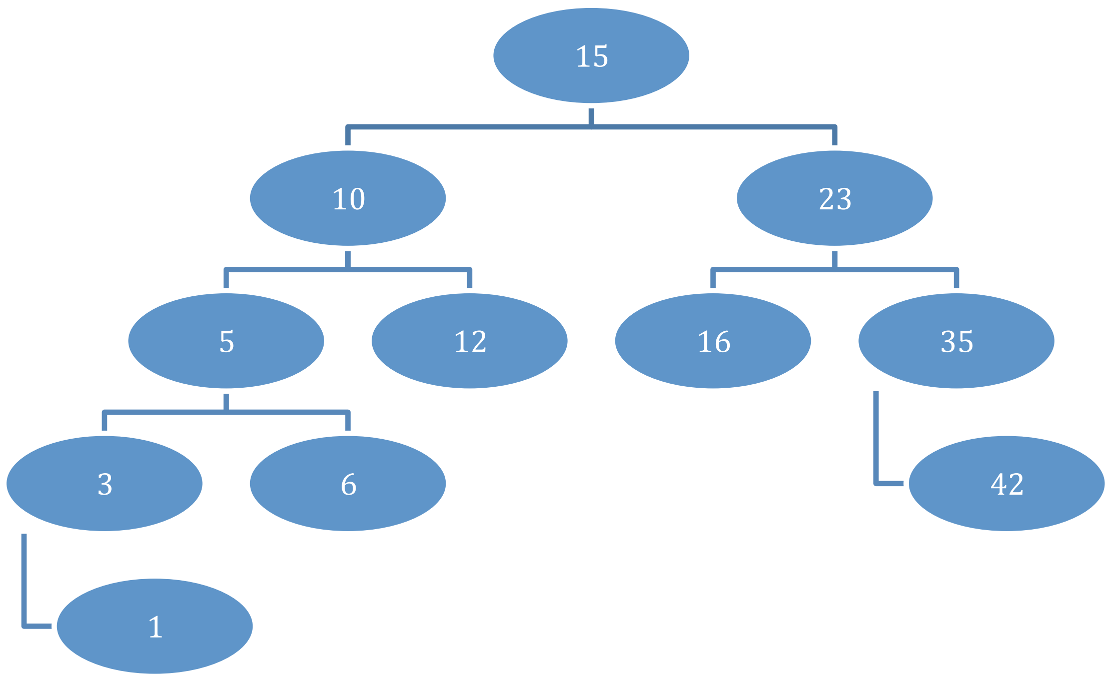
CS532 Homework 2
Archana Machireddy

Question 1

The tree I created is:



1 is the left child of 3, and 42 is the right child of 35.

```
t = hw2.Tree()
t.root = hw2.Node(15)
t.Insert(hw2.Node(10))
t.Insert(hw2.Node(5))
t.Insert(hw2.Node(6))
t.Insert(hw2.Node(12))
t.Insert(hw2.Node(3))
t.Insert(hw2.Node(1))
t.Insert(hw2.Node(23))
t.Insert(hw2.Node(35))
t.Insert(hw2.Node(42))
t.Insert(hw2.Node(16))

print('In-order walk at root node:')
print(t.root.InOrderWalk())

print('In-order walk at node 5:')
print(t.root.left.left.InOrderWalk())
```

```python
print('Print node 16:')
print(t.root.right.left.key)

print('Search for node 10 in the tree from root:')
print(t.root.Search(10).key)
print('Search for node 6 in the tree from node 10:')
print(t.root.left.Search(6).key)

print('Minimum key in the entire tree:')
print(t.root.Min().key)
print('Minimum key in the tree under node 23:')
print(t.root.right.Min().key)
print('Minimum key in the tree under node 6:')
print(t.root.left.left.right.Min().key)

print('Maximum key in the entire tree:')
print(t.root.Max().key)
print('Successor to the root is:')
print(t.root.Succ().key)
print('Successor to node 6 is:')
print(t.root.left.left.right.Succ().key)
print(t.root.Search(6).Succ().key)

print('Deleting node 23:')
print('Before deletion : %s' %(t.root.InOrderWalk()))
t.Delete(t.root.Search(23))
print('After deletion : %s' %(t.root.InOrderWalk()))

print('Height of the entire tree')
print(t.root.Height(t.root.Search(15)))
print('Height of the tree from node 5')
print(t.root.Height(t.root.Search(5)))

print('Deleting node 5:')
print('Before deletion using str function: %s' %(str(t.root)))
t.Delete(t.root.Search(5))
print('After deletion using str function: %s' %(str(t.root)))
```

```
Archanas-MacBook-Pro:HW2 archana$ python test_hw2.py
In-order walk at root node:
135610121516233542
In-order walk at node 5:
1356
Print node 16:
16
Search for node 10 in the tree from root:
10
Search for node 6 in the tree from node 10:
6
Minimum key in the entire tree:
1
Minimum key in the tree under node 23:
16
Minimum key in the tree under node 6:
6
Maximum key in the entire tree:
42
Successor to the root is:
16
Successor to node 6 is:
10
10
Deleting node 23:
Before deletion : 135610121516233542
After deletion : 1356101215163542
Height of the entire tree
5
Height of the tree from node 5
3
Deleting node 5:
Before deletion using str function: ((((1)3)5(6))10(12))15((16)35(42))
After deletion using str function: ((((1)3)6)10(12))15((16)35(42))
```

The delete function provided does not work when the node to the right of the node to be deleted is a leaf node. For example the given code does not work to delete node 5 or 10 in the above tree. Line 42 of the given code is '`y.right.parent = y`'. While deleting 5, y is 6, which make y.right as None. So accessing y.right.parent gives an error as there is no parent to None. For this reason I added in the check '`if y.right is not None:`'. When y is already None, its child's parents need not be set to None.

Modified delete code:

```python
def Delete(self,z):
    if z.left is None:
        self.Transplant(z,z.right)
    elif z.right is None:
```

```
        self.Transplant(z,z.left)
    else:
        y = z.right.Min()
        if y.parent is not None:
            self.Transplant(y,y.right)
            y.right = z.right
            if y.right is not None:
                y.right.parent = y
        self.Transplant(z,y)
        y.left = z.left
        y.left.parent = y
```

Question 2

```
def __str__(self):
    s = ""
    if self.left is not None:
        s += '('
        s += str(self.left)
        s += ')'
    s += str(self.key)
    if self.right is not None:
        s += '('
        s += str(self.right)
        s += ')'
    return s
```

Question 3

```
def Insert_new(self,z):
    y = None
    x = self.root

    if x is None:
        self.root = z

    while x is not None:
        y = x
        if z.key < x.key:
            x = x.left
        else:
            x = x.right
    z.parent = y
    if z.key < y.key:
        y.left = z
```

```
    else:
        y.right = z
```

Question 4

Inserting them in ascending order will result in a tree with all elements strictly on the rightmost branch as each inserted element will the greater than all the ones inserted so far. (Order : 1,2,3,4,5,6,7)

Inserting them in descending order will result in a tree with all elements strictly on the leftmost branch as each inserted element will the lesser than all the ones inserted so far. (Order : 7,6,5,4,3,2,1)

To get a strictly balanced tree, first the middle element needs to be inserted (4). The elements remaining on the left are 1,2 and 3. The middle element among these, i.e. 2, needs to be inserted next, followed by 1 and 3. Similarly on the right side, first 6 needs to be inserted followed by 5 and 7. (Order : 4,2,6,3,5,1,7) This is because of the property of the binary search tree that the key at a given node is greater than all the keys to its left and smaller than all the keys to its right. Inserting in this order will give a strictly balance tree of height 3 on both sides.

```
Archanas-MBP:HW2 archana$ python test_hw2.py
Inserting in ascending order: 1(2(3(4(5(6(7))))))
Inserting in descending order: ((((((1)2)3)4)5)6)7
Inserting in balanced order: ((1)2(3))4((5)6(7))
```

Question 5

```
def Height(self,x):
    if x is None:
        return 0
    else:
        left_height = self.Height(x.left)
        right_height = self.Height(x.right)

    return 1 + max(left_height,right_height)
```

As we are visiting each node in the tree once, a tree with n nodes will have time complexity O(n). As we are going through all the nodes the lower bound is $\Omega(n)$ and upper bound is O(n), therefore it is $\Theta(n)$ too.

Question 6

```
def BuildTree1023():
    numbers = random.sample(range(1,1024), 1023)
```

```
    t = hw2.Tree()
    for i in range(len(numbers)):
        t.Insert(hw2.Node(numbers[i]))
    return t.root.Height(t.root)

def BuildTrees():
    height = np.zeros(1000)
    for i in range(1000):
        height[i] = BuildTree1023()
    print('Average height of the tree is %f'%np.mean(height))
```

```
Archanas-MacBook-Pro:HW2 archana$ python test_hw2.py
Average height of the tree is 22.011000
```

The smallest height it can have is 10.
In a tree with $n=2^m-1$ nodes, the minimum height of the binary tree will be $h(n) = m = \log_2(n)$.

The average height differs from the optimal height by 10 units as the insertion of nodes is random, and will not give the perfect balanced tree.