1. If $f(n) = O\big(g(n)\big)$, then, by the formal definition of Big-Oh, this is only true when there exists positive constants c and n_0 such that $f(n) \leq c \cdot g(n)$ for $n \geq n_0$.

   If $f(n) = \Omega\big(g(n)\big)$, then, by the formal definition of Big-Omega, this is only true when there exists positive constants c and n_0 such that $f(n) \geq c \cdot g(n)$ for $n \geq n_0$.

   Therefore, if both of the above statements are true, then we can conclude that $c_1 \cdot g(n) \leq f(n) \leq c_2 \cdot g(n)$ for $n \geq n_0$.

   This final statement is the definition of Big-Theta, which only holds true if g(n) is both the lower bound (Big-Omega) and upper bound (Big-Oh) of f(n), for some constants c_1 and c_2, respectively. If either of these statements were false, which they are not – as we have shown, then $f(n) \neq \Theta(g(n))$.

2. Let $f_1(n) = \Omega(g_1(n))$ and $f_2(n) = \Omega(g_2(n))$. From this, it follows that there are constants c_1 and c_2, both positive integers, such that $f_1(n) \geq c_1 \cdot g_1(n)$ and $f_2(n) \geq c_2 \cdot g_2(n)$ for all $n \geq n_0$.
   It is inherently true that $f_1(n) + f_2(n) \geq f_1(n) + f_2(n)$.
   It then follows that $f_1(n) + f_2(n) \geq c_1 \cdot g_1(n) + c_2 \cdot g_2(n)$
   $f_1(n) + f_2(n) \geq \text{minimum}(c_1, c_2) \cdot g_1(n) + \text{minimum}(c_1, c_2) \cdot g_2(n)$
   $f_1(n) + f_2(n) \geq \text{minimum}(c_1, c_2) \cdot \big(g_1(n) + g_2(n)\big)$
   $f_1(n) + f_2(n) = \Omega(g_1(n) + g_2(n))$, for $c = \text{minimum}(c_1, c_2)$ and $n \geq n_0$, which is the definition of Big-Omega, as shown below.
   $f(n) = \Omega\big(g(n)\big)$ if $f(n) \geq c \cdot g(n)$