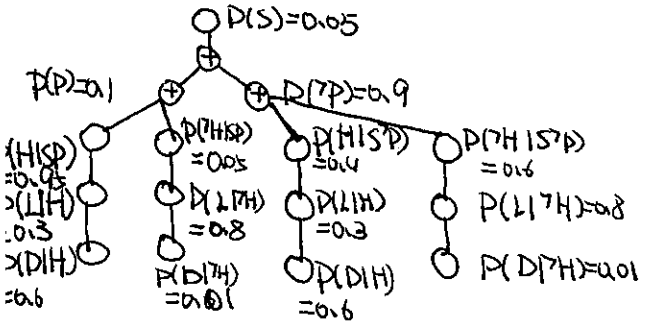
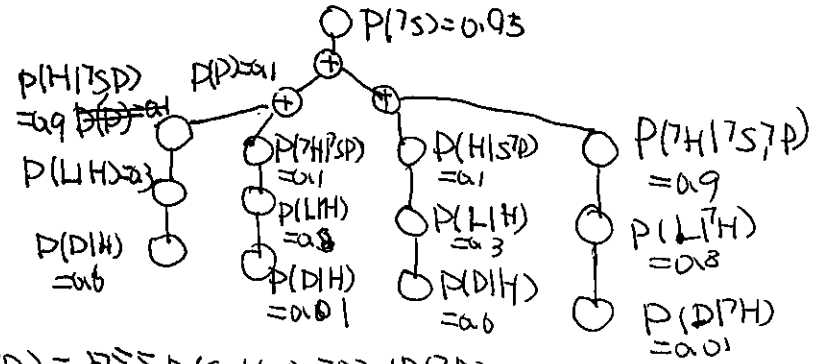
① Exact Results :

1) $P(Sick \mid L, D) = \sum_H \sum_P P(S, H, P \mid L, D) = \sum_H \sum_P P(S, H, P, L, D) \mid P(L, D) = \alpha \sum_H \sum_P P(S, H, P, L, D)$

$= \sum_H \sum_P P(S) P(P) P(H \mid SP) P(L \mid H) P(D \mid H) = \alpha \langle 0.5 \times \begin{pmatrix} (0.1 \times 0.95 \times 0.3 \times 0.6) + (0.1 \times 0.05 \times 0.8 \times 0.01) \\ + (0.4 \times 0.3 \times 0.6 \times 0.9) + (0.9 \times 0.6 \times 0.8 \times 0.01) \end{pmatrix}, 0.95 \times \begin{pmatrix} (0.95 \times 0.9 \times 0.3 \times 0.6) + (0.95 \times 0.1 \times 0.8 \times 0.1) \\ + (0.05 \times 0.1 \times 0.3 \times 0.6) + (0.9 \times 0.8 \times 0.05 \times 0.01) \end{pmatrix} \rangle = \alpha \langle 0.004313, 0.0370127 \rangle$.

$= \text{Normalize}(\alpha \langle 0.004313, 0.0370127 \rangle) = \langle 0.104367\ldots, 0.895632\ldots \rangle \approx \langle 0.104, 0.896 \rangle$

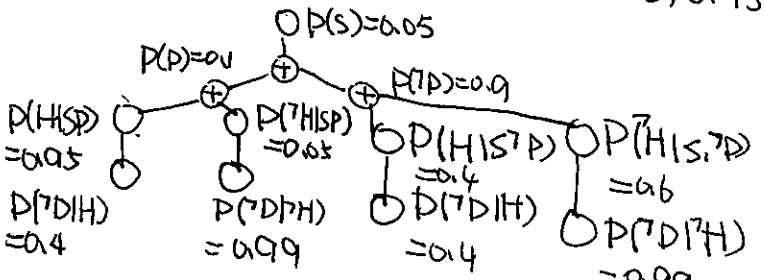


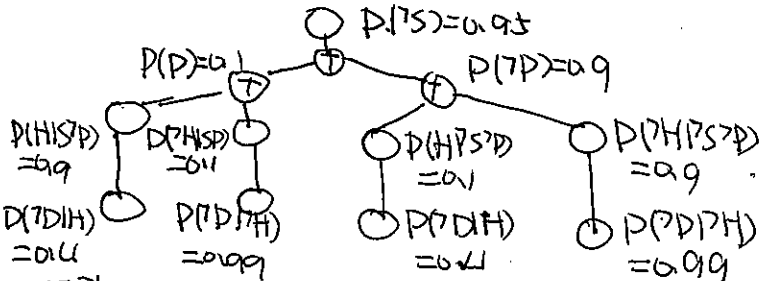


2) $P(Sick \mid Doctor = false) = \sum_H \sum_P P(S, H, P \mid \neg D) = \alpha \sum_H \sum_P P(S, H, P, \neg D) \mid P(\neg D) = \alpha \sum_H \sum_P P(S, H, P, \neg D)$

$= \alpha \sum_H \sum_P P(S) P(D) P(H \mid SP) P(\neg D \mid H) = \alpha \langle 0.05 \times \begin{pmatrix} 0.1 \times 0.95 \times 0.4 + 0.1 \times 0.05 \times 0.99 \\ + 0.9 \times 0.4 \times 0.4 + 0.9 \times 0.6 \times 0.99 \end{pmatrix}, 0.95 \times \begin{pmatrix} 0.1 \times 0.9 \times 0.4 + 0.1 \times 0.1 \times 0.4 \\ + 0.9 \times 0.1 \times 0.6 + 0.9 \times 0.9 \times 0.99 \end{pmatrix} \rangle$

$= \alpha \langle 0.0360775, 0.838961 \rangle = \langle 0.041223, 0.958776\ldots \rangle \approx \langle 0.041, 0.959 \rangle$



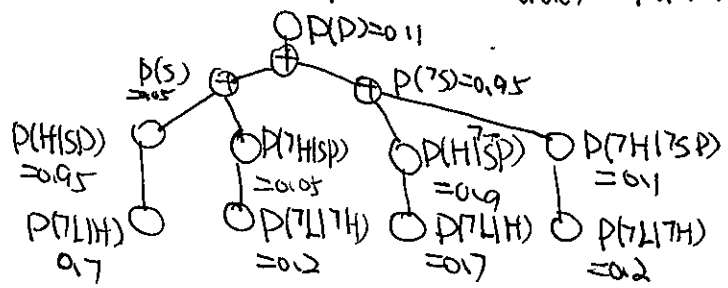


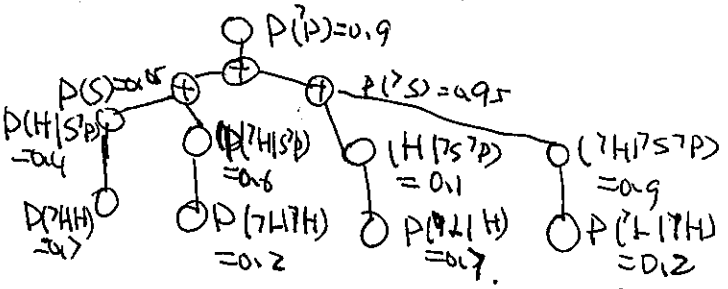
3) $P(Publ \mid Lecture = false) = P(P \mid \neg L) = \sum_H \sum_S P(H S P \mid \neg L) = \sum_H \sum_S (H S P \neg L \mid P(\neg L)) = \alpha \sum_H \sum_S P(H S P \neg L)$

$= \alpha \sum_H \sum_S P(P) P(S) P(H \mid S, P) P(\neg L \mid H) = \alpha \langle 0.1 \times \begin{pmatrix} 0.05 \times 0.95 \times 0.7 + 0.05 \times 0.05 \times 0.3 \\ + 0.95 \times 0.9 \times 0.7 + 0.95 \times 0.1 \times 0.3 \end{pmatrix}, 0.9 \times \begin{pmatrix} 0.05 \times 0.4 \times 0.7 + 0.05 \times 0.6 \times 0.3 \\ + 0.95 \times 0.1 \times 0.7 + 0.95 \times 0.9 \times 0.3 \end{pmatrix} \rangle$

$= \alpha \langle 0.065125, 0.231757 \rangle = \langle 0.219368\ldots, 0.780631\ldots \rangle \approx \langle 0.2194, 0.7806 \rangle$

~~4) $P(Publ \mid Lecture = false, Doctor = true) = P(P \mid \neg L, D) =$~~





4) $P(Publ \mid Lecture = false, Doctor = true) = P(P \mid \neg L, D) = \sum_S \sum_H P(D S H \mid \neg L, D) = \sum_S \sum_H P(P, S, H \mid \neg L, D) \mid P(\neg L, D)$

$= \alpha \sum_S \sum_H P(P, S, H, \neg L, D) = \alpha \langle 0.1 \times \begin{pmatrix} 0.05 \times 0.95 \times 0.6 \times 0.7 + 0.05 \times 0.05 \times 0.6 \times 0.3 \\ + 0.95 \times 0.4 \times 0.1 \times 0.7 + 0.95 \times 0.6 \times 0.99 \times 0.3 \end{pmatrix}, 0.9 \times \begin{pmatrix} 0.05 \times 0.4 \times 0.6 \times 0.7 + 0.05 \times \\ 0.6 \times 0.99 \times 0.1 + 0.95 \times 0.1 \times 0.6 \\ \times 0.7 + 0.95 \times 0.9 \times 0.99 \times 0.3 \end{pmatrix} \rangle$

$= \alpha \langle 0.0292905, 0.201177 \rangle = \text{Normalize } \alpha \langle 0.0292905, 0.201177 \rangle$

$= \langle 0.127095\ldots, 0.872904\ldots \rangle \approx \langle 0.1271, 0.8729 \rangle$.

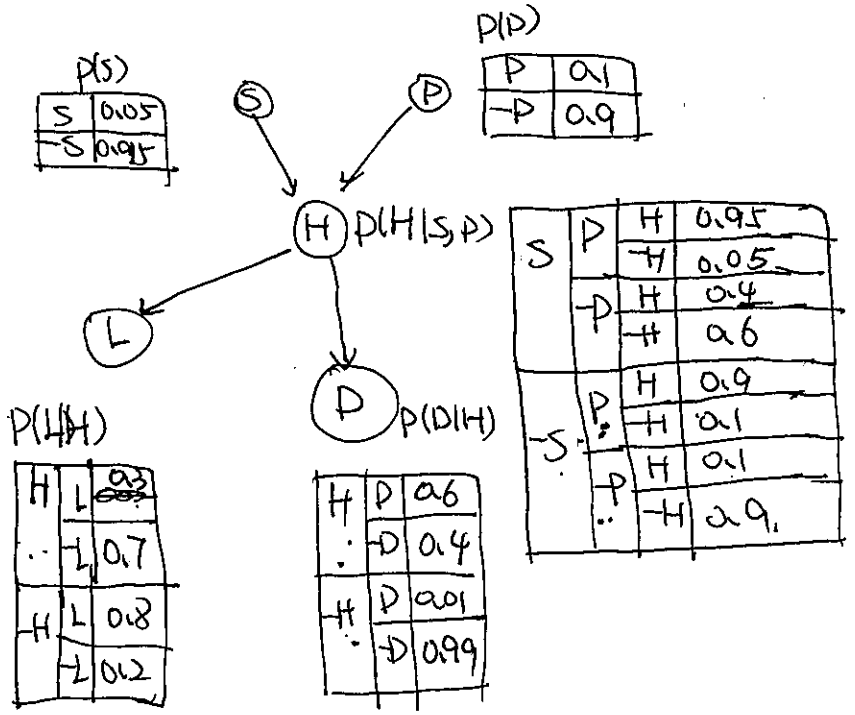
2) Approximate Results: With Rejection Sampling

Random 10 samples

s	p	h	l	d
54	60	39	63	90
5	83	42	1	56
59	99	2	19	31
25	46	22	50	84
94	59	52	16	52
86	6	43	97	24
73	5	93	55	53
11	77	3	70	56
91	84	56	46	18
15	36	38	50	30

Random 20 samples

	s	p	h	l	d
1	7	6	11	75	25
2	38	91	48	44	54
3	39	91	90	72	46
4	45	73	38	15	71
5	25	54	4	82	90
6	86	56	28	98	83
7	93	4	3	28	53
8	100	67	68	77	60
9	3	94	32	21	37
10	100	55	86	17	35
11	65	28	96	18	45
12	61	1	93	86	19
13	89	40	59	79	76
14	45	59	85	76	61
15	4	43	45	13	34
16	43	67	60	32	17
17	31	63	81	13	3
18	66	45	62	54	99



Random 10 samples: $(-S,-P,-H,L,D)$, $(S,-P,-H,-L,-D)$, $(-S,-P,H,L,-D)$

$SPS(x_1 \cdots x_n) = P(x_1 \cdots x_n)$ $(-S,-P,-H,L,-D)$, $(-S,P,H,-L,-D)$; $(-S,-P,H,-L,D)$

$\lim_{N\to\infty} \frac{NPS(x_1 \cdots x_n)}{N} = SPS(x_1 \cdots x_n)$ $(-S,P,H,-L,D)$; $(-S,-P,H,-L,D)$, $(-S,-P,H,L,-D)$

$= P(x_1 \cdots x_n)$ $(-S,-P,-H,L,-D)$.

Random 20 samples: $(-S,P,H,-L,-D)$, $(-S,-P,-H,L,-D)$, $(-S,-P,-H,L,-D)$

$(-S,-P,-H,-L,-D)$, $(-S,-P,H,-L,D)$, $(-S,-P,-H,L,-D)$ $(-S,P,-H,L,-D)$

$(-S,-P,-H,L,-D)$, $(S,-P,H,L,-D)$, $(-S,-P,-H,-L,-D)$, $(-S,-P,-H,-L,-D)$

$(-S,P,H,-L,D)$, $(-S,-P,-H,L,-D)$, $(-S,-P,-H,L,-D)$ $(-S,-P,-H,L,-D)$.

For $P(Pub|lecture=false)$, 10 sample rejection just remain: $(S,-P,-H,-L,-D)$

$(-S,P,H,-L,-D)$, $(-S,-P,H,-L,D)$, $(-S,P,H,-L,D)$, $(-S,-P,H,-L,D)$

inside $P(Pub=true|lecture=false) = \frac{2}{5} = 0.4$ ∴ $\hat{P}(x|e) = \alpha NPS(x,e) = \frac{NPS(x,e)}{NPS(e)}$

$\alpha \frac{P(x,e)}{P(e)} = P(x|e)$. ∴ the approximate results is $\langle 0.4, 0.6\rangle$ the exact is $\langle 0.22, 0.78\rangle$

20 sample: $(-S,P,H,-L,-D)$, $(-S,-P,-H,-L,-D)$, $(-S,-P,H,-L,-D)$, $(-S,-P,H,-L,D)$, $(-S,-P,-H,-L,-D)$

$(-S,P,H,-L,D)$, $(-S,-P,-H,-L,-D)$, ~~$\hat{P}(x|e) = P(Pub=true|lecture=false) = \frac{2}{7} \approx 0.286$~~

∴ the approximate result is $\langle 0.286, 0.714\rangle$

For (Pub | Lecture=false, Doctor=true) 10 sample : $(-S,-P,H,-L,D)$, $(-S,P,H,-L,D)$

$(-S,-P,H,-L,D)$ $\hat{P}(Pub=true \mid Lecture=false, Doctor=true) = \frac{1}{3} \approx 0.33$

∴the approximate result is $\langle 0.33, 0.67 \rangle$ the exact result is $\langle 0.1271, 0.8729 \rangle$.

20 sample : $(-S,-P,H,-L,D)$ $(-S,P,H,-L,D)$

$\hat{P}(Pub=true \mid Lecture=false, Doctor=true) = 0.5$

the approximate result is $\langle 0.5, 0.5 \rangle$.

The problem is refuse too much sample with the evidence variable incesed.

the Question 4 not have the good result with rejection sampling.

the exact results: $P(S \mid \neg L, D) = \langle 0.104, 0.896 \rangle$

$P(S \mid \neg D) = \langle 0.041, 0.959 \rangle$

$P(P \mid \neg L) = \langle 0.219, 0.781 \rangle$

$P(P \mid \neg L, D) = \langle 0.127, 0.873 \rangle$

the approximate results: $P(P \mid \neg L)$ (10 sample) $= \langle 0.4, 0.6 \rangle$

$P(P \mid \neg L)$ (20 sample) $= \langle 0.286, 0.714 \rangle$

$P(P \mid \neg L, D)$ (10 sample) $= \langle 0.33, 0.67 \rangle$

$P(P \mid \neg L, D)$ (20 sample) $= \langle 0.5, 0.5 \rangle$