

In order to determine whether a given undirected graph is a tree, we must ensure that the two properties of a tree (as mentioned in the problem) are present: acyclicity and connectivity. To detect if there are any cycles in the graph, we can perform a breadth-first search (BFS) or depth-first search (DFS) to iterate over the entire graph. For every visited vertex, we must check all adjacent vertices for two conditions: whether the adjacent vertex has already been visited **and** if the adjacent vertex is **not** the parent of the vertex we are visiting. If both of these conditions are true, then there is a cycle present in the graph and, thus, is not a tree. The above search will also determine the connectivity of the graph, since any vertex that is not visited by a BFS or DFS is not connected to the graph. The pseudo code below performs this determination using a depth-first search. As long as it is implemented using an adjacency list, it will run in $O(n + m)$ time where n is the number of vertices and m is the number of edges. Were it implemented using an adjacency matrix, it would run in $O(n^2)$ time.

Input: v - vertex

Output: *true* if graph G is a tree, *false* otherwise

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1  Algorithm: DetectCycleDFS
2  Mark  $v$  as visited
3  For each vertex  $w$  adjacent to  $v$ 
4      If  $w$  has not yet been visited
5          If DetectCycleDFS( $w$ ) returns true
6              Return true
7          End
8      End
9      Else if  $w$  has been visited and is not the parent  $v$ 
10         Return true
11     End
12 End
13 Return false

```

Input: $G = (V, E)$ - undirected graph to check

Output: *true* if graph G is a tree, *false* otherwise

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1  Algorithm: Tree?
2  Create new array of booleans of size  $V$ 
3  Initialize each bool to false
4  For each unvisited vertex  $v$  in  $V$ 
5      If DetectCycleDFS( $v$ ) returns true
6          Return true
7      End
8  End
9  Return false

```