

1. Given $a_n = 2a_{n-1} + 2^n$ where $a_0 = 0$; for $n > 0$. Prove $a_n = n2^n$ for $n \geq 0$.

a. B/C: If $a_n = 2a_{n-1} + 2^n = n2^n$, then a_0 must equal 0.

$$a_0 = 0 \times 2^0$$

$$a_0 = 0 \times 1$$

$$a_0 = 0$$

Proven.

b. I/S: Let $n = k$ and, therefore, $k \geq 0$. Suppose $a_k = 2a_{k-1} + 2^k = k2^k$.

$$a_{k+1} = 2a_k + 2^{k+1}$$

$$a_{k+1} = 2 \times k2^k + 2^{k+1}$$

$$a_{k+1} = k2^{k+1} + 2^{k+1}$$

$$a_{k+1} = (k+1)2^{k+1}$$

Proven for $k+1$.

2. Given $b_n = 4b_{n-2}$ where $b_0 = 1$ and $b_1 = 6$. Prove $b_n = 2^{n+1} - (-2)^n$ for $n \geq 0$.

a. B/C: If $b_n = 4b_{n-2} = 2^{n+1} - (-2)^n$, then b_0 must equal 1 and b_1 must equal 6.

$$b_0 = 2^{0+1} - (-2)^0$$

$$b_0 = 2^1 - (-2)^0$$

$$b_0 = 2 - 1$$

$$b_0 = 1$$

Proven.

$$b_1 = 2^{1+1} - (-2)^1$$

$$b_1 = 2^2 - (-2)^1$$

$$b_1 = 4 - -2$$

$$b_1 = 6$$

Proven.

b. I/S: Let $n = k$ and $k \geq 0$. Suppose $b_n = 4b_{n-2} = 2^{n+1} - (-2)^n$ for $n = 0, 1, \dots, k$.

$$b_k = 4b_{k-2}$$

$$b_k = 4 \times (2^{k-2+1} - (-2)^{k-2})$$

$$b_k = 2^2 \times (2^{k-1} - (-2)^{k-2})$$

$$b_k = 2^{k-1+2} - (-2)^{k-2+2}$$

$$b_k = 2^{k+1} - (-2)^k$$

Proven for k .

$$b_{k+1} = 4b_{k+1-2}$$

$$b_{k+1} = 4b_{k-1}$$

$$b_{k+1} = 4 \times (2^{k-1+1} - (-2)^{k-1})$$

$$b_{k+1} = 2^2 \times (2^k - (-2)^{k-1})$$

$$b_{k+1} = 2^{k+2} - (-2)^{k-1+2}$$

$$b_{k+1} = 2^{k+2} - (-2)^{k+1}$$

Proven for $k+1$.