CS532 Homework 5 - Critique
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Question 1

In b(i,j,w), i and j are the indices referring to a start and end position, and w is the maximum allowed weight. The function gives the best value for items i through j, having weight at most w.

Question 2

The base case is whether or not to include the first item, which has index 0. If the weight of the first item(w_0) is more than the maximum allowed weight(w), it cannot be included, and therefore the best value for items having weight at most w is still 0. If the weight of the first item is less than or equal to the maximum allowed weight, it can be included, and therefore the best value for items having weight at most w is now equal to the value of the first item v_0.

Question 3

If there are n items, and S is the subset of items resulting in best value having weight at most w, if item i was removed from these items, the remaining items must be the best set of items weighting at most $w – w_i$ that can be picked from the original n-1 items, excluding item i.

In order to obtain the best subset from a list on n items, for each item we have to make the choice of including or excluding it. The value of best subset of items (S_i) that has total weight w is either:

1. The value of best subset of S_{i-1} that has total weight w (first term: b(0,i-1,w)) (i.e. exclude the item), or
2. The value of best subset of S_{i-1} that has total weight $w – w_i$ plus the value of item i v_i (second term: b(0,i-1, $w – w_i$)+ v_i) (i.e. include the item)

We take the max as we want the best overall value for the items. If including the item gives a higher overall value, we go with second choice else we exclude the item.

Question 4

At each step in recursion there are two choices, include the item or exclude the item from the list of best combination of items. For each choice there is one sub-problem to solve.

Question 5

The two core requirements of dynamic programming are optimal substructure and overlapping sub-problems. A problem exhibits optimal substructure, if an optimal solution to the problem contains within it the optimal solution to its sub-problems. This problem exhibits the optimal sub-structure property. If there are n items, and S is the subset of items resulting in best value having weight at most w, if item i is removed from this subset, the remaining items must be the best set of items weighting at most w – w_i that can be picked from the original n-1 items, excluding item i.

While solving for subsets of the original items, we calculate the best value for a few subsets repeatedly; this gives rise to overlapping sub-problems. Therefore, 0-1 knapsack problem can be solved using dynamic programming.

Question 6

The sub-problems need to be ordered. In bottom-up we solve smaller problems and use the results of the smaller problems to compute results of bigger problems. Here for each item i, we use the values calculated excluding that item for different weights and these values would have already been filled by the time we go to item i. So we need to solve the smaller problems first to easily calculate solutions for bigger problems.

Question 7

Nested list is a simple data structure to store the intermediate results. Code to initialize it to all zeros:

```
cost = [[0 for i in range(w+1)] for j in V]
```

Question 7 Critique

I called two-dimensional array as a nested list. I didn’t mention that we use only a 2D array, as the first term is always going to be 0. The second term varies form 0 to one less than the number of items, and the weight parameter varies from 0 to the maximum weight of the knapsack.

Question 8

```
def knapsack1(W,V,w):
    cost = [[0 for i in range(w+1)] for j in V]
    for i in range(len(V)):
        for j in range(w+1):
            if i == 0:
                if W[i] <= j:
                    cost[i][j] = V[i]
                else:
                    cost[i][j] = 0
            elif W[i] <= j:
                cost[i][j] = max(cost[i-1][j],cost[i-1][j-W[i]]+V[i])
            else:
                cost[i][j] = cost[i-1][j]
    return cost[i][j]
```

Question 8 Critique

I was returning cost[i][j] as in my code by the end they would have reach the final cell. But specifying the final cell explicitly is better (`cost[len(V)-1][w]`). I put the i == 0 case inside the loop. I should have left it out side the loop as we are just going over it once for each j, instead of it checking it now for every i and j.

```
def knapsack1(W,V,w):
    cost = [[0 for i in range(w+1)] for j in V]
    for j in range(w+1):
        cost[0][j] = 0
        if W[0] <= j:
            cost[0][j] = V[0]
        for i in range(1,len(V)):
            cost[i][j] = cost[i-1][j]
            if W[i] <= j:
                cost_inc = cost[i-1][j-W[i]]+V[i]
                if cost_inc > cost[i][j]:
                    cost[i][j] = cost_inc
    return cost[len(V)-1][w]
```

Question 9

Solution 1: Backtracking from the calculated cost matrix

```
def knapsack2(W,V,w): #1
    cost = [[0 for i in range(w+1)] for j in V]
    for i in range(len(V)):
        for j in range(w+1):
            if i == 0:
                if W[i] <= j:
                    cost[i][j] = V[i]
                else:
                    cost[i][j] = 0
            else:
                cost[i][j] = cost[i-1][j]
                if W[i] <= j:
                    cost_inc = cost[i-1][j-W[i]]+V[i]
                    if cost_inc > cost[i][j]:
                        cost[i][j] = cost_inc


    cost_final = cost[len(V)-1][w]
    items = []
    cur_w = w
    for i in range(len(V),-1,-1):
        if i == 0:
            if cur_w >= W[i]:
                items.append(i)
            break
        if cost_final != cost[i-1][cur_w]:
            items.append(i)
            cost_final = cost_final - V[i]
            cur_w = cur_w - W[i]
    return (cost[len(V)-1][w],items)
```

Solution 2: Using a nested list to store if the item is included in the best solution and backtracking on this list

```
def knapsack2(W,V,w):#2
    cost = [[0 for i in range(w+1)] for j in V]
    items = [[0 for i in range(w+1)] for j in V]
    for i in range(len(V)):
        for j in range(w+1):
            if i == 0:
                if W[i] <= j:
                    cost[i][j] = V[i]
                    items[i][j] = 1
                else:
                    cost[i][j] = 0
            else:
                cost[i][j] = cost[i-1][j]
                if W[i] <= j:
                    cost_inc = cost[i-1][j-W[i]]+V[i]
                    if cost_inc > cost[i][j]:
                        cost[i][j] = cost_inc
                        items[i][j] = 1
    cost_final = cost[len(V)-1][w]
    final_items = []
    cur_w = w
    for i in range(len(V)-1,-1,-1):
        if items[i][cur_w] == 1:
            final_items.append(i)
            cur_w = cur_w - W[i]
    return (cost[len(V)-1][w],final_items)
```

Question 9 Critique

Updating the list in place is a good way to arrive at the solution (include[i][w] = include[i-1][w-w_array[i]] + [i]). I was just marking the items included in each iteration and then tracing back.

Question 10

```
def knapsack3_sub(W,V,w): # Core top down code
    n = len(V)-1
    if cache[(n,w)][0] != -1:
        return cache[(n,w)]

    if n == 0:
        if W[n] <= w:
            cache[(n,w)] = (V[n],0)
            return cache[(n,w)]
        else:
            cache[(n,w)] = (0,0)
            return cache[(n,w)]
    if W[n] > w:
        cache[(n,w)] = (knapsack3(W[:n],V[:n],w),0)
        return cache[(n,w)]
    else:
        x1 = knapsack3(W[:n],V[:n],w)[0]
        x2 = V[n] + knapsack3(W[:n],V[:n],w-W[n])[0]
        if x2 > x1:
            cache[(n,w)] = (x2,1)
        else:
            cache[(n,w)] = (x1,0)
        return cache[(n,w)]


def knapsack3(W,V,w): # Code to return best value and list of indices
```

```
of items included
    best,seq = knapsack3_sub(W,V,w)
    cost_final = cache[len(V)-1,w][0]
    final_items = []
    cur_w = w
    for i in range(len(V)-1,-1,-1):
        if cache[i,cur_w][1] == 1:
            final_items.append(i)
            cur_w = cur_w - W[i]
    return (best,final_items)
```

Question 10 Critique

I am caching both the best value and the best list of items.

Question 11

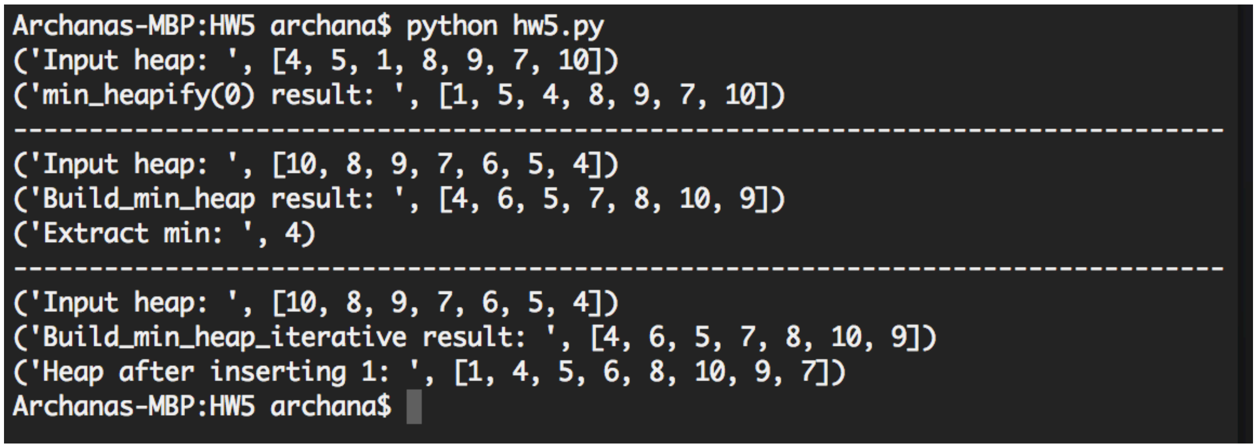
```
def parent(self,i):
    return int((i-1)//2)

def left(self,i):
    return 2*i+1

def right(self,i):
    return 2*i+2
```

Question 12

```
def min_heapify(self,i):
    l = self.left(i)
    r = self.right(i)
    if l <= self.heap_size-1 and self.A[l] < self.A[i]:
        smallest = l
    else:
        smallest = i
    if r <= self.heap_size-1 and self.A[r] < self.A[smallest]:
        smallest = r
    if smallest != i:
        w = self.A[i]
        self.A[i] = self.A[smallest]
        self.A[smallest] = w
        self.min_heapify(smallest)

def build_min_heap(self):
    self.heap_size = self.length
    for i in range((self.length//2)-1,-1,-1):
        self.min_heapify(i)

def heap_extract_min(self):
    if self.heap_size < 1:
        print("heap overflow")
    min = self.A[0]
    self.A[0] = self.A[self.heap_size-1]
    del self.A[self.heap_size-1]
    self.heap_size = self.heap_size-1
    self.min_heapify(0)
    return min
```

Code to test the above functions:

```
b=[4,5,1,8,9,7,10]
h1=heap(b)
print('Input heap: ', b)
h1.min_heapify(0)
print('min_heapify(0) result: ',b)

print('-'*80)

b=[10, 8, 9, 7, 6, 5, 4]
h1=heap(b)
print('Input heap: ', b)
h1.build_min_heap()
print('Build_min_heap result: ',b)
m=h1.heap_extract_min()
print('Extract min: ',m)

print('-'*80)

b=[10, 8, 9, 7, 6, 5, 4]
h1=heap(b)
print('Input heap: ', b)
h1.build_min_heap_iterative()
print('Build_min_heap_iterative result: ',b)

h1.min_heap_insert(1)
print('Heap after inserting 1: ',b)
```



```
Archanas-MBP:HW5 archana$ python hw5.py
('Input heap: ', [4, 5, 1, 8, 9, 7, 10])
('min_heapify(0) result: ', [1, 5, 4, 8, 9, 7, 10])
--------------------------------------------------------------------------------
('Input heap: ', [10, 8, 9, 7, 6, 5, 4])
('Build_min_heap result: ', [4, 6, 5, 7, 8, 10, 9])
('Extract min: ', 4)
--------------------------------------------------------------------------------
('Input heap: ', [10, 8, 9, 7, 6, 5, 4])
('Build_min_heap_iterative result: ', [4, 6, 5, 7, 8, 10, 9])
('Heap after inserting 1: ', [1, 4, 5, 6, 8, 10, 9, 7])
Archanas-MBP:HW5 archana$
```

Question 12 Critique

I didn't initialize the heap properly. I didn't define the heap with a maximum length. I directly gave a list as an input. As my initialization was different, I cannot set the array initially the way it is done in the solutions.

Question 13

```python
def min_heapify_iterative(self,i):
    while i <= self.heap_size:
        l = self.left(i)
        r = self.right(i)
        if  l <= self.heap_size-1 and self.A[l] < self.A[i]:
            smallest = l
        else:
            smallest = i
        if r <= self.heap_size-1 and self.A[r] < self.A[smallest]:
            smallest = r
        if smallest != i:
            w = self.A[i]
            self.A[i] = self.A[smallest]
            self.A[smallest] = w
            i = smallest
        else:
            break
```

Question 13 Critique

I didn't include build_min_heap_iterative in the pdf, but it is there in the hw5.py file.

Question 14

```
def min_heap_insert(self,key):
    self.heap_size = self.heap_size + 1
    self.A.append(-float('Inf'))
    if key < self.A[self.heap_size-1]:
        print('New key smaller than current key')
    self.A[self.heap_size-1] = key
    i = self.heap_size-1
    while i > 0 and self.A[self.parent(i)] > self.A[i]:
        w = self.A[i]
        self.A[i] = self.A[self.parent(i)]
        self.A[self.parent(i)] = w
        i = self.parent(i)
```

Question 14 Critique

For insert we only need to traverse the path from the newly inserted node towards the root, to find its correct position. For this we need to repeatedly compare the present key and its parent and exchange if it is smaller than the parent and continue else terminate. But by doing so I didn't follow the condition that the solution must be similar to question 13. I also didn't initialize the heap with a maximum length. So didn't check if the heap size had reached the maximum. I was just appending the list. I should have appended the list with +Inf and checked if the key was greater than –Inf.