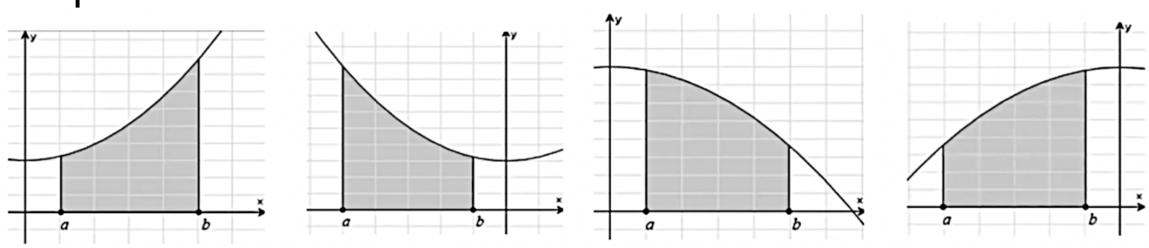


Increasing Concave Up — Decreasing Concave Up — Decreasing Concave Down — Increasing Concave Down

For a strictly increasing function, the right Riemann sum is an ________________

For a strictly decreasing function, the right Riemann sum is an ________________
Concavity is not considered.

Midpoint Riemann Sum

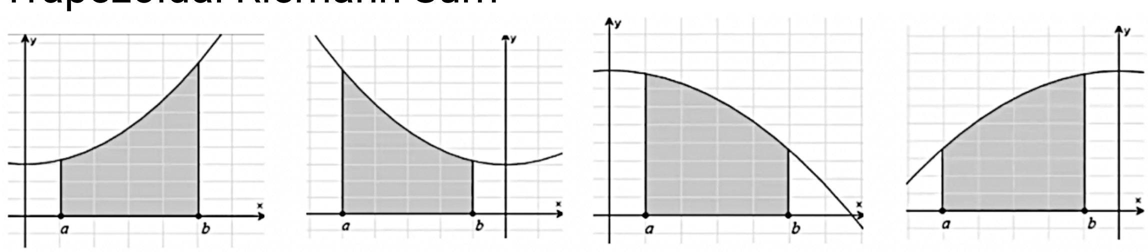


Increasing Concave Up — Decreasing Concave Up — Decreasing Concave Down — Increasing Concave Down

On a concave up interval, the midpoint Riemann sum is an ________________

On a concave down interval, the midpoint Riemann sum is an ________________
Increasing and decreasing is not considered.

Trapezoidal Riemann Sum



Increasing Concave Up — Decreasing Concave Up — Decreasing Concave Down — Increasing Concave Down

On a concave up interval, the trapezoidal Riemann sum is an ________________

On a concave down interval, the trapezoidal Riemann sum is an ________________
Increasing and decreasing is not considered.

WHAT SHOULD WE TAKE AWAY?

- Right, left, midpoint Riemann sums and trapezoidal sums are estimates.
- Concavity is used to determine if the trapezoidal sum or the midpoint Riemann sum is an over or underestimate.
- If the function is strictly increasing or decreasing on a specified interval, it is possible to determine if the right or left Riemann sum is an under or overestimate.

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6.3 Riemann Sums, Summation Notation and Definite Integral Notation

6.3: Daily Video 1

In this video, we will connect the limit of a Riemann sum to a definite integral.

2.C *Connecting Representations*

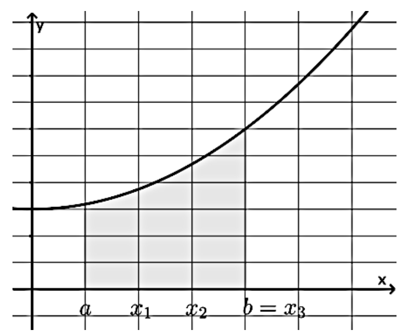
Identify a re-expression of mathematical information presented in a given representation.

WHAT WILL WE LEARN?

- In section 6.2, we estimated the area under the curve using rectangles.
- In this video, we will explore a method that is exact and the notation that corresponds.

Right Riemann Sum

Given a continuous function *f*, using rectangles, approximate the area under the curve bounded by the x-axis and x = *a* and x = *b*.



The Limit of a Riemann Sum

$$\lim_{n\to\infty}\sum_{k=1}^{n} f\left(x_k\right)\Delta x =$$

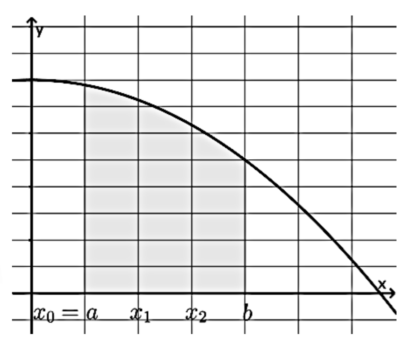
Integral Notation

The integral from *a* to *b* of *f*(*x*) with respect to *x*.

$$\int_{\underbrace{a}_{\text{lower limit}}}^{\overbrace{b}^{\text{upper limit}}} \overbrace{\underbrace{f\left(x\right)}_{\text{height}}}^{\text{integrand}}\ \underbrace{dx}_{\text{base}}$$

Left Riemann Sum

Given a continuous function *f*, using rectangles, approximate the area under the curve bounded by the x-axis and x = *a* and x = *b*.



The Limit of a Riemann Sum $\lim_{n\to\infty}\sum_{k=1}^{n} f\left(x_{k-1}\right)\Delta x = \int_a^b f\left(x\right)dx$

WHAT SHOULD WE TAKE AWAY?

Definite Integrals

We now have a process for calculating the area under the curve, integration.

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BC Unit 6: Integration and Accumulation of Change

6.3 Riemann Sums, Summation Notation and Definite Integral Notation

6.3: Daily Video 2

In this video, we will practice problems involving limits of Riemann sums and definite integrals.

2.C *Connecting Representations*

Identify a re-expression of mathematical information presented in a given representation.

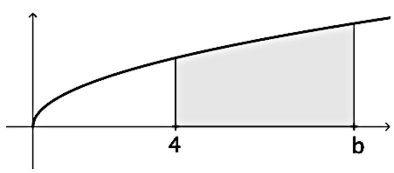
WHAT WILL WE LEARN?

Working with Representations
We will work through problems that connect the limit of a Riemann sum to a definite integral.

$$\lim_{n\to\infty}\sum_{k=1}^{n} f(x_k)\Delta x = \int_a^b f(x)\,dx$$

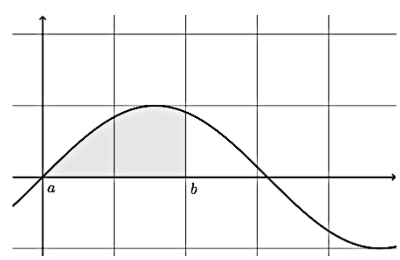
Example

For what value of b, does the integral $\int_4^b \sqrt{x}\,dx$ equal $\lim_{n\to\infty}\sum_{k=1}^{n}\left[\left(\sqrt{4+\frac{5}{n}k}\right)\left(\frac{5}{n}\right)\right]$?

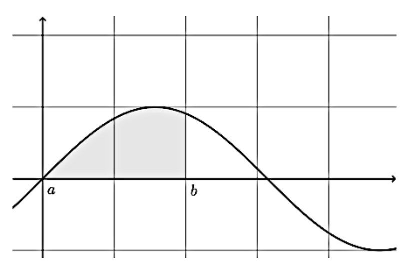


Left or Right?

$$\int_0^2 \sin x\,dx = \lim_{n\to\infty}\sum_{k=1}^{n}\left[\left(0+\sin\left(\frac{2k}{n}\right)\right)\left(\frac{2}{n}\right)\right]$$



$$\int_0^2 \sin x\,dx = \lim_{n\to\infty}\sum_{k=1}^{n}\left[\left(0+\sin\left(\frac{2(k-1)}{n}\right)\right)\left(\frac{2}{n}\right)\right]$$



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Example

The function f is given by $f\left(x\right)=e^{x}$

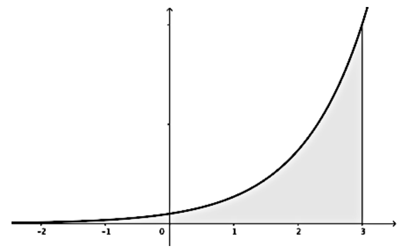
Which is equivalent to the area of the shaded region?

(A) $\lim\limits_{n\to\infty}\sum\limits_{k=1}^{n}\left[\left(e^{-2+\frac{3k}{n}}\right)\left(\frac{5}{n}\right)\right]$

(B) $\lim\limits_{n\to\infty}\sum\limits_{k=1}^{n}\left[\left(e^{-2+\frac{k}{n}}\right)\left(\frac{3}{n}\right)\right]$

(C) $\lim\limits_{n\to\infty}\sum\limits_{k=1}^{n}\left[\left(e^{-2+\frac{3k}{n}}\right)\left(\frac{3}{n}\right)\right]$

(D) $\lim\limits_{n\to\infty}\sum\limits_{k=1}^{n}\left[\left(e^{-2+\frac{5k}{n}}\right)\left(\frac{5}{n}\right)\right]$



WHAT SHOULD WE TAKE AWAY?

Working with Definite Integrals

- The Riemann sum is an approximation answer.
- The limit of an approximate Riemann sum can be interpreted as a definite integral.
- In later topics, we will learn ways to evaluate definite integrals analytically.

BC Unit 6: Integration and Accumulation of Change

6.4 The Fundamental Theorem of Calculus and Accumulation Functions

6.4: Daily Video 1

In this video, we will discuss how the Fundamental Theorem of Calculus connects differentiation and integration.

1.D *Implementing Mathematical Processes*

Identify an appropriate mathematical rule or procedure based on the relationship between concepts or processes to solve problems.

WHAT WILL WE LEARN?

The Fundamental Theorem of Calculus
In this video we are going to connect two major topics of calculus, namely differentiation and integration.

The Fundamental Theorem of Calculus:

If f is a continuous function on an interval containing a, then $\frac{d}{dx}\left(\int_a^x f(t)dt\right) = f(x)$, where x is in the interval.

Important: Differentiation and integration are ________________________________

Example

Let g be the function defined by $g(x) = \int_0^x (3t^2+1)dt$. Find $g'(x)$.

Example

Let F be the function defined by $F(x) = \int_{\pi}^{x}\left(\sin\frac{t^3}{4}\right)dt$. Find $F'(x)$.

Example

Let f be the function given by $f(x) = \int_{-5}^{x} g(t)dt$ where g is a differentiable function. The table has selected values of f, g, g'. What is $f'(3)$?

x	1	3
$f(x)$	3	1
$g(x)$	0	2
$g'(x)$	4	5

WHAT SHOULD WE TAKE AWAY?

The Fundamental Theorem of Calculus connects differentiation and integration, stating that the two are inverse operations of each other.

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BC Unit 6: Integration and Accumulation of Change

6.4 The Fundamental Theorem of Calculus and Accumulation Functions

6.4: Daily Video 2

In this video, we will practice solving problems by applying the Fundamental Theorem of Calculus.

1.D *Implementing Mathematical Processes*

Identify an appropriate mathematical rule or procedure based on the relationship between concepts or processes to solve problems.

WHAT WILL WE LEARN?

In this video we are going practice solving more complicated problems involving the Fundamental Theorem of Calculus and the Chain Rule (Topic 3.1).

If f is a continuous function on an interval containing a, then $\frac{d}{dx}\left(\int_a^x f(t)dt\right)=f(x)$, where x is in the interval.

If f is a continuous function on an interval containing a, then $\frac{d}{dx}\left(\int_a^{g(x)} f(t)dt\right)=f(g(x))\cdot g'(x)$, where x is in the interval.

Example

Let h be the function defined by $h(x)=\int_{-5}^{\sin x}(3t^2+1)dt$. Find $h'(x)$.

Example

Let F be the function defined by $F(x)=\int_0^{3x^2}(\ln(2t^3+5))dt$. Find $F'(x)$.

Connecting Back to Velocity

We know that velocity, *v(t),* is the rate of change of position, *s(t)* with respect to time. In section 6.1, we learned that the area bounded by a velocity graph, *v(t)* and the *t* axis gives us information about position.

$$s(t)=\int_a^t v(x)dx$$

Then if we apply the FTC... $s'(t)=$

So the rate of change of position is velocity.

WHAT SHOULD WE TAKE AWAY?

- The Fundamental Theorem of Calculus connects differentiation and integration, stating that the two are inverse operations of each other.
- When there is a composition of functions, you will need to apply the Chain Rule.

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BC Unit 6: Integration and Accumulation of Change

6.5 Interpreting the Behavior of Accumulation Functions Involving Area

6.5: Daily Video 1

In this video, we will apply the Fundamental Theorem of Calculus to analyze functions defined by an integral.

2.D *Connecting Representations*

Identify how mathematical characteristics or properties of functions are related in different representations.

WHAT WILL WE LEARN?

Analyzing Functions
We are going to use derivatives to analyze the behavior of functions written as an integral.

The Fundamental Theorem of Calculus tells us $\frac{d}{dx}\left(\int_a^x f(t)dt\right)=f(x)$.

Using the First Derivative to Analyze the Behavior of a Function

Characteristic of the first derivative	**Behavior of the original function**
$f'(x)>0$	$f(x)$ is
$f'(x)<0$	$f(x)$ is
$f'(x)$ increasing	$f(x)$ is
$f'(x)$ decreasing	$f(x)$ is

Example (no calculator)

Let f be the function given by $f(x)=\int_0^x ((2t+1)(t-3))dt$

Analyze $f(x)$:

On what interval(s) is *f* increasing?

On what interval(s) is *f* decreasing?

At what values of x, does f have any relative extrema?

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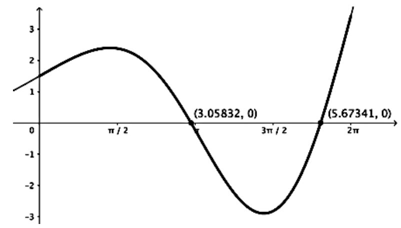
Example (with calculator)

A function g is given by $g(x) = \int_0^x \left(t\sin\left(\sqrt{t^2+4}\right)+1.5\right)dt$ for $0 \le x \le 2\pi$.

On what interval(s) is g increasing?



On what interval(s) is g increasing?

WHAT SHOULD WE TAKE AWAY?

Analyzing Functions Defined by an Integral

- When the function that is being analyzed is defined as an integral, apply the Fundamental Theorem of Calculus to find the derivative.
- In the next video, we will practice analyzing functions defined as an integral and presented as a graph.

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6.5 Interpreting the Behavior of Accumulation Functions Involving Area

6.5: Daily Video 2

In this video, we will apply the Fundamental Theorem of Calculus to analyze functions defined by an integral presented graphically.

2.D *Connecting Representations*

Identify how mathematical characteristics or properties of functions are related in different representations.

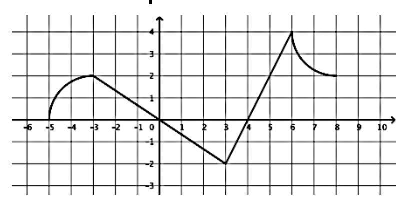
WHAT WILL WE LEARN?

Analyzing Functions

We will use the derivative to analyze functions written as an integral by describing when the function is increasing/decreasing, concave up or down, as well as where the function has relative extrema or points of inflection.

Analyzing the Behavior of a Function

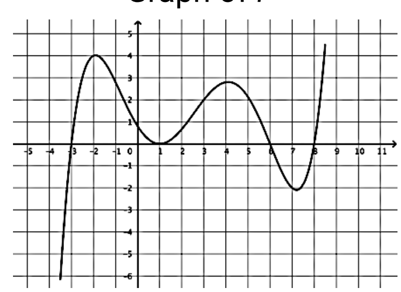
Graph of *f*



The function *f* is defined on the closed interval [-5, 8]. The graph of *f* consists of two line segments and two quarter circles as shown in the figure.

Let *g* be the function defined by $g(x)=\int_{-5}^{x} f(t)\,dt$. On what interval(s) is *g* both increasing and concave down?

Graph of *f*



The function *f* is shown on the closed interval [-3.5, 8.5] in the figure. Let *g* be the function defined by $g(x)=\int_{-5}^{x} f(t)\,dt$.

How many points of inflection does *g* have?

WHAT SHOULD WE TAKE AWAY?

Analyzing Functions Defined by an Integral

- When the function that is being analyzed is defined as an integral, apply the Fundamental Theorem of Calculus to find the derivative.
- Use the characteristics of the derivative to interpret the behavior of the function.

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6.6 Applying Properties of Definite Integrals

6.6: Daily Video 1

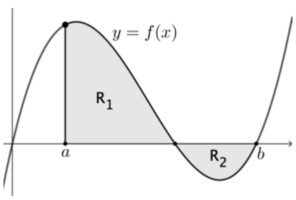
In this video, we will discuss and apply the properties of definite integrals.

3.D *Justification*

Apply an appropriate mathematical definition, theorem, or test.

WHAT WILL WE LEARN?

- We will learn about 5 properties of definite integrals.
- We will then apply the properties to calculate definite integrals represented graphically and algebraically.



The definite integral represents the cumulative signed area between the function and the *x*-axis bounded by $x = a$ and $x = b$.

Properties of Definite Integrals

Integral of a **constant times** a function:

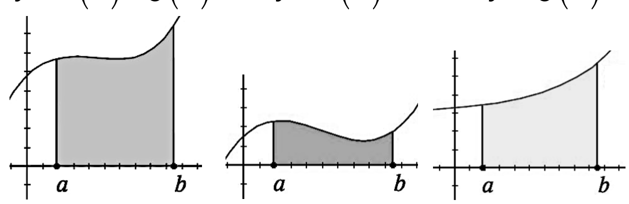
$$\int_a^b k\cdot f(x)dx = k\int_a^b f(x)dx$$

where *k* is any constant.

Integral of the **sum** of two functions:

$$\int_a^b \left(f(x)+g(x)\right)dx = \int_a^b f(x)dx + \int_a^b g(x)dx\,.$$

$y = f(x)+g(x)$ $\qquad$ $y = f(x)$ $\qquad$ $y = g(x)$

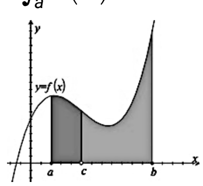


Reversal of limits of integration:

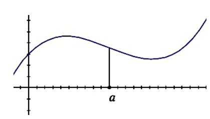
$$\int_a^b f(x)dx = -\int_b^a f(x)dx$$

Integral of the definite integral over **adjacent intervals**: $\int_a^c f(x)dx + \int_c^b f(x)dx = \int_a^b f(x)dx$

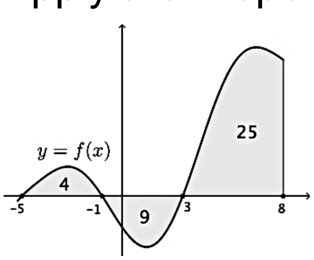
where *c* is any constant such that $a < c < b$.



Same limits of integration: If *f* is defined at $x = a$ then $\int_a^a f(x)dx = 0$



Apply the Properties – Graphical Examples



$$\int_{-5}^{3} 2f(x)dx =$$

$$\int_3^3 f(x)dx + \int_8^{-1} f(x)dx =$$

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Apply the Properties – Algebraic Examples

Given $\int_{-1}^{4} f(x)\,dx = 7$ and $\int_{-1}^{4} g(x)\,dx = 3$. Evaluate $\int_{-1}^{4} \left(5f(x) - 2g(x)\right)dx$.

Given $\int_{-3}^{0} f(x)\,dx = 8$ and $\int_{0}^{2} f(x)\,dx = -5$. Evaluate $4\int_{-3}^{2} f(x)\,dx$.

Given $\int_{6}^{-5} g(x)\,dx = -10$ and $\int_{6}^{8} g(x)\,dx = 7$. Evaluate $\int_{-5}^{8} 2g(x)\,dx$.

BC Unit 6: Integration and Accumulation of Change

6.6 Applying Properties of Definite Integrals

6.6: Daily Video 2

In this video, we will work with functions that have removable or jump discontinuities.

3.D *Justification* Apply an appropriate mathematical definition, theorem, or test.

WHAT WILL WE LEARN? We will calculate definite integrals using areas and properties of definite integrals extended to functions with removable or jump discontinuities.

Remember the Properties of Definite Integrals

Integral of a **constant times** a function:
$\int_a^b k\cdot f(x)dx = k\int_a^b f(x)dx$ where k is any constant.

Integral of the **sum** of two functions:
$\int_a^b (f(x)+g(x))dx = \int_a^b f(x)dx + \int_a^b g(x)dx$.

Reversal of limits of integration:
$\int_a^b f(x)dx = -\int_b^a f(x)dx$

Same limits of integration: If f is defined at $x = a$ then $\int_a^a f(x)dx = 0$

Integral of the definite integral over **adjacent intervals**: $\int_a^c f(x)dx + \int_c^b f(x)dx = \int_a^b f(x)dx$ where c is any constant such that $a < c < b$.

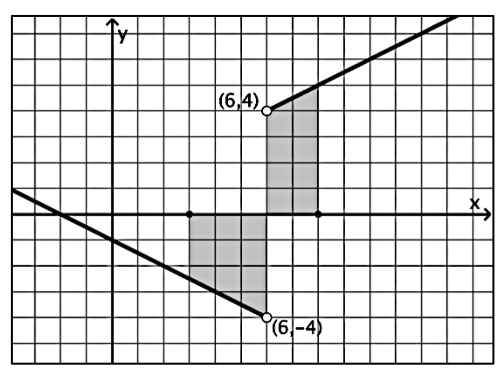
What Makes a Function Integrable?

A function, *f*, is integrable when:

- *f* is continuous on a closed interval [a, b]
- *f* is bounded on the closed interval [a, b] and has at most a finite number of discontinuities over [a, b].

Jump Discontinuity Example

If $f(x) = \dfrac{(x-6)\left(\frac{1}{2}x+1\right)}{|x-6|}$, then find the value of $\int_3^8 f(x)dx$.



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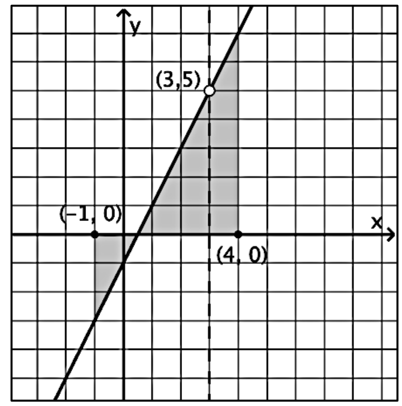
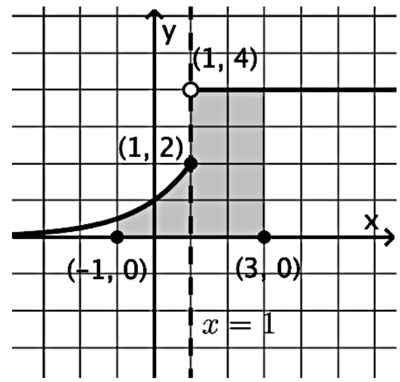
Removable Discontinuity Example

If $g(x)=\dfrac{(2x-1)(x-3)}{x-3}$, then find the value of $\int_{-1}^{4} g(x)\,dx$.





Example – Piecewise Function

If $f(x)=\begin{cases} 2^x & x \le 1 \\ 4 & x > 1 \end{cases}$, then find the value of $\int_{-1}^{3} f(x)\,dx$.

WHAT SHOULD WE TAKE AWAY?

What Makes a Function Integrable?

A function, *f,* is integrable when:

- *f* is continuous on a closed interval [a, b]
- *f* is bounded on the closed interval [a, b] and has at most a finite number of discontinuities over [a, b]
- We can calculate definite integrals using areas and properties of definite integrals of functions with removable or jump discontinuities.

BC Unit 6: Integration and Accumulation of Change

6.6 Applying Properties of Definite Integrals

6.6: Daily Video 3

In this video, we will practice calculating definite integrals using geometric areas and properties of definite integrals.

3.D *Justification* Apply an appropriate mathematical definition, theorem, or test.

WHAT WILL WE LEARN?

- We will learn about the integral of a constant property.
- Work through an algebraic definite integral problem.
- Work through definite integrals connected to a graph that will require using geometric area formulas to calculate definite integrals.

Remember the Properties of Definite Integrals

Remember the Properties of Definite Integrals

Integral of a **constant times** a function:

$\int_a^b k \cdot f(x)dx = k\int_a^b f(x)dx$ where k is any constant.

Integral of the **sum** of two functions:

$\int_a^b (f(x)+g(x))dx = \int_a^b f(x)dx + \int_a^b g(x)dx$.

Reversal of limits of integration:

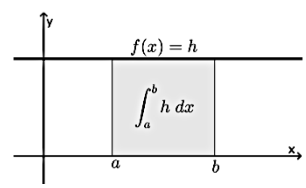
$\int_a^b f(x)dx = -\int_b^a f(x)dx$

Same limits of integration: If f is defined at $x = a$ then $\int_a^a f(x)dx = 0$

Integral of the definite integral over **adjacent intervals**: $\int_a^c f(x)dx + \int_c^b f(x)dx = \int_a^b f(x)dx$ where c is any constant such that $a < c < b$.

Integral of a Constant

If $f(x)=h$, where h is some constant, then $\int_a^b h dx =$ ________________ .



Algebraic Example

If $\int_{-4}^{1}(5f(x)-3)dx = 30$ and $\int_{8}^{-4} f(x)dx = 14$, then $\int_1^8 f(x)dx$?

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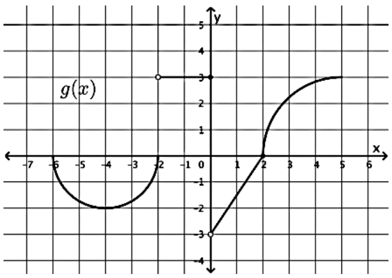
Graphical Example

The graph of the function g on the interval $-6 \leq x \leq 5$ consists of a semi-circle, 2 line segments and a quarter circle and shown. Use the graph to calculate the definite integrals.



$\int_{-2}^{2} g(x)\,dx =$

$\int_{0}^{5} g(x)\,dx =$

$\int_{-6}^{0} g(x)\,dx =$

$\int_{2}^{-6} g(x)\,dx =$

WHAT SHOULD WE TAKE AWAY?

Properties of Integrals

We have learned some properties of integrals that apply in algebraic settings as well as in graphical applications.

6.7 The Fundamental Theorem of Calculus and Definite Integrals

6.7: Daily Video 1

In this video, we will explore what is meant by antiderivative and apply the Fundamental Theorem of Calculus to evaluate definite integrals.

3.D *Justification*

Apply an appropriate mathematical definition, theorem, or test.

WHAT WILL WE LEARN?

The Fundamental Theorem of Calculus

There are two parts to the Fundamental Theorem of Calculus (FTC).

The Fundamental Theorem of Calculus tells us $\frac{d}{dx}\left(\int_a^x f(t)\,dt\right) = f(x)$.

Consequently, differentiation and integration are inverse operations.

If a function *f* is continuous on the closed interval [a,b] and *F* is an antiderivative of *f* on the interval [a,b], then $\int_a^b f(x)\,dx = F(b) - F(a)$ or $\int_a^b F'(x)\,dx = F(b) - F(a)$

Some call this the **Integral Evaluation** part of the theorem.

Evaluate the Definite Integral

$\int_0^{\frac{\pi}{3}} \cos(x)\,dx$

The antiderivative for $\cos(x) =$ ________________

because $\frac{d}{dx}(\qquad\qquad) = \cos(x)$

$\int_{-1}^{4} 6x^2\,dx$

The antiderivative for $6x^2 =$ ________________

because $\frac{d}{dx}(\qquad\qquad) = 6x^2$

$\int_1^9 \sqrt{x}\,dx$

The antiderivative for $\sqrt{x} = x^{\frac{1}{2}}$ is ________________

because $\frac{d}{dx}(\qquad\qquad) = x^{\frac{1}{2}}$

WHAT SHOULD WE TAKE AWAY?

We can evaluate a definite integral using the Fundamental Theorem of Calculus.

$$\int_a^b f(x)\,dx = F(b) - F(a) \text{ or } \int_a^b F'(x)\,dx = F(b) - F(a)$$

The antiderivative of a function *f* is a function *F* whose derivative is *f*, $F'(x) = f(x)$.

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6.7 The Fundamental Theorem of Calculus and Definite Integrals

6.7: Daily Video 2

In this video, we will apply the Fundamental Theorem of Calculus to solve problems.

3.D *Justification* — Apply an appropriate mathematical definition, theorem, or test.

WHAT WILL WE LEARN?

The Fundamental Theorem of Calculus tells us $\frac{d}{dx}\left(\int_a^x f(t)dt\right) = f(x)$.

If a function *f* is continuous on the closed interval [a,b] and *F* is an antiderivative of *f* on the interval [a,b], then $\int_a^b f(x)dx = F(b) - F(a)$ or $\int_a^b F'(x)dx = F(b) - F(a)$

Apply the FTC

$\int_2^x (3t^2 - 2)dt$

If *G*(*x*) is an antiderivative for *f*(*x*) and *G*(5) = −8 then what is an expression that is equal to *G*(11)?

Selected values of the twice differentiable function *f* and its derivative *f′* are given in the table.

What is the value of $\int_0^5 f'(x)dx$?

x	0	2	3	5
$f(x)$	-15	-8	4	7
$f'(x)$	12	8	3	-2

WHAT SHOULD WE TAKE AWAY?

We can evaluate a definite integral using the Fundamental Theorem of Calculus.

$$\int_a^b f(x)dx = F(b) - F(a) \text{ or } \int_a^b F'(x)dx = F(b) - F(a)$$

The antiderivative of a function *f* is a function *F* whose derivative is *f*, $F'(x) = f(x)$.

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6.7 The Fundamental Theorem of Calculus and Definite Integrals

6.7: Daily Video 3

In this video, we will apply the Fundamental Theorem of Calculus to solve problems presented graphically as well as using technology.

3.D *Justification*

Apply an appropriate mathematical definition, theorem, or test.

WHAT WILL WE LEARN?

In this video, we will apply the FTC in situations that will require the use of technology. We will also solve problems that are presented graphically.

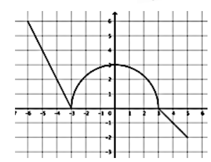
If a function f is continuous on the closed interval [a,b] and F is an antiderivative of f on the interval [a,b], then $\int_a^b f(x)dx = F(b) - F(a)$ or $\int_a^b F'(x)dx = F(b) - F(a)$

Apply the FTC

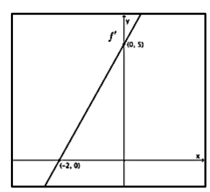
Let f be a differentiable function such that $f(4) = 3.462$ and $f'(x) = \ln\sqrt{x^2+5}$. What is the value of $f(7)$?

The graph of g consists of line segments and a semicircle as shown in the diagram.

What is the value of $\int_{-5}^{4} g'(x)dx$?



The graph of f', the derivative of f, is the line shown. If $f(-2) = 3$ then what is $f(0)$?



WHAT SHOULD WE TAKE AWAY?

- We applied the Fundamental theorem of Calculus in a variety of situations in this section.
- Don't forget that at times it is appropriate to choose to use technology.
- Moving forward you will be learning about the indefinite integral and learning a variety of new techniques.

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