Homework 5

Due 09/13/16

September 8, 2016

1. Use the formal definition of Big-Oh to prove the extension of the Envelopment Property of Addition to more than two functions; that is, if $f_1(n)$, $f_2(n)$, ..., $f_x(n)$, and $g_1(n)$, ..., $g_x(n)$ are functions of n such that $g_1(n) = O(f_1(n))$, $g_2(n) = O(f_2(n))$, etc., then

$$\sum_{i=1}^{x} g_i(n) = O\left(\sum_{i=1}^{x} f_i(n)\right),$$

   for all $x \geq 2$.

2. Use the properties of Big-Theta presented in class (*not the formal definition*) to prove that if $f(n) = 561n\lg(n)+17.9n\sqrt{n}+1024$, $g(n) = \Theta(f(n))$, and $h(n) = O(f(n))$, then $g(n)h(n) = O(n^3)$. You may assume that $\ln n = O(\sqrt{n})$ and $n\sqrt{n} = \Omega(1)$. *Hint*: start by proving that $f(n) = \Theta(n\sqrt{n})$.