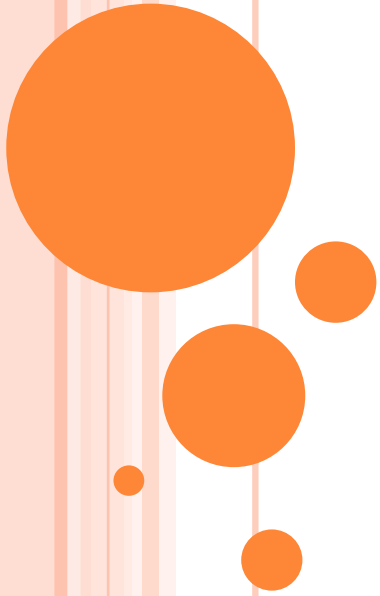


Secret Key: stream ciphers & block ciphers

Computer & Network Security

Secret Key cryptography

Alice and Bob share

- A crypto protocol E
- A secret key K
- They communicate using E with key K
- Adversary knows E, knows some exchanged messages but ignores K

Two approaches:

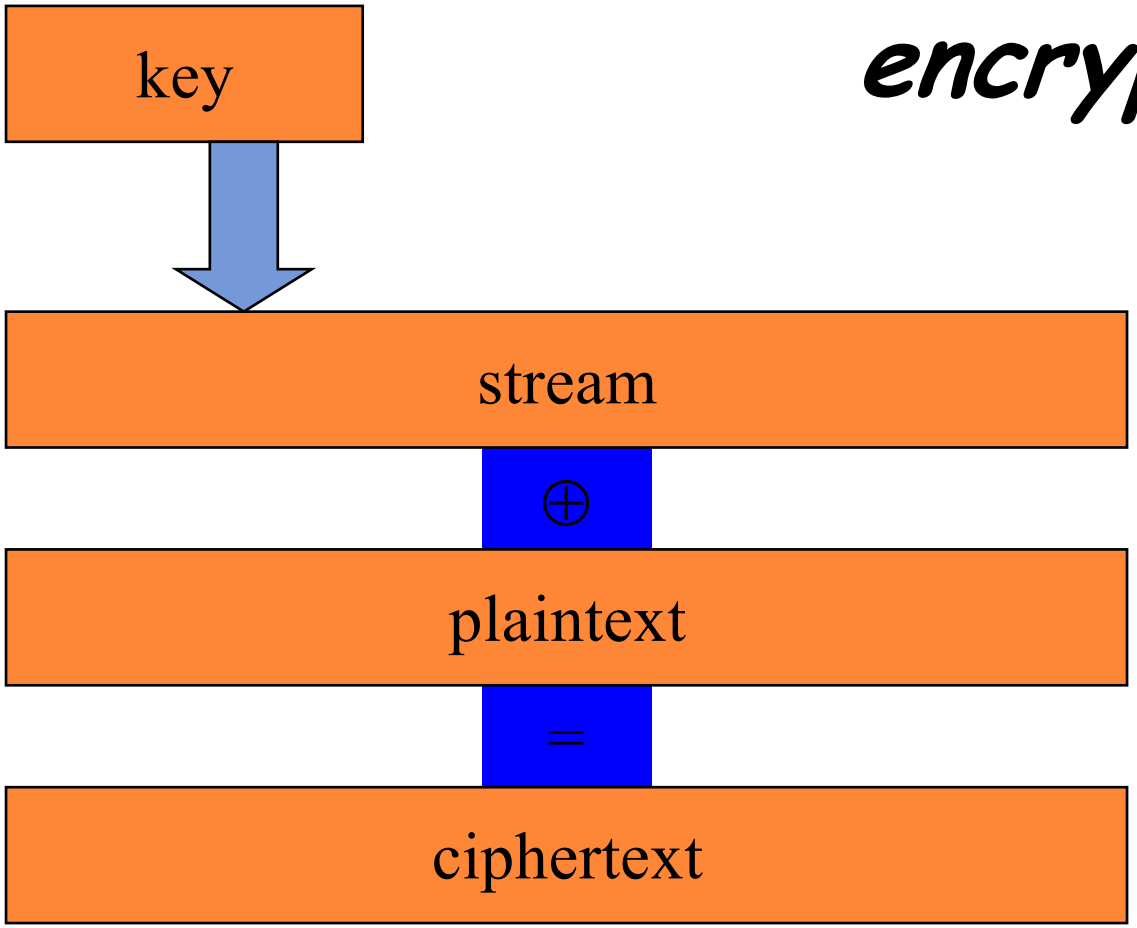
- Stream Cipher
- Block ciphers

Stream Ciphers

Idea: try to simulate *one-time pad*

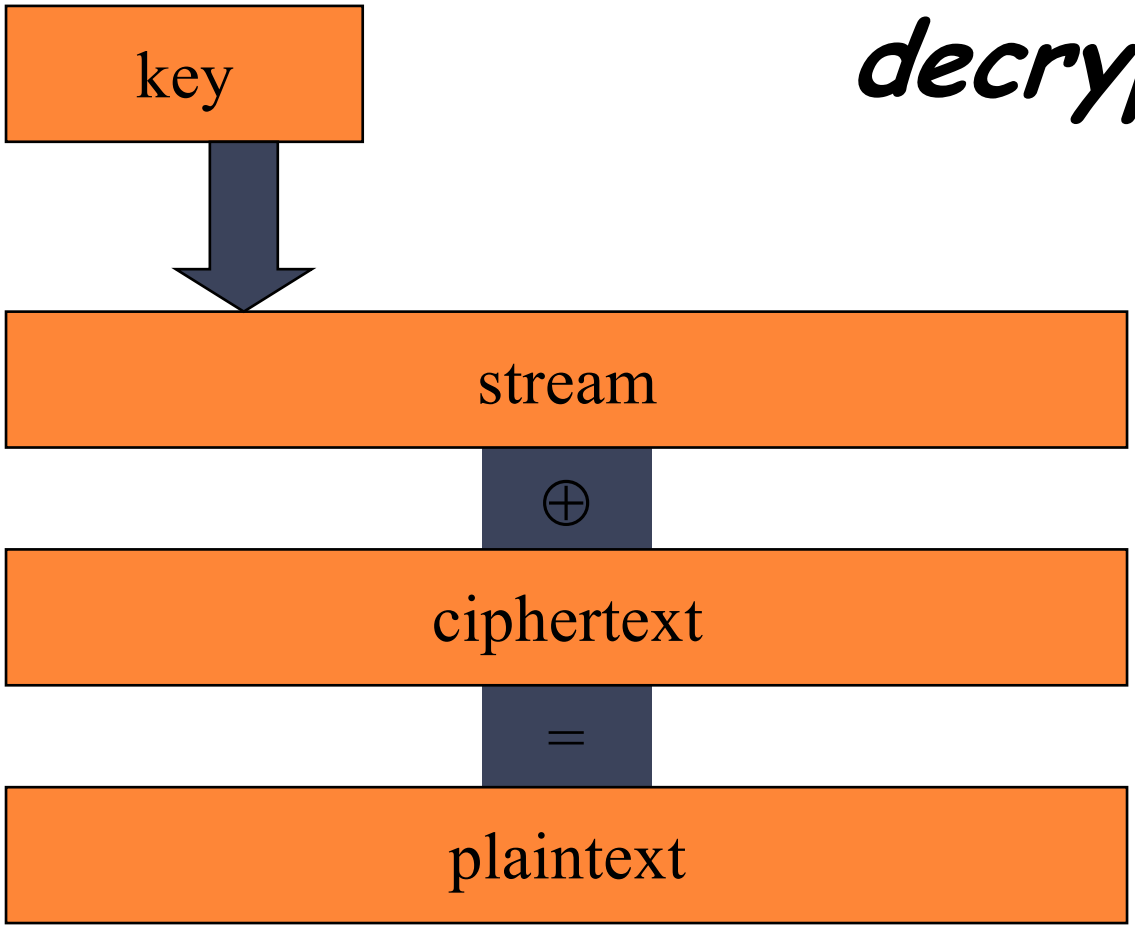
- define a secret key ("seed")
- using the seed generate a byte stream (*Keystream*): *i*-th byte is function of
  - only key (*synchronous* stream cipher), or
  - both key and first *i*-1 bytes of ciphertext (*asynchronous* stream cipher)
- obtain ciphertext by bitwise XORing plaintext and keystream

Synchronous Stream Cipher



encryption

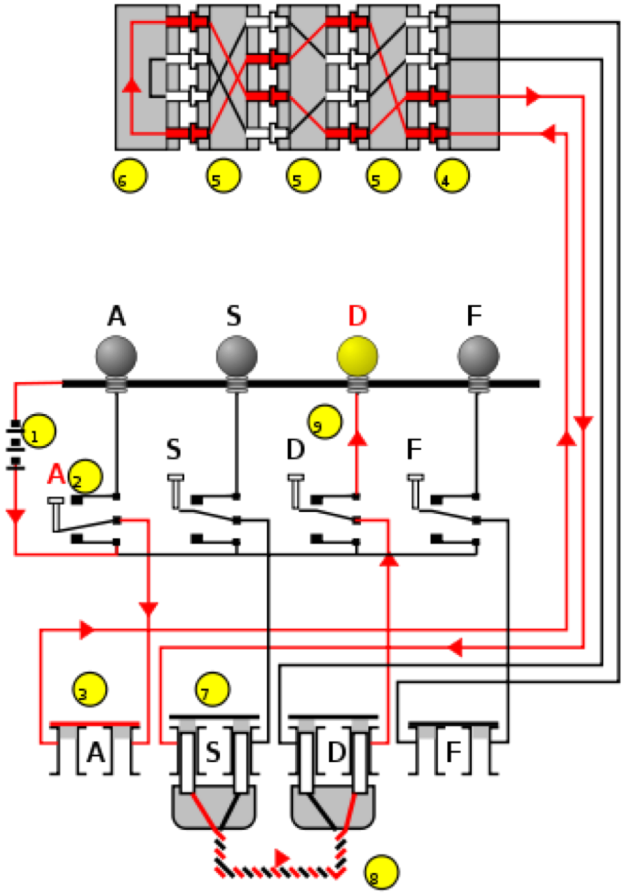
Synchronous Stream Cipher



decryption

Streams Ciphers in practice

- Many codes before 1940
- Enigma - II world war (Germany)
- A5 – GSM (encryption cell phone-base station)
- WEP - used in Ethernet 802.11 (wireless)
- RC-4 (Ron's Code)



Enigma wiring diagram showing current flow. The A key is encoded to the D lamp. D yields A, but A never yields A; this property was due to a patented feature unique to the Enigmas, and could be exploited by cryptanalysts in some situations.

A5/1

- Stream cipher (1987) used to provide over-the-air communication privacy in the GSM cellular telephone standard
- There was a terrific row between the NATO signal intelligence agencies in the mid 1980s over whether GSM encryption should be strong or not
- Used in Europe and in the United States. A5/2 was a deliberate weakening of the algorithm for certain export regions
- Initially kept secret, but the general design was leaked in 1994, and the algorithms were entirely reverse engineered in 1999 by Marc Briceno
  - A number of serious weaknesses in the cipher have been identified

RC-4

Ronald Linn Rivest

- RC: Ron's Code
  - (Ron = Ronald Rivest, MIT, born in 1947 in NY state)
- Considered safe: 1987 - 1994 kept secret, after '94 extensively studied
- Good for exporting (complying with US restrictions)
- Easy to program, fast
- Very popular: Lotus Notes, SSL, Wep etc.
- RC4's weak key schedule can give rise to a variety of serious problems

RC4: PROPERTIES

- variable key length (byte)
- synchronous
- starting from the key, it generates an apparently random permutation
- eventually the sequence will repeat
- however, long period > 10^{100} (in this way it simulates one-time-pad)
- very fast: 1 byte of output requires 8-16 instructions

RC-4 INITIALIZATION

Goal: generate a (pseudo)random permutation of the first 256 natural numbers

1. j=0
2. $S_0=0, S_1=1, \ldots, S_{255}=255$
3. Assume a key of 256 bytes $k_0,\ldots,k_{255}$ (if the key is shorter, repeat)
4. for i=0 to 255 do
    1. $j = (j + S_i + k_i) \bmod 256$
    2. exchange S_i and S_j

In this way we obtain a permutation of 0, 1, …,255, the resulting permutation is a function of the key

RC-4 KEY-STREAM GENERATION

Input: permutation S of 0, 1, …, 255

1. i = 0, j = 0
2. while (true) // we'll not cycle forever, isn't it?
3. i = (i + 1) mod 256
4. j = (j + S_i) mod 256
5. exchange S_i and S_j
6. t = (S_i + S_j) mod 256
7. k = S_t // compute XOR

at every iteration compute the XOR between k and next byte of plaintext (or ciphertext)

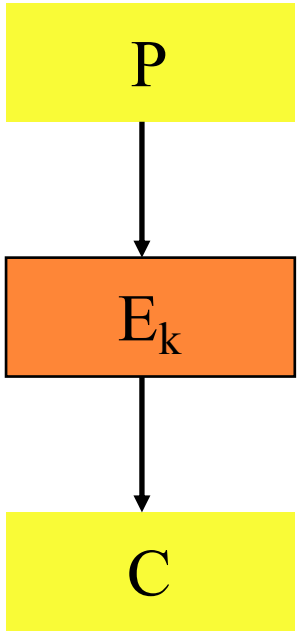
Block Ciphers

Given

- a block P of text of h bits (h fixed)
- a key k of fixed # of bits

a cryptographic protocol E_k produces a block C of h bits, function of P and k

Note: lengths of both block and key (# of bits) are fixed (not necessarily equal)



Real World Block Ciphers

- DES, 3-DES - (1976; 64 bit block, 56 bit key)
- RC-2 (1987)
  - designed for exporting cryptography within IBM Lotus Notes
  - 64 bit block, variable key size, vulnerable to an attack using 2^{34} chosen plaintexts
- IDEA (1991)
  - 64 bit block, 128 bit key
  - Strong, only weakened variants have been broken
- Blowfish (1993)
  - 64-bit block size and a variable key length from 32 up to 448 bits
  - Still strong

Real World Block Ciphers

- RC5 (1994)
  - variable block size - 32, 64 or 128 bits - key size (0 to 2040 bits) and number of rounds (0 to 255). The original suggested choice of parameters were a block size of 64 bits, a 128-bit key and 12 rounds.
  - Distributed.net has brute-forced RC5 messages encrypted with 56-bit and 64-bit keys, and is working on cracking a 72-bit key; as of February 2014, 3.112% of the keyspace has been searched (it was 1.488% at March 2011). At the current rate, it will take approximately 287 years to test every possible remaining key
    - distributed.net (or Distributed Computing Technologies, Inc. or DCTI) is a worldwide distributed computing effort that is attempting to solve large scale problems using otherwise idle CPU or GPU time. It is a non-profit organization
- AES (Rijndael, 2001)
  - 128 bit block, 128-256 bit key
  - Very strong

Symmetric Block Ciphers

Standard

out in

DES AES

Historic Note

DES (data encryption standard) is a symmetric block cipher using 64 bit blocks and a 56 bit key.

Developed at IBM, approved by the US government (1976) as a standard. Size of key (56 bits) was apparently small enough to allow the NSA (US national security agency) to break it exhaustively even back in 70's.

In the 90's it became clear that DES is too weak for contemporary hardware & algorithmics (Matsui "linear attack", requires only 2^{43} known plaintext/ciphertext pairs; in 1999 Deep Crack and distributed.net break a DES key in 22 hours and 15 minutes)

LINEAR CRYPTANALYSIS

Wikipedia: *based on finding affine approximations to the action of a cipher.*

"There are two parts to linear cryptanalysis.

- The first is to construct linear equations relating plaintext, ciphertext and key bits that have a high bias; that is, whose probabilities of holding (over the space of all possible values of their variables) are as close as possible to 0 or 1.
- The second is to use these linear equations in conjunction with known plaintext-ciphertext pairs to derive key bits."

Historic Note (cont.)

The US government NIST (National Inst. of standards and technology) announced a call for an advanced encryption standard in 1997.

This was an international open competition. Overall, 15 proposals were made and evaluated, and 6 were finalists. Out of those, a proposal named Rijndael, by Daemen and Rijmen (two Belgians), was chosen in February 2001.

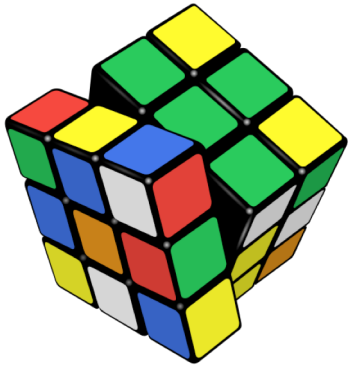
AES finalist	positive	negative
Rijndael	86	10
Serpent	59	7
Twofish	31	21
RC6	23	37
MARS	13	84

AES - Advanced Encryption Standard

- Symmetric block cipher (block size: 128 bits)
- Key lengths: 128, 192, or 256 bits
- Approved US standard (2001)
- Finite fields algebra

Group and Fields

AES
Advanced Encryption Standard



+, 0, and -a
are only notations!

REVIEW - GROUPS

Def (group): A set G with a binary operation $+$ (addition) is called a *commutative* (or *Abelian*) *group* if

1. $\forall\ a, b \in G,\ a + b \in G$ [G is closed under $+$]
2. $\forall\ a, b, c \in G,\ (a+b)+c = a+(b+c)$ [associative]
3. $\forall\ a, b \in G,\ a + b = b + a$ [commutative]
4. $\exists\ 0 \in G,\ \forall\ a \in G,\ a + 0 = a$ [identity element]
5. $\forall\ a \in G,\ \exists\ -a \in G,\ a + (-a) = 0$ [inverse element]

SUB-GROUPS

- Def.: Let $(G, +)$ be a group, $(H,+)$ is a sub-group of $(G,+)$ if it is a group, and $H \subseteq G$.

- Claim: Let $(G, +)$ be a finite group, and $H \subseteq G$. *If* H is closed under $+$, then $(H,+)$ is a sub-group of $(G,+)$.
  - in other words: $(H,+)$ is not a group only if H is not closed under $+$

- **Lagrange theorem**: if G is finite and $(H,+)$ is a sub-group of $(G,+)$ then $|H|$ divides $|G|$

CONGRUENCE

- two naturals a and b are said to be congruent modulo n (n is a positive integer)

$$a \equiv b \pmod{n}$$

  if $|a - b|$ is multiple of n, or, equivalently, the integer divisions of a and n and of b and n yield the same remainder

- the congruence relation is reflexive, symmetric and transitive, hence it is an equivalence relation
- the quotient set Z_n is the set of n classes of equivalence, congruent to 0, 1, …, n-1
  - $-1 \equiv n - 1 \pmod{n}$, $-2 \equiv n - 2 \pmod{n}$, etc.

PROPERTIES OF CONGRUENCES

- invariance over addition

$$a \equiv b \pmod{n} \Leftrightarrow (a+c) \equiv (b+c) \pmod{n} \qquad \forall a, b, c \in \mathbb{N}, \forall n \in \mathbb{N}_0$$

- invariance over multiplication

$$a \equiv b \pmod{n} \Rightarrow a \cdot c \equiv b \cdot c \pmod{n} \qquad \forall a, b, c \in \mathbb{N}, \forall n \in \mathbb{N}_0$$

- invariance over exponentiation

$$a \equiv b \pmod{n} \Rightarrow a^k \equiv b^k \pmod{n} \qquad \forall a, b \in \mathbb{N}, \forall k \in \mathbb{N}, \forall n \in \mathbb{N}_0$$

Order of Elements

- Let a^n denote $a + \ldots + a$ (n times)
  - if operator thought as multiplicative: a^n denotes $aaa\ldots a$ (n times)
- We say that a is of *order* n if $a^n = 0$, and for any $m < n$, $a^m \neq 0$
  - all elements of finite groups have finite order
  - $a^n = 1$ for multiplicative operator
- Z_m = set of natural numbers mod m
  - elements of Z_m: classes of equivalence of congruent integers (mod m)
- Z_m^* = set of natural numbers mod m that are relatively prime (co-prime) to m (multiplicative group of Z_m)
- $\phi(m)$ = Euler's *totient* function = $|Z_m^*|$
- **Euler theorem**: For all a in Z_m^*, $a^{\phi(m)} = 1 \bmod m$. Hence $a^{k\phi(m)+1} = a \bmod m$, $k \geq 0$
  - Can be extended to Z_m, where $m = pq$, with p and q prime: $a^{k\phi(m)+1} = a \bmod m$, $k \geq 0$

CHALLENGE (JUNE 2017)

compute

$$2^{200} \bmod 127$$

without a scientific calculator….

Cyclic Groups

- Claim: let G be a group and a be an element of order n. The set

  $\langle a \rangle = \{1, a, \ldots, a^{n-1}\}$ is a sub-group of G.
- a is called the *generator* of $\langle a \rangle$.
- If G is generated by a, then G is called *cyclic*, and a is called a primitive element of G.
- Theorem: for any prime p, the multiplicative group of Z_p (namely, Z_p^*) is cyclic

EXAMPLES

- Z and addition (0 identity; -a inverse of a) is a group
- Z_n and addition mod n is a group (0 identity; -a inverse of a)
- Z and multiplication is NOT a group (inverse exists only for 1 and -1)
- Set of non null rational numbers and multiplication is a group
- Z_n [a mod n] and multiplication IS NOT a group (0 has not inverse)
- Z_n – {0} [a mod n] and multiplication IS NOT ALWAYS a group
  - n = 6 then {1,2,3,4,5} is not close (2*3= 0 mod 6)
  - n prime then it is a group
- Z_n^* [a mod n] and multiplication if MCD(a,n) = 1 is a group (1 is identity)
- And if as + nt = 1 mod n then s is inverse of a
  - n = 15 then {1,2,4,7,8,11,13,14}
  - n = 5 {1,2,3,4} (in fact all numbers are prime with 5)

REVIEW - RINGS

+, · ,0, 1 and -a are only notations!

Def (ring): A set F with two binary operations + (addition) and · (multiplication) is called a commutative *ring* with identity if

1 $\forall\ a,b \in F,\ a+b \in F$

2 $\forall\ a,b,c \in F,\ (a+b)+c=a+(b+c)$

3 $\forall\ a,b \in F,\ a+b=b+a$

4 $\exists\ 0 \in F,\ \forall\ a \in F,\ a+0=a$

5 $\forall\ a \in F,\ \exists\ -a \in F,\ a+(-a)=0$

6 $\forall\ a,b \in F,\ a \cdot b \in F$

7 $\forall\ a,b,c \in F,\ (a \cdot b) \cdot c=a \cdot (b \cdot c)$

8 $\forall\ a,b \in F,\ a \cdot b=b \cdot a$

9 $\exists\ 1 \in F,\ \forall\ a \in F,\ a \cdot 1=a$

10 $\forall\ a,b,c \in F, a \cdot (b+c)=a \cdot b+a \cdot c$

REVIEW - FIELDS

+, ·, 0, 1, -a and a^{-1} are only notations!

Def (field): A set F with two binary operations + (addition) and · (multiplication) is called a *field* if

1 $\forall\ a,b \in F,\ a+b \in F$

2 $\forall\ a,b,c \in F,\ (a+b)+c=a+(b+c)$

3 $\forall\ a,b \in F,\ a+b=b+a$

4 $\exists\ 0 \in F,\ \forall\ a \in F,\ a+0=a$

5 $\forall\ a \in F,\ \exists\ -a \in F,\ a+(-a)=0$

6 $\forall\ a,b \in F,\ a \cdot b \in F$

7 $\forall\ a,b,c \in F,\ (a \cdot b) \cdot c=a \cdot (b \cdot c)$

8 $\forall\ a,b \in F,\ a \cdot b=b \cdot a$

9 $\exists\ 1 \in F,\ \forall\ a \in F,\ a \cdot 1=a$

10 $\forall\ a,b,c \in F, a \cdot (b+c)=a \cdot b+a \cdot c$

11 $\forall\ a \neq 0 \in F,\ \exists\ a^{-1} \in F,\ a \cdot a^{-1}=1$

REVIEW - FIELDS

A field is a commutative ring with identity where each non-zero element has a multiplicative inverse

$$\forall\ a \neq 0 \in F,\ \exists\ a^{-1} \in F,\ a \cdot a^{-1} = 1$$

Equivalently, $(F,+)$ is a commutative (additive) group, and $(F \setminus \{0\}, \cdot)$ is a commutative (multiplicative) group. (with $\cdot$ distributive over $+$)

EXAMPLES

Z_n with addition and multiplication is a ring but not always a field

- n = 15 NO ({1,2,3,4,…,14} is not a group with respect to multiplication)
- n = 5 YES ({1,2,3,4} is a group w.r.t. multiplication)

Polynomials over Fields

Let $f(x) = a_n \cdot x^n + a_{n-1} \cdot x^{n-1} + a_{n-2} \cdot x^{n-2} + \ldots + a_1 \cdot x + a_0$ be a polynomial of degree n in one variable x over a field F (namely $a_n, a_{n-1}, \ldots, a_1, a_0 \in F$).

Theorem: The equation $f(x) = 0$ has at most n solutions in F

Remark: The theorem does not hold over rings with identity.

For example, in Z_{24} the equation $6 \cdot x = 0$ has six solutions (0,4,8,12,16,20)

[2 has not a mult. inverse]

Polynomial Remainders

Let $f(x) = a_n \cdot x^n + a_{n-1} \cdot x^{n-1} + a_{n-2} \cdot x^{n-2} + \ldots + a_1 \cdot x + a_0$

$g(x) = b_m \cdot x^m + b_{m-1} \cdot x^{m-1} + b_{m-2} \cdot x^{m-2} + \ldots + b_1 \cdot x + b_0$

be two polynomials over F such that $m < n$ (or $m = n$).

Theorem: There is a unique polynomial $r(x)$ of degree $< m$ over F such that

$$f(x) = h(x) \cdot g(x) + r(x).$$

Remark: r(x) is called the remainder of $f(x)$ modulo $g(x)$.

Maple 8 Worksheet File

```
> rem(4*x^5 + 3*x^2 + 1 , x^3+2 , x);
```

$$1 - 5x^2$$

```
> gcd(4*x^5 + 3*x^2 + 1 , x^3+2 );
```

$$1$$

Finite Fields

Def (finite field): A field $(F,+,\cdot)$ is called a finite field if the set F is finite.

Example: Z_p denotes $\{0,1,\ldots,p-1\}$. We define $+$ and $\cdot$ as addition and multiplication modulo p, respectively.

One can prove that $(Z_p,+,\cdot)$ is a field iff p is prime.

Q.: Are there any finite fields except $(Z_p,+,\cdot)$?

Galois Fields GF(p^k)

Theorem: For every prime power p^k ($k = 1, 2, \ldots$) there is a unique finite field containing p^k elements. These fields are denoted by GF(p^k).

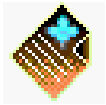
There are no finite fields with other cardinalities.

Évariste Galois (1811-1832)

Polynomials over Finite Fields

Polynomial equations and factorizations in finite fields can be different than over the rationals.

Examples from an XMAPLE session:



Maple 8
Worksheet File

```
factor(x^6-1);  # over the rationals
```

$$(x-1)(x+1)(x^2+x+1)(x^2-x+1)$$

```
Factor(x^6-1) mod 7; # over Z7
```

$$(x+1)(x+3)(x+2)(4+x)(x+5)(x+6)$$

```
factor(x^4+x^2+x+1); # over the rationals
```

$$x^4+x^2+x+1$$

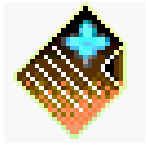
```
Factor(x^4+x^2+x+1) mod 2; # over Z2
```

$$(x+1)(x^3+x^2+1)$$

Irreducible Polynomials

A polynomial is irreducible in GF(p) if it does not factor over GF(p). Otherwise it is reducible.

Examples:



Maple 8 Worksheet File

```
Factor(x^5+x^4+x^3+x+1) mod 5;
```

$$(x+2)(x^3+3x+2)(x+4)$$

```
Factor(x^5+x^4+x^3+x+1) mod 2;
```

$$x^5+x^4+x^3+x+1$$

The same polynomial is reducible in Z_5 but irreducible in Z_2.

Implementing GF(p^k) arithmetic

Theorem: Let f(x) be an irreducible polynomial of degree k over Z_p.

The finite field $GF(p^k)$ can be realized as the set of degree k-1 polynomials over Z_p, with addition and multiplication done modulo f(x).

Example: Implementing $GF(2^5)$

By the theorem the finite field $GF(2^5)$ can be realized as the set of degree 4 polynomials over Z_2, with addition and multiplication done modulo the irreducible polynomial $f(x)=x^5+x^4+x^3+x+1$.

The coefficients of polynomials over Z_2 are 0 or 1.
So a degree k polynomial can be written down by k+1 bits.
For example, with k=4:

$x^3+x+1 \longleftrightarrow (0,1,0,1,1)$

$x^4+ x^3+x+1 \longleftrightarrow (1,1,0,1,1)$

Implementing $GF(2^5)$

Addition: bit-wise XOR (since 1+1=0)

$$
\begin{array}{lll}
 & x^3+x+1 & (0,1,0,1,1) \\
+ & & \\
 & x^4+x^3+x & (1,1,0,1,0) \\
\hline
 & x^4 \quad +1 & (1,0,0,0,1)
\end{array}
$$

Implementing GF(2^5)

Multiplication: Polynomial multiplication, and then remainder modulo the defining polynomial f(x):

```
> g(x):=(x^4+x^3+x+1)*(x^3+x+1);
```

$$g(x) := (x^4 + x^3 + x + 1)(x^3 + x + 1)$$

```
> f(x):=x^5+x^4+x^3+x+1;
```

$$f(x) := x^5 + x^4 + x^3 + x + 1$$

```
> rem(g(x),f(x),x);
```

$$1 + 3x^4 + x^3 + 2x$$

```
> % mod 2;
```

$$1 + x^4 + x^3$$

$$(1,1,0,1,1) * (0,1,0,1,1)$$

$$= (1,1,0,0,1)$$

For small size finite field, a lookup table is the most efficient method for implementing multiplication.

Implementing $GF(2^5)$ in XMAPLE

Irreducible polynomial

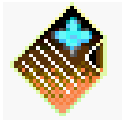
```
> G32:=GF(2,5,x^5+x^4+x^3+x+1):
> a := G32[ConvertIn](x);
```

$$a := x$$

```
> b := G32[`^`](a,8):                    # colon at end of
  statement supresses printing
  c := G32[`^`](a,9):
  G32[ConvertOut](b);      # canonical
  representation, higher momonials to the left
  G32[ConvertOut](c);
```

$$x^3 + x^2 + x + 1$$

$$x^4 + x^3 + x^2 + x$$



Maple 8
Worksheet File

More GF(2^5) Operations in XMAPLE

```
> d := G32[`+`](b,c):
  G32[ConvertOut](d);
```

$$x^4 + 1$$

```
> G32[isPrimitiveElement](d);
```

$$true$$

```
> e:=G32[`^`](a,-1):
  G32[ConvertOut](e);
```

$$x^4 + x^3 + x^2 + 1$$

```
> G32[`*`](a,e);
```

$$1$$

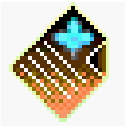
```
> for i from 1 to 32 do
    f:= G32[`^`](a,i):
    print(f, G32[isPrimitiveElement](f))
    end do:
```

$$x, true$$
$$x^2, true$$
$$x^3, true$$
$$x^4, true$$
$$1 + x + x^3 + x^4, true$$
$$1 + x^2 + x^3, true$$
$$x + x^3 + x^4, true$$

Addition: b+c

test primitive element

e <--inverse of a
Multiplication: a*e



Maple 8
Worksheet File

Loop for finding primitive elements

AES - Advanced Encryption Standard

- Symmetric block cipher
- Key lengths: 128, 192, or 256 bits

Rationale

- Resistance to all known attacks
- Speed and code compactness
  - good for devices with limited computing power, e.g. smart cards
- Simplicity

AES Specifications

- Input & output block length: 128 bits.
- State: 128 bits, arranged in a 4-by-4 matrix of bytes.

$A_{0,0}$	$A_{0,1}$	$A_{0,2}$	$A_{0,3}$
$A_{1,0}$	$A_{1,1}$	$A_{1,2}$	$A_{1,3}$
$A_{2,0}$	$A_{2,1}$	$A_{2,2}$	$A_{2,3}$
$A_{3,0}$	$A_{3,1}$	$A_{3,2}$	$A_{3,3}$

Each byte is viewed as an element in $GF(2^8)$

Input/Output: $A_{0,0}$, $A_{1,0}$, $A_{2,0}$, $A_{3,0}$, $A_{0,1}$,...

AES Specifications

- Key length: 128, 196, 256 bits.

Cipher Key Layout: n = 128, 196, 256 bits, arranged in a 4-by-n/32 matrix of bytes.

$K_{0,0}$	$K_{0,1}$	$K_{0,2}$	$K_{0,3}$	$K_{0,4}$	$K_{0,5}$
$K_{1,0}$	$K_{1,1}$	$K_{1,2}$	$K_{1,3}$	$K_{1,4}$	$K_{1,5}$
$K_{2,0}$	$K_{2,1}$	$K_{2,2}$	$K_{2,3}$	$K_{2,4}$	$K_{2,5}$
$K_{3,0}$	$K_{3,1}$	$K_{3,2}$	$K_{3,3}$	$K_{3,4}$	$K_{3,5}$

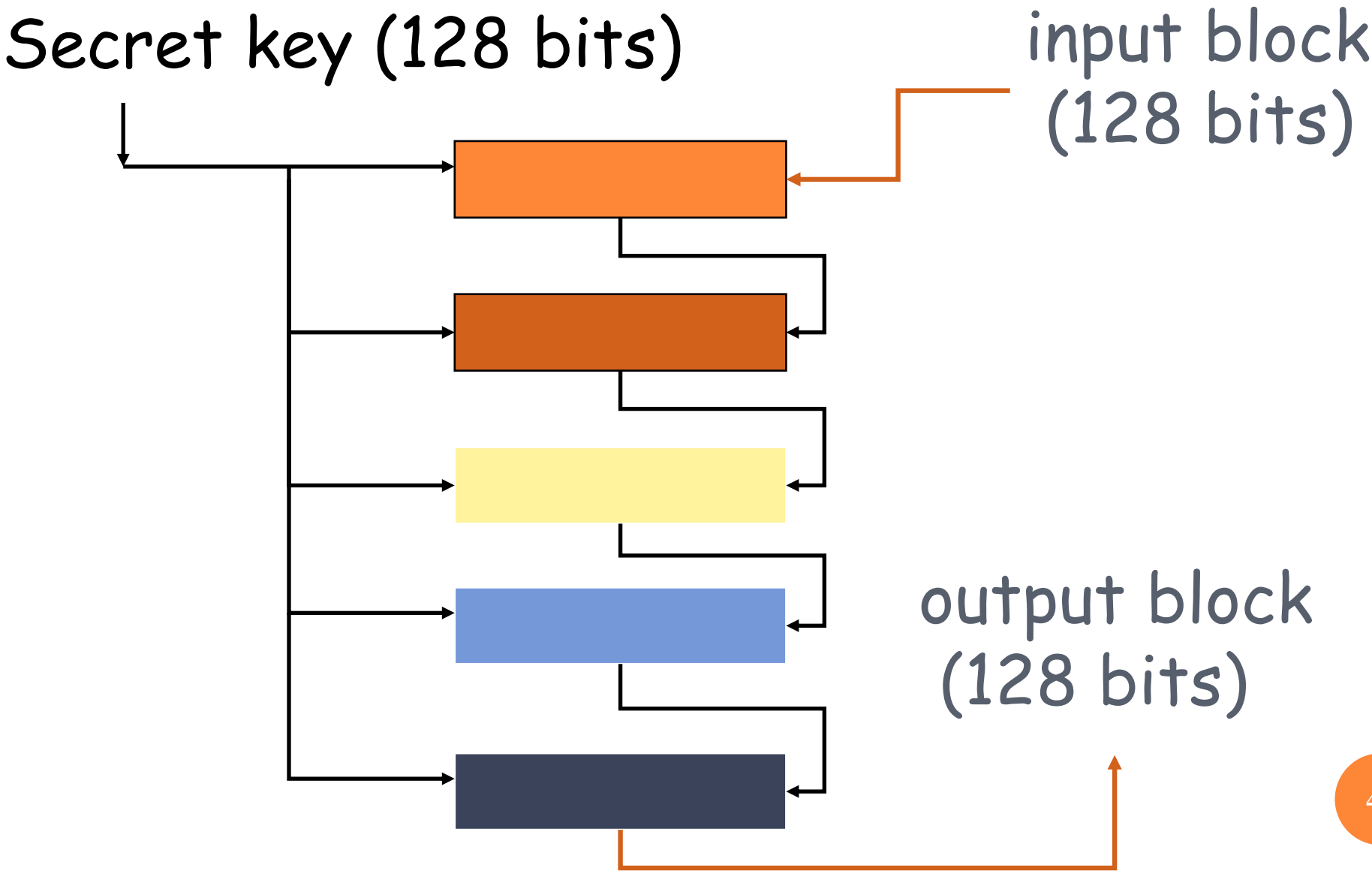
Initial layout: $K_{0,0}$, $K_{1,0}$, $K_{2,0}$, $K_{3,0}$, $K_{0,1}$,...

AES Specifications

High level code

- AES(State, Key)
  - KeyExpansion(Key, ExpandKey)
  - AddRoundKey(State, ExpandKey[0])
  - for (i = 1; i < R; i++) do
    Round(State, ExpandKey[i]);
  - FinalRound(State, ExpandKey[R]);

Encryption: Carried out in rounds



Rounds in AES

128 bits AES uses 10 rounds, no shortcuts known for 6 rounds

- The secret key is expanded from 128 bits to 10 round keys, 128 bits each.
- Each round changes the state, then XORs the round key. (for longer keys, add one round for every extra 32 bits)

Each rounds complicates things a little. Overall it seems infeasible to invert without the secret key (but easy given the key).

AES Specifications: One Round

Transform the state by applying:

$A_{0,0}$	$A_{0,1}$	$A_{0,2}$	$A_{0,3}$
$A_{1,0}$	$A_{1,1}$	$A_{1,2}$	$A_{1,3}$
$A_{2,0}$	$A_{2,1}$	$A_{2,2}$	$A_{2,3}$
$A_{3,0}$	$A_{3,1}$	$A_{3,2}$	$A_{3,3}$

1. Substitution
2. Shift rows
3. Mix columns
4. XOR round key

Substitution (S-Box)

Substitution operates on every Byte separately: $A_{i,j} \leftarrow A_{i,j}^{-1}$
(multiplicative inverse in GF(2^8) which is highly non linear)

If $A_{i,j} = 0$, don't change $A_{i,j}$

Clearly, the substitution is invertible.

Cyclic Shift of Rows

$A_{0,0}$	$A_{0,1}$	$A_{0,2}$	$A_{0,3}$
$A_{1,3}$	$A_{1,0}$	$A_{1,1}$	$A_{1,2}$
$A_{2,2}$	$A_{2,3}$	$A_{2,0}$	$A_{2,1}$
$A_{3,1}$	$A_{3,2}$	$A_{3,3}$	$A_{3,0}$

no shift

shift 1 position

shift 2 positions

shift 3 positions

Clearly, the shift is invertible.

Mixing Columns

Every state column is considered as a Polynomial over GF(2^8)

Multiply with an invertible polynomial
03 $x^3 + 01x^2 + 01x + 02 \pmod{x^4 + 1}$
Inv = 0B x^3 + 0D x^2 +09 x + 0E

Round: SubBytes(State)
ShiftRows(State)
MixColumns(State)
AddRoundKey(State,ExpandedKey[i])

Key Expansion

- Generate a "different key" per round
- Need a 4 x 4 matrix of values (over $GF(2^8)$) per round
- Based upon a non-linear transformation of the original key.
- Details available: *The Design of Rijndael*, Joan Daemen and Vincent Rijmen, Springer

Breaking AES

Breaking 1 or 2 rounds is easy.

It is not known how to break 5 rounds.

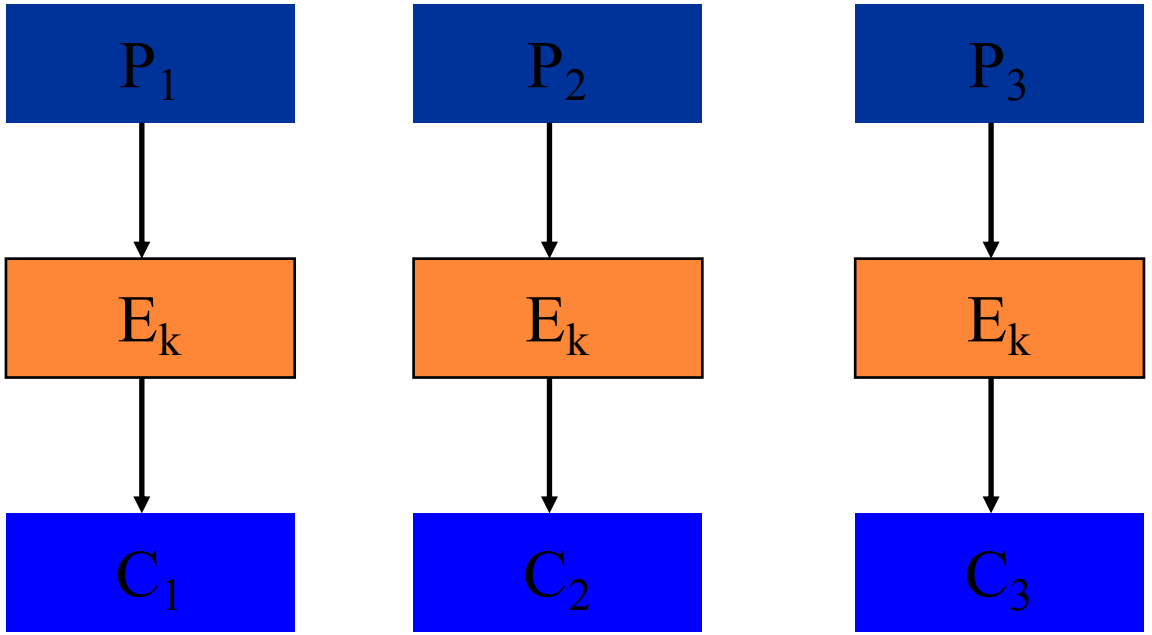
Breaking the full 10 rounds AES efficiently (say 1 year on existing hardware, or in less than 2^{128} operations) is considered impossible ! (a good, tough challenge…)

BLOCK CIPHER MODES OF OPERATION

- block ciphers operate on blocks of fixed length, often 64 or 128 bits
- because messages may be of any length, and because encrypting the same plaintext under the same key always produces the same output,

several modes of operation have been invented which allow block ciphers to provide confidentiality for messages of arbitrary length

ECB Mode Encryption (Electronic Code Book)



encrypt each plaintext block separately

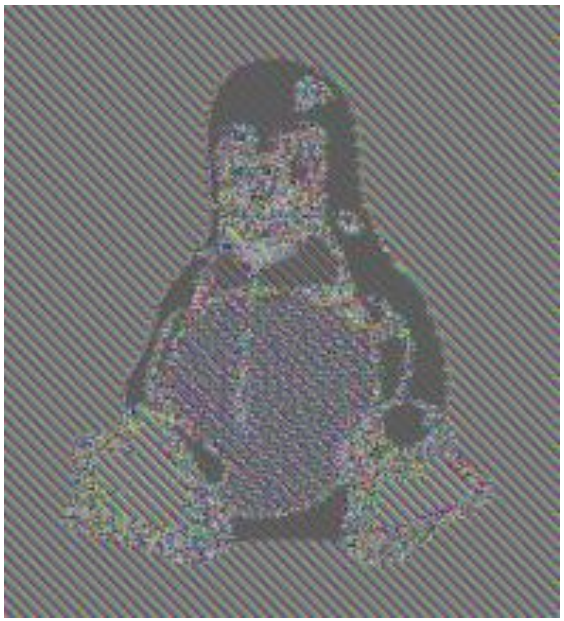
Properties of ECB

- Simple and efficient
- Parallel implementation possible
- Does not conceal plaintext patterns
- Active attacks are possible (plaintext can be easily manipulated by removing, repeating, or interchanging blocks).

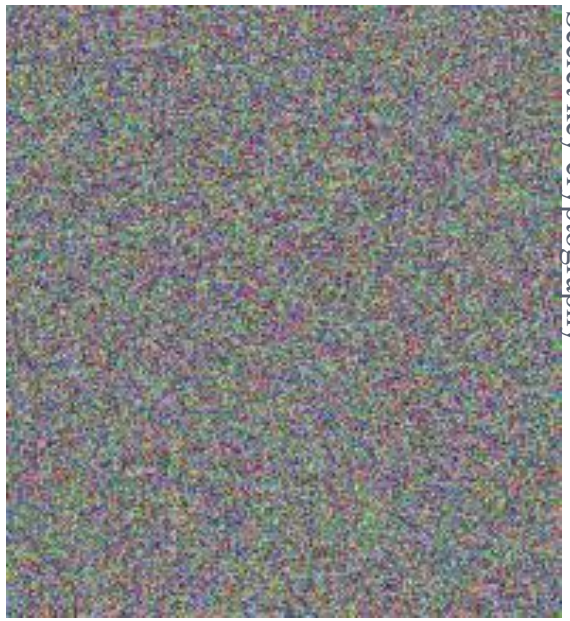
ECB: PLAINTEXT REPETITIONS

plaintext

ciphertext ECB

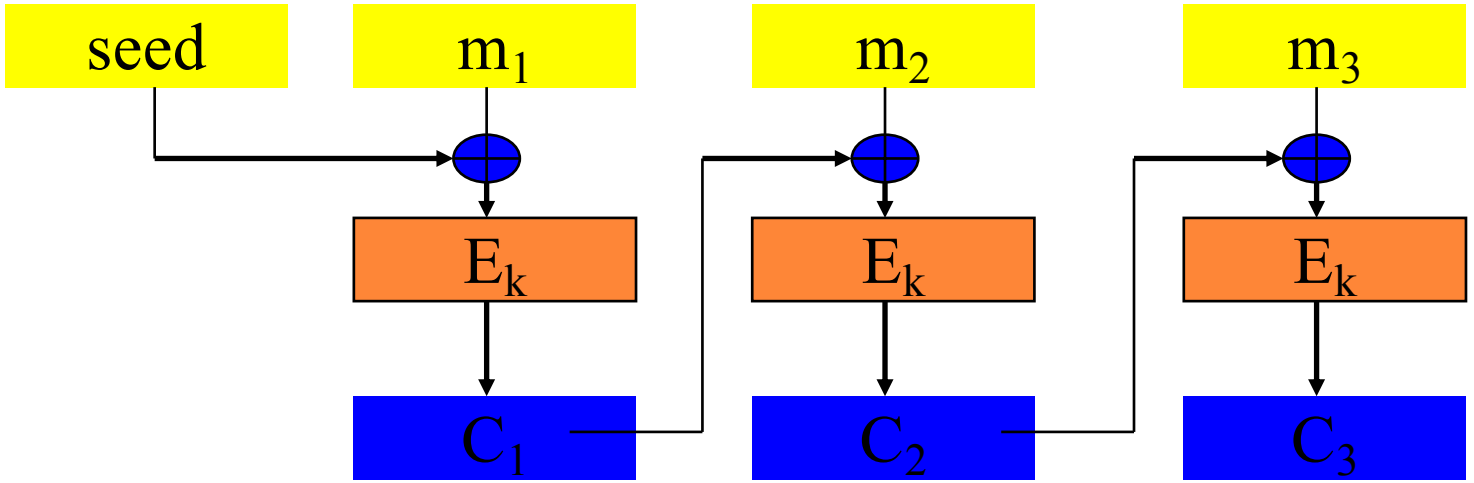


good ciphertext



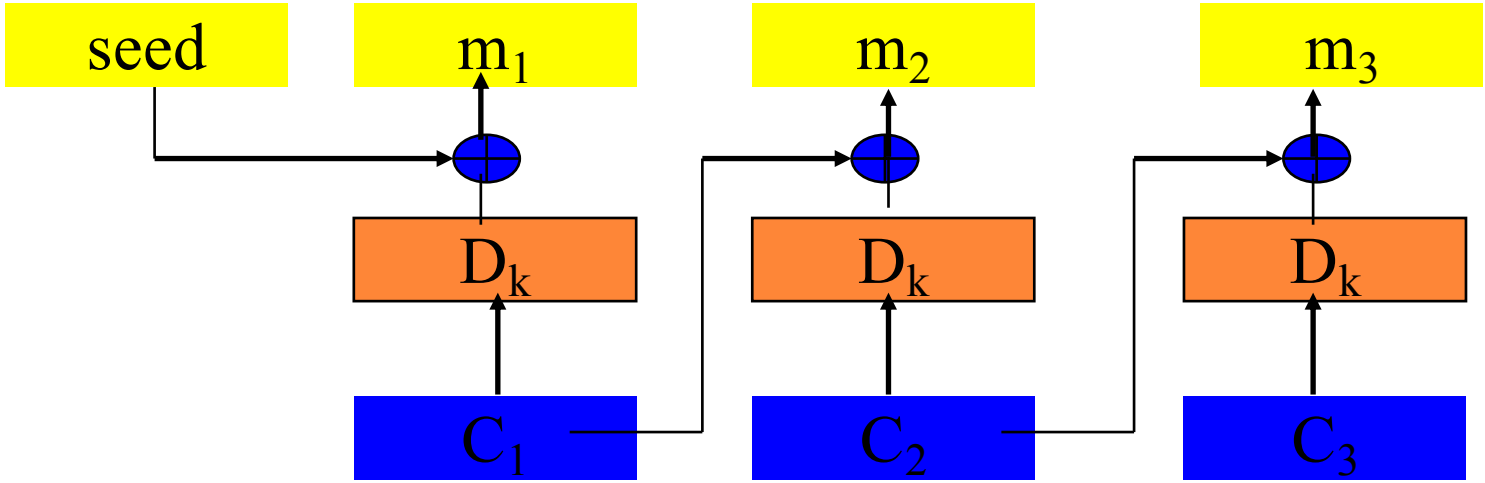
CBC (Cipher Block Chaining) mode

IBM, 1976



- Previous ciphertext is XORed with current plaintext before encrypting current block
- Seed is used to start the process; it can be sent without encryption
- Seed = 0 safe in most but NOT all cases (e.g. assume the file with salaries is sent once a month, with the same seed we can detect changes in the salaries) therefore a random seed is better

CBC (CIPHER BLOCK CHAINING): DECRYPTION



Problem

IF a transmission error changes one bit of $C_{(i-1)}$

THEN block m_i changes in a predictable way (this can be exploited by adversary)

BUT there are unpredictable changes in $m_{(i-1)}$;

Solution: always use error detecting codes (for example CRC) to check quality of transmission

Properties of CBC

- Asynchronous stream cipher
- Errors in one ciphertext block propagate
  - a one-bit change to the ciphertext causes complete corruption of the corresponding block of plaintext, and inverts the corresponding bit in the following block of plaintext
- Conceals plaintext patterns
- No parallel implementation known
  - no parallel encryption
  - what about decryption?
- Plaintext cannot be easily manipulated
- Standard in most systems: SSL, IPSec etc.

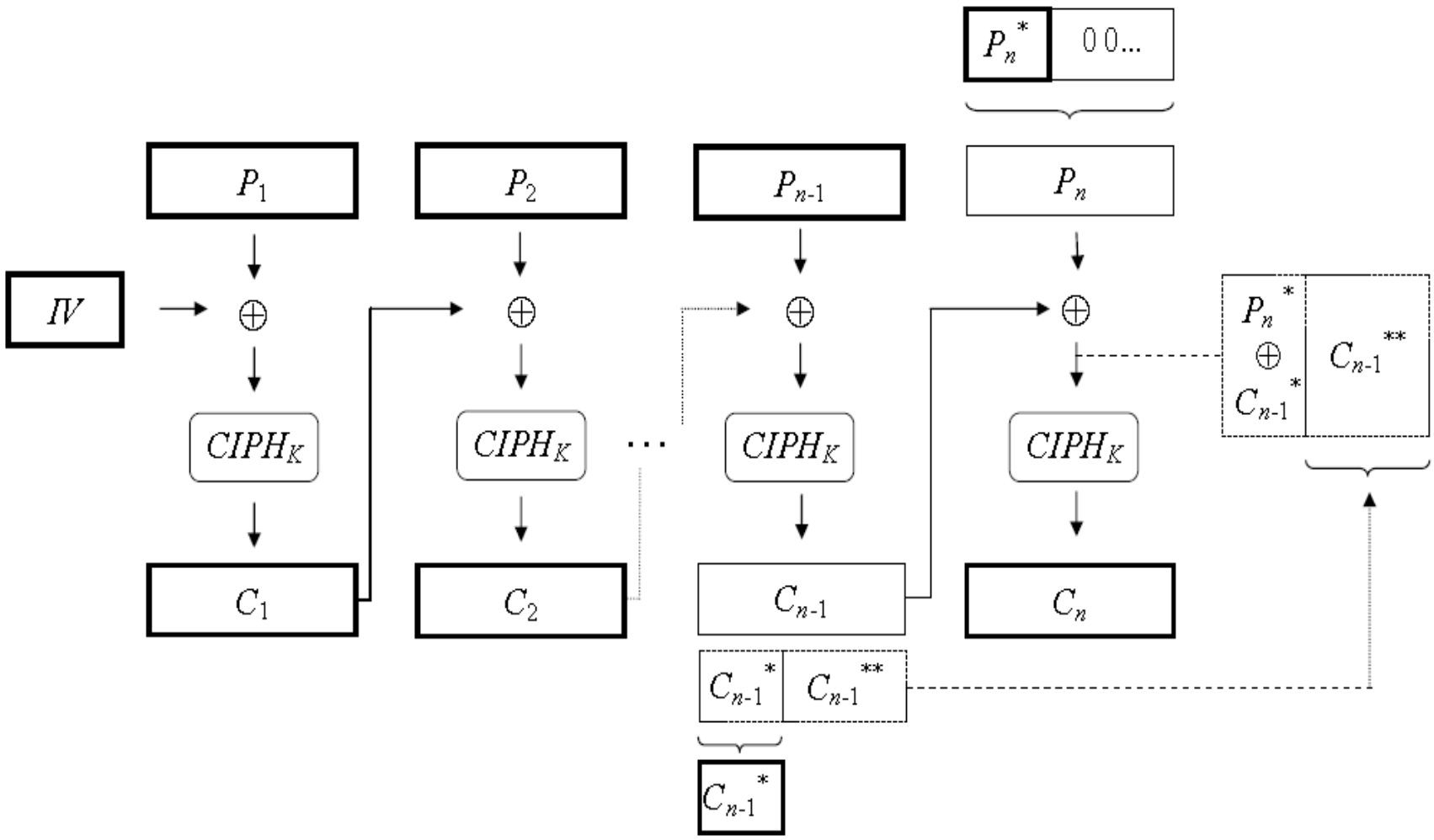
MORE ON CBC

- message must be padded to a multiple of the cipher block size
  - one way to handle this issue is ciphertext stealing
- a plaintext can be recovered from just two adjacent blocks of ciphertext
  - as a consequence, decryption can be parallelized
  - usually a message is encrypted once, but decrypted many times

CIPHERTEXT STEALING

- general method that allows for processing of messages that are not evenly divisible into blocks
  - without resulting in any expansion of the ciphertext
  - at the cost of slightly increased complexity
- consists of altering processing of the last two blocks of plaintext, resulting in a reordered transmission of the last two blocks of ciphertext (and no ciphertext expansion)
- suitable for ECB and CBC
- from NIST website http://csrc.nist.gov/groups/ST/toolkit/BCM/documents/ciphertext%20stealing%20proposal.pdf

CIPHERTEXT STEALING SCHEME



ENCRYPTION/DECRYPTION

Encryption procedure

- If the plaintext length is not a multiple of the block size, pad it with enough zero bits until it is.
- Encrypt the plaintext using the Cipher Block Chaining mode.
- Swap the last two ciphertext blocks.
- Truncate the ciphertext to the length of the original plaintext.

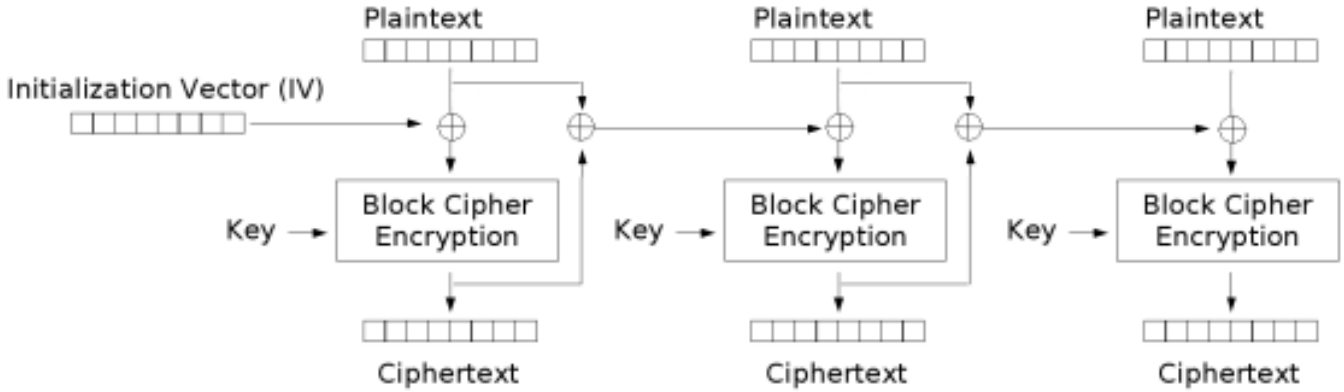
Decryption procedure

- If the ciphertext length is not a multiple of the block size, say it is n bits short, then pad it with the last n bits of the block cipher decryption of the last full ciphertext block.
- Swap the last two ciphertext blocks.
- Decrypt the ciphertext using the Cipher Block Chaining mode.
- Truncate the plaintext to the length of the original ciphertext.

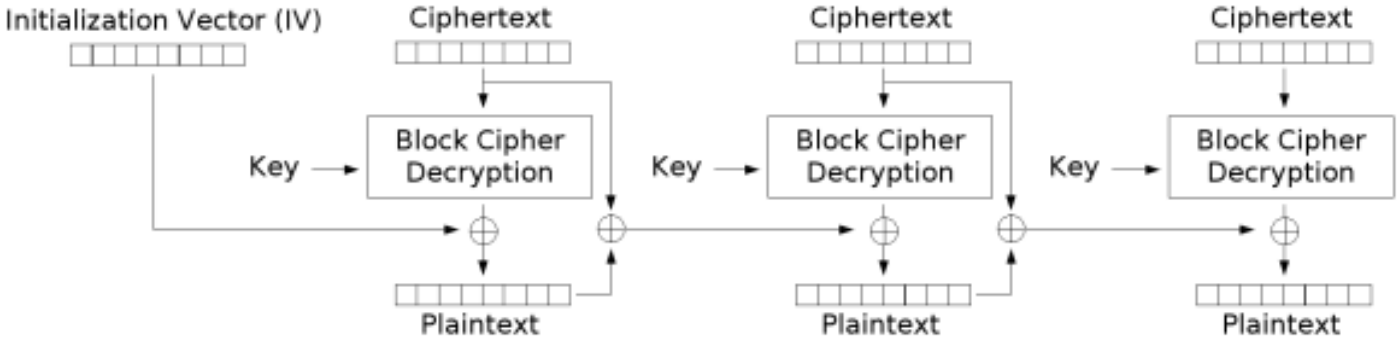
PROPAGATING CIPHER-BLOCK CHAINING (PCBC)

- designed to extend or propagate a single bit error both in encryption and decryption
- used in Kerberos v4 and WASTE, most notably, but otherwise is not common

PCBC SCHEME



Propagating Cipher Block Chaining (PCBC) mode encryption



Propagating Cipher Block Chaining (PCBC) mode decryption

MORE ON PCBC

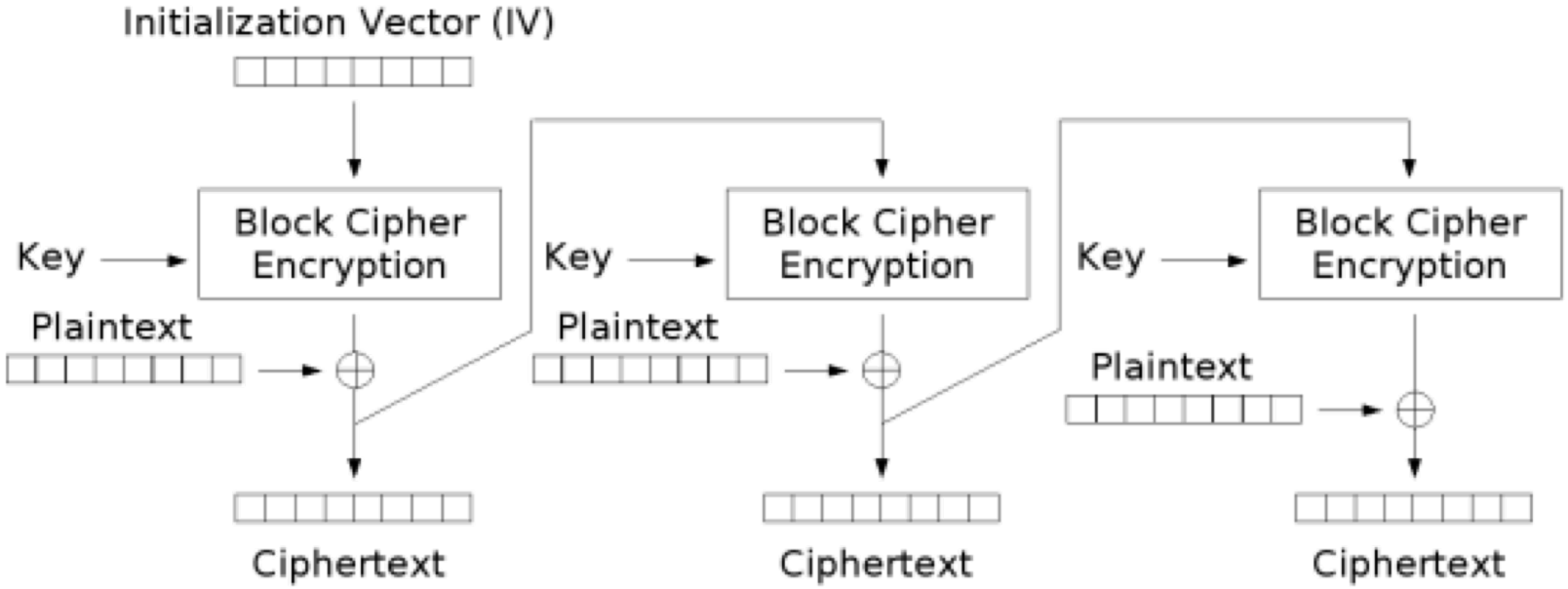
- on a message encrypted in PCBC mode, if two adjacent ciphertext blocks are exchanged, this does not affect the decryption of subsequent blocks (this does not happen to CBC)
  - $I_{i+2}=C_{i+1}\oplus(D_K(C_{i+1})\oplus I_{i+1})$
  - $I_{i+1}=C_i\oplus(D_K(C_i)\oplus I_i)$
  - $I_{i+2}=C_{i+1}\oplus D_K(C_{i+1})\oplus C_i\oplus D_K(C_i)\oplus I_i$

  (I_j denotes the feedback input to column j)

- for this reason, PCBC is not used in Kerberos v5

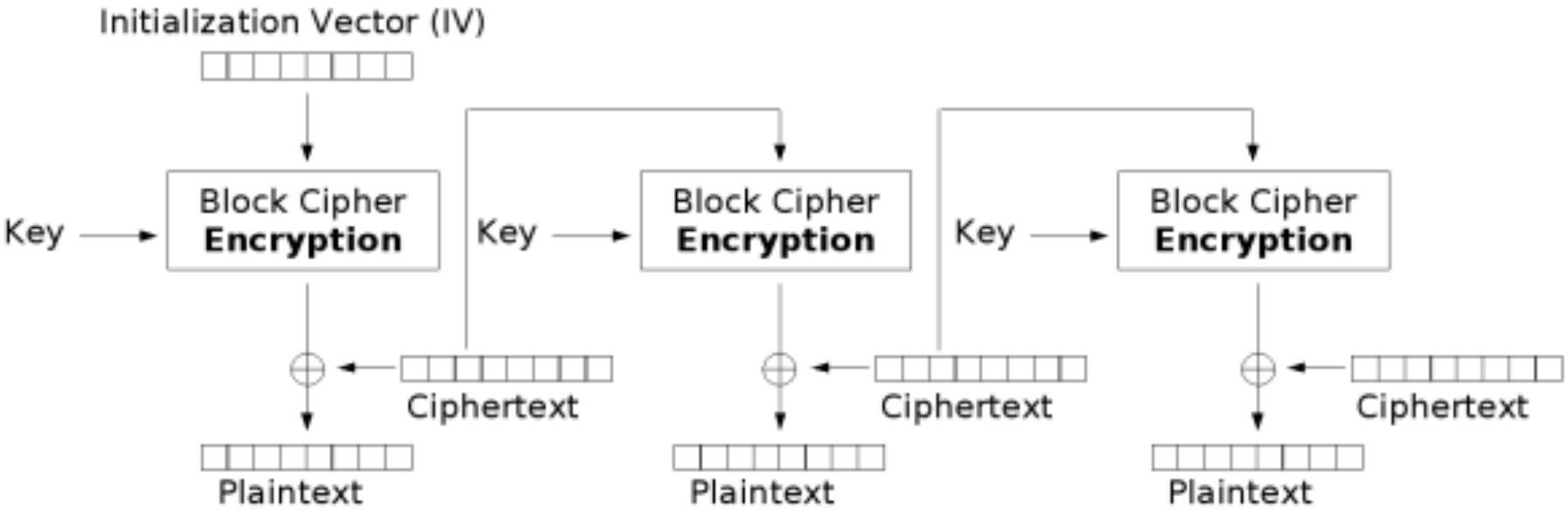
CIPHER FEEDBACK (CFB)

- similar to CBC, makes a block cipher into an asynchronous stream cipher
  - i.e., supports some re-synchronizing after error, if input to encryptor is given thru a shift-register



Cipher Feedback (CFB) mode encryption

CFB DECRYPTION

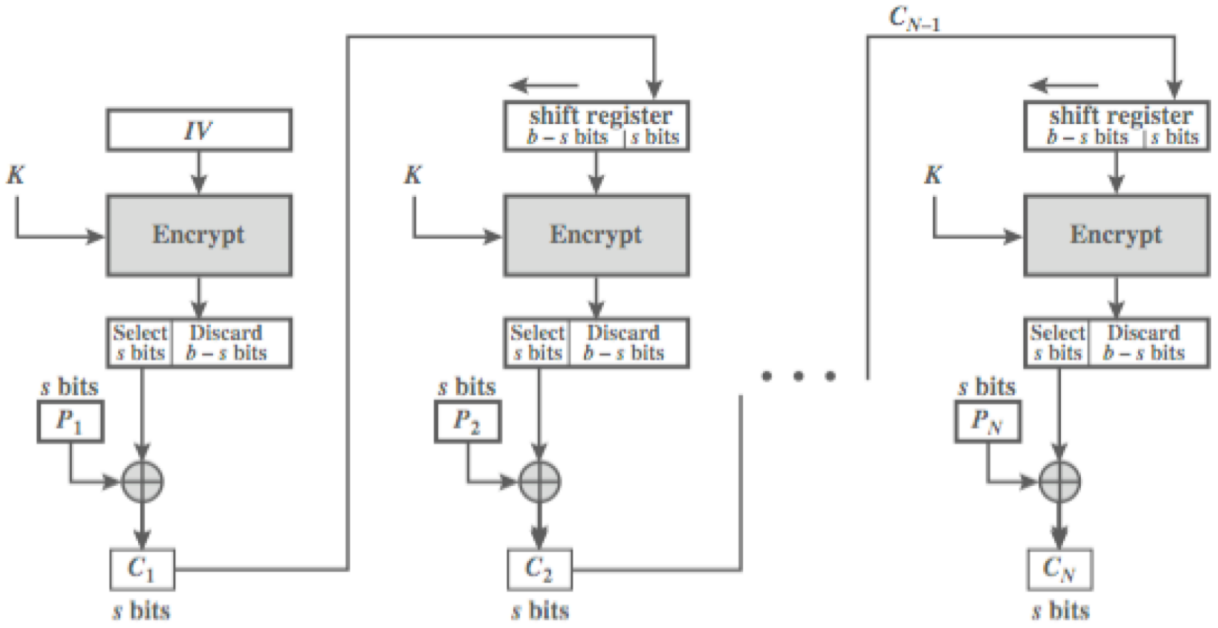


Cipher Feedback (CFB) mode decryption

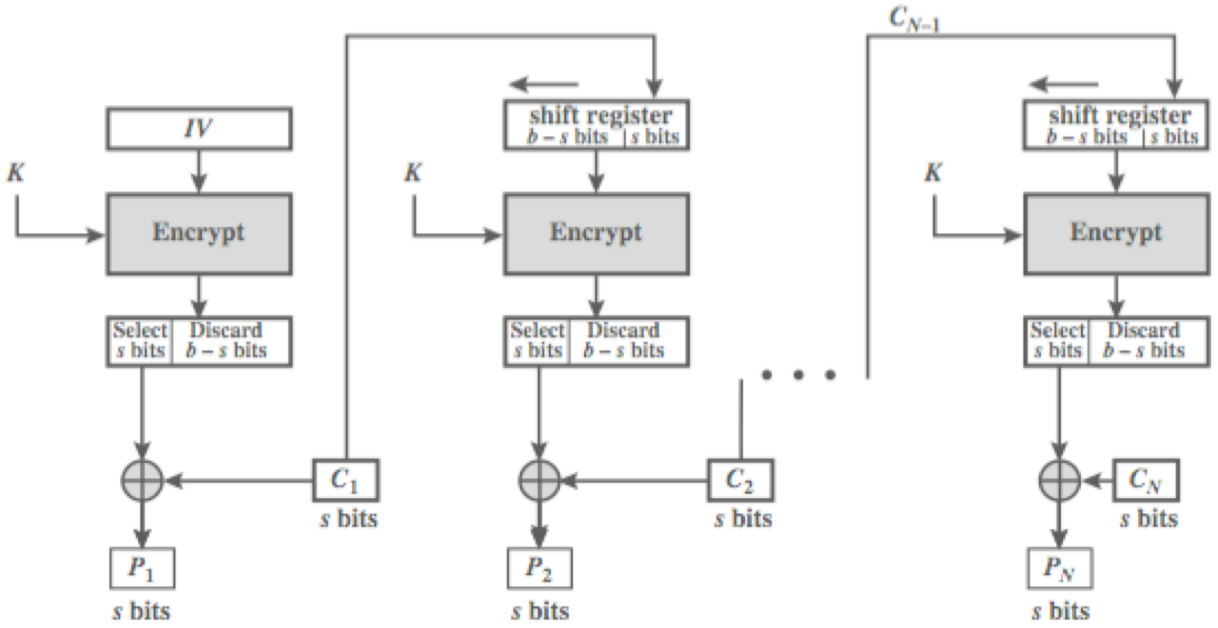
CFB

- Like CBC mode, changes in the plaintext propagate forever in the ciphertext, and encryption cannot be parallelized.
  - Also like CBC, decryption can be parallelized.
- When decrypting, a one-bit change in the ciphertext affects two plaintext blocks: a one-bit change in the corresponding plaintext block, and complete corruption of the following plaintext block. Later plaintext blocks are decrypted normally.
- CFB shares two advantages over CBC mode: the block cipher is only ever used in the encrypting direction, and the message does not need to be padded to a multiple of the cipher block size.

CFB WITH SHIFT-REGISTER



(a) Encryption



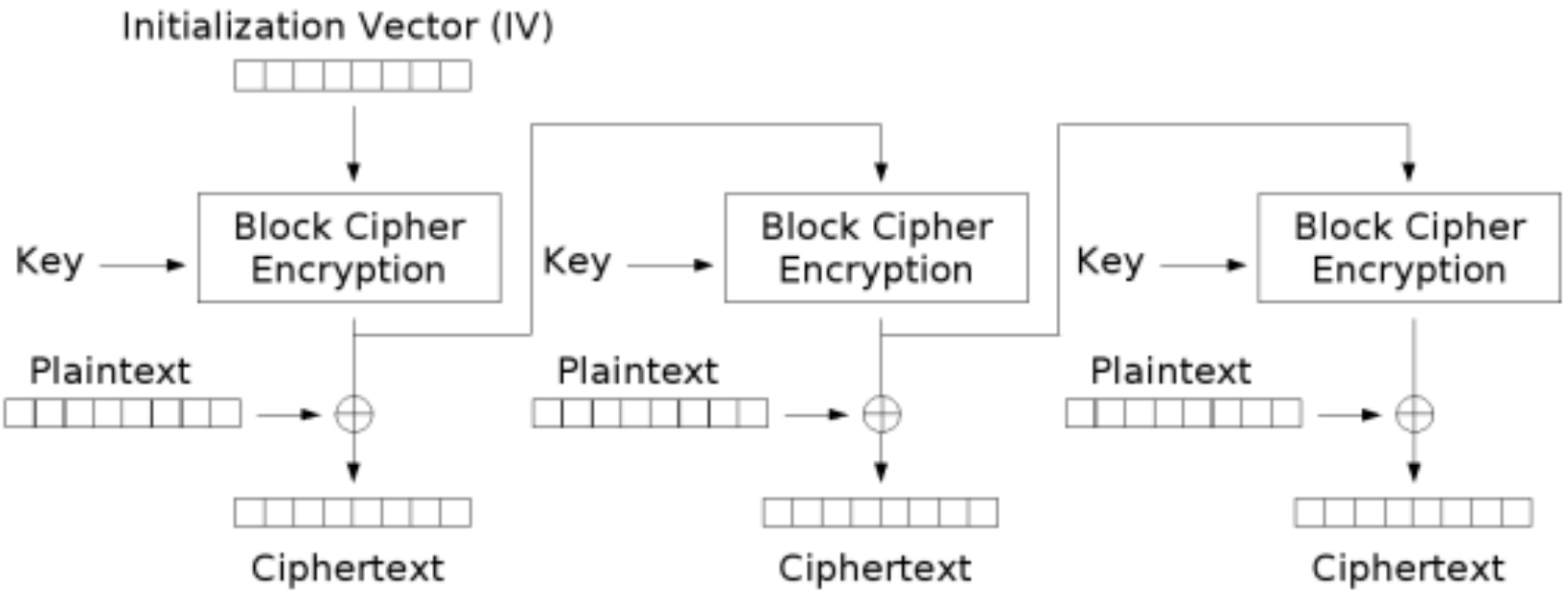
(b) Decryption

s =1, 8, 64 or 128

OFB Mode (Output FeedBack)

- Makes a block cipher into a synchronous stream cipher: it generates keystream blocks, which are then XORed with the plaintext blocks to get the ciphertext.
- Flipping a bit in the ciphertext produces a flipped bit in the plaintext at the same location. This property allows many error correcting codes to function normally even when applied before encryption.

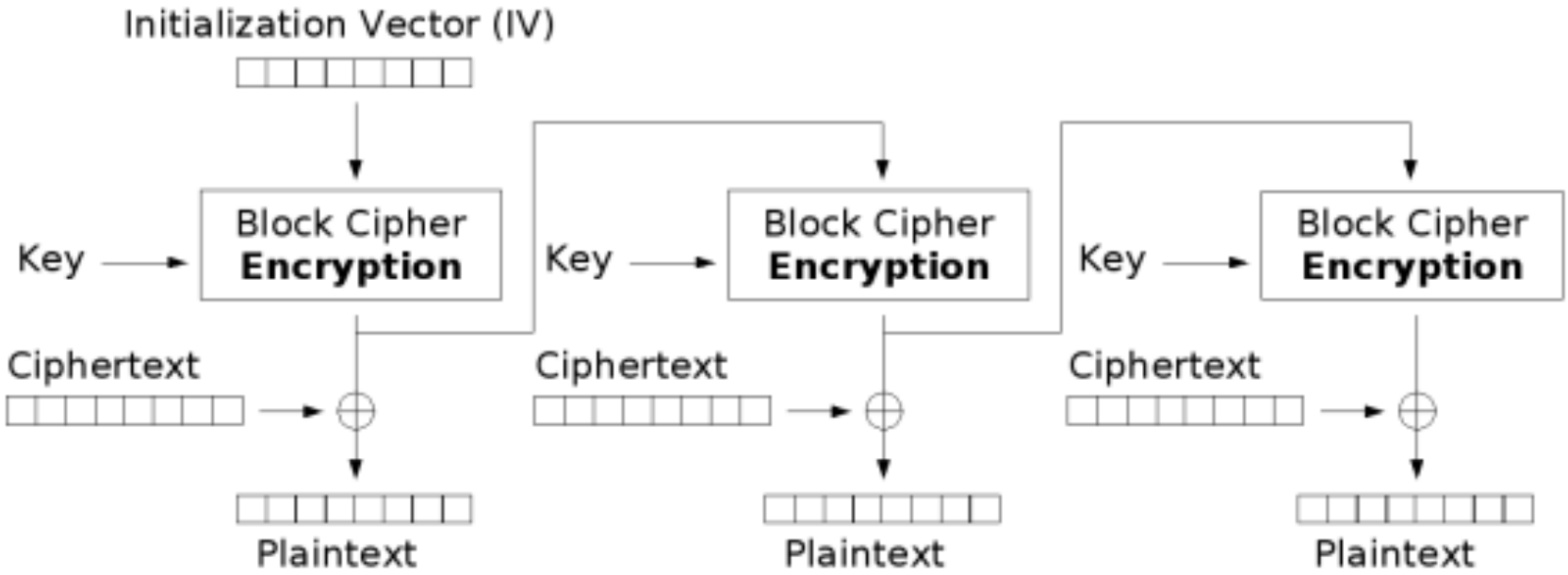
OFB SCHEME



Output Feedback (OFB) mode encryption

An initialization vector IV is used as a "seed" for a sequence of data blocks

OFB DECRYPTION



Output Feedback (OFB) mode decryption

OFB PROPERTIES

Because of the symmetry of the XOR operation, encryption and decryption are exactly the same

$$C_i = P_i \oplus O_i$$
$$P_i = C_i \oplus O_i$$
$$O_i = E_k(O_{i-1})$$
$$O_0 = \mathrm{IV}$$

OFB MODE

Discussion

- If E_k is public (known to the adversary) then initial seed must be encrypted (why?)
- If E is a cryptographic function that depends on a secret key then initial seed can be sent in clear (why?)
- Initial seed must be modified for EVERY new message - even if it is protected and unknown to the adversary (in fact if the adv knows a pair message, initial seed then he can encode every message - why?)
- Extension: it can be modified in such a way that only k bits are used to compute the ciphertext (k-OFB)

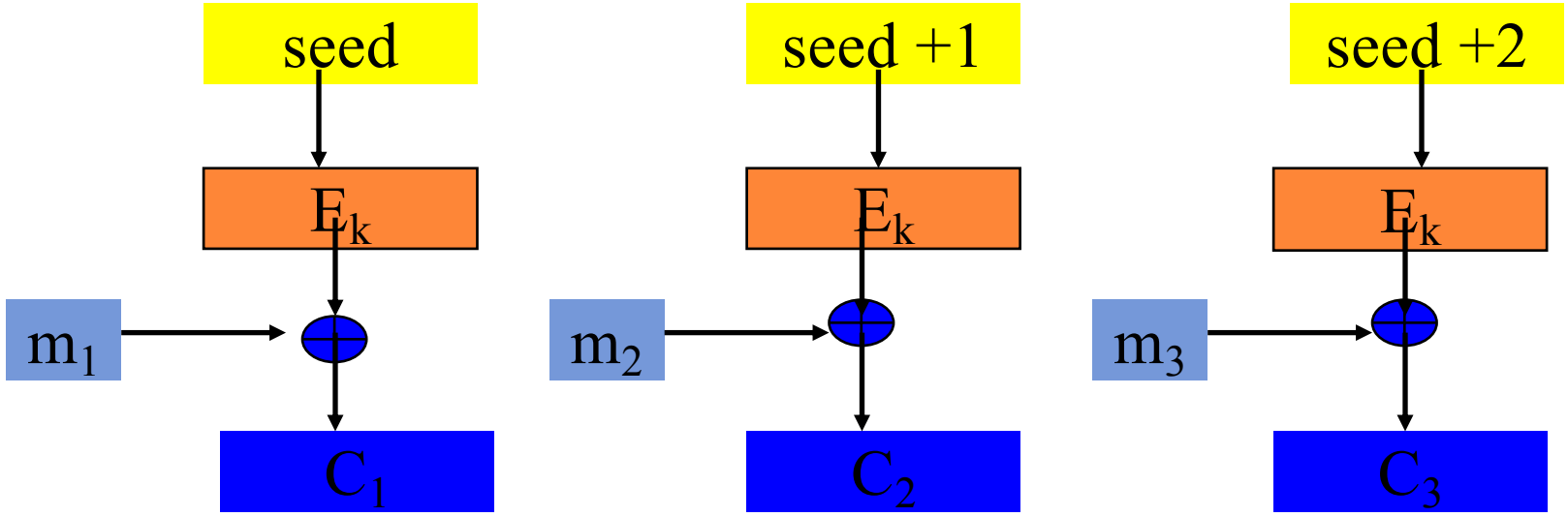
Properties of OFB

- Synchronous stream cipher
- Errors in ciphertext do not propagate
- Pre-processing is possible
- Conceals plaintext patterns
- No parallel implementation known
- Active attacks by manipulating plaintext are possible

CTR (Counter Mode)

- also known as Integer Counter Mode (ICM) and Segmented Integer Counter (SIC) mode
- turns a block cipher into a stream cipher: it generates the next keystream block by encrypting successive values of a "counter"
  - counter can be any function which produces a sequence which is guaranteed not to repeat for a long time, although an actual counter is the simplest and most popular.
- the usage of a simple deterministic input function raised controversial discussions
- has similar characteristics to OFB, but also allows a random access property during decryption
- well suited to operation on a multi-processor machine where blocks can be encrypted in parallel

CTR (Counter Mode)



Similar to OFB

- There are problems in repeated use of same seed (like OFB)
- CTR vs OFB: using CTR you can decrypt the message starting from block i for any i (i.e. You do not need to decrypt from the first block as in OFB)

Initialization vector (IV)

- Most modes (except ECB) require an initialization vector, or IV
  - sort of "dummy block" to kick off the process for the first real block, and also to provide some randomization for the process.
  - no need for the IV to be secret, in most cases, but it is important that it is never reused with the same key.
- For CBC and CFB, reusing an IV leaks some information about the first block of plaintext, and about any common prefix shared by the two messages.
- In CBC mode, the IV must, in addition, be unpredictable at encryption time
  - see TLS CBC IV attack
- For OFB and CTR, reusing an IV completely destroys security.

AES Proposed Modes

- CTR (Counter) mode (OFB modification): Parallel implementation, offline pre-processing, provable security, simple and efficient
- OCB (Offset Codebook) mode - parallel implementation, offline preprocessing, provable security (under specific assumptions), authenticity
- also see Block Cipher Modes, by NIST http://csrc.nist.gov/groups/ST/toolkit/BCM/index.html 2001 - 2010

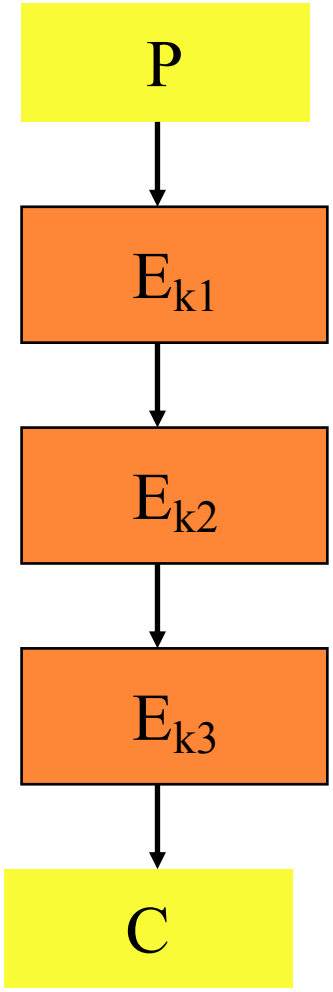
Strengthening a Given Cipher

- Design multiple key lengths – AES
- Key Whitening - the DES-X idea
  - Key whitening consists of steps that combine the data with portions of the key (most commonly using a simple XOR) before the first round and after the last round of encryption.
  - First use by Ron Rivest for strengthen DES, in 1984

$$\text{DES-X}(M) = K_2 \oplus \text{DES}_K(M \oplus K_1)$$

  - apparently key size becomes 184 (= 56 + 64×2), but its strength is 119 ($|K_1| = |K_2| = 64$)
- Iterated ciphers – Triple DES (3-DES), triple IDEA and so on

Triple Cipher - Diagram



Iterated Ciphers

- Plaintext undergoes encryption repeatedly by underlying cipher
- Ideally, each stage uses a different key
- In practice triple cipher is usually

  $C = E_{k1}(E_{k2}(E_{k1}(P)))$ [EEE mode] or

  $C = E_{k1}(D_{k2}(E_{k1}(P)))$ [EDE mode]

  EDE is more common in practice

Two or Three Keys

- Sometimes only two keys are used in 3-DES
- Identical key must be at beginning and end
- Legal advantage (export license) due to smaller overall key size
- Used as a KEK (key-encrypting key) in the BPI (Baseline Privacy Interface) protocol which secures the DOCSIS cable modem standard
  - DOCSIS: international standard that permits the addition of high-speed data transfer to an existing Cable TV system. It is employed by many cable television operators to provide Internet access over their existing hybrid fiber coaxial infrastructure

Adversary's goal

- Final goal: find the secret key
- Partial goals:
  - Reduce the # of possible keys
  - Detect patterns in the text
  - Decode part of the text
  - Modify the ciphertext obtaining a plausible text (even without breaking the cipher; even without knowing which modifications)

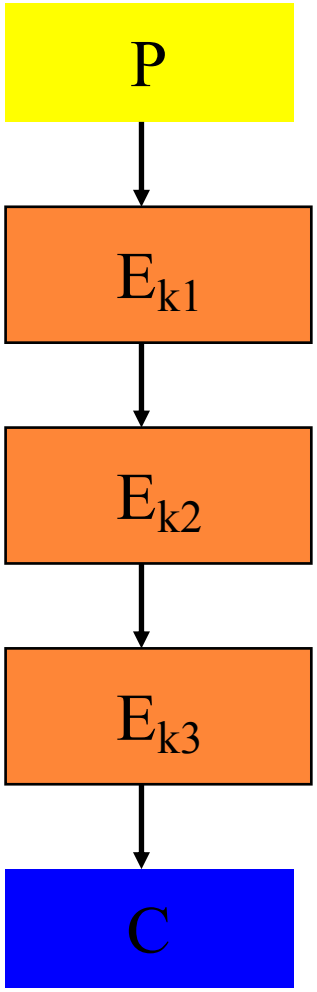
DOUBLE DES: MEET-IN-THE-MIDDLE ATTACK

Cipher twice with two different keys? NO!

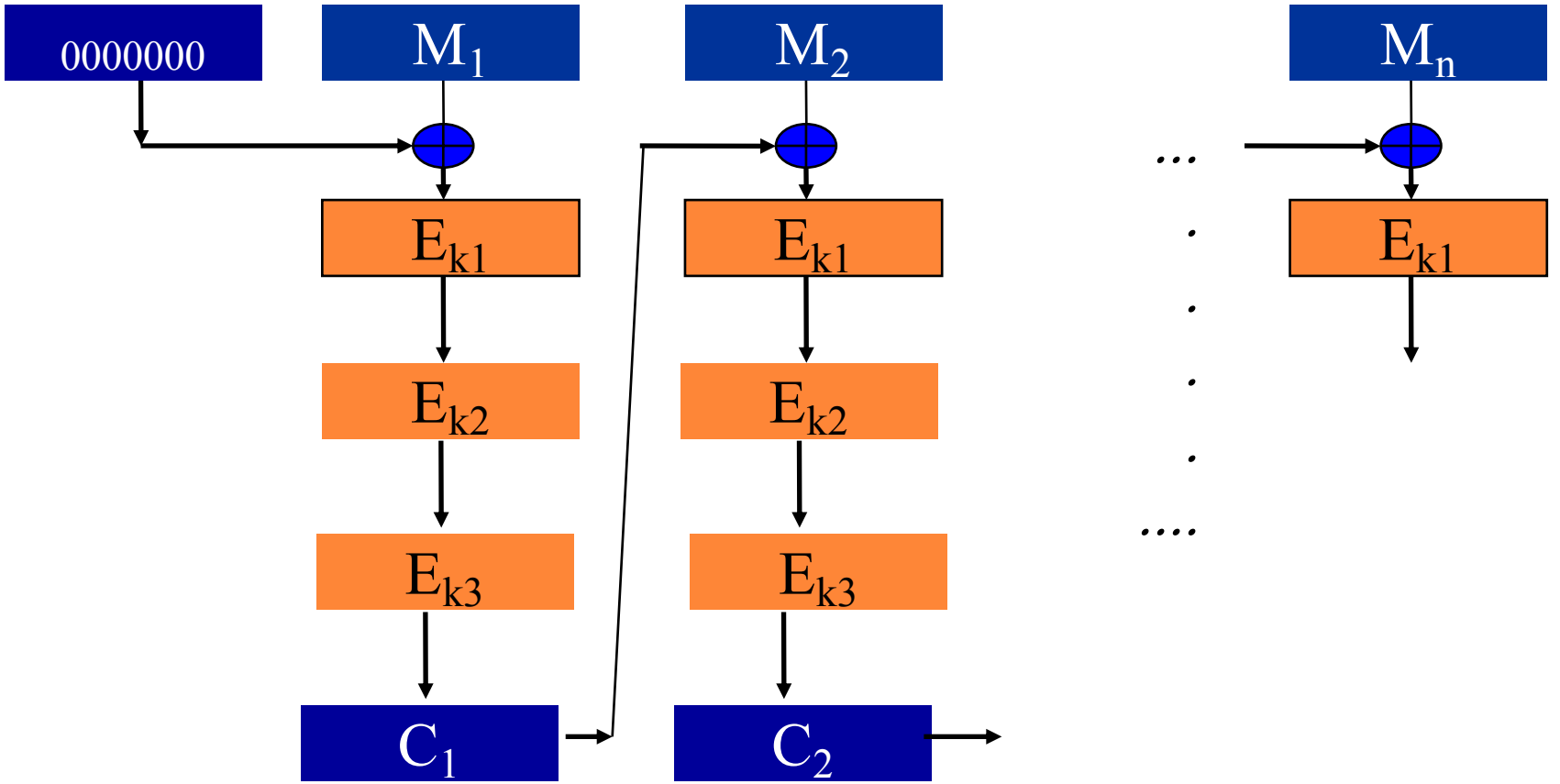
Meet-in-the-middle attack. Requirements

- Known plaintext/ciphertext pairs
- 2^n encryptions + 2^n decryptions (2 keys of n bit), instead of 2^{2n} brute-force
- 2^n memory space
- Idea: try all possible 2^n encryptions of the plaintext and all possible 2^n decryptions of the ciphertext. Encryptions stored into a lookup table.
- Check for a pair of keys that transform the plaintext in the ciphertext. Test pair on other pairs plaintext/ciphertext
- Note: the method can be applied to all block codes

TRIPLE ENCODING



TRIPLE ENCODING AND CBC

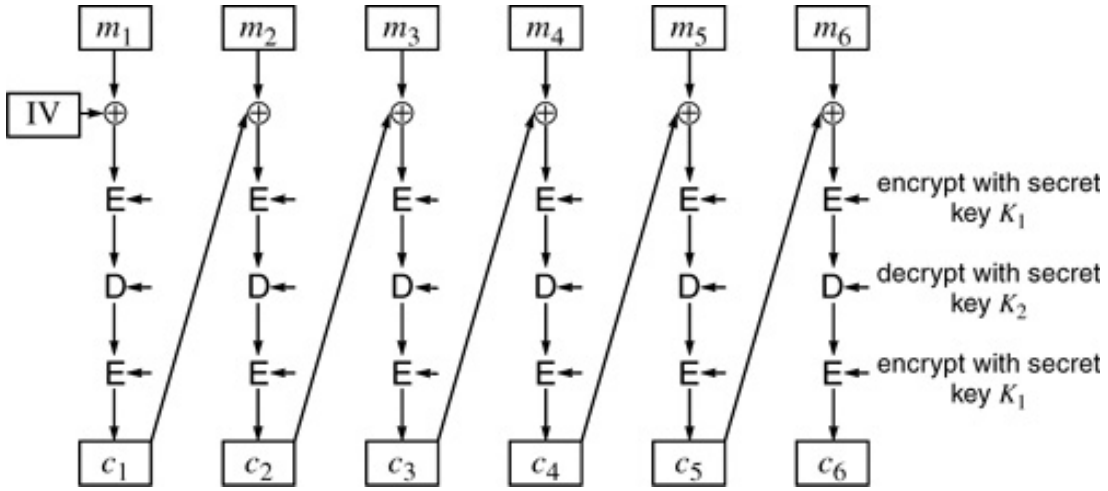


In the picture: External CBC: code (using triple encoding) each block ; then concatenate

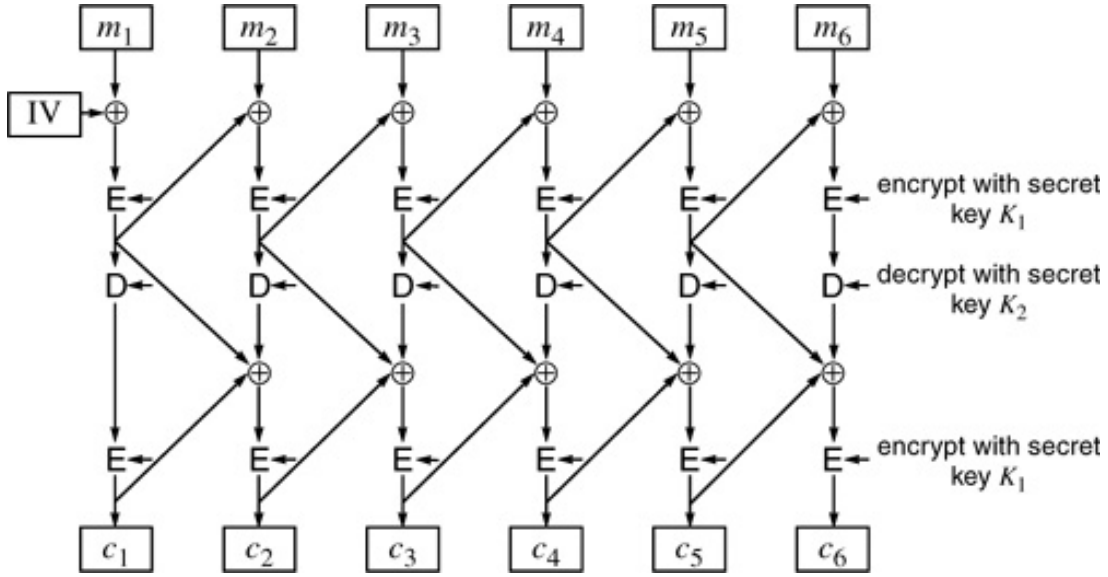
Other possibility: Internal CBC (the concatenation is internally made, before 1st encryption and after decryption). More secure, but less used

EDE CBC INTERNAL VS EXTERNAL

Outside EDE CBC



Inside EDE CBC



WHY CBC OUTSIDE?

1. Bit Flipping
   - CBC Outside: One bit flip in the cipher text causes that block of plain text and next block garbled ⇒ Self-Synchronizing
   - CBC Inside: One bit flip in the cipher text causes more blocks to be garbled.
2. Pipelining
   - More pipelining possible in CBC inside implementation.
3. Flexibility of Change:
   - CBC outside: Can easily replace CBC with other feedback modes (ECB, CFB, ...)

Exercises

- Evaluate error propagation in CBC e OFB
  - Show how an adversary can modify a block as he/she prefers assuming that the remaining part of the message is modified
  - Discuss the security of this and techniques for avoiding such attacks
- CBC and OFB use and initial seed that must be known to both the sender and the receiver
  - Assume that the initial seed is sent in the clear (so it is known to the adversary). Show how the adversary is able to modify part of the message. Conclusion: either the initial seed is fixed in advance or it must be encrypted and sent before the message
  - Break OFB if you use the same key and the same initial seed more than once

HOMEWORK

- What is a practical method of finding a triple of keys that maps a given plain text to a given cipher text using EDE?
- Hint

1. You have only one (m, c) pair
2. Worst case is to have 3 nested loops for trying all k1, k2, k3 $\Rightarrow 2^{56} \times 2^{56} \times 2^{56} = 2^{168}$ steps but requires storing only 1 intermediate result.
3. How can you reduce the number of steps using more storage for intermediate results.