Homework 18 sample solution

Due 11/18/16

November 15, 2016

The algorithm below is a reduction that uses the solution to the Bandersnatch (BS) problem to solve the JubJub (JJ) problem. You may assume that this reduction is correct.

```
   Input: data: array of positive integers
   Input: n: size of data
   Output: JubJub(data)
 1 Algorithm: JubJubReduction
 2 cap = max(data)
 3 for i = 1 to n do
 4 |   x = min(data)
 5 |   if x > cap then
 6 |   |   return false
 7 |   end
 8 |   Bandersnatch(data)
 9 end
10 return true
```

Suppose we know that BS has a worst-case complexity that is bounded above by $O(B(n))$ and below by $\Omega(b(n))$, while the worst-case complexity of JJ is known to be $O(J(n))$ and $\Omega(j(n))$, where $B(n)$, $b(n)$, $J(n)$, and $j(n)$ are all $\Omega(n^{10})$. Answer the following questions about the JubJubReduction algorithm.

1. What is the worst-case time complexity of JubJubReduction?

   JubJubReduction takes $O(nB(n)$ time. The call to Bandersnatch in line 8, takes $O(B(n))$ time, which dominates the $\Theta(n)$ complexity for line 4 and $\Theta(1)$ complexity for lines 5 and 6. Since each iteration of the for loop takes $O(B(n))$, the total time for the for loop is $O(nB(n))$, which dominates the $\Theta(n)$ complexity of line 2 and $\Theta(1)$ complexity of line 10.

2. Which of the following four statements must be true based on JubJubReduction? Please justify your answer.

   (a) BS is $O(J(n)/n)$.

   (b) BS is $\Omega(j(n)/n)$.

   (c) JJ is $O(nB(n))$.

(d) JJ is $\Omega(nb(n))$.

The correct statements are b and c. JJ must be $O(nB(n))$ because JubJubReduction is an $O(nB(n))$ algorithm that solves JJ; the optimum algorithm cannot be any worse than this. BS must be $\Omega(j(n)/n)$ because if it were less than $j(n)/n$, JubJubReduction would be an algorithm that solves JJ in less than $j(n)$ time, which is impossible based on the complexity of JJ.