Homework 12

Due 10/14/16

October 11, 2016

Stirling Numbers of the Second Kind, represented as $S(n, k)$, count (among other things) the number of ways to divide n objects into exactly k nonempty piles. For example, there are 7 ways to divide four objects into two piles:

$$\begin{array}{cc}
\{1,2,3\} & \{4\} \\
\{1,2,4\} & \{3\} \\
\{1,3,4\} & \{2\} \\
\{2,3,4\} & \{1\} \\
\{1,2\} & \{3,4\} \\
\{1,3\} & \{2,4\} \\
\{1,4\} & \{2,3\}
\end{array}$$

Stirling Numbers of the Second Kind can be calculated using the following recurrence relation:

$$S(n,k) = \begin{cases} 1, & \text{if } k = 1 \text{ or } k = n \\ kS(n-1,k) + S(n-1,k-1), & \text{otherwise} \end{cases}$$

Use this recurrence to answer the questions below. If you are interested why this recurrence is true, you may see the footnote[1] below; *however*, you do not need to understand why this works in order to solve the problem.

1. Describe a naïve recursive algorithm that computes $S(n, k)$ using this recurrence.

2. What map implementation would you use for a memoized dynamic programming solution to this problem and why?

3. Describe a memoized dynamic programming solution to this problem.

[1] When adding one element to go from $n-1$ to n, the new element can either go in a pile by itself or be added to an existing pile. There are $S(n-1, k-1)$ ways to put these $n-1$ elements into $k-1$ piles, and we can create the k^{th} pile by putting the new element by itself. On the other hand, there are $S(n-1, k)$ ways to put the $n-1$ elements into k piles, and we can add the new element to any of the k piles. There's only one way to put everything into one pile, and the only way to put n elements into n piles is to put everything in its own pile.