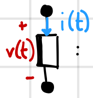
ELETTROTECNICA

By Edoardo

TENSIONE : $v(t)$ VOLT e relativi multipli e sottomultipli

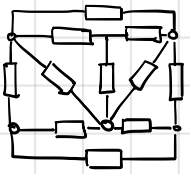
CORRENTE : $i(t)$ AMPERE "



: BIPOLO Posizione segni e verso sono arbitrarie, ma per convenzione mettiamo + sul morsetto in cui "entra" la corrente

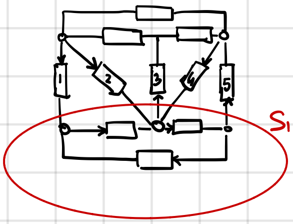
$p(t) = v(t) \cdot i(t)$ = potenza istantanea (≥ 0 entrante grazie alla convenzione precedente)

Un circuito è un insieme di bipoli connessi



LEGGI DI KIRCHOF

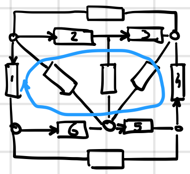
LKC: Tracciando una superficie tagliante sul circuito, la somma algebrica delle correnti entranti e uscenti è zero



$S_1 : i_1 + i_2 - i_3 + i_4 - i_5 = 0$

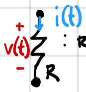
Valida per ogni superficie

LKT: Tracciando un cammino chiuso sul circuito la somma delle tensioni concordi al verso del cammino è uguale alla somma di quelle discordi



$V_2 + V_3 = V_1 + V_4 + V_5 + V_6 \Rightarrow V_2 + V_3 - V_1 - V_4 - V_6 = 0$

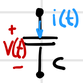
Valido per ogni cammino chiuso



: RESISTORE

$v(t) = R\,i(t)$ Legge di Ohm

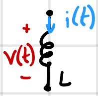
$p(t) = v(t) \cdot i(t) = R\,i^2(t)$



: CONDENSATORE

$i(t) = C\, dv/dt$ (Farad)

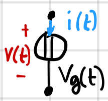
$\mathcal{E}(t)$ = energia $= \int_{-\infty}^{t} p(\tau)\,d\tau = \int_{-\infty}^{t} v(\tau)\,i(\tau)\,d\tau = C \int_{-\infty}^{t} v\,dv = \frac{1}{2} C v^2(t)$



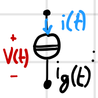
: INDUTTORE

$v(t) = L\, di/dt$ (Ω sec = Henry)

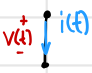
$\mathcal{E}(t) = \int_{-\infty}^{t} p(\tau)\,d\tau = L \int_{-\infty}^{t} i\,di = \frac{1}{2} L\, i^2(t)$



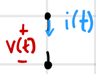
: GENERATORE INDIPENDENTE DI TENSIONE $v(t) = V_g(t)$



: GENERATORE INDIPENDENTE DI CORRENTE $i(t) = i_g(t)$



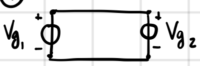
: CORTO CIRCUITO $v(t) = 0$



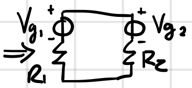
: $i(t) = 0$

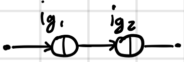
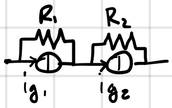
CASI ASSURDI

①

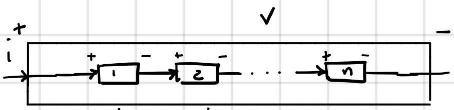


Secondo LKT $V_{g_1} = V_{g_2}$, ma non lo sono ⇒ manca la resistenza per equipararli



②  Caso analogo con LKC ⇒ 

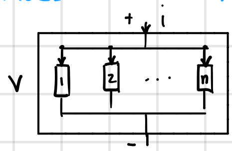
CONNESSIONE IN SERIE



$i = i_1 = i_2 = \dots = i_n$

$V = V_1 + V_2 + \dots + V_n$

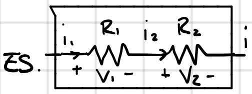
CONNESSIONE IN PARALLELO



$i = i_1 + i_2 + \dots + i_n$

$V = V_1 = V_2 = \dots = V_n$

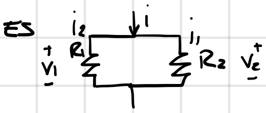
ES.



$$\begin{cases} i = i_1 = i_2 \\ V = V_1 + V_2 = R_1 i_1 + R_2 i_2 = i(R_1 + R_2) \end{cases}$$

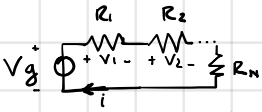
Due resistori in serie sono uguali ad un resistore con resistenza la somma delle resistenze

ES



$$\begin{cases} V = V_1 = V_2 \\ i = i_1 + i_2 = \frac{V_1}{R_1} + \frac{V_2}{R_2} = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)V = (G_1 + G_2)V = \left(\frac{R_1 + R_2}{R_1 R_2}\right)V = \end{cases}$$

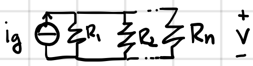
ES



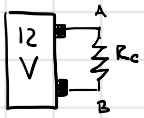
$V_g = V_1 + V_2 + \dots + V_n = i(R_1 + R_2 \dots R_n)$

$i = V_g / R_1 + R_2 + \dots + R_n \Rightarrow \boxed{V_n = R_n \cdot V_g / R_1 + R_2 \dots + R_n}$ Partizione di tensione

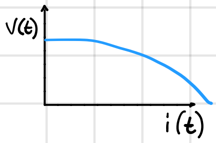
ES



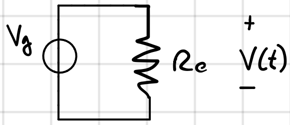
$$\begin{cases} i_g = i_1 + i_2 + \dots + i_n = G_1 V_1 + G_2 V_2 + \dots + G_n V_n = (G_1 + G_2 + \dots + G_n)V \\ V = V_1 = V_2 \dots = V_n \end{cases} \Rightarrow \boxed{i_n = G_n V = \frac{G_n}{G_1 + G_2 + \dots G_n} \cdot i_g}$$ Partizione di corrente



caso reale

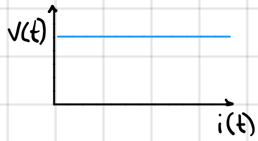


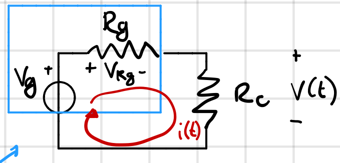
$\Longrightarrow$



$V_g = 12V = V(t)$

componenti ideali



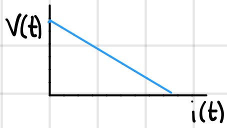


Aggiungiamo le "perdite" $\longrightarrow R_g$

$-V_g + V_{Rg} + V = 0 \Longrightarrow V = V_g - V_{Rg}$

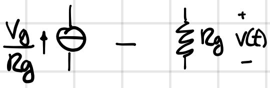
simili

equivalenti



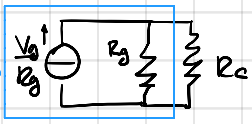
$V_{Rg} = R_g \cdot i \Rightarrow V = V_g - R_g i$

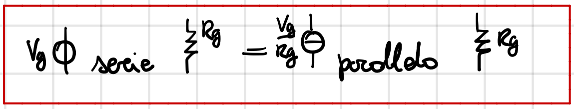
Esplicitiamo la i:

$i(t) = \underbrace{\frac{V_g}{R_g}} - \underbrace{\frac{V(t)}{R_g}}$ $\Longrightarrow$ 

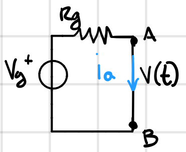
entrambi correnti

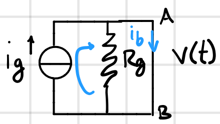
poiché le correnti si sommano, i due componenti sono in parallelo





ES





$V(t) = 0$ perché corto circuito, ia deve essere uguale ad ib

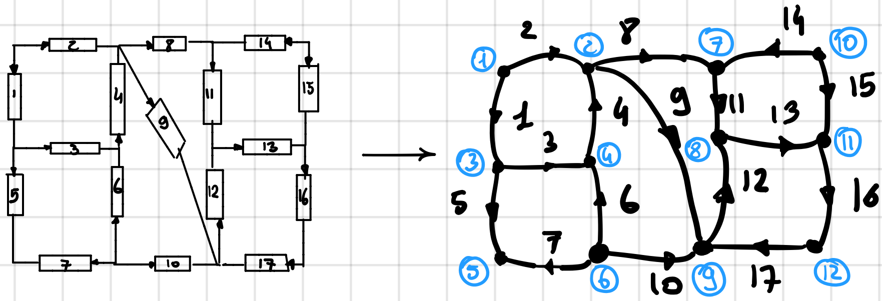
$$\left.\begin{aligned} i_a &= \frac{V_g}{R_g} \\ i_b &= i_g \end{aligned}\right\} \Longrightarrow i_g = V_g/R_g$$

R bipoli

Trovare i e v per ogni componente ⇒ 2R incognite ⇒ 2R equazioni Indipendenti

$\begin{cases} \text{R eq di definizione} \\ \text{R da LKT e LKC} \end{cases}$ NB bisogna controllare che siano indipendenti

ES

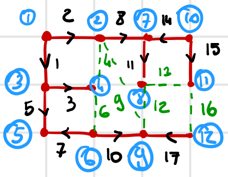


grafo del circuito orientato

17 rami

12 nodi

Prendo i rami che toccano tutti i nodi senza fare un percorso chiuso



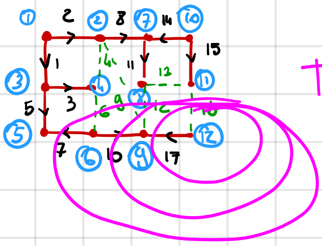
albero   Rami albero $= N - 1$ (11)

coalbero   Rami coalbero $= R - (N-1)$

Se all'albero aggiungo un ramo del coalbero ottengo un percorso chiuso chiamato maglia fondamentale

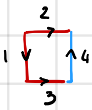
Posso ottenere tante maglie fondamentali quanti sono i rami del coalbero

Posso individuare delle superfici chiuse che tagliano un solo ramo dell'albero e quanti voglio del coalbero

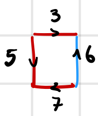


Tagli fondamentali   Tanti quanti sono i rami dell'albero

Es maglia (LKT)



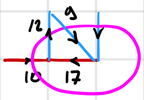
$\Rightarrow V_4 - V_2 + V_1 + V_3 = 0$



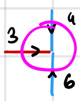
$V_6 - V_3 + V_5 - V_7 = 0$

$V_{coalbero} + \overset{\text{matrice}}{[A]}\, V_{albero} = 0$ $[A]$ 0 se non c'è il ramo, 1 se concorde a V_c, −1 se discorde

Es. taglio (LKC)



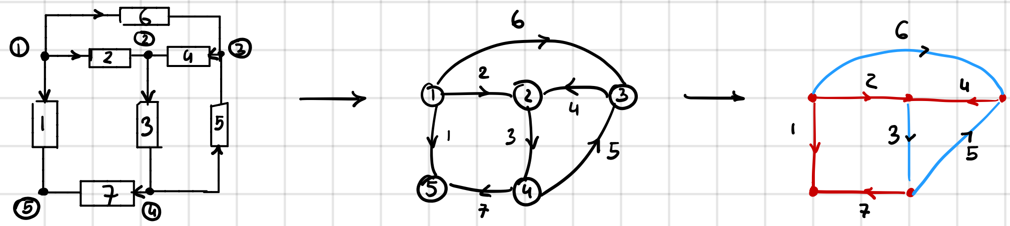
$i_{10} + i_9 - i_{12} + i_{16} = 0$



$i_3 - i_4 + i_6 = 0$

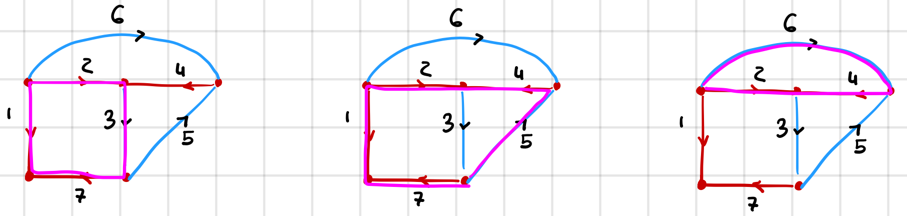
$I_a + [B] I_c = 0$

Esempio



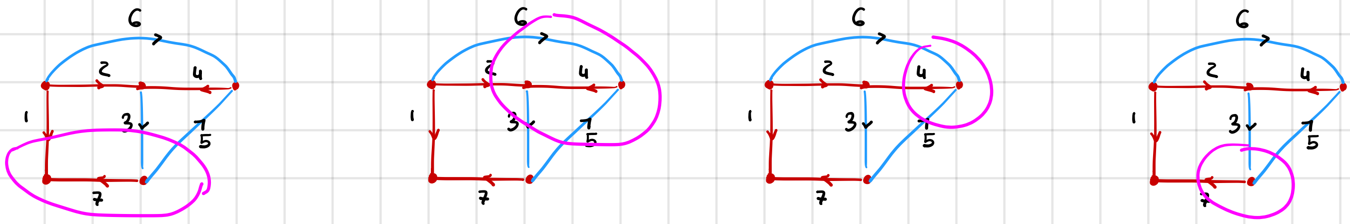
$R_a = 4 = N-1$
$R_c = 3 = R-(N-1)$

3 maglie



$M3: V_3 - V_1 + V_2 + V_7 = 0$ $M5: V_5 + V_1 - V_2 + V_4 - V_7 = 0$ $M6: V_6 - V_2 + V_4 = 0$

4 tagli



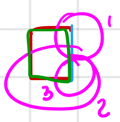
$T1: i_1 + i_3 - i_5 = 0$ $T2: i_2 - i_3 + i_5 + i_6 = 0$ $T4: i_4 - i_5 - i_6 = 0$ $T7: i_7 - i_3 + i_5 = 0$

$$\begin{bmatrix} V_3 \\ V_5 \\ V_7 \end{bmatrix} + \underbrace{\begin{bmatrix} -1 & 1 & 0 & 1 \\ 1 & -1 & 1 & -1 \\ 0 & -1 & 1 & 0 \end{bmatrix}}_{A} \begin{bmatrix} V_1 \\ V_2 \\ V_4 \\ V_7 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

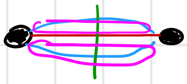
$$\begin{bmatrix} i_1 \\ i_2 \\ i_4 \\ i_7 \end{bmatrix} + \underbrace{\begin{bmatrix} 1 & -1 & 0 \\ -1 & 1 & 1 \\ 0 & -1 & -1 \\ -1 & 1 & 0 \end{bmatrix}}_{B} \begin{bmatrix} i_3 \\ i_5 \\ i_6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$[A] = -[B]^T$

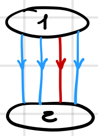
- Una maglia fondamentale relativa ad un ramo del coalbero contiene tutti i rami dell'albero i cui tagli contengono quel ramo



- Un taglio fondamentale relativo ad un ramo dell'albero contiene tutti i rami del coalbero le cui maglie contengono quel ramo



- Due rami a_i e c_j tali che il taglio relativo ad a_i comprende c_j. Allora il segno della corrente su c_j nel taglio fondamentale è opposto alla tensione su a_i



Dati $[V]$ e $[I]$ i vettori delle tensioni e correnti (albero e coalbero):

$$[V]^T[I] = \begin{bmatrix} V_a \\ V_c \end{bmatrix}^T \begin{bmatrix} I_a \\ I_c \end{bmatrix} = V_a^T I_a + V_c^T I_c$$

$(A^T = B)$

$$V_c = -A V_a \quad I_a = -B I_c \Rightarrow -V_a^T B I_c - [A V_a]^T I_c = -V_a^T B I_c - V_a^T A^T I_c = -V_a^T B I_c + V_a^T B I_c = 0$$

$\boxed{\sum_1^R V_k i_k = 0 = \sum_1^R P_k}$ principio fondamentale della conservazione della potenza $\quad P = \text{potenza} = V \cdot i$

Consideriamo due circuiti diversi ① e ② ma con lo stesso grafo. $\rightarrow [V_1]^T[I_2] = 0$ (oppure $[V_2]^T[I_1] = 0$) Teorema di Tellegen

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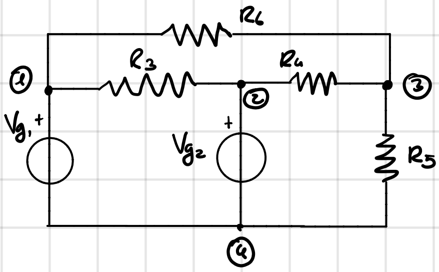
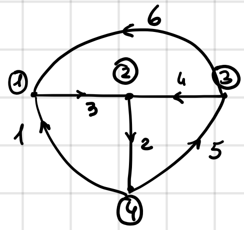
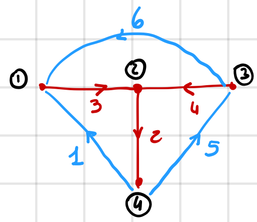
Consideriamo un circuito con resistori (R_k) e generatori di tensione (V_{g_k})







(versi presi a caso!)

$V_{g_1} = 1\,V \quad V_{g_2} = 2\,V$

$R_3 = R_4 = R_6 = 1\,\Omega$

$R_5 = 2\,\Omega$

Eq. componenti

$V_1 = -V_{g_1} = -1$

$V_2 = V_{g_2} = 2$

$V_3 = R_3 i_3$

$V_4 = R_4 i_4$

$V_5 = R_5 i_5$

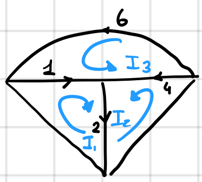
$V_6 = R_6 i_6$

Eq. maglie

$V_1 + V_2 + V_3 = 0 = -V_{g_1} + V_{g_2} + R_3 i_3 \Rightarrow R_3 i_3 = V_{g_1} - V_{g_2}$

$V_5 + V_2 + V_4 = 0 = R_5 i_5 + V_{g_2} + R_4 i_4 \Rightarrow R_4 i_4 + R_5 i_5 = -V_{g_2}$

$V_6 + V_3 - V_4 = 0 = R_6 i_6 + R_3 i_3 - R_4 i_4 \Rightarrow R_3 i_3 - R_4 i_4 + R_6 i_6 = 0$



Supponiamo scorrino delle correnti nelle maglie con lo stesso verso del ramo del coalbero

$$\left.\begin{matrix} I_1 = i_1 \\ I_2 = i_5 \\ I_3 = i_6 \end{matrix}\right\} \Rightarrow \begin{cases} i_3 = I_1 + I_3 \\ i_2 = I_1 + I_2 \\ i_4 = I_2 - I_3 \end{cases}$$

Metodo Base Maglie

Sostituiamo alle eq. di maglia:

$$\left.\begin{cases} R_3(I_1 + I_3) = V_{g_1} - V_{g_2} \\ R_4(I_2 - I_3) + R_5 I_2 = -V_{g_2} \\ R_3(I_1 + I_3) - R_4(I_2 - I_3) + R_6 I_3 = 0 \end{cases}\right\} \Rightarrow \begin{matrix} I_1 + I_3 = 1 - 2 \\ I_2 - I_3 + 2I_2 = -2 \\ I_1 + I_3 - I_2 + I_3 + I_3 = 0 \end{matrix}$$

$\Downarrow$

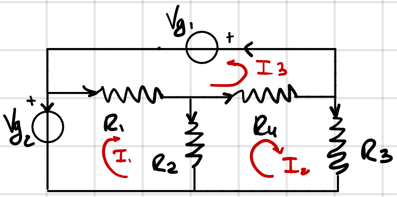
$$\begin{cases} R_3 I_1 + R_3 I_3 = V_{g_1} - V_{g_2} \\ (R_4 + R_5) I_2 - R_4 I_3 = -V_{g_2} \\ R_3 I_1 - R_4 I_2 + (R_3 + R_4 + R_6) I_3 = 0 \end{cases}$$

forma matriciale $\Rightarrow$

$$\begin{bmatrix} R_3 & 0 & R_3 \\ 0 & (R_4 + R_5) & -R_4 \\ R_3 & -R_4 & (R_3 + R_4 + R_6) \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{array}{cl} \begin{bmatrix} V_{g_1} - V_{g_2} \\ -V_{g_2} \\ 0 \end{bmatrix} & \begin{matrix} \leftarrow M_1 \\ \leftarrow M_2 \\ \leftarrow M_3 \end{matrix} \end{array}$$

Sulla diagonale troviamo tutte le resistenze (somma) comprese in ogni singola maglia (es. [1,1] → maglia 1 → R_3)
[x,y] contiene le resistenze in comune tra la maglia x e la maglia y (es. [1,2] → 0)
Il segno è negativo se le correnti sono discordi!
Per i termini noti si mette il segno negativo se la corrente entra dal morsetto positivo.
Con queste regole possiamo evitare di fare tutti i passaggi precedenti
Risolvendo il sistema troviamo I_1 I_2 e I_3

ES



Settiamo le 3 correnti di maglia come vogliamo

$$\begin{bmatrix} (R_1+R_2) & -R_2 & R_1 \\ -R_2 & (R_2+R_3+R_4) & R_4 \\ R_1 & R_4 & (R_1+R_4) \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} V_{g_2} \\ 0 \\ -V_{g_1} \end{bmatrix}$$

Sostituendo i valori numerici troviamo I_1, I_2 e I_3

$i_{g_2} = -I_1$ $\qquad V_{R_1} = R_1(I_1+I_3)$ ecc.

$i_{R_1} = I_1 + I_3$

$i_{R_2} = I_1 - I_2$

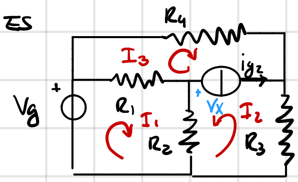
$i_{R_4} = I_2 + I_3$

$i_{R_3} = I_2$

$i_{g_1} = I_3$

Se ho un generatore di corrente posso considerarlo come un generatore di tensione V_x incognita ma ho anche una corrente i_g nota e il tutto rimane "inalterato"

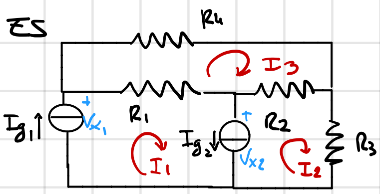
ES



$$\begin{bmatrix} (R_1+R_2) & R_2 & -R_1 \\ R_2 & (R_2+R_3+R_4) & 0 \\ -R_1 & 0 & (R_1+R_4) \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} V_{g_1} \\ V_x \\ V_x \end{bmatrix}$$

con $i_{g_2} = -I_2 - I_3$

ES

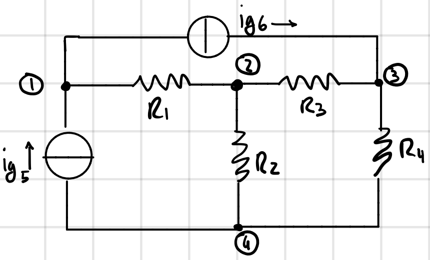


$$\begin{bmatrix} R_1 & 0 & -R_1 \\ 0 & (R_2+R_3) & -R_2 \\ -R_1 & -R_2 & (R_1+R_2+R_4) \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} V_{x1} - V_{x2} \\ V_{x2} \\ 0 \end{bmatrix}$$

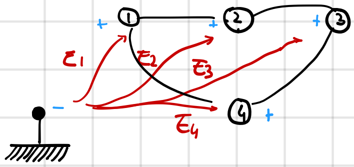
con:
$i_{g_1} = I_1$
$i_{g_2} = I_1 - I_2$

$V_{g_1} = -V_{x_1}$ ...

Consideriamo un circuito con resistori e generatori di corrente

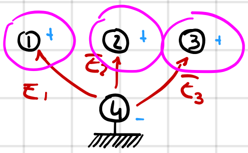


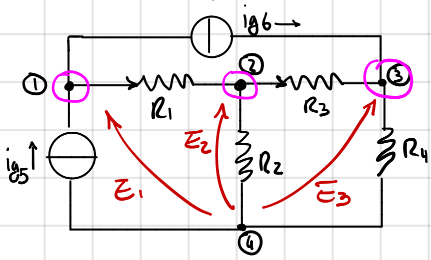
$\longrightarrow$



Così posso trovare le tensioni in funzione delle altre

$\to$ a questo punto uso direttamente un nodo come punto di riferimento $\longrightarrow$





Posso ora considerare i 3 tagli fondamentali

1) $-i_{g5} + i_{g6} + \left(\frac{E_1-E_2}{R_1}\right) = 0 \longrightarrow G_1(E_1-E_2) = i_{g5} - i_{g6}$

2) $\left(\frac{E_2-E_1}{R_1}\right) + E_2/R_2 + \left(\frac{E_2-E_3}{R_3}\right) = 0 \longrightarrow G_1(E_2-E_1) + G_2E_2 + G_3(E_2-E_3) = 0$

3) $-i_{g6} + \frac{E_3-E_2}{R_3} + \frac{E_3}{R_4} = 0 \longrightarrow G_3(E_3-E_2) + G_4E_3 = i_{g6}$

$$G_1E_1 - G_1E_2 = i_{g5} - i_{g6}$$
$$-G_1E_1 + (G_1+G_2+G_3)E_2 - G_3E_3 = 0$$
$$-G_3E_2 + (G_3+G_4)E_3 = i_{g6}$$

$$\Rightarrow \underbrace{\begin{bmatrix} G_1 & -G_1 & 0 \\ -G_1 & G_1+G_2+G_3 & -G_3 \\ 0 & -G_3 & G_3+G_4 \end{bmatrix}}_{A} \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix} = \underbrace{\begin{bmatrix} i_{g5} - i_{g6} \\ 0 \\ i_{g6} \end{bmatrix}}_{B}$$

A: sulla diagonale ho la somma delle conduttanze collegate ai nodi
A[x,y] conduttanza tra nodo x e y con segno negativo

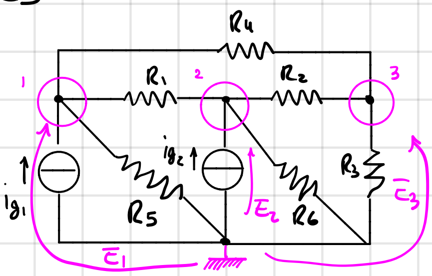
Metodo Base Nodi

B: correnti entranti (positive) e uscenti (negative) nel nodo

$i_1 = (E_1 - E_2)G_1 \qquad V_1 = R_1 i_1$

$i_3 = (E_2 - E_3)G_3 \quad \ldots$

ES

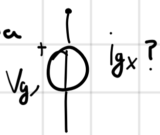


$$\begin{bmatrix} (G_1+G_4+G_5) & -G_1 & -G_4 \\ -G_1 & (G_1+G_2+G_6) & -G_2 \\ -G_4 & -G_2 & (G_2+G_3+G_4) \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix} = \begin{bmatrix} i_{g1} \\ i_{g2} \\ 0 \end{bmatrix}$$

Quando conviene usare il metodo base nodi o base maglie?

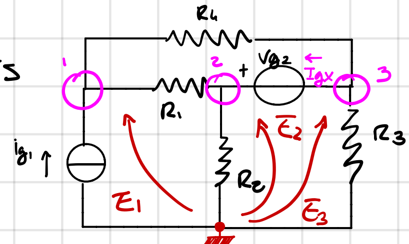
Se nodi > maglie $\longrightarrow$ metodo maglie, altrimenti nodi

Se ho dei generatori di tensione posso vederli come generatori di corrente con i_{gx} incognita: caso analogo al metodo su base maglia



$V_g =$ combinazioni di E_i

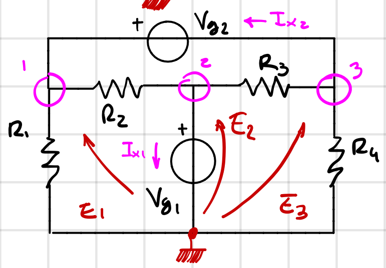
ES



$$\begin{bmatrix} (G_1+G_4) & -G_1 & -G_4 \\ -G_1 & (G_1+G_2) & 0 \\ -G_4 & 0 & (G_3+G_4) \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix} = \begin{bmatrix} I_{g_1} \\ I_{gx} \\ -I_{gx} \end{bmatrix}$$

con $V_{g_2} = E_2 - E_3$

ES



$$\begin{bmatrix} (G_1+G_2) & -G_2 & 0 \\ -G_2 & (G_2+G_3) & -G_3 \\ 0 & -G_3 & (G_3+G_4) \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix} = \begin{bmatrix} I_{x_2} \\ -I_{x_1} \\ -I_{x_2} \end{bmatrix}$$

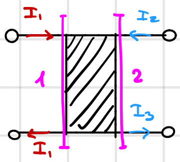
con:

$E_2 = V_{g_1}$

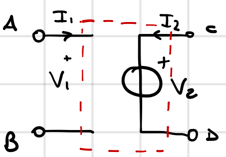
$E_1 - E_3 = V_{g_2}$

controllare verso correnti.

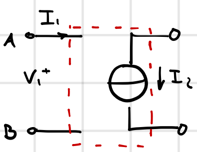
RETI 2 PORTE



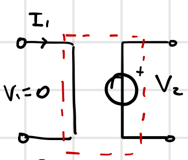
porte: corrente entrante - corrente uscente (un bipolo x una porta)



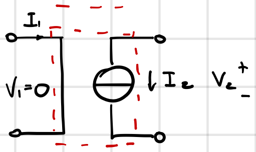
$V_1 = \alpha V_2 \longrightarrow$ V_1 possiamo considerarlo come un generatore di tensione controllato IN TENSIONE ovvero dipendente da V_2



$V_1 = r I_2 \longrightarrow$ generatore di tensione controllato IN CORRENTE

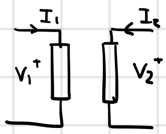


$I_1 = g V_2 \longrightarrow$ generatore di corrente controllato IN TENSIONE

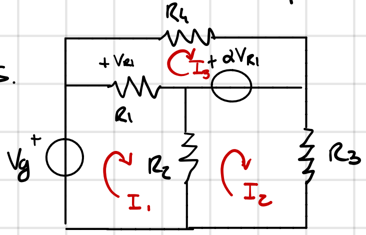


$I_1 = K I_2 \longrightarrow$ generatore di corrente controllato IN CORRENTE

Possiamo considerare il quadripolo come una coppia di bipoli



ES.

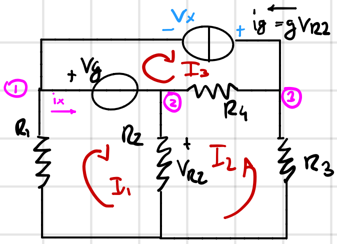


$$\begin{bmatrix} (R_1+R_2) & -R_2 & -R_1 \\ -R_2 & (R_2+R_3) & 0 \\ -R_1 & 0 & (R_1+R_4) \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} V_g \\ -\alpha V_{R1} \\ \alpha V_{R1} \end{bmatrix}$$

(base maglia)

bisogna trovare V_{R1} ... $V_{R1} = R_1(I_1 - I_3)$

ES



$$\begin{bmatrix} (R_1+R_2) & R_2 & 0 \\ R_2 & (R_2+R_3+R_4) & R_4 \\ 0 & R_4 & R_4 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} -V_g \\ 0 \\ V_x+V_g \end{bmatrix}$$

(base maglie)

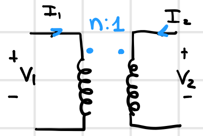
$i_g = gV_{R2} \longrightarrow V_{R2} = R_2(I_1+I_2) \longrightarrow i_g = -I_3 \longrightarrow \boxed{-I_3 = gR_2(I_1+I_2)}$ oppure ...

$$\begin{bmatrix} G_1 & 0 & 0 \\ 0 & (G_2+G_4) & -G_4 \\ 0 & -G_4 & (G_3+G_4) \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} i_g - i_x \\ i_x \\ -i_g \end{bmatrix}$$

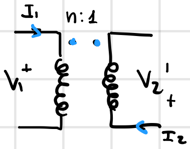
$$\boxed{\begin{array}{l} E_1 - E_2 = V_g \\ i_g = gV_{R2} = gE_2 \end{array}}$$

(base nodi)

TRASFORMATORE IDEALE

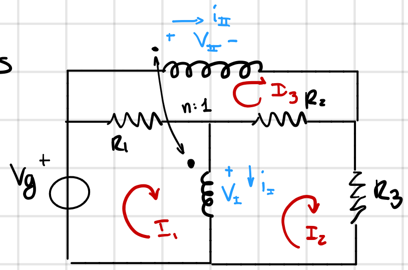


I pallini indicano come le grandezze sono accoppiate, in questo caso $\begin{cases} V_1 = nV_2 \\ I_1 = -\frac{1}{n} I_2 \end{cases}$



$$\begin{cases} V_1 = -nV_2 \\ I_1 = \frac{1}{n} I_2 \end{cases}$$

ES



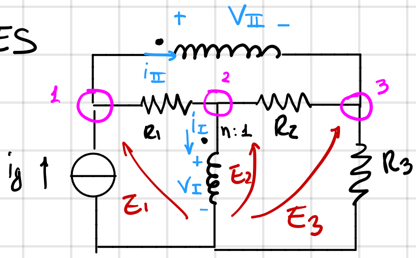
Denominiamo prima di tutto le grandezze dei trasformatori!

$$\begin{bmatrix} R_1 & 0 & -R_1 \\ 0 & (R_2+R_3) & -R_2 \\ -R_1 & -R_2 & (R_1+R_2) \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} V_g - V_I \\ V_I \\ -V_{II} \end{bmatrix}$$

(base maglie)

con: $\begin{cases} V_1 = nV_2 \\ i_I = -\frac{1}{n} i_{II} \end{cases} \cup \begin{cases} i_I = I_1 - I_2 \\ i_{II} = I_3 \end{cases}$

ES



$$\begin{bmatrix} G_1 & -G_1 & 0 \\ -G_1 & (G_1+G_2) & -G_2 \\ 0 & -G_2 & (G_2+G_3) \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix} = \begin{bmatrix} i_g - i_{II} \\ -i_I \\ i_{II} \end{bmatrix}$$

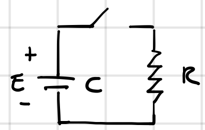
(base nodi)

con $\begin{cases} V_I = nV_{II} \\ i_I = -\frac{i_{II}}{n} \end{cases} \cup \begin{cases} V_I = E_2 \\ V_{II} = E_1 - E_3 \end{cases}$

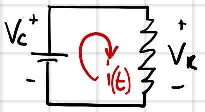
induttore: $V_L(t) = L\frac{di_L}{dt}$
condensatore: $i_c(t) = C\frac{dV_c}{dt}$

} se sono presenti tali componenti ⟶ sistema differenziale di ordine n con n pari al numero di eq. differenziali (# componenti diff.)

ES. Scaricamento



All'istante $t=0$ l'interruttore si chiude ed inizia a scorrere corrente fino a quando E diventa zero a causa delle perdite.

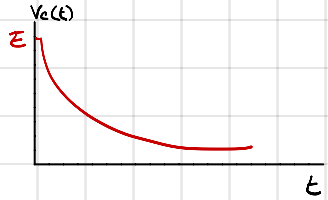


$t \geq 0 \longrightarrow i(t) = \frac{V_R(t)}{R}$ ma $V_R(t) = V_C(t)$ poiché sono in parallelo

$i(t) = -C\frac{dV_c}{dt}$ (perché entra dal morsetto negativo)

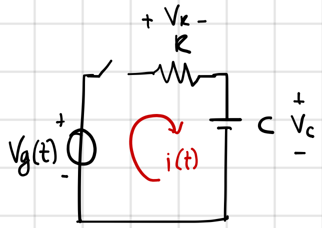
$\Rightarrow -RC\, dV_c/dt = V_c(t) \longrightarrow RC\, dV_c/dt + V_c(t) = 0$ eq. differenziale ⟶ sol: $V_c(t) = Ae^{\alpha t}$

$\rightarrow RC\frac{d}{dt}(Ae^{\alpha t}) + Ae^{\alpha t} = RCA\alpha e^{\alpha t} + Ae^{\alpha t} = 0 \longrightarrow \alpha = -1/RC \Rightarrow V_c(t) = Ae^{-t/RC}$



$V(0) = A = E \longrightarrow \boxed{V_c(t) = Ee^{-t/RC}}$

ES Caricamento

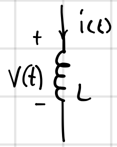


$t \geq 0$: $\begin{cases} V_R(t) + V_c(t) = V_g(t) \\ V_g(t) = E \\ V_R(t) = R\,i(t) \\ i(t) = C\, dV_c/dt \end{cases} \Rightarrow RC\, dV_c/dt + V_c(t) = E \longrightarrow$ sol: $V_c(t) = V_{omogenea} + V_{part.}$

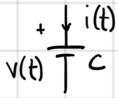
$V_{omogenea} \longrightarrow Ae^{\alpha t}$ (come prima)
$V_{part} \longrightarrow E$ (a tentativi, ponendo $V_c = E$)

} $\longrightarrow V_c(t) = Ae^{\alpha t} + E \quad \alpha = -1/RC$

$V_c(0) = A + E = 0 \longrightarrow A = -E \Rightarrow \boxed{V_c(t) = E(1 - e^{-t/RC})}$



$v(t) = L\, di/dt$



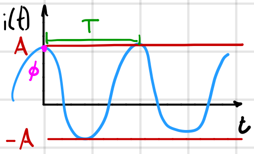
$i(t) = C\, dv/dt$

Se le eccitazioni sono costanti:

induttore: $v(t) = 0$ ⟶ si comporta come

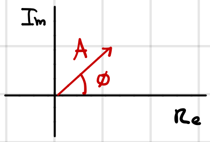
conduttore: $i(t) = 0$ ⟶ si comporta come 

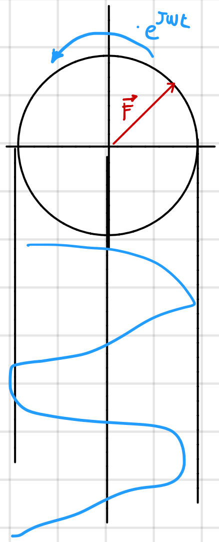
CORRENTE ALTERNATA



$\boxed{i(t) = A\cos(wt + \phi)}$ $\qquad T = 2\pi/w$

Dato w le grandezze del circuito dipenderanno solo da A e ϕ. ⇒





associamo quindi alla nostra curva un vettore: $\vec{F} = Ae^{J\phi}$ = Fasore

Più semplicemente: $f(t) = A\cos(wt + \phi) \longrightarrow \vec{F} = A(x + JY)$ con: $\begin{cases} X = \cos(\phi) \\ Y = \sin(\phi) \end{cases}$

NB. $A\sin(wt + \phi) = A\cos(wt + \phi - \pi/2)$

Per tornare indietro:

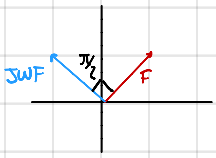
$f(t) = Re[\vec{F}e^{Jwt}]$, ricordando che $e^{Jwt} = \cos wt + J\sin wt \rightarrow Re[F(\cos wt + J\sin wt)]$

⇒ prendo solo parte senza J

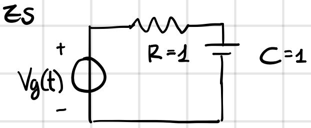
Se ho una grandezza $e(t)$ (sinusoidale) e dei componenti con memoria (condensatori ecc) avrò anche de/dt

$e(t) = \vec{F} \qquad e(t) \longrightarrow e(t) = Re[Fe^{Jwt}] = \vec{F}$

$de/dt = Re[Fe^{Jwt}/dt] = Re[FJwe^{Jwt}] = Jw\vec{F}$



Es



$V_g(t) = 2\cos(2t + \pi/4)$

$\underbrace{Rc\, dVc/dt}_{R} + \underbrace{V_c(t)}_{C} = \underbrace{V_g(t)}_{g}$ dobbiamo trovare Vc con i fasori

$\vec{V_g} = 2e^{J\pi/4} = \sqrt{2}(1 + J)$

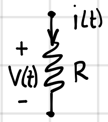
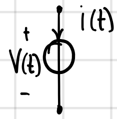
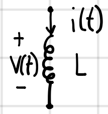
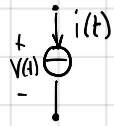
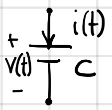
$\rightarrow Rc\, Re[Jw\vec{V_c}e^{Jwt}] + Re[\vec{V_c}e^{Jwt}] = Re[\vec{V_g}e^{Jwt}]$

$\rightarrow RcJw\vec{V_c} + \vec{V_c} = \vec{V_g}$

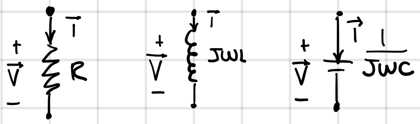
$\rightarrow \vec{V_c} = \vec{V_g}/(1 + JwRc) = \sqrt{2}/5\,(3 - J)$

$\Rightarrow V_c(t) = Re[\vec{V_c}e^{Jwt}] = \frac{\sqrt{2}}{5}Re[(3-J)(\cos 2t + J\sin 2t)] = \frac{\sqrt{2}}{5}(3\cos 2t + \sin 2t)$

É possibile applicare i fasori direttamente alle componenti (se hanno grandezze sinusoidali)

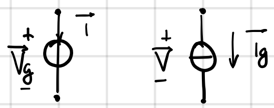
	$v(t) = R\,i(t) \longrightarrow \vec{V} = R\vec{I}$ ①		$V(t) = V_g(t) \longrightarrow \vec{V} = \vec{V}_g$ ④
	$v(t) = L\,{di}/{dt} \longrightarrow \vec{V} = L j\omega \vec{I}$ ②		$i(t) = i_g(t) \longrightarrow \vec{I} = \vec{I}_g$ ⑤
	$i(t) = C\,{dV}/{dt} \rightarrow \vec{I} = C j\omega \vec{V}$ ③		

NB nei casi 2 e 3 le formule che ricaviamo possono essere ricondotte alla legge di ohm



} 3 tipi di resistori fittizi = IMPEDENZE $z\left(= R, j\omega L, \frac{1}{j\omega C}\right) = \frac{V}{I}$

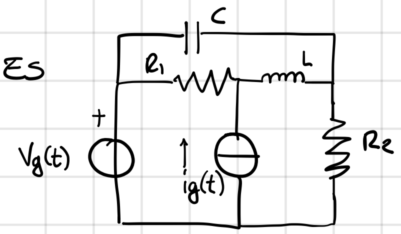
Nel caso dei generatori:



} non più dipendenti da t

Possiamo lavorare come i normali circuiti!!!

Es



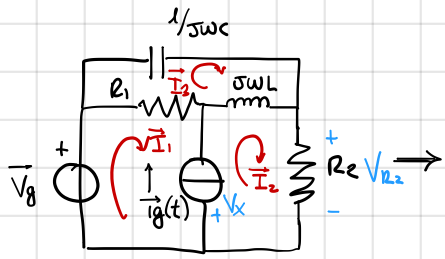
$R = 1\,\Omega \qquad L = 2H \qquad C = 1F \qquad \omega = 1$

$V_g(t) = \cos t$

$i_g(t) = \sin t \qquad$ Troviamo $V_{R2}(t)$

$\longrightarrow \vec{V}_g = 1$

$\longrightarrow \vec{I}_g = -j$



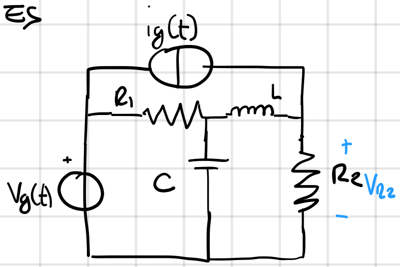
$$\longrightarrow \begin{bmatrix} R_1 & 0 & -R_1 \\ 0 & (R_2 + j\omega L) & -j\omega L \\ -R_1 & -j\omega L & \left(\frac{1}{j\omega C} + j\omega L + R_1\right) \end{bmatrix} \begin{bmatrix} \vec{I}_1 \\ \vec{I}_2 \\ \vec{I}_3 \end{bmatrix} = \begin{bmatrix} \vec{V}_g + \vec{V}_x \\ -V_x \\ 0 \end{bmatrix}$$

con: $\vec{I}_g = \vec{I}_2 - \vec{I}_1$

(base maglia)

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & (1+j2) & -j2 \\ -1 & -j2 & (1+j2-j) \end{bmatrix} \begin{bmatrix} \vec{I}_1 \\ \vec{I}_2 \\ \vec{I}_3 \end{bmatrix} = \begin{bmatrix} 1+\vec{V}_x \\ -\vec{V}_x \\ 0 \end{bmatrix} \Rightarrow \vec{V}_{R2} = R_2 \vec{I}_2 \Rightarrow V_{R2}(t) = \mathrm{Re}\left[\vec{V}_{R2}\, e^{j\omega t}\right]$$

ES



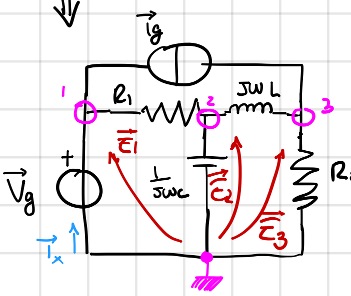
$\omega = 1$

$V_g(t) = 2\cos(t + \pi/4)$

$i_g(t) = \sin(t)$

$R = 1\ \Omega \quad L = 1\ H \quad C = 2F$ Trovare $V_{R2}(t)$

$\Downarrow$



$\vec{V}_g = 2e^{J\pi/4} = 2(\cos\pi/4 + J\sin\pi/4) = 2\left(\frac{\sqrt{2}}{2} + J\frac{\sqrt{2}}{2}\right) = \sqrt{2}(1+J)$

$\vec{I}_g = -J$

$$\begin{bmatrix} G_1 & -G_1 & 0 \\ -G_1 & (Y_1 + 1/J\omega L + J\omega C) & -1/J\omega L \\ 0 & -1/J\omega L & (1/J\omega L + G_2) \end{bmatrix} \begin{bmatrix} \vec{E}_1 \\ \vec{E}_2 \\ \vec{E}_3 \end{bmatrix} = \begin{bmatrix} \vec{I}_{gx} - \vec{I}_g \\ 0 \\ \vec{I}_g \end{bmatrix} \quad \text{con: } \vec{E}_1 = \vec{V}_g$$

(base nodi)

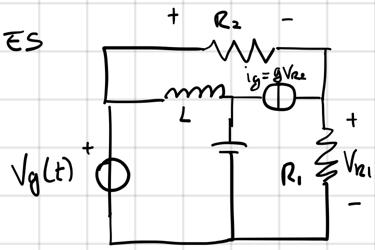
$\vec{V}_{R2} = \vec{E}_3 \longrightarrow V_{R2}(t) = Re[\vec{V}_{R2}\, e^{J\omega t}]$

Quando abbiamo componenti "controllati" (es. $V_g(t) = aV_1$) nel dominio dei fasori non cambia nulla

ES. $V_g(t) = r\, i_1(t) \longrightarrow \vec{V}_g = r\vec{I}_1$ (generatori)

ES $\begin{cases} V_1 = nV_2 \\ i_1 = -1/n\, i_2 \end{cases} \longrightarrow \begin{cases} \vec{V}_1 = n\vec{V}_2 \\ \vec{I}_1 = -1/n\, \vec{I}_2 \end{cases}$ (trasformatori)

ES

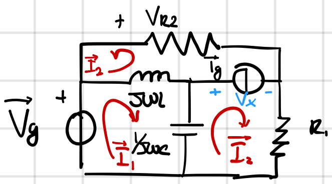


$V_g(t) = \text{cost}$

$R = 1\ \Omega$ Trovare $V_{R1}(t)$

$L = 2H$, $C = 1F$

$g = 2$

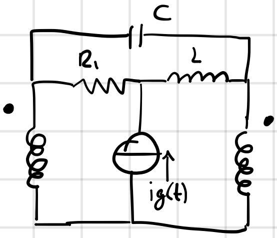


$\vec{V}_g = 1$

$$\begin{bmatrix} (J\omega L + 1/J\omega C) & -1/J\omega C & J\omega L \\ -1/J\omega C & (R_1 + 1/J\omega C) & 0 \\ J\omega L & 0 & (R_2 + J\omega L) \end{bmatrix} \begin{bmatrix} \vec{I}_1 \\ \vec{I}_2 \\ \vec{I}_3 \end{bmatrix} = \begin{bmatrix} \vec{V}_g \\ -\vec{V}_x \\ -\vec{V}_x \end{bmatrix} \quad \text{con: } \vec{I}_g - g\vec{V}_{R2} = g(-R_2 I_3)$$

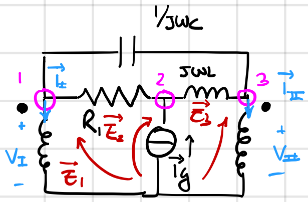
(base maglie)

ES



$i_g(t) = \sin t$

$R = 1\,\Omega \quad n = 2 \quad L = 1\,H \quad C = 2F$ Trovare $V_{R_1}(t)$



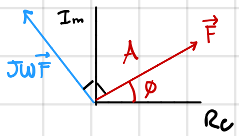
$\vec{I}_g = -j$

$\omega = 1$

$$\begin{bmatrix} (G_1 + j\omega C) & -G_1 & -j\omega C \\ -G_1 & (G_1 + 1/j\omega L) & -1/j\omega L \\ -j\omega C & -1/j\omega L & (j\omega C + 1/j\omega L) \end{bmatrix} \begin{bmatrix} \vec{E}_1 \\ \vec{E}_2 \\ \vec{E}_3 \end{bmatrix} = \begin{bmatrix} -\vec{I}_I \\ \vec{I}_g \\ -\vec{I}_{II} \end{bmatrix}$$

$$\begin{cases} \vec{V}_I = n \vec{V}_{II} \\ \vec{I}_I = -1/n\, \vec{I}_{II} \end{cases} \longrightarrow \begin{cases} \vec{E}_1 = n E_3 \\ I_I = -1/n\, \vec{I}_{II} \end{cases}$$

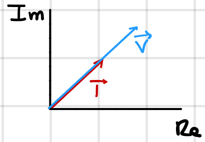
$$\vec{V}_{R_1} = E_1 - E_2 \longrightarrow V_{R_1}(t) = Re[\vec{V}_{R_1} e^{j\omega t}]$$



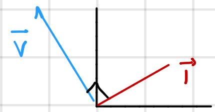
$e(t) = \vec{F}$

$de/dt = j\omega\vec{F}$

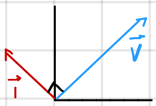
$\vec{V} = R\vec{I}$



$\vec{V} = j\omega L\vec{I}$

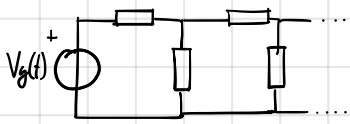


$\vec{V} = \frac{1}{j\omega C}\vec{I}$



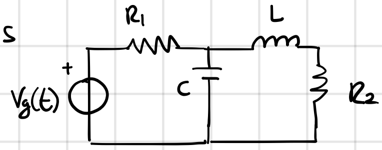
$\Longrightarrow$ Metodo Grafico dei Fasori

Può essere usato in circuiti del tipo:



detti a scala

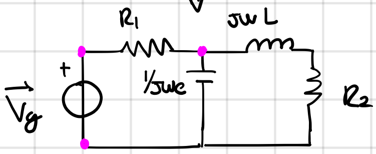
ES



$V_g(t) = \frac{1}{2}\cos t$

$R_1 = 1\,\Omega \quad R_2 = 2\,\Omega \quad C = 2F \quad L = 1H$

$\Downarrow$



$\vec{V}_g = 1 \quad \omega = 1$

Determinare $V_{R2}(t)$

Supponiamo di conoscere $V_{R2} = 1$

conosciamo quindi anche $i_{R2} = V_{R2}/R_2 = \frac{1}{2}$

che è anche la corrente che scorre sull'induttore

$\rightarrow \vec{V}_L = j\omega L\,\vec{I}_{R2} = j\,\vec{I}_{R2}$

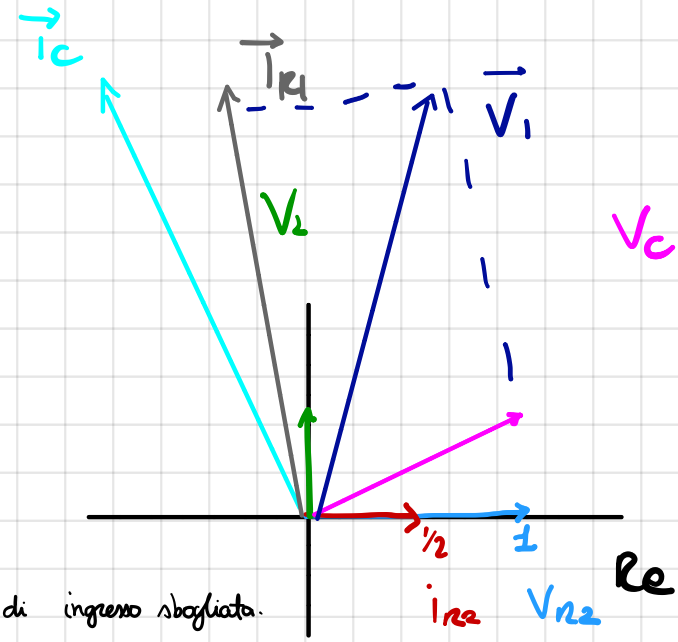
$\Rightarrow \vec{V}_C = \vec{V}_L + \vec{V}_{R2}$

$\Rightarrow \vec{I}_C = j\omega C\,\vec{V}_C = j2\,\vec{V}_C$

$\Rightarrow \vec{I}_{R_1} = \vec{I}_C + \vec{I}_{R2}$

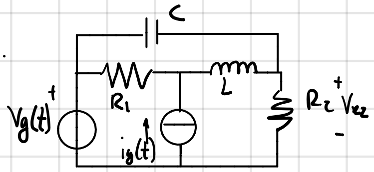
$\vec{V}_{R1} = R_1\,\vec{I}_{R1}$

$\rightarrow$ LKT $\rightarrow V_1 = V_{R1} + V_C$



Ho così tutte le relazioni tra i componenti, ma con la tensione di ingresso sbagliata.

Se abbiamo più pulsazioni ω diverse nel circuito posso applicare la legge della sovrapposizione degli effetti: lavoro nel circuito "con un ω alla volta" annullando gli altri e trovo la risposta nel tempo per ognuno e poi le sommo

Es. 

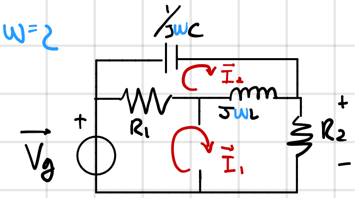
$R = 1\,\Omega \quad C = 1F \quad L = 2H$

$V_g(t) = \cos 2t$

$i_g(t) = \sin 1t$

Trovare $V_{R2}(t)$

$\omega = 2$

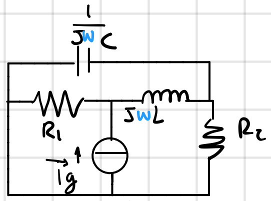


$\vec{V}_g = 1$

"Annullo" il generatore di corrente con un circuito aperto

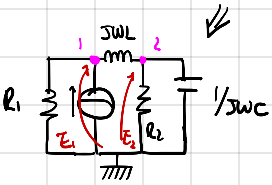
$$\begin{bmatrix} (R_1 + j\omega L + R_2) & -(R_1 + j\omega L) \\ -(R_1 + j\omega L) & (R_1 + j\omega L + \frac{1}{j\omega C}) \end{bmatrix} \begin{bmatrix} \vec{I}_1 \\ \vec{I}_2 \end{bmatrix} = \begin{bmatrix} \vec{V}_g \\ 0 \end{bmatrix} \Rightarrow \vec{V}'_{R2} = R_2 \vec{I}_1 \Rightarrow V'_{R2}(t) = \mathrm{Re}\left[e^{j\omega t}\,\vec{V}'_{R2}\right]$$

$\omega = 1$



$\vec{I}_g = -j$

"Annullo" il generatore di tensione con un corto circuito



(equivalente)

$$\begin{bmatrix} (G_1 + \frac{1}{j\omega L}) & -\frac{1}{j\omega L} \\ -\frac{1}{j\omega L} & (G_2 + j\omega C + \frac{1}{j\omega L}) \end{bmatrix} \begin{bmatrix} \vec{E}_1 \\ \vec{E}_2 \end{bmatrix} = \begin{bmatrix} \vec{I}_g \\ 0 \end{bmatrix} \Rightarrow \vec{V}''_{R2} = \vec{E}_2 \longrightarrow V''_{R2}(t) = \mathrm{Re}\left[e^{j\omega t}\,\vec{V}''_{R2}\right]$$

$$\Longrightarrow V_{R2}(t) = V'_{R2}(t) + V''_{R2}(t)$$

Cosa succede alla potenza $p(t) = v(t)\,i(t)$ con grandezze sinusoidali?

$$v(t) = V\cos(\omega t + \varphi_v) \longrightarrow \vec{V} = V e^{j\varphi_v}$$
$$i(t) = I\cos(\omega t + \varphi_i) \longrightarrow \vec{I} = I e^{j\varphi_i}$$

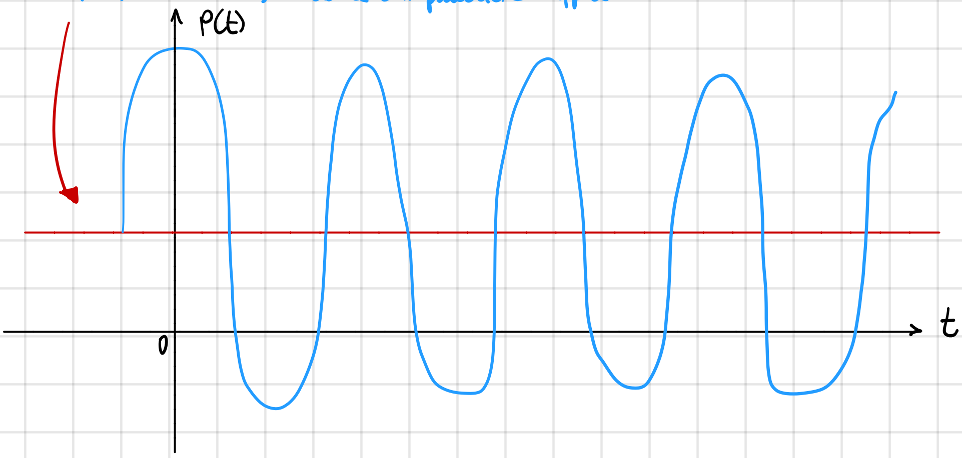
$$v(t) = \frac{\vec{V}e^{j\omega t} + \vec{V}^* e^{-j\omega t}}{2} \qquad i(t) = \frac{\vec{I}e^{j\omega t} + \vec{I}^* e^{-j\omega t}}{2}$$

complessi coniugati

$$p(t) = v(t)\cdot i(t) = \frac{\vec{V}e^{j\omega t} + \vec{V}^* e^{-j\omega t}}{2}\cdot\frac{\vec{I}e^{j\omega t} + \vec{I}^* e^{-j\omega t}}{2} = \frac{\vec{V}\vec{I}e^{j2\omega t}}{4} + \underbrace{\frac{\vec{V}\vec{I}^*}{4} + \frac{\vec{V}^*\vec{I}}{4}}_{\text{complessi coniugati ①}} + \frac{\vec{V}^*\vec{I}^* e^{-j2\omega t}}{4}$$

complessi coniugati ②

$$= \underbrace{\tfrac{1}{2}\mathrm{Re}[\vec{V}\vec{I}^*]}_{\text{① costante}} + \underbrace{\tfrac{1}{2}\mathrm{Re}[\vec{V}\vec{I}e^{j2\omega t}]}_{\text{② sinusoidale con pulsazione doppia}}$$



Quando è negativa la potenza è in uscita, in entrata altrimenti
La parte costante rappresenta la potenza media o potenza Attiva P_a

$$P_a = \tfrac{1}{2}\mathrm{Re}[\vec{V}\vec{I}^*] = \tfrac{1}{2}\mathrm{Re}[Z\vec{I}\vec{I}^*] = \tfrac{1}{2}\mathrm{Re}[Z I^2] = \frac{I^2}{2}\mathrm{Re}[Z] \quad (\vec{V} = Z\vec{I}) \rightarrow \text{resistori}$$

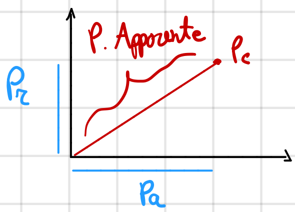
La P_a misura quindi la potenza assorbita dalla parte resistiva del circuito

$$P_a = \tfrac{1}{2}\mathrm{Re}[V e^{j\varphi_v} I e^{-j\varphi_i}] = \frac{VI}{2}\mathrm{Re}[e^{j(\varphi_v - \varphi_i)}] = \frac{VI}{2}\mathrm{Re}[\cos(\varphi_v - \varphi_i) + j\sin(\varphi_v - \varphi_i)] = \frac{VI}{2}\cos(\varphi_v - \varphi_i)$$

Se $\varphi_v = \varphi_i$ la potenza attiva è massima, se V e I sono "in quadratura", ovvero $\cos(\dots) = 0$, la potenza attiva è nulla

$$\tfrac{1}{2}\vec{V}\vec{I}^* = \underbrace{\tfrac{1}{2}\mathrm{Re}[\vec{V}\vec{I}^*]}_{P_a} + j\underbrace{\tfrac{1}{2}\mathrm{Im}[\vec{V}\vec{I}^*]} = \text{Potenza Complessa}$$

$$\tfrac{1}{2}\mathrm{Im}[\vec{V}\vec{I}^*] = \tfrac{1}{2}\mathrm{Im}[Z\vec{I}\vec{I}^*] = \tfrac{1}{2}I^2\,\mathrm{Im}[Z] = \text{Potenza Reattiva}$$



$$P_r = \tfrac{1}{2}\mathrm{Im}[V e^{j\varphi_v} I e^{j\varphi_i}] = \tfrac{1}{2}VI\sin(\varphi_v - \varphi_i)$$

Se I e V sono in quadratura P_r è massima, se sono in fase è zero

Un circuito è lineare se effetto$(t) = K$ causa(t)

Sovrapposizione degli effetti:

$$\left.\begin{array}{l} c_1(t) \to e_1(t) \\ c_2(t) \to e_2(t) \end{array}\right\} \to a c_1(t) + b c_2(t) \to a e_1(t) + b e_2(t)$$

Causalità: $c(t) \to e(t)$

Permanenza/Invarianza nel tempo:

$$c(t) \to e(t)$$
$$c(t_2) \to e(t_2)$$

Passività:

$$E = \int_0^t p(t)\,dt$$

$v(t) = R\,i(t)$

$$\begin{aligned} P(t) &= \tfrac{1}{2} Re[\vec{V}\vec{I}^*] + \tfrac{1}{2} Re[\vec{V}\vec{I} e^{j2\omega t}] \\ &= \tfrac{1}{2} Re[R\vec{I}\vec{I}^*] + \tfrac{1}{2} Re[R\vec{I}\vec{I}^* e^{j2\omega t}] \\ &= \frac{RI^2}{2} + \frac{RI^2}{2} Re[e^{j2\omega t}] = \frac{RI^2}{2} + \frac{RI^2}{2} Re[\cos 2\omega t + j\sin 2\omega t] \\ &= \boxed{\frac{RI^2}{2} + \frac{RI^2}{2}\cos 2\omega t} \end{aligned}$$

$V(t) = L\,di/dt$

$\to \vec{V} = j\omega L \vec{I}$

$$\begin{aligned} P(t) &= \tfrac{1}{2} Re[\vec{V}\vec{I}^*] + \tfrac{1}{2} Re[\vec{V}\vec{I} e^{j2\omega t}] \\ &= \tfrac{1}{2} Re[j\omega L\vec{I}\vec{I}^*] + \tfrac{1}{2} Re[j\omega L\vec{I}\vec{I} e^{j2\omega t}] \\ &= \frac{I^2}{2} \cancel{Re[j\omega L]} + \tfrac{1}{2}\omega L\, Re[j\vec{I}\vec{I} e^{j2\omega t}] \\ &= \frac{\omega L}{2} Re[I^2 e^{j\frac{\pi}{2}} e^{j2\varphi_i} e^{j2\omega t}] = \boxed{\tfrac{1}{2}\omega L I^2 \cos(2\omega t + 2\varphi_i + \pi/2)} \end{aligned}$$

$$\boxed{P_R = \tfrac{1}{2} j\omega L I^2}$$

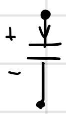
$\mathcal{E}_L(t) = \frac{1}{2} L i^2(t)$ $\qquad i(t) = I\cos(\omega t + \varphi_i)$

$$\mathcal{E}_{L\,media} = \frac{1}{T_m}\int_0^{T_m} \tfrac{1}{2} L i^2(\tau)\,d\tau$$

$$\to \mathcal{E}_m = \tfrac{1}{2} L I^2 \int_0^{T_m} \cos^2(\omega t + \varphi_i)\,d\tau \to \cos^2\alpha = \frac{1+\cos 2\alpha}{2} \to \tfrac{1}{4} L I^2 \left[\frac{1}{T_m}\int d\tau + \frac{1}{T_m}\int \cancel{\cos 2}(\dots)\,d\tau\right]$$

$$= \mathcal{E}_m = \tfrac{1}{4} L I^2$$

$$L I^2 = \frac{2P_R}{\omega} \to \mathcal{E}_m = \frac{1}{2}\frac{P_R}{\omega} \to P_R = 2\omega \mathcal{E}_m$$



$i(t) = C\frac{dv}{dt} \qquad \vec{I} = j\omega C\vec{V}$

$$p(t) = \tfrac{1}{2} Re[\vec{V}\vec{I}^*] + \tfrac{1}{2} Re[\vec{V}\vec{I} e^{j2\omega t}] = \tfrac{1}{2} Re[-j\vec{V}\omega C\vec{V}^*] + \tfrac{1}{2} Re[\vec{V} j\omega C\vec{V} e^{j2\omega t}] =$$

$$= \tfrac{1}{2}\omega C V^2 \cancel{Re[-j]} + \tfrac{1}{2}\omega C\, Re[j\vec{V}\vec{V} e^{j2\omega t}] = \tfrac{1}{2}\omega C\, Re[jV^2 e^{j2\varphi_v} e^{j2\omega t}] = \tfrac{1}{2}\omega C V^2 Re[e^{j(2\omega t + 2\varphi_v + \pi/2)}]$$

$j = e^{j\pi/2}$

$$= \boxed{\tfrac{1}{2}\omega C V^2 \cos(2\omega t + 2\varphi_v + \pi/2)}$$

$$P_R = \tfrac{1}{2} Im[\vec{V}\vec{I}^*] = \tfrac{1}{2} Im[-\vec{V} j\omega C\vec{V}^*] = \boxed{-\frac{j\omega C V^2}{2}}$$

$$\mathcal{E}_c(t) = \frac{1}{2} C v^2(t)$$

$v(t) = V\cos(\omega t + \varphi_v)$

$$\mathcal{E}_{cm} = \frac{1}{T_m}\int_0^{T_m} \mathcal{E}_c(\tau)\,d\tau = \frac{C}{2T_m}\int_0^{T_m} v^2(\tau)\,d\tau = \frac{C}{2T_m} V^2 \int_0^{T_m} \cos^2(\omega\tau + \varphi_v)\,d\tau =$$

$\cos^2\alpha = \frac{1+\cos 2\alpha}{2}$

nel lungo termine

$$= \frac{C}{2T_m}V^2\left[\int_0^{T_m} \frac{1}{2}\,dt + \frac{1}{2}\cancel{\int \cos 2(\omega t + \varphi_v)\,dt}\right] = \frac{CV^2}{4}$$

$\rightarrow$ Sostituisco CV^2 in P_R $\longrightarrow$ $P_R = \frac{-j\omega 4\mathcal{E}_m}{2} = -j2\omega\mathcal{E}_{cm}$

$$P_c = \frac{1}{2}\vec{V}\vec{I}^* \qquad \vec{V} \quad \vec{I} \rightarrow \boxed{Z}$$

$$= \frac{1}{2}\sum_1^n \vec{V}_k\vec{I}_k = \frac{1}{2}[\vec{V}]^T[\vec{I}^*]$$

$$[\vec{V}_{coalbero}] = [A][\vec{V}_{albero}]$$

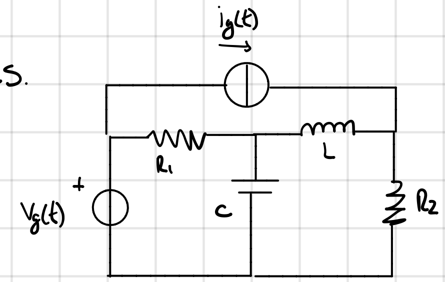
$$[\vec{I}_{albero}] = -[A]^T[\vec{I}_{coalbero}]$$

$$\Longrightarrow \frac{1}{2}\left[\frac{[V_a]}{[V_c]}\right]^T\left[\frac{[I_a^*]}{[I_c^*]}\right] = \frac{1}{2}[V_a]^T[I_a^*] + \frac{1}{2}[V_c]^T[I_c^*]$$

$$= -\frac{1}{2}[V_a]^T[A]^T[I_c^*] + \frac{1}{2}[V_a]^T[A]^T[I_c^*] = 0$$

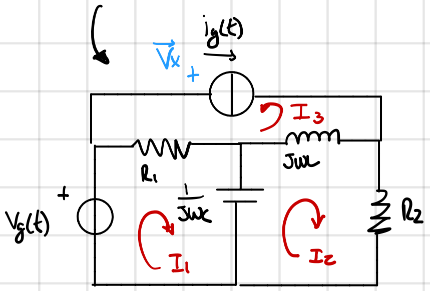
La P_c del sistema è nulla: $P_c = P_a + jP_R = 0$

ES.



$V_g(t) = 2\cos t \qquad i_g(t) = \sin t$

$R = 1\,\Omega \qquad L = 2H \qquad C = 1F$



$\vec{V}_g = 2$

$\vec{I}_g = -j$

$$\begin{bmatrix} \left(R_1 + \frac{1}{j\omega C}\right) & -\frac{1}{j\omega C} & R_1 \\ -\frac{1}{j\omega C} & \left(\frac{1}{j\omega C} + j\omega L + R_2\right) & j\omega L \\ R_1 & j\omega L & (R_1 + j\omega L) \end{bmatrix}\begin{bmatrix}\vec{I}_1 \\ \vec{I}_2 \\ \vec{I}_3\end{bmatrix} = \begin{bmatrix}\vec{V}_g \\ 0 \\ \vec{V}_x\end{bmatrix}$$

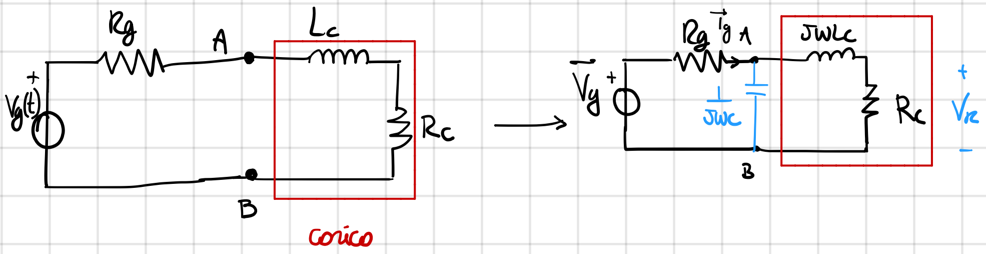
con: $\vec{I}_3 = -\vec{I}_g$

$$P_{aR_2} = \frac{1}{2}\mathrm{Re}[\vec{V}_{R2}\vec{I}_2^*] = \frac{1}{2}R_2\,\mathrm{Re}[\vec{I}_2\vec{I}_2^*] = \frac{1}{2}R_2 I_2^2$$

$$P_{aR_1} = \frac{1}{2}R_1|\vec{I}_1 + \vec{I}_3|^2$$

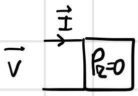
$$P_{cV_g} = \frac{1}{2}[\vec{V}_g\vec{I}_1^*]$$

$$P_{ci_g} = \frac{1}{2}[\vec{V}_x\vec{I}_g]$$



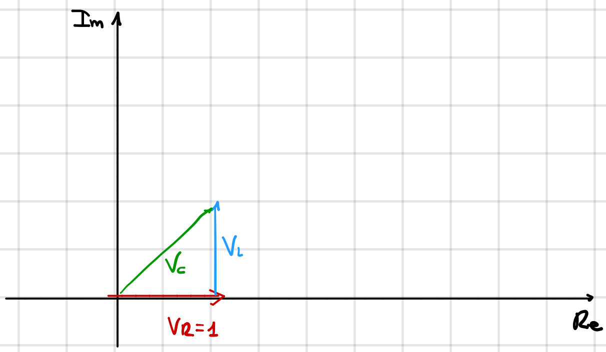
L'obbiettivo del condensatore è non sperperare la Pr del generatore → Pr carico = 0

Problema del RIFASAMENTO



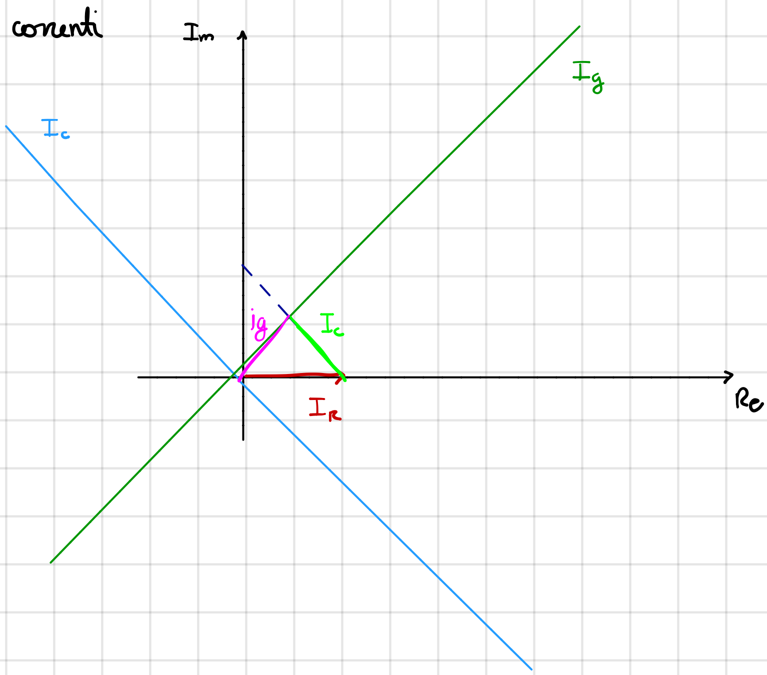
se $\vec{V} \parallel \vec{I}$

Tensioni

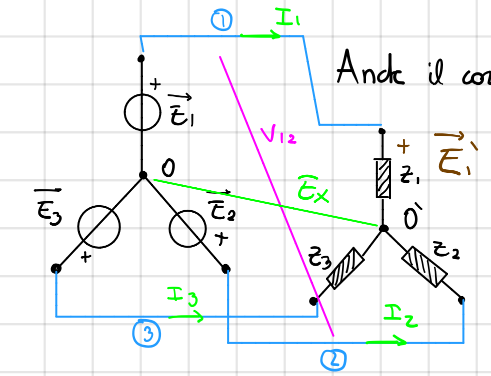


- $\vec{V}_R = 1$
- → $\vec{I}_R = \frac{R_c}{\vec{V}_R}$
- → $V_L = JWL_c \vec{I}_R$
- → $V_c = V_L + V_R$
- → $I_c = JWC V_c$ → $I_c \perp V_c$
- → $I_g \parallel V_c$
- → $I_g = I_c + I_R$ (devono formare una linea chiusa)

correnti

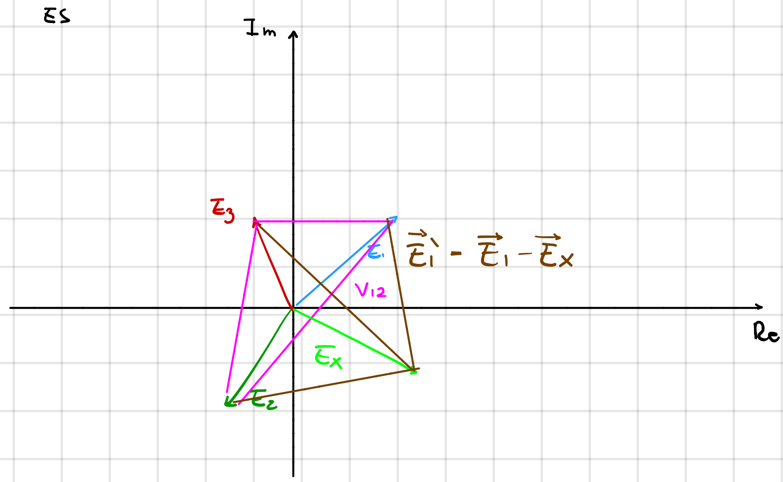


SISTEMI TRIFASE



Anche il carico è "a stella"

ES

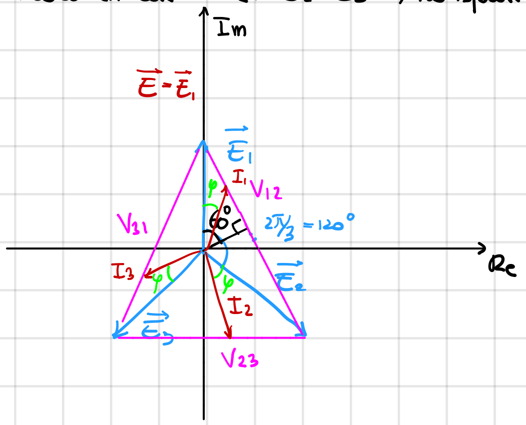


$V_{12} = E_1 - E_2$

Metodo base nodi:

$$\vec{E}_x(G_1+G_2+G_3) - \vec{E}_1(G_1) - \vec{E}_2(G_2) - \vec{E}_3 G_3 = 0 \longrightarrow \vec{E}_x = \frac{\vec{E}_1 G_1 + \vec{E}_2 G_2 + \vec{E}_3 G_3}{G_1+G_2+G_3}$$

Nel caso in cui $E_1 = E_2 = E_3$, ma sfasati di $\frac{2\pi}{3}$ $\Longrightarrow$ Sistema trifase SIMMETRICO



$$\vec{E}_k = \vec{E}\, e^{-j(k-1)2\pi/3} \qquad k=1,2,3$$

$$\frac{V_{12}}{2} : \sin 60° = E : \sin 90 \longrightarrow V_{12} = E\sqrt{3}$$

Supponiamo anche che $Z_1 = Z_2 = Z_3 = Z$ $\Longrightarrow$ Sistema trifase simmetrico EQUILIBRATO

$$\vec{E}_x = (\vec{E}_1 + \vec{E}_2 + \vec{E}_3)\, G / 3G \qquad \vec{I}_k = \vec{E}_k \cdot G$$

$$\vec{E}_k = Z \vec{I}_k \qquad P_a = \frac{1}{2} V I \cos\varphi$$

$$P_a = \sum_1^3 \frac{1}{2} \mathrm{Re}[\vec{E}_k \vec{I}_k^*] = \sum \frac{E^2}{2} \mathrm{Re}[Y^*] = \frac{3}{2} I^2 \mathrm{Re}[Z] = \frac{3}{2} E I \cos\varphi$$

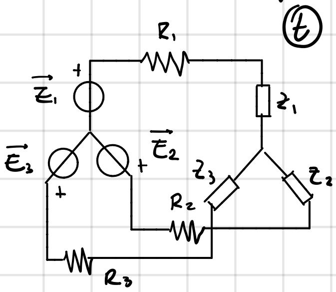
$$P_r = \sum \frac{1}{2} \mathrm{Im}[\vec{E}_k \vec{I}_k^*] = \frac{3}{2} E I \sin\varphi$$

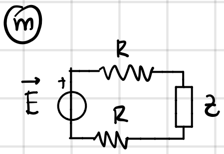
$$p(t) = \sum p_k(t) = \sum \frac{1}{2} \mathrm{Re}[\vec{E}_k \vec{I}_k^*] + \underbrace{\sum \frac{1}{2} \mathrm{Re}[\vec{E}_k I_k e^{j2\omega t}]}$$

$$\sum \frac{1}{2} \mathrm{Re}[Y E_k^2 e^{j2\omega t}] = \sum \frac{1}{2} \mathrm{Re}[Y E^2 e^{-j(k-1)4\pi/3} e^{j2\omega t}] = \frac{1}{2} \mathrm{Re}[Y \vec{E}^2 e^{j2\omega t} \underbrace{\cancel{\textstyle\sum e^{-j(k-1)4\pi/3}}}_{0}] = 0$$

$$\Longrightarrow \boxed{P(t) = \text{cost} = P_a}$$

Introduciamo resistenze di perdita





$R_1 = R_2 = R_3 = R$

$P_{ut} = \frac{3}{2} E I \cos\varphi$ utile

$P_{pt} = \frac{3}{2} I^2 R$ persa

$P_{um} = \frac{1}{2} E I \cos\varphi$

$P_{pm} = \frac{1}{2} I^2 R$

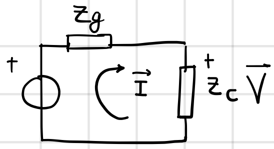
$$\frac{P_{ut}}{P_{pt}} = \frac{P_{um}}{P_{pm}}$$

impedenza di linea $Z_L = R_L + jX_L$

$$P_p = \frac{1}{2} I^2 R_L \quad \longrightarrow \quad P_p = \frac{1}{2} \frac{4 P_u^2 R_L}{V^2 \cos^2\varphi}$$

$$P_u = \frac{1}{2} V I \cos\varphi \Rightarrow I = \frac{2 P_u}{V \cos\varphi}$$

$V_g(t) \longrightarrow \vec{V}_g$



$Z_g = R_g + jX_g$

$Z_c = \underbrace{R_c}_{} + jX_c$

$$P_a = \frac{1}{2} \mathrm{Re}\left[\vec{V}\vec{I}^*\right] = \frac{1}{2} \mathrm{Re}\left[Z_c \vec{I}\vec{I}^*\right] = \frac{I^2}{2} \mathrm{Re}\left[Z_c\right] = \frac{I^2}{2} \cdot R_c$$

$$\vec{V}_g = \vec{I}(Z_g + Z_c) \longrightarrow \vec{I} = \vec{V}_g / Z_c + Z_g \longrightarrow I = \vec{V}_g / (R_c + R_g) + j(X_c + X_g) \longrightarrow P_a = \frac{1}{2} \frac{R_c \cdot V_g^2}{(R_c + R_g)^2 + (X_c + X_g)^2}$$

Trovare Z_c per avere il massimo trasferimento di P_a — Teorema del max trasferimento di P_a

$\longrightarrow$ derivata di P_a $\longrightarrow$ per semplificare deriviamo P_a^{-1}

$$F = \frac{(R_g + R_c)^2 + (X_g + X_c)^2}{V_g^2 \, R_c}$$

① $$\left.\partial F / \partial X_c\right|_{R_c = \mathrm{cost}} = \frac{2(X_g + X_c)}{R_c} = 0 \longrightarrow \boxed{X_c = -X_g}$$

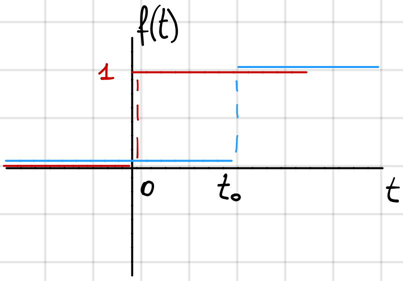
② $$\left.\partial F / \partial R_c\right|_{X_c = \mathrm{cost}} = \partial F / \partial R_c \frac{(R_g + R_c)^2}{R_c} = 2(R_g + R_c)R_c - (R_g + R_c)^2 = 2\cancel{R_g}R_c + 2R_c^2 - R_g^2 - 2\cancel{R_g}R_c = 0 \longrightarrow \boxed{R_c = R_g}$$

$$\Rightarrow \boxed{Z_c = Z_g^*}$$

$$P = \frac{1}{2} \frac{R_g V_g^2}{(2R_g)^2} = V_g^2 / 8R_g = P_d \text{ potenza disponibile}$$

$$\eta \text{ rendimento} = P_u / P_{erogata} = \frac{1}{2} R I^2 \Big/ \frac{1}{2}(R_g + R_c) I^2 = \frac{1}{2}$$

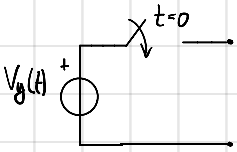
ECCITAZIONI NON SINUSOIDALI

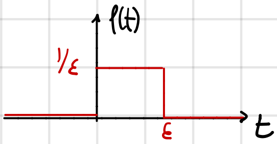


$$f(t) = \begin{cases} 0 & t<0 \\ 1 & t>0 \end{cases} \qquad u_{-1}(t) \text{ gradino}$$

Se l'ampiezza non è 1 → $a\,u_{-1}(t)$

$$f(t) = \begin{cases} 0 & t<t_0 \\ 1 & t>t_0 \end{cases} \qquad u_{-1}(t-t_0) \text{ gradino ritardato}$$

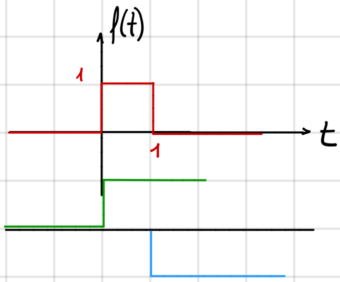




$$\lim_{\varepsilon \to 0} \frac{1}{\varepsilon} \cdot \varepsilon = 1$$

$$\Longrightarrow \begin{cases} A\,u_0(t) & \text{non unitario} \\ u_0(t-t_0) & \text{ritardato} \end{cases}$$

$$\int_{-\infty}^{T} u_0(t)\,dt = \begin{cases} 0 & T<0 \\ 1 & T>0 \end{cases}$$



$$f(t) = u_{-1}(t) - u_{-1}(t-1)$$

$$f(t) = t\,u_{-1}(t)$$

Trasformata di Laplace

$f(t) \longrightarrow F(s)$

$$F(s) = \mathcal{L}[f(t)] = \lim_{T \to \infty} \int_0^T f(t)\,e^{-st}\,dt$$

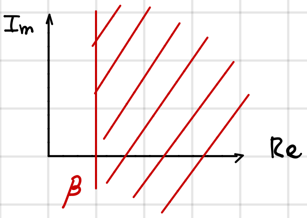
Proprietà:

1) $\left.\begin{array}{l} f_1(t) \to F_1(s) \\ f_2(t) \to F_2(s) \end{array}\right\}$ $a f_1(t) + b f_2(t) \longrightarrow a F_1(s) + b F_2(s)$

2) $f(t) \to F(s)$
$df/dt \longrightarrow sF(s) - f(0)$

3) $f(t) \longrightarrow F(s)$
$f(t-t_0) \longrightarrow e^{-st_0} F(s)$

$$F(s)=\lim_{T\to\infty}\int_0^T f(t)\,e^{-st}\,dt$$



$\mathrm{Re}(s) > \beta$

ascissa di convergenza

TEOREMA VALORE INIZIALE $\quad \lim_{t\to 0} f(t) = \lim_{s\to\infty}[sF(s)]$

TEOREMA VALORE FINALE $\quad \lim_{t\to\infty} f(t) = \lim_{s\to 0}[sF(s)]$

TRASFORMATE

		β
$u_{-1}(t) \longrightarrow$	$\frac{1}{s}$	>0
$u_0(t) \longrightarrow$	1	$>-\infty$
$e^{-s_0t}u_{-1}(t) \longrightarrow$	$\frac{1}{s+s_0}$	$>\mathrm{Re}[-s_0]$
$t^n e^{-s_0t}u_{-1}(t) \longrightarrow$	$\frac{n!}{(s+s_0)^{n+1}}$	
$t\,u_{-1}(t) \longrightarrow$	$\frac{1}{s^2}$	
$(t-t_0)u_{-1}(t-t_0) \longrightarrow$	$\frac{e^{-st_0}}{s^2}$	

$f(t) = A\cos(\omega_0 t+\varphi)\,u_{-1}(t)$ sottinteso sempre (se non presente)

$$= \frac{1}{2}\left(A e^{j(\omega_0 t+\varphi)} + A e^{-j(\omega_0 t+\varphi)}\right] u_{-1}(t) = \left[\frac{A}{2}e^{j(\omega_0 t+\varphi)} + \frac{A}{2}e^{-j(\omega_0 t+\varphi)}\right]u_{-1}(t)$$

$$= \frac{A}{2}e^{j\omega_0 t}e^{j\varphi}u_{-1}(t) + \frac{A}{2}e^{-j\omega_0 t}e^{-j\varphi}u_{-1}(t)$$

$$\longrightarrow F(s) = \frac{A}{2}e^{j\varphi}\frac{1}{s-j\omega_0} + \frac{A}{2}e^{-j\varphi}\frac{1}{s+j\omega_0} = \cancel{\frac{A}{2}}\,2\,\mathrm{Re}\left[\frac{e^{j\varphi}}{s-j\omega_0}\right] = A\,\mathrm{Re}\left[\frac{e^{j\varphi}(s+j\omega_0)}{s^2+\omega_0^2}\right]$$

$$= \frac{A}{s^2+\omega_0^2}\mathrm{Re}\left[(\cos\varphi+j\sin\varphi)(s+j\omega_0)\right] = \frac{A}{s^2+\omega_0^2}\left[s\cos\varphi - \omega_0\sin\varphi\right]$$

ANTITRASFORMATA

Tornare nel dominio del tempo dal dominio di Laplace.

$F(s) = \frac{N(s)}{D(s)} \qquad N(s) = a_m s^m + a_{m-1}s^{m-1} \dots a_1 s + a_0 \qquad D(s) = b_n s^n + \dots + a_1 s + a_0$

Polo funzione complessa: punto s_0 per il quale $\lim_{s\to s_0}|F(s)| = \infty$

Prendiamo un punto s_0 NON Polo:

$F(s) = \sum_0^\infty c_i (s-s_0)^i$ = sviluppo di Taylor con $c_i = \frac{1}{i!}\frac{d^i F(s)}{ds^i}\Big|_{s_0}$

Ora prendiamo una funzione in cui s_0 Polo molt. r $\rightarrow$ Non possiamo usare Taylor

$G(s) = (s-s_0)^r F(s) \longrightarrow G(s) = \sum c_i (s-s_0)^i$ con $c_i = \frac{1}{i!}\frac{d^i G(s)}{ds^i}\Big|_{s=s_0}$

$$F(s) = \underbrace{\frac{c_0}{(s-s_0)^r} + \frac{c_1}{(s-s_0)^{r-1}} + \frac{c_2}{(s-s_0)^{r-2}} + \dots + \frac{c_{r-1}}{s-s_0}}_{①} + \underbrace{\sum_r^\infty c_i (s-s_0)^{i-r}}_{②}$$

① Per $s \to 0$ tendono a ∞ $\Longrightarrow$ Parte singolare P_{S_0}

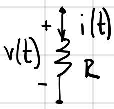
② Parte reale P_{R_0}

$$P_{S\infty} = c_0 s^r + c_1 s^{r-1} \dots c_{r-1}$$

$$\boxed{F(s) = \overset{\text{Poli}}{\underset{\text{finiti}}{\sum P_{S_i}}} + \overset{\text{Polo}}{\underset{\text{infinito}}{P_{S\infty}}} + \text{Cost}} \longrightarrow \text{Se grado}(N) > \text{grado}(D)$$

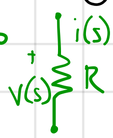
Polo semplice: $\dfrac{c_{r-1}}{s-s_0} \longrightarrow c_{r-1}\, e^{s_0 t}\, u_{-1}(t)$

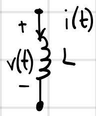
$$\frac{c}{(s-s_0)^k} \longrightarrow \frac{c\, t^{n-1}}{(n-1)!}\, e^{s_0 t}\, u_{-1}(t)$$



$v(t) = R\, i(t) \longrightarrow V(s) = R\, i(s)$ ①

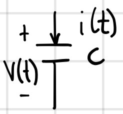
Resistore fittizio





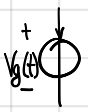
$v(t) = L\, di/dt \longrightarrow V(s) = L(s\, i(s) - i(0)) = sL\, i(s) - L\, i(0)$ ②

$i(s) = V(s)/sL + i(0)/s$



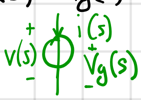
$i(t) = C\, dv/dt \longrightarrow i(s) = C(sV(s) - V(0)) = sCV(s) - CV(0)$ ③

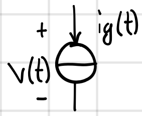
$V(s) = \dfrac{i(s)}{sC} + V(0)/s$



$v(t) = v_g(t) \longrightarrow V(s) = V_g(s)$ ④

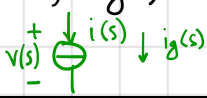
generatore fittizio





$i(t) = i_g(t) \longrightarrow i(s) = i_g(s)$ ⑤

generatore fittizio



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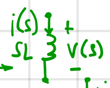
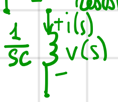
2) Supponiamo sia scarico: $i(0) = 0 \longrightarrow V(s) = sL\, i(s)$ $\quad i(s) = V(s)/sL \longrightarrow$ resistore fittizio

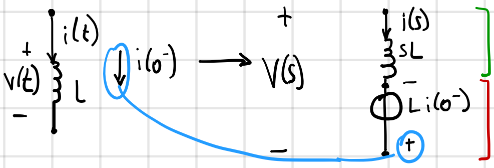


3) Supponiamo sia scarico: $v(0) = 0 \longrightarrow V(s) = i(s)/sC$ $\quad i(s) = sCV(s) \longrightarrow$ resistore fittizio



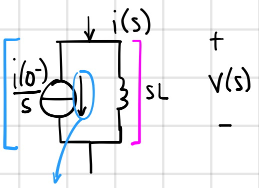
Nel dominio di Laplace sono mantenute le leggi di Kirchoff. Come per i fasori possiamo lavorare come al solito riportando poi il risultato nel dominio del tempo

2) Se carico: $V(s) = \boxed{sL\, i(s)} - \boxed{L\, i(0^-)}$ $\qquad i(s) = \boxed{\dfrac{V(s)}{sL}} + \boxed{i(0^-)/s}$



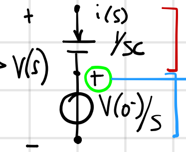
esce dal morsetto positivo

oppure

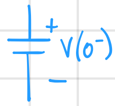


stesso verso di partenza

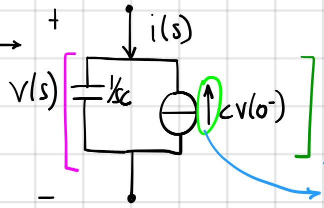
3) Se carico: $V(s) = \boxed{\frac{i(s)}{sC}} + \boxed{\frac{V(0^-)}{s}}$ $\longrightarrow$

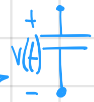


stesso verso del caso di partenza



oppure $i(s) = \boxed{sCV(s)} - \boxed{CV(0^-)}$ $\longrightarrow$

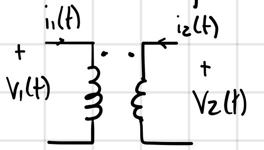




dal segno negativo a quello positivo del caso di partenza

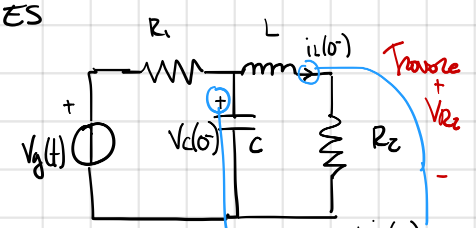
Nel caso di generatori controllati $V_g(t) = a\,v(t) \longrightarrow V_g(s) = a\,V(s)$

Nel caso del trasformatore ideale:



$$\begin{cases} V_1(t) = n V_2(t) \\ i_1(t) = -\frac{1}{n} i_2(t) \end{cases} \longrightarrow \begin{cases} V_1(s) = n V_2(s) \\ i_1(s) = -\frac{1}{n} i_2(s) \end{cases}$$

ES

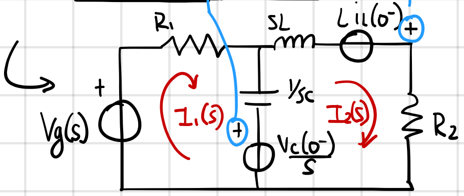


$V_g(t) = u_{-1}(t)\ V$

$R_1 = R_2$, $L = 1$, $C = 1$

$V_c(0^-) = 2V$

$i_L(0^-) = 1$



$V_g(s) = \frac{1}{s}$

$$\begin{bmatrix} (R_1 + \frac{1}{sC}) & -\frac{1}{sC} \\ -\frac{1}{sC} & (R_2 + sL + \frac{1}{sC}) \end{bmatrix} \begin{bmatrix} I_1(s) \\ I_2(s) \end{bmatrix} = \begin{bmatrix} V_g(s) - \frac{V_c(0^-)}{s} \\ \frac{V_c(0^-)}{s} + L\,i_L(0^-) \end{bmatrix}$$

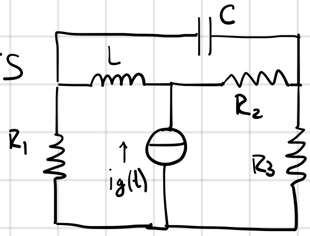
base maglie

$$\begin{cases} (1+\frac{1}{s})I_1(s) - \frac{1}{s}I_2(s) = \frac{1}{s} - \frac{2}{s} \xrightarrow{\cdot s} (s+1)I_1(s) - I_2(s) = -1 \\ -\frac{1}{s}I_1(s) + (1+s+\frac{1}{s})I_2(s) = \frac{2}{s} + 1 \xrightarrow{\cdot s} -I_1(s) + (s^2+s+1)I_2(s) = 2+s \xrightarrow{\cdot(s+1)} -(s+1)I_1(s) + (s+1)(s^2+s+1)I_2(s) = (s+1)(2+s) \end{cases}$$

$\Longrightarrow$ sottraggo la seconda dalla prima $\Longrightarrow [(s+1)(s^2+s+1) - 1]\,I_2(s) = (s+1)(s+2) - 1 \longrightarrow I_2(s) = \frac{s^2+3s+1}{s(s^2+2s+2)}$

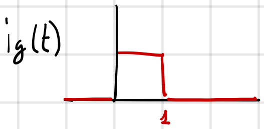
$V_{R2}(s) = R_2\,I_2(s)$

ES

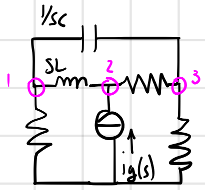


Trovare $+\; V_{R_2}(t)\; -$

$R=1 \quad C=1 \quad L=1$



$\Rightarrow i_g(t) = u_{-1}(t) - u_{-1}(t-1) \Rightarrow \frac{1}{s} - \frac{e^{-s}}{s} = i_g(s)$

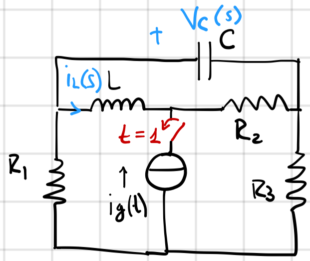


$$\begin{bmatrix} \left(\frac{1}{R_1}+\frac{1}{sL}+sC\right) & -\frac{1}{sL} & -sC \\ -\frac{1}{sL} & \left(\frac{1}{sL}+\frac{1}{R_2}\right) & -\frac{1}{R_2} \\ -sC & -\frac{1}{R_2} & \left(\frac{1}{R_2}+\frac{1}{sL}+sC\right) \end{bmatrix} \begin{bmatrix} E_1(s) \\ E_2(s) \\ E_3(s) \end{bmatrix} = \begin{bmatrix} 0 \\ i_g(s) \\ 0 \end{bmatrix}$$

base nodi

$V_{R_3}(s) = E_3(s) \longrightarrow$ porto nel tempo $\longrightarrow V_{R_3}(t)$

Se:

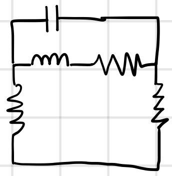


$0 < t < 1$

$i_g(t) = u_{-1}(t) \longrightarrow i_g(s) = \frac{1}{s} \Longrightarrow V_{R_3}(s) \rightarrow V_{R_3}(t)$

$t > 1$

$i_g(t) = 0 \Longrightarrow$

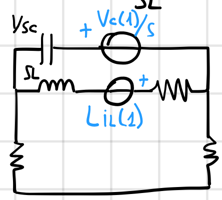


con induttore e condensatori carichi

$\Longrightarrow$ in $0<t<1$ devo trovare anche $V_C(s)$ e $i_L(s)$

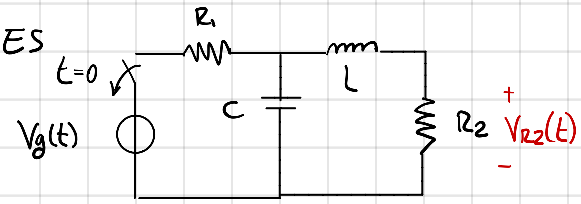
$V_C(s) = E_1(s) - E_3(s) \longrightarrow V_C(t) \longrightarrow V_C(1^+)$

$i_L(s) = \frac{E_1(s) - E_2(s)}{sL} \longrightarrow i_L(t \longrightarrow i_L(1^+)$



Il risultato finale dovrà essere uguale a quello in base nodi precedente

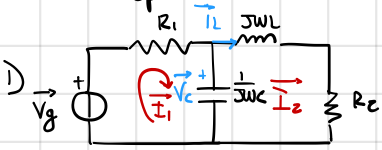
ES



$R=1 \quad C=1 \quad L=2 \quad V_g(t) = \sin t$

- $t<0$ metodo fasori
- $t>0$ laplace

1)



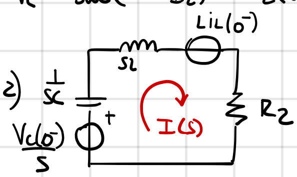
$\vec{V_g} = -j$

$$\left.\begin{aligned} &\left(R_1 + \frac{1}{j\omega C}\right)\vec{I_1} - \frac{1}{j\omega C}\vec{I_2} - \vec{V_g} \\ &-\frac{1}{j\omega C}\vec{I_1} + \left(R_2 + j\omega L + \frac{1}{j\omega C}\right)\vec{I_2} = 0 \end{aligned}\right\} \vec{V_{R_2}} = R_2 \vec{I_2} \longrightarrow V_{R_2}(t) = Re\left[\vec{V_{R_2}} e^{j\omega t}\right]$$

$\vec{I_L} = \vec{I_2} \longrightarrow i_L(t) - Re\left[\vec{I_L} e^{j\omega t}\right]$

$\vec{V_C} = \frac{1}{j\omega C}(\vec{I_1} - \vec{I_2}) = I_2(j\omega L + R_2) \longrightarrow V_C(t) = Re\left[\vec{V_C} e^{j\omega t}\right]$

2)



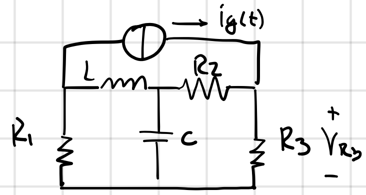
(R_1 scompare)

$\longrightarrow \left(R_2 + sL + \frac{1}{sC}\right) I(s) = \frac{V_C(0^-)}{s} + L i_L(0^-)$

$\longrightarrow (s + 2s^2 + 1) I(s) = V_C(0^-) + 2 s\, i_L(0^-)$

$\Longrightarrow V_{R_2}(s) = R_2 I(s)$

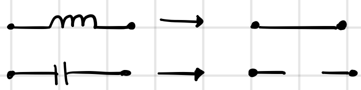
ES

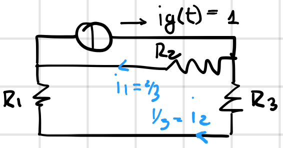


$i_g(t) = \begin{cases} 1 & t<0 \\ 0 & t>0 \end{cases}$ $\quad R=1 \quad L=1 \quad C=1$

• $t<0$

$v_L(t) = L\, di/dt = 0$ perché i costante
$i_c(t) = C\, dv/dt = 0$

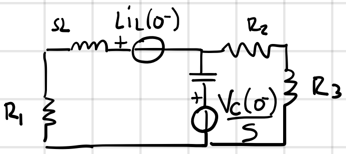
 corto circuito, circuito aperto



$V_{R_3}(t) = R_3 i_2(t) = 1/3$
$i_L(t) = i_1(t) = 2/3$
$V_c(t) = -V_{R_1} i_2 = -1/3$

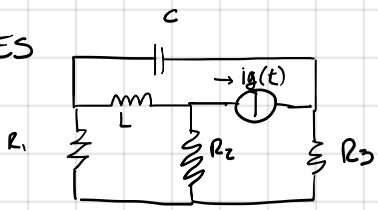
(si può fare anche in base nodi/maglie)

• $t>0$



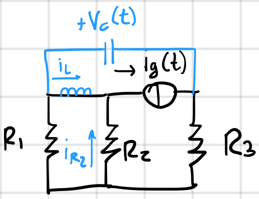
$V_{R_3}(t)$?

ES

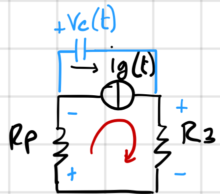


$i_g(t) \begin{cases} 1 & t<0 \\ 0 & t>0 \end{cases}$ $\quad R = 1\ \Omega \quad L = 1H \quad C = 1F$

• $t<0$



$R_p = \frac{1}{R_1} + \frac{1}{R_2} = \frac{R_1+R_2}{R_1R_2} \longrightarrow$

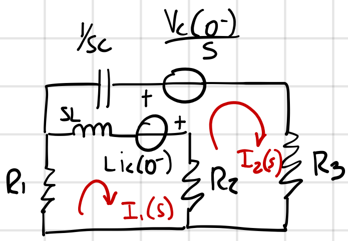


$\Rightarrow V_{R_3}(t) = R_3\, i_g(t) = 1V$

$\boxed{V_c(t)} = -V_{R_p} - V_{R_3} = -2-1 = \boxed{-3V}$

$\boxed{i_L(t) = \frac{1}{2} A}$

• $t>0$

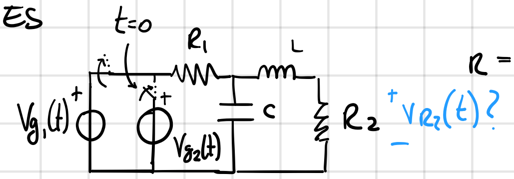


$$\begin{bmatrix} (R_1+R_2+sL) & -R_2-sL \\ -R_2-sL & (R_2+R_3+sL+\frac{1}{sC}) \end{bmatrix} \begin{bmatrix} I_1(s) \\ I_2(s) \end{bmatrix} = \begin{bmatrix} Li_L(0^-) \\ -Li_L(0^-) - \frac{V_c(0^-)}{s} \end{bmatrix}$$

base maglie

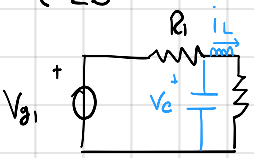
Trovo $I_2(s) \longrightarrow V_{R_3}(s) = R_3 I_2(s)$

ES



$R = 1\Omega \quad L = 1H \quad C = 1F \qquad V_{g_1}(t) = 2V \quad V_{g_2}(t) = 1V$

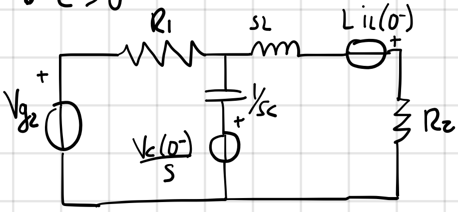
• $t < 0$



$V_{R_2}(t) = V_{g_1}(t)\,\frac{R_2}{R_1+R_2} = 1V$

$$\boxed{\begin{aligned} i_L(t) &= \frac{V_{g_1}(t)}{R_1+R_2} = \frac{1}{2} \\ V_C(t) &= V_{R_2}(t) = 1V \end{aligned}}$$

• $t > 0$



$V_{g_2}(s) = 1/s \longrightarrow$ Metodo base maglie $\longrightarrow \ldots$

R, V_g, i_g costanti $\left.\begin{aligned} e &= \text{eccitazioni} \\ u &= \text{risposta} \end{aligned}\right] \rightarrow u = k \cdot e$

Nel tempo $\longrightarrow u(t) = k \cdot e(t)$

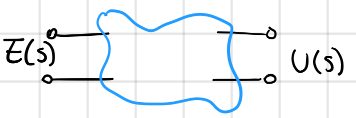
Se sono presenti elementi con memoria $\longrightarrow$ Fasori o Laplace

eccitazioni variabili R, C, L nel dominio $s \longrightarrow u(s) = F(s)\,E(s)$

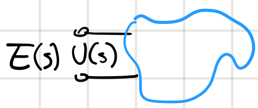
$\hookrightarrow$ funzione di rete (razionale e reale in s)

$$F(s) = \frac{u(s)}{E(s)}$$

Con più eccitazioni: $u(s) = \underbrace{\sum_{1}^{N_g} F_k(s)E_k(s)}_{\text{generatori}} + \underbrace{\sum^{N_C} H_k(s)\,V_{C_k}(0^-)}_{\text{condensatori}} + \underbrace{\sum_{1}^{N_L} K_k(s)\,i_{L_k}(0^-)}_{\text{induttori}}$



oppure



Nel secondo caso E(s) e U(s) devono essere di tipo diverso: tensione e corrente o viceversa e le funzioni di rete sono dette di ingresso.

A secondo del tipo di E(s) e U(s) la funzione di ingresso può essere un'ammettenza o una impedenza d'ingresso

Nel primo caso parliamo di funzione di trasferimento

U \ E	V	I
V	F(s) in tensione	Impedenza di trasferimento o di ingresso
I	Ammettenza di trasferimento o di ingresso	F(s) in corrente

Prendiamo il caso in cui $E(s) = 1 \longrightarrow U(s) = E(s)$

$E(s) = 1$ se $e(t) = u_0(t)$ impulso demendore $\longrightarrow u(t) = h(t) \longrightarrow F(s) = \mathcal{L}[h(t)]$

Se $h(t)$ è nota:

$$\left.\begin{array}{l} e(t) \rightarrow E(s) \\ h(t) \longrightarrow F(s) \end{array}\right\} \longrightarrow u(s) = F(s)E(s) \longrightarrow u(t) = \mathcal{L}^{-1}[U(s)]$$

$$u(t) = \int_0^t h(t-\tau)e(\tau)\,d\tau$$

Un circuito si dice stabile se tutte le risposte impulsive tendono a zero per $t \to \infty \Rightarrow \lim_{t\to\infty} h(t) = 0$

$$U(s) = \mathcal{L}[h(t)] = \frac{N(s)}{D(s)}$$

- polo $s = \alpha_i$ con molteplicità r, reale

$$\Rightarrow \underbrace{\frac{a_i}{(s-\alpha_i)^n}}_{\text{n-esimo fratto semplice}} \overset{\mathcal{L}^{-1}}{\Rightarrow} a_i \cdot \frac{t^{n-1}}{(n-1)!} \cdot e^{\alpha_i t} u_{-1}(t)$$

$$\left[\begin{array}{l} \alpha_i < 0 \longrightarrow 0 \\ \alpha_i > 0 \longrightarrow \infty \\ \alpha_i = 0 \quad \begin{cases} n = 1 & \text{non diverge} \\ n > 1 & \longrightarrow \infty \end{cases} \end{array}\right.$$

- poli $s = s_i$ e $s = s_i^*$ complessi coniugati con molteplicità r

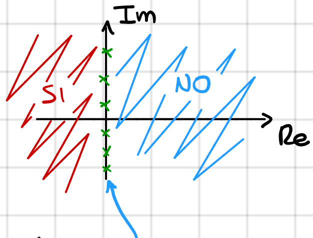
$$\Rightarrow \frac{A_i}{(s-s_i)^n} + \frac{A_i^*}{(s-s_i^*)} \Rightarrow \left(A_i \cdot \frac{t^{n-1}}{(n-1)!} \cdot e^{s_i t} + A_i^* \cdot \frac{t^{n-1}}{(n-1)!} \cdot e^{s_i^* t}\right) u_{-1}(t)$$

$$A_i = |A| e^{j\varphi_i} \Longrightarrow |A_i| e^{j\varphi_i} \cdot \frac{t^{n-1}}{(n-1)!} e^{(\sigma_i + j\omega_i)t} + |A_i^*| e^{-j\varphi_i} \frac{t^{n-1}}{(n-1)!} e^{(\sigma_i - j\omega_i)t}$$

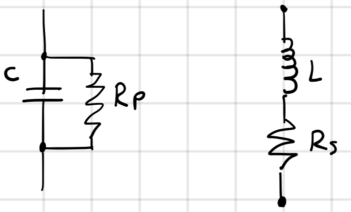
$$= \ldots = 2|A_i| \frac{t^{n-1}}{(n-1)!} e^{\sigma_i t} \cos(\varphi_i + j\omega_i t)$$

$$\left[\begin{array}{l} \sigma_i < 0 \longrightarrow 0 \\ \sigma_i > 0 \longrightarrow \infty \\ \sigma_i = 0 \quad \begin{cases} n = 1 & \text{limitato} \\ n > 1 & \longrightarrow \infty \end{cases} \end{array}\right.$$

Un circuito è stabile se i poli della funzione di rete sono nel semipiano sinistro



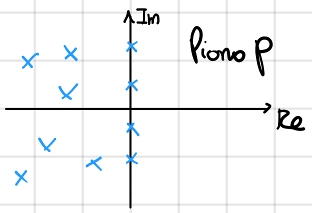
Per i circuiti passivi possiamo ammettere poli sull'asse immaginario.



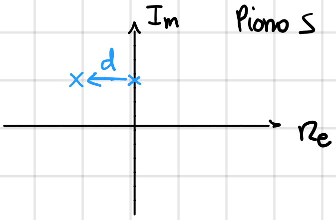
impedenze:

$$Z_L = R_s + sL = L\left(\frac{R_s}{L}+s\right) \qquad Y_C = \frac{1}{R_P} + sC = C\left(\frac{1}{R_PC}+s\right)$$

$$\left\{ \frac{R_s}{L} = \frac{1}{R_PC} = d \right. \qquad \left\{ \begin{array}{l} Z_L = L(d+s) \\ Y_C = C(d+s) \end{array} \right. \quad \left\{ p = d+s \right. \qquad \left\{ \begin{array}{l} Z_L = pL \\ Y_C = pC \end{array} \right.$$



$\longrightarrow s = p - d$



Abbiamo detto che $U(s) = \underbrace{\sum F_k(s)E_k(s)}_{①} + \underbrace{\sum H_k(s)V_{C_k}(0^-) + \sum K_k(s)\, i_{L_k}(0^-)}_{②}$

La parte ① esiste se esistono generatori indipendenti: risposta forzata
La parte ② esiste se esistono condizioni iniziali diverse da zero: risposta libera

Supponiamo di avere un eccitazione $e(t) = A\cos(\omega t+\varphi)\, u_{-1}(t)$ con $t>0$ in un circuito stabile:
allora $U(s) = F(s)E(s)$

$\rightarrow$ poli $U(s)$ = poli $\underset{\downarrow\; s_i = \sigma_i + j\omega_i}{F(s)}$ + poli $\underset{\downarrow\; s = \pm j\omega}{E(s)}$ $\qquad \sigma_i < 0$

$$U(s) = U_t(s) + \frac{R}{s-j\omega} + \frac{R^*}{s+j\omega} \longrightarrow u(t) = u_t(t) + A_1\cos(\omega t+\varphi_1)\, u_{-1}(t)$$

per $t\to\infty$ tende a 0 — risposta transitoria

per $t\to\infty$ limitata — risposta permanente

$\rightarrow u(t) = u_t(t) + u_p(t)$

(Il metodo dei fasori, se la risposta transitoria tende a zero, restituisce la risposta permanente)

Supponiamo $e(t) = A\cos(\omega_0 t + \varphi)\, u_{-1}(t)$, $t = 0$

con i fasori : $\vec{E} = A e^{j\varphi}$

- $e(t) = \frac{\vec{E}}{2} e^{j\omega_0 t} + \frac{\vec{E}^*}{2} e^{-j\omega_0 t}$

- $E(s) = \frac{\vec{E}/2}{s - j\omega_0} + \frac{\vec{E}^*/2}{s + j\omega_0}$

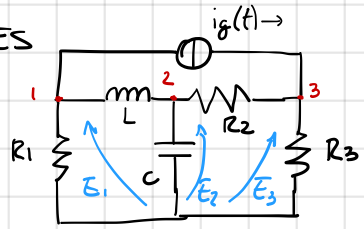
residui

- $U(s) = F(s)\, E(s) = F(s) \left[\frac{\vec{E}/2}{s - j\omega_0} + \frac{\vec{E}^*/2}{s + j\omega_0} \right]$

- $U_p(s) = \frac{\vec{U}/2}{s - j\omega_0} + \frac{\vec{U}^*/2}{s + j\omega_0}$

$\hookrightarrow \vec{U}/2 = U(s)(s - j\omega_0)\big|_{s = j\omega_0} = F(j\omega_0) \cdot \vec{E}/2 \longrightarrow \vec{U} = \underbrace{F(j\omega_0)}_{\text{funzione di rete}} \overbrace{\vec{E}}^{\text{fasore eccitazione}}$

ES



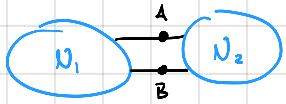
$R = 1\ \Omega \quad L = 1 \quad C = 2$

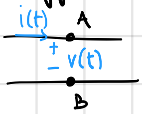
$i_g(t) = \sin t\ u_{-1}(t) \qquad \omega_0 = 1$

$$\begin{bmatrix} \left(\frac{1}{R_1} + \frac{1}{sL}\right) & -\frac{1}{sL} & 0 \\ -1/sL & \left(\frac{1}{R_2} + 1/sL + sC\right) & -\frac{1}{R_2} \\ 0 & -1/R_2 & \left(\frac{1}{R_2} + \frac{1}{R_3}\right) \end{bmatrix} \begin{bmatrix} E_1(s) \\ E_2(s) \\ E_3(s) \end{bmatrix} = \begin{bmatrix} -i_g(s) \\ 0 \\ i_g(s) \end{bmatrix}$$

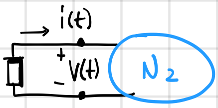
base nodi

Supponiamo di avere un circuito formato da due parti:



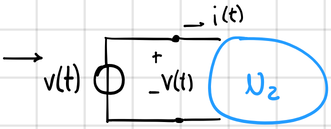


Mettiamo un bipolo al posto di N_1:



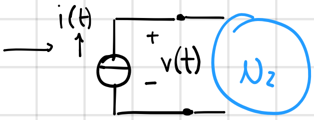
al momento però le caratteristiche del bipolo sono incognite: delle 2R equazioni ce ne manca una!

Usando quelle che abbiamo $\longrightarrow$ $a\,v(t) + b\,i(t) + c = 0$ con a, b, c incognite

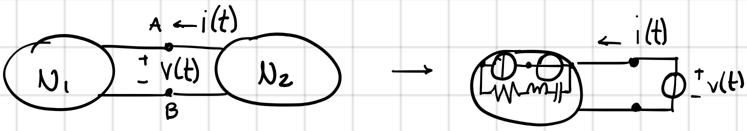


$\longrightarrow$ $i(t) = -\frac{c}{b} - \frac{a}{b}v(t)$

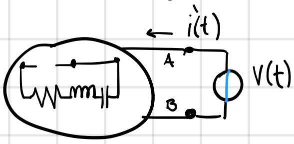
Teorema di sostituzione



$\longrightarrow$ $v(t) = -\frac{c}{a} - \frac{b}{a}i(t)$

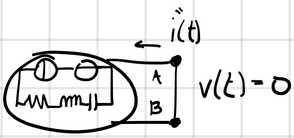


1) Attiviamo il generatore esterno e annulliamo quelli interni:



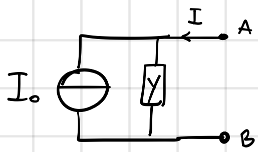
$I' = y \cdot v$

2) Facciamo l'inverso:



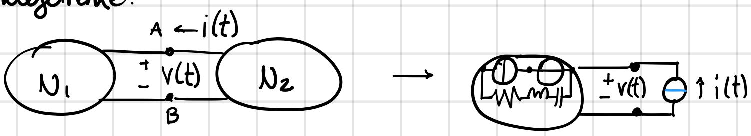
$i''(t) = i_0(t)$

$i(t) = i'(t) + i''(t)$ $\longrightarrow$ $I = yV + I_0$



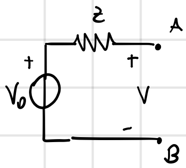
circuito equivalente secondo Norton
(Teorema di Norton)

Analogamente:



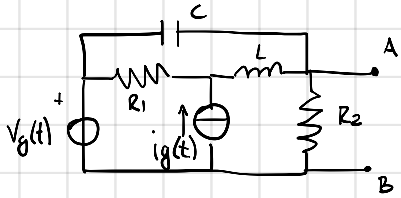
1) $V' = zI$
2) $V'' = V_0$

$\Big\}$ $V = zI + V_0$



circuito equivalente secondo Thevenin
(Teorema di Thevenin)

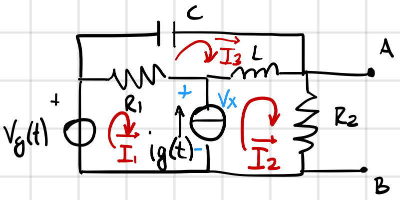
ES



$V_g(t) = \sin t$ $\quad R = 1 \quad L = 2 \quad C = 1$

$i_g(t) = \text{cost} \quad \omega = 1$

Circuito equivalente Thevenin

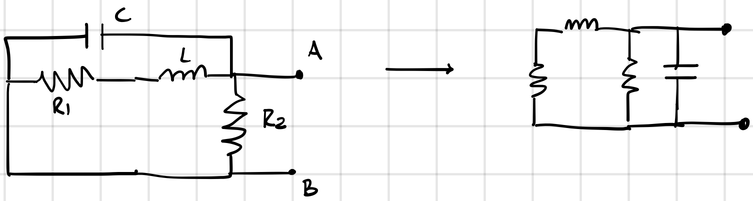


$$\begin{bmatrix} R_1 & 0 & -R_1 \\ 0 & (R_2 + jwL) & -jwL \\ -R_1 & -jwL & (R_1 + jwL + \frac{1}{jwC}) \end{bmatrix} \begin{bmatrix} \vec{I}_1 \\ \vec{I}_2 \\ \vec{I}_3 \end{bmatrix} = \begin{bmatrix} \vec{V}_g - \vec{V}_x \\ \vec{V}_x \\ 0 \end{bmatrix} \qquad \vec{I}_2 - \vec{I}_1 = \vec{i}_g$$

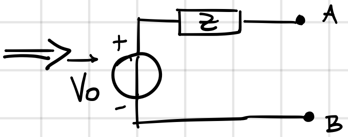
$\vec{V}_g = -j \longrightarrow$ Trovo $\vec{I}_2 \longrightarrow \vec{V}_o = \vec{I}_2 R_2$

$\vec{i}_g = 1$

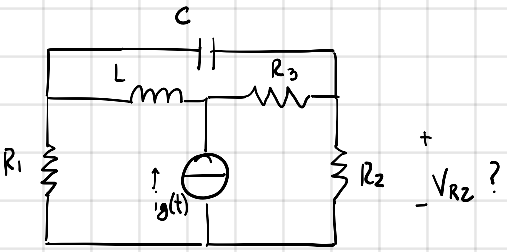
Annullo i generatori (passaggi invertiti rispetto a prima):



$Z = (R_1 + jwL) \mathbin{//} R_2 \mathbin{//} \frac{1}{jwC}$



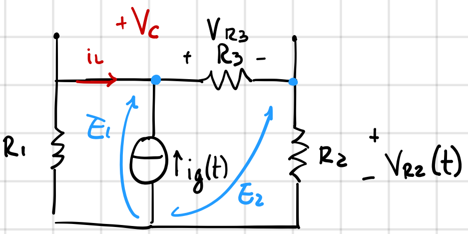
ES



$R = 1 \quad L = 1 \quad C = 1$

$$i_g(t) = \begin{cases} 1 & t < 0 \\ 0 & t > 0 \end{cases}$$

$t < 0$



$$\begin{bmatrix} \left(\frac{1}{R_1} + \frac{1}{R_3}\right) & -1/R_3 \\ -1/R_3 & \left(\frac{1}{R_2} + \frac{1}{R_3}\right) \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \end{bmatrix} = \begin{bmatrix} i_g \\ 0 \end{bmatrix}$$

base nodi

Devo trovare i_L e V_c da usare poi in $t > 0$

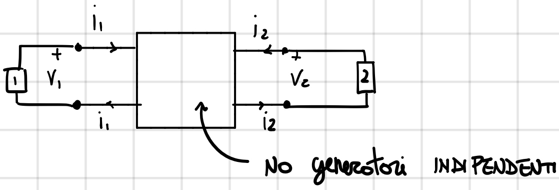
- $i_L = i_{R_1} = E_1 / R_1$
- $V_c = V_{R3} = E_1 - E_2$

Se nell'eq. $av + bi + c = 0$

$b=0 \longrightarrow V = -c/a$ perdo l'informazione sulla corrente

$a=0 \longrightarrow i = -c/b$ perdo l'informazione sulla tensione

$c=0 \longrightarrow$ Non ci sono generatori indipendenti

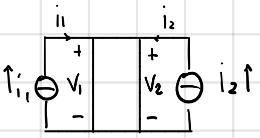


$$\longrightarrow \begin{cases} \boxed{a_{11}V_1 + a_{12}V_2} + \boxed{b_{11}i_1 + b_{12}i_2} = 0 \\ \boxed{a_{21}V_1 + a_{22}V_2} + \boxed{b_{21}i_1 + b_{22}i_2} = 0 \end{cases} \qquad \boxed{[A][V]} + \boxed{[B][i]} = [0]$$

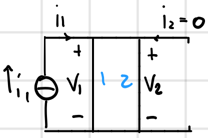
$$[V] = -[A]^{-1}[B][i] = [Z][i] \quad \text{solo se } [A] \text{ è invertibile}$$

$$[I] = -[B]^{-1}[A][V] = [Y][V] \quad \text{solo se } [B] \text{ è invertibile}$$

1) $\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$



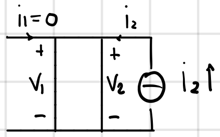
Facciamo lavorare un generatore alla volta:



$$\begin{cases} V_1 = Z_{11}i_1 + \cancel{Z_{12}i_2} & \longrightarrow Z_{11} = \frac{V_1}{i_1}\Big|_{i_2=0} \\ V_2 = Z_{21}i_1 + \cancel{Z_{22}i_2} & \longrightarrow Z_{21} = V_2/i_1 \Big|_{i_2=0} \end{cases}$$

impedenza ingresso 1

impedenza trasferimento



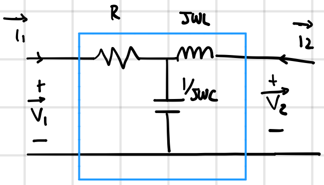
$$\begin{cases} V_1 = \cancel{Z_{11}i_1} + Z_{12}i_2 & \longrightarrow Z_{12} = V_1/i_2 \Big|_{i_1=0} \\ V_2 = \cancel{Z_{21}i_1} + Z_{22}i_2 & \longrightarrow Z_{22} = V_2/i_2 \Big|_{i_1=0} \end{cases}$$

impedenza di trasferimento

impedenza di ingresso 2

$\Rightarrow [Z]$ se esiste è detta matrice delle impedenze a vuoto

Es.



(fasori)

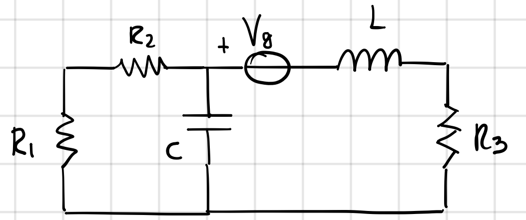
$Z_{11} = \vec{V}_1/\vec{I}_1 \Big|_{\vec{I}_2=0} = R + \frac{1}{j\omega C}$

$Z_{21} = \vec{V}_2/\vec{I}_1 \Big|_{\vec{I}_2=0} = 1/j\omega C$

$Z_{21} = \vec{V}_1/\vec{I}_2 \Big|_{\vec{I}_1=0} = \frac{1}{j\omega C}$

$Z_{22} = \vec{V}_2/\vec{I}_2 \Big|_{\vec{I}_1=0} = j\omega L + \frac{1}{j\omega C}$

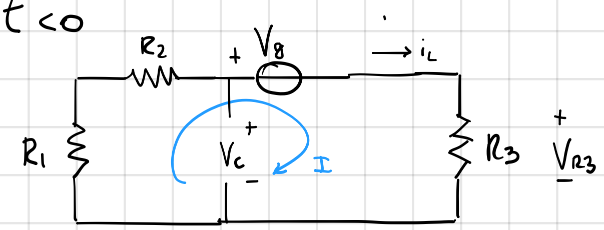
Compito Marzo



$+$ $V_{R3}(t)$? $-$

$R = 1 \quad C = 1 \quad L = 1 \quad V_g = \begin{cases} 1 & t<0 \\ 0 & t>0 \end{cases}$

$t<0$



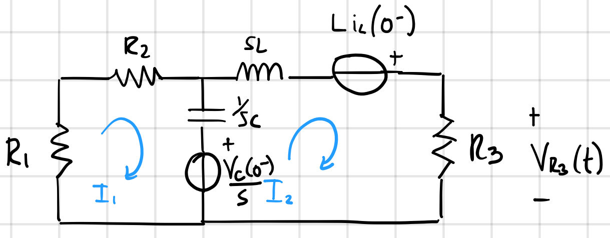
$(R_1+R_2+R_3)I = -V_g \longrightarrow I = -1/3$

$i_L = I = -1/3$

$V_c = -(-1/3 - 1/3) = 2/3$ (somma tensioni mezza maglia sinistra)

$V_{R3} = -1/3$

$t>0$



$$\begin{bmatrix} \left(R_1+R_2+\frac{1}{sC}\right) & -1/sC \\ -1/sC & \left(R_3+1/sC+sL\right) \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} -\frac{V_c(0^-)}{s} \\ \frac{V_c(0^-)}{s} + Li_L(0^-) \end{bmatrix}$$

base maglia

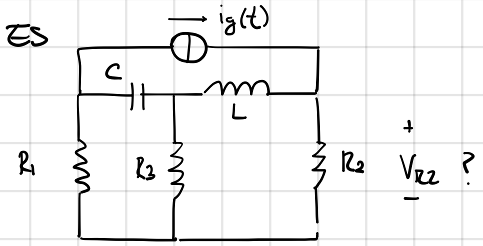
$$\begin{bmatrix} 2+1/s & -1/s \\ -1/s & 1+1/s+s \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} -2/3s \\ 2/3s - 1/3 \end{bmatrix}$$

$$I_2 = \frac{\det \begin{vmatrix} 2+1/s & -2/3s \\ -1/s & 2/3s - 1/3 \end{vmatrix}}{\det \begin{vmatrix} 2+1/s & -1/s \\ -1/s & 1+1/s+s \end{vmatrix}} = \frac{3-2s}{6s^2+9s+9}$$

$$V_{R3} = \frac{1}{3}\mathcal{L}^{-1}\left[\frac{3-2s}{2s^2+3s+3}\right] = \frac{1}{3}\left[\frac{K_1}{s-p_1} + \frac{K_2}{s-p_2}\right] \quad \text{con} \quad p_{12} = \frac{-3\pm\sqrt{15}}{4}$$

Trovo K_1 e K_2 (Vedi automatica)

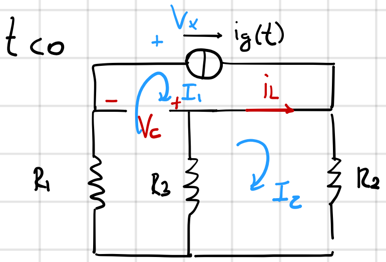
ES



$C = L = R = 1$

$$i_g(t) = \begin{cases} 1 & t<0 \\ 0 & t>0 \end{cases}$$

$t<0$



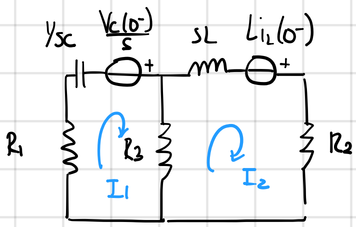
$$\begin{bmatrix} R_1+R_3 & -R_3 \\ -R_3 & R_2+R_3 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} V_x \\ 0 \end{bmatrix} \rightarrow \begin{cases} 2I_1 - I_2 = V_x \rightarrow 2 - I_2 = V_x \longrightarrow V_x = 3/2 \\ -I_1 + 2I_2 = 0 \longrightarrow I_2 = 1/2 \end{cases}$$

$V_c = V_x = 3/2$

$i_L = I_2 = 1/2$

$V_{R2} + I_2 R_3 - I_1 R_3 = 0 \longrightarrow V_{R2} = 1/2$

$t>0$

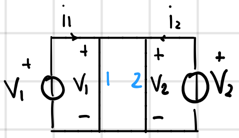


$$\begin{bmatrix} (R_1+R_3+1/sC) & -R_3 \\ -R_3 & (sL+R_2+R_3) \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} V_c/s \\ L\, i_L \end{bmatrix}$$

$\longrightarrow I_2 = \frac{2s+3}{4s^2+8s+4} = \frac{\frac{3+2s}{4}}{s^2+2s+1} \longrightarrow (s+1)^2$ $\qquad V_{R2}(t) = R_2\, \mathcal{L}^{-1}[I_2(s)] = \mathcal{L}^{-1}\left[\frac{\frac{3+2s}{4}}{(s+1)^2}\right]$

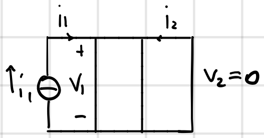
$I_2 = \frac{K_1}{(s+1)} + \frac{K_2}{(s+1)^2} \longrightarrow$ Trovo K_1 e K_2 $\Rightarrow K_1 = 1 \quad K_2 = 1 \longrightarrow I_2(s) = \frac{1/4}{s+1} + \frac{1}{(s+1)^2}$

$\longrightarrow I_2(t) = -1/4\, e^{-t} + t e^{-t} \Rightarrow V_{R2}(t) = I_2(t)$



$$\begin{cases} I_1 = y_{11}V_1 + y_{12}V_2 \\ I_2 = y_{21}V_1 + y_{22}V_2 \end{cases}$$

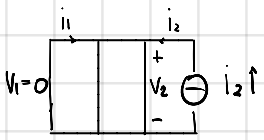
1) Annulliamo V_2:



$y_{11} = I_1/V_1 \,|_{V_2=0}$ ommettenza di ingresso 1

$y_{21} = I_2/V_1 \,|_{V_2=0}$ ommettenza di trasferimento

2) Annulliamo V_1:

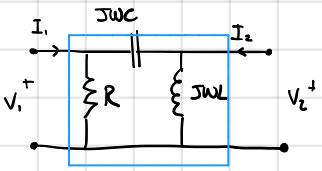


$y_{22} = I_2/V_2 \,|_{V_1=0}$ ommettenza ingresso 2

$y_{12} = I_1/V_2 \,|_{V_1=0}$ ommettenza trasferimento

$$\begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} = [y]$$ = motrice ommettenze di corto circuito

ES



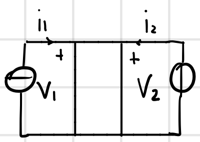
$y_{11} = I_1/V_1 \,|_{V_2=0} = \frac{1}{R} + j\omega C$

$y_{21} = I_2/V_1 \,|_{V_2=0} = -j\omega C$

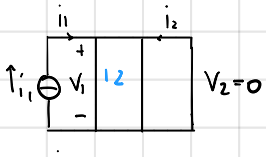
$y_{12} = I_1/V_2 \,|_{V_1=0} = -j\omega C$

$y_{22} = I_2/V_2 \,|_{V_1=0} = j\omega C + 1/j\omega L$

$[z]$, $[y]$ $\begin{cases} V_1 = h_{11} I_1 + h_{12} V_2 \\ I_2 = h_{21} I_1 + h_{22} V_2 \end{cases} \longrightarrow [H]$ = matrice ibrida



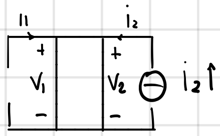
1) Annulliamo V_2



$h_{11} = V_1/I_1 \,|_{V_2=0}$ impedenza ingresso 1

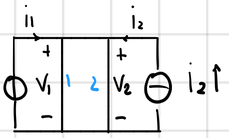
$h_{21} = I_2/I_1 \,|_{V_2=0}$ funzione trasferimento in corrente

2) Annulliamo I_1



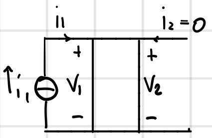
$h_{12} = V_1/V_2 \,|_{I_1=0}$ funzione trasferimento in tensione

$h_{22} = I_2/V_2 \,|_{I_1=0}$ ammettenza ingresso 2



$\begin{cases} I_1 = g_{11} V_1 + g_{12} I_2 \\ V_2 = g_{21} V_1 + g_{22} I_2 \end{cases} \longrightarrow [G]$

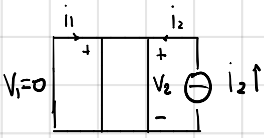
1) Annulliamo I_2



$g_{11} = I_1/V_1 \,|_{I_2=0}$ ammettenza ingresso 1

$g_{21} = V_2/V_1 \,|_{I_2=0}$ funzione trasferimento in tensione

2) Annulliamo V_1

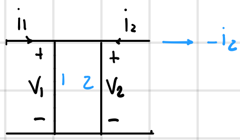


$g_{22} = V_2/I_2 \,|_{V_1=0}$ impedenza ingresso 2

$g_{12} = I_2/I_1 \,|_{V_1=0}$ funzione trasferimento in corrente

---

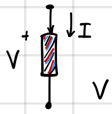
$\begin{cases} V_2 = A' V_1 + B' I_1 \\ -I_2 = C' V_1 + D' I_1 \end{cases}$



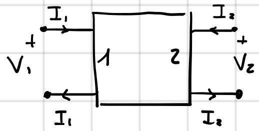
$\longrightarrow [T'] = \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix}$ = matrice di trasmissione diretta

$\begin{cases} V_1 = A V_2 - B I_2 \\ I_1 = C V_2 - D I_2 \end{cases} \longrightarrow [T] = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$ = matrice di trasmissione inversa

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = [T] \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

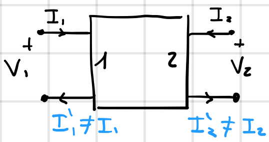


$V = Z \cdot I$ $\Longrightarrow$



$[V] = [Z][I]$

Se abbiamo:



non si tratta di rete 2 porte (dipende dal circuito interno) e non valgono le precedenti relazioni

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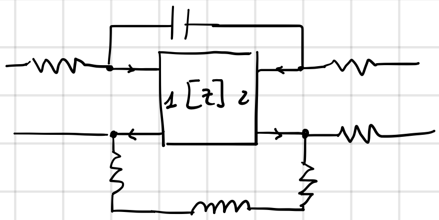
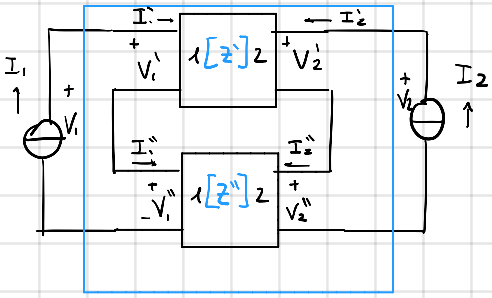
Prendiamo una rete 2 porte in un circuito:



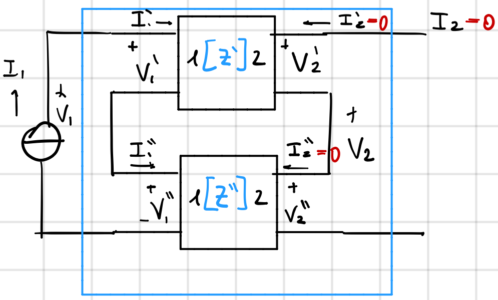
Non è detto che sia ancora una rete due porte e quindi non è detto sia possibile usare le relazioni

---



Le due componenti sono reti 2porte prese singolarmente
Così collegate lo sono ancora?

Annullo I_2:

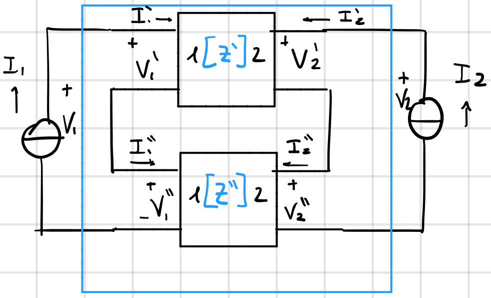


Se le condizioni in rosso sono soddisfatte si tratta ancora di reti 2porte

Analogamente si può fare annullando I_1.

Verificato che sono ancora reti 2 porte avremo:

$$\begin{bmatrix} V_1' \\ V_2' \end{bmatrix} = [z'] \begin{bmatrix} I_1' \\ I_2' \end{bmatrix} \quad e \quad \begin{bmatrix} V_1'' \\ V_2'' \end{bmatrix} = [z''] \begin{bmatrix} V_1'' \\ V_2'' \end{bmatrix}$$



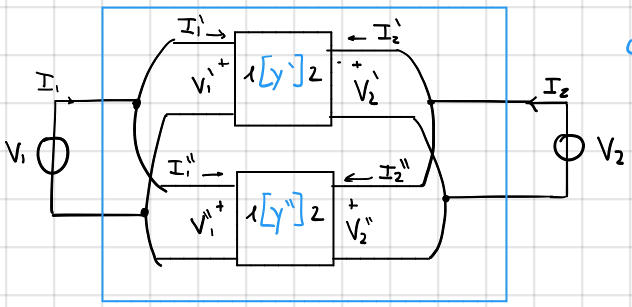
connessione serie-serie

$I_1' = I_1'' = I_1 \qquad I_2' = I_2'' = I_2$

$V_1' + V_1'' = V_1 \qquad V_2' + V_2'' = V_2$

$$\rightarrow \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} V_1' \\ V_2' \end{bmatrix} + \begin{bmatrix} V_1'' \\ V_2'' \end{bmatrix} = [z'] \begin{bmatrix} I_1' \\ I_2' \end{bmatrix} + [z''] \begin{bmatrix} I_1'' \\ I_2'' \end{bmatrix} \rightarrow \begin{bmatrix} I_1' \\ I_2' \end{bmatrix} = \begin{bmatrix} I_1'' \\ I_2'' \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \left\{ [z'] + [z''] \right\} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = [z] \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$ come i bipoli



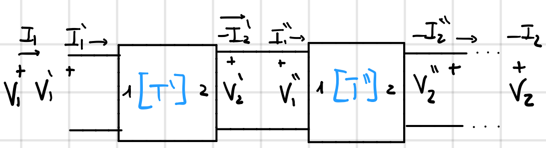
connessione parallelo-parallelo

Se lavorano ancora come reti 2 porte posso usare le matrici.

$$\begin{bmatrix} I_1' \\ I_2' \end{bmatrix} = [y'] \begin{bmatrix} V_1' \\ V_2' \end{bmatrix} \qquad \begin{bmatrix} I_1'' \\ I_2'' \end{bmatrix} = [y''] \begin{bmatrix} V_1'' \\ V_2'' \end{bmatrix}$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} I_1' \\ I_2' \end{bmatrix} + \begin{bmatrix} I_1'' \\ I_2'' \end{bmatrix} = [y'] \begin{bmatrix} V_1' \\ V_2' \end{bmatrix} + [y''] \begin{bmatrix} V_1'' \\ V_2'' \end{bmatrix} \rightarrow \begin{bmatrix} V_1' \\ V_2' \end{bmatrix} = \begin{bmatrix} V_1'' \\ V_2'' \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \rightarrow \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \left\{ [y'] + [y''] \right\} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = [y] \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

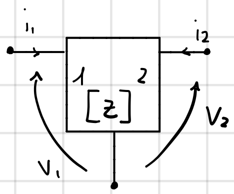
Reti in cascata:



Sicuramente si comportano come reti 2 porte

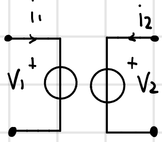
$$\begin{bmatrix} V_1' \\ I_1' \end{bmatrix} = [T'] \begin{bmatrix} V_1' \\ -I_2' \end{bmatrix} \qquad \begin{bmatrix} V_1'' \\ I_1'' \end{bmatrix} = [T''] \begin{bmatrix} V_2'' \\ -I_2'' \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} V_1' \\ I_1' \end{bmatrix} = [T'] \begin{bmatrix} V_2' \\ -I_2' \end{bmatrix} \rightarrow \begin{bmatrix} I_1'' \\ V_1'' \end{bmatrix} = \begin{bmatrix} -I_2' \\ V_2'' \end{bmatrix} \rightarrow \begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = [T'] \begin{bmatrix} V_1'' \\ I_1'' \end{bmatrix} = [T'][T''] \begin{bmatrix} V_2'' \\ -I_2'' \end{bmatrix} = [T] \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

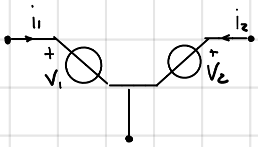


Rete sbilanciata $\longrightarrow$ si comporta sicuramente come una rete 2 porte

$$\begin{cases} V_1 = z_{11} i_1 + z_{12} i_2 \\ V_2 = z_{21} i_1 + z_{22} i_2 \end{cases} \longrightarrow$$



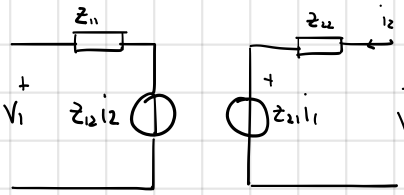
bilanciata



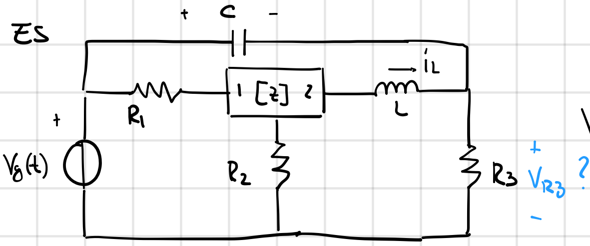
sbilanciata

Nel caso [Y] i generatori sarebbero di corrente.

[Z] :



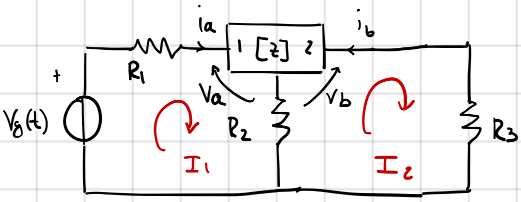
Es



$R = 1 \quad L = 1 \quad C = 1$

$$V_g(t) = \begin{cases} 1 & t<0 \\ 0 & t>0 \end{cases} \qquad [Z] = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$$

$t<0$



$$\begin{cases} V_a = 1\, i_a + 1\, i_b \\ V_b = 1\, i_a + 2\, i_b \end{cases} \qquad \begin{bmatrix} (R_1+R_2) i_1 - R_2 i_2 = V_g - V_a \\ -R_2 i_2 + (R_2+R_3) i_2 = V_b \end{bmatrix} \quad \text{con} \quad \begin{cases} i_a = i_1 \\ i_b = -i_2 \end{cases}$$

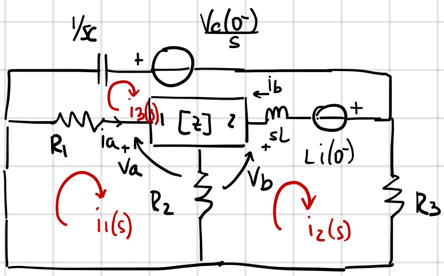
base maglie

$V_{R3} = R_3 i_2 \longrightarrow$ Trovo i_2

Calcolo le grandezze degli elementi con memoria:

- $i_L = i_2$
- $V_C = V_g - V_{R3}$

$t>0$

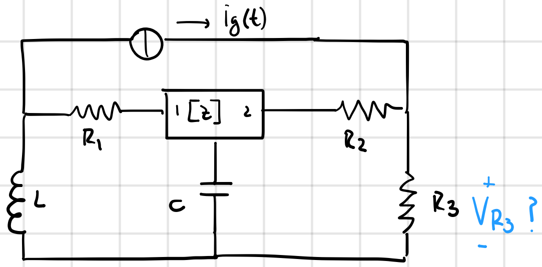


$$(R_1+R_2)\, i_1(s) - R_2 i_2(s) - R_1 i_3(s) = -V_a$$
$$-R_2 i_1(s) + (R_2+R_3+sL)\, i_2(s) - sL\, i_3(s) = V_b + L\, i_L(0^-)$$
$$-R_1 i_1(s) - sL\, i_2(s) + (R_1 + sL + 1/sC)\, i_3(s) = -\frac{V_C(0^-)}{s} - L\, i_L(0^-) - V_b + V_a$$

base maglie

$$\begin{cases} V_a = i_a + i_b \\ V_b = i_a + 2 i_b \end{cases} \quad \text{con} \quad \begin{cases} i_a = i_1 - i_3 \\ i_b = i_3 - i_2 \end{cases}$$

ES.

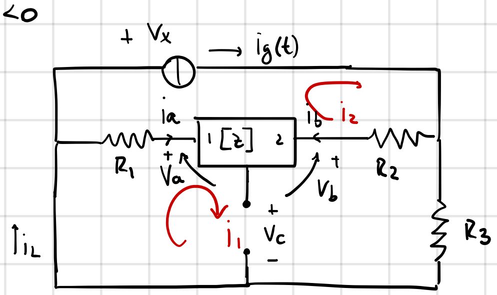


$R = C = L = 1$

$$i_g(t) = \begin{cases} 1 & t<0 \\ 0 & t>0 \end{cases}$$

$$[Z] = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$$

$t<0$



$$(R_1+R_2+R_3)\,i_1 - (R_1+R_2)\,i_2 = -V_a + V_b$$
$$-(R_1+R_2)\,i_1 + (R_1+R_2)\,i_2 = -V_b + V_a - V_x$$

base maglie

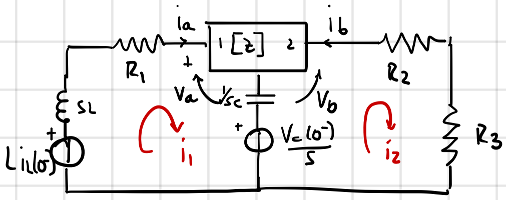
$$\begin{cases} V_a = i_a + i_b \\ V_b = -i_a + i_b \end{cases}$$

$i_2 = i_1$

$V_{R_3} = i_1 R_3$

$V_c = -R_1 i_1 - V_a$

$t>0$

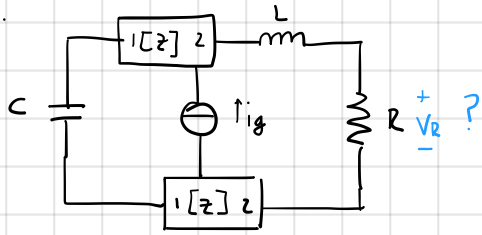


$$\left(R_1 + sL + \tfrac{1}{sC}\right) i_1(s) - \tfrac{1}{sC}\, i_2(s) = L\,i_L(0^-) - V_a - \frac{V_c(0^-)}{s}$$
$$-\tfrac{1}{sC}\, i_1(s) + \left(R_2 + R_3 + \tfrac{1}{sC}\right) i_2(s) = \frac{V_c(0^-)}{s} + V_b$$

$$\begin{cases} V_a = i_a + i_b \\ V_b = -i_a + i_b \end{cases}$$

$$\begin{cases} i_a = i_1 \\ i_b = -i_2 \end{cases}$$

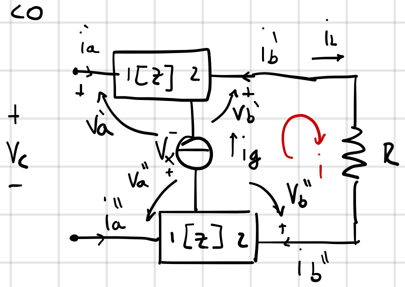
ES.



$$[z] = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} \qquad R = C = L = 1$$

$$i_g(t) \begin{cases} 1 & t<0 \\ 0 & t>0 \end{cases}$$

$t<0$



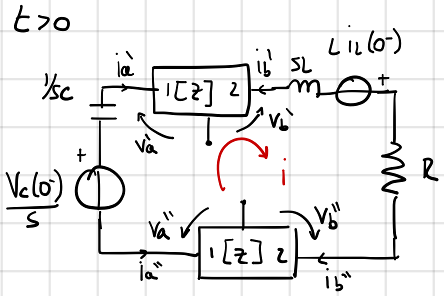
$$R\,i = -V_b'' - V_x + V_b'$$

$$\begin{cases} V_a' = 2 i_a' + i_b' \\ V_b' = i_a' + i_b' \end{cases} \qquad \begin{cases} V_a'' = 2 i_a'' + i_b'' \\ V_b'' = i_a'' + i_b'' \end{cases} \qquad (i = i_g)$$

$$\begin{cases} i_a' = 0 = i_a'' \\ i_b' = -i \\ i_b'' = i \end{cases} \longrightarrow \quad V_b' = -i \qquad V_b'' = i$$

$$\begin{cases} i_L = i \\ V_C = V_a' - V_x - V_a'' \end{cases}$$

$t>0$



$$\left(R + sL + \frac{1}{sC}\right) i = -V_a' + V_b' + L\,i_L(0^-) - V_b'' + V_a'' + \frac{V_c(0^-)}{s}$$

$$\begin{cases} i_a' = i = i_b'' \\ i_b' = -i = i_a'' \end{cases}$$

1) Thevenin e Norton
2) Grafo circuito e proprietà
3) Massima potenza attiva
4) Rifasamento p. 324
5) Funzione di rete e proprietà
6) Potenza attiva, reattiva e complessa
7) Linearità, permanenza, causalità, passività ecc.