1. <u>Invariant</u>: after iteration i, sum $= \mathrm{i}^2$.
   <u>1st Iteration</u>: Before the first iteration, sum $= 0$. During the first iteration, sum $= 0 + (2 \times 1) - 1 = 1$. Thus, sum $= \mathrm{i}^2$ after iteration 1, since $1^2 = 1$.
   <u>Kth Iteration</u>: Suppose that sum $= \mathrm{k}^2$ after the kth iteration. In the $(\mathrm{k}+1)^{\mathrm{th}}$ iteration, sum $= \mathrm{k}^2 + 2 \times (\mathrm{k}+1) - 1 = \mathrm{k}^2 + 2\mathrm{k} + 1$. This polynomial may be factored into $(\mathrm{k}+1) \times (\mathrm{k}+1)$, or $(\mathrm{k}+1)^2$, which proves that sum $= \mathrm{i}^2$ for the $(\mathrm{k}+1)^{\mathrm{th}}$ iteration.
   When $\mathrm{n} = 0$, sum $= 0$ is returned and is correct, since $0^2 = 0$. For all other cases, by the loop invariant, sum $= \mathrm{i}^2$ after every iteration of the for loop, so sum $= \mathrm{n}^2$ after the nth iteration, and thus this function returns n^2 whenever the loop iterates.

2. <u>B/C</u>: When $\mathrm{n} = 0$, QuickPow returns 1 which is correct, since QuickPow outputs 2^{n} and $2^0 = 1$.
   <u>I/S</u>: Given some particular but arbitrary value x in n such that $\mathrm{x} \geq 0$, if P(x) is true for all $x \leq k$, $\mathrm{QuickPow(k)} = 2^{\mathrm{k}}$ must be true. QuickPow(k) must, for any value of k – either even or odd – return 2^{k}.
   If k is <u>even</u>, then $\mathrm{t} = \mathrm{QuickPow}\left(\frac{\mathrm{k}}{2}\right)$ which, under assumption of the induction hypothesis, is equal to $2^{\frac{\mathrm{k}}{2}}$. If $\mathrm{t} = 2^{\frac{\mathrm{k}}{2}} = \sqrt[2]{2^{\mathrm{k}}}$, QuickPow returns $\mathrm{t}^2 = \left(\sqrt[2]{2^{\mathrm{k}}}\right)^2 = 2^{\mathrm{k}}$.
   If k is <u>odd</u>, then $\mathrm{t} = \mathrm{QuickPow}\left(\frac{\mathrm{k}-1}{2}\right)$ which, under assumption of the induction hypothesis, is equal to $2^{\frac{\mathrm{k}-1}{2}}$. If $\mathrm{t} = 2^{\frac{\mathrm{k}-1}{2}} = \sqrt[2]{2^{\mathrm{k}-1}}$, QuickPow returns $2\mathrm{t}^2 = 2 \times (\sqrt[2]{2^{\mathrm{k}-1}})^2 = 2 \times 2^{\mathrm{k}-1} = 2^{\mathrm{k}}$.

   Given $\mathrm{QuickPow(k)} = 2^{\mathrm{k}}$, it must follow that $\mathrm{QuickPow(k+1)} = 2^{\mathrm{k}+1}$, for some $\mathrm{k} \geq x$.
   If $\mathrm{k}+1$ is <u>even</u>, then $\mathrm{t} = \mathrm{QuickPow}\left(\frac{\mathrm{k}+1}{2}\right)$ which, under assumption of the induction hypothesis, is equal to $2^{\frac{\mathrm{k}+1}{2}}$. If $\mathrm{t} = 2^{\frac{\mathrm{k}+1}{2}} = \sqrt[2]{2^{\mathrm{k}+1}}$, QuickPow returns $\mathrm{t}^2 = \left(\sqrt[2]{2^{\mathrm{k}+1}}\right)^2 = 2^{\mathrm{k}+1}$.
   If $\mathrm{k}+1$ is <u>odd</u>, then $\mathrm{t} = \mathrm{QuickPow}\left(\frac{\mathrm{k}+1-1}{2}\right)$ which, under assumption of the induction hypothesis, is equal to $2^{\frac{\mathrm{k}}{2}}$. If $\mathrm{t} = 2^{\frac{\mathrm{k}}{2}} = \sqrt[2]{2^{\mathrm{k}}}$, QuickPow returns $2\mathrm{t}^2 = 2 \times (\sqrt[2]{2^{\mathrm{k}}})^2 = 2 \times 2^{\mathrm{k}} = 2^{\mathrm{k}+1}$.

   Therefore, for all values x in n such that $\mathrm{x} \geq 0$, $\mathrm{QuickPow(x)} = 2^{\mathrm{x}}$ since for all values k and $\mathrm{k}+1$ in n, QuickPow returns 2^{k} and $2^{\mathrm{k}+1}$, respectively.