Homework 19

Due 11/23/16

November 17, 2016

1. The Subset Sum Decision problem (SSD) accepts an array of integers $data$ and target t and returns whether $data$ contains some subset that sums up to t. Prove that $SSD \in NP$.

2. The Subset Sum problem (SS) accepts the same input as SSD, but it returns a subset that sums to t rather than true or false. The following algorithm describes a reduction from SS to SSD:

```
Input: $data$: array of integers
Input: $n$: size of $data$
Input: $t$: target value
Output: $S$: subset of $data$ that sums to $t$, or $\emptyset$ if no such set exists
Algorithm: SSReduction
if $\neg SSD(data, t)$ then
    return $\emptyset$
end
$sub = 0$
$top = n$
while $sub < top$ do
    if $\neg SSD(data[1..(top - 1)], t)$ then
        /* Add $data[top]$ to subset */
        $sub = sub + 1$
        Swap $data[top]$ and $data[sub]$
    else
        /* Delete $data[top]$ from array */
        $top = top - 1$
    end
end
return $data[1..sub]$
```

What do we know about the worst-case complexity of SS if the worst-case complexity of SSD is $O(S(n))$ and $\Omega(s(n))$?