

A large, light gray Fibonacci spiral is centered on the page, starting from a small square in the upper right quadrant and expanding outwards. The spiral is composed of a series of quarter-circles connected by straight lines, creating a smooth, logarithmic curve. The text of the document is centered within the upper half of the spiral.

*Homework 4*

# COMPUTER AIDED VERIFICATION

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## QUESTION I

Apply the resolution algorithm to the following formula in CNF form to show that it is satisfiable. (The subscripts to the clauses are just for ease of reference.

$$(\neg a \vee \neg b)_1 \wedge (a \vee c)_2 \wedge (b \vee c)_3 \wedge (a \vee \neg b \vee \neg d)_4 \wedge (b \vee d)_5 \wedge (b \vee \neg c \vee \neg d)_6$$

Use the resolution order  $a < b < c < d$ . Derive a satisfying assignment

SOLUTION:

$$S_0 = (\neg a \vee \neg b) \wedge (a \vee c) \wedge (b \vee c) \wedge (a \vee \neg b \vee \neg d) \wedge (b \vee d) \wedge (b \vee \neg c \vee \neg d)$$

$$S_1 = (\neg b \vee c) \wedge (\neg b \vee \neg d) \wedge (b \vee c) \wedge (b \vee d) \wedge (b \vee \neg c \vee \neg d)$$

$$S_2 = (c) \wedge (c \vee d) \wedge (\neg d \vee c) \wedge (\neg c \vee \neg d)$$

$$S_3 = (\neg d) \wedge (\neg d)$$

$$S_4 = \text{True}$$

Assignments

$$\eta_4 = \emptyset$$

$$\eta_3 = \{\neg d\}$$

$$\eta_2 = \{c, \neg d\}$$

$$\eta_1 = \{b, c, \neg d\}$$

$$\eta_0 = \{\neg a, b, c, \neg d\}$$

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## QUESTION 2

Apply the resolution algorithm to the following formula in CNF form to show that it is unsatisfiable.

$$(\neg a \vee \neg b)_1 \wedge (a \vee c)_2 \wedge (b \vee c)_3 \wedge (a \vee \neg b \vee \neg d)_4 \wedge (b \vee d)_5 \wedge (b \vee \neg c \vee \neg d)_6 \wedge (\neg b \vee \neg c \vee d)_7.$$

Use the resolution order  $a < b < c < d$ . Draw a resolution tree that shows how the empty clause may be deduced from the given clauses

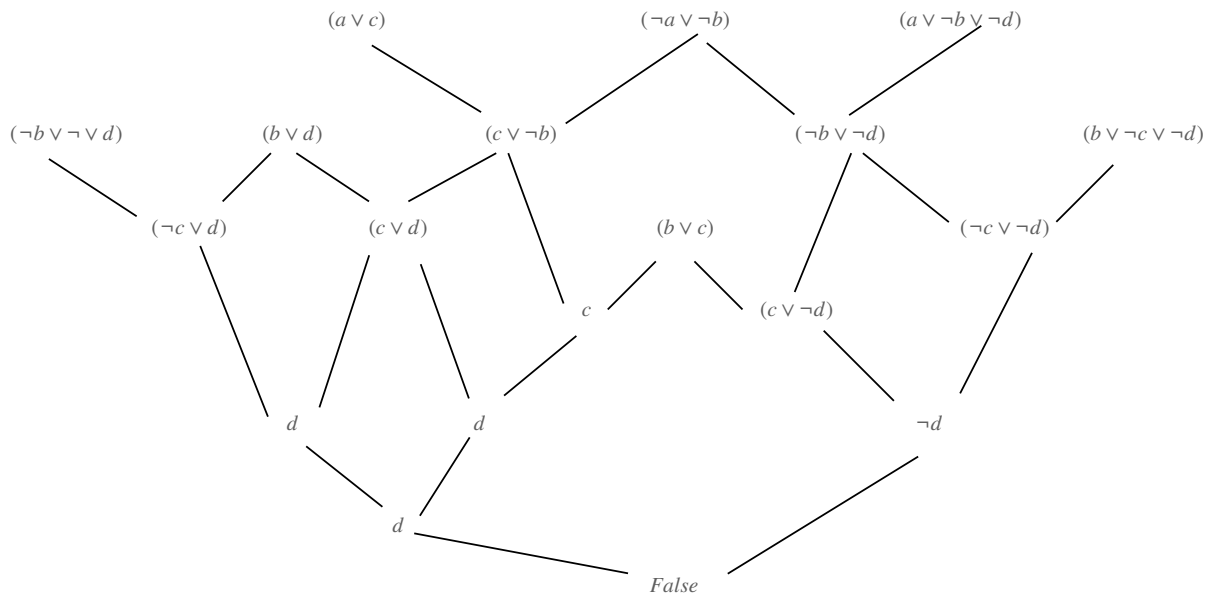
SOLUTION:

$$S_0 = (\neg a \vee \neg b) \wedge (a \vee c) \wedge (b \vee c) \wedge (a \vee \neg b \vee \neg d) \wedge (b \vee d) \wedge (b \vee \neg c \vee \neg d) \wedge (\neg b \vee \neg c \vee d)$$

$$S_1 = (c) \wedge (c \vee d) \wedge (\neg d \vee c) \wedge (\neg c \vee \neg d) \wedge (\neg c \vee d)$$

$$S_2 = (\neg d \wedge d)$$

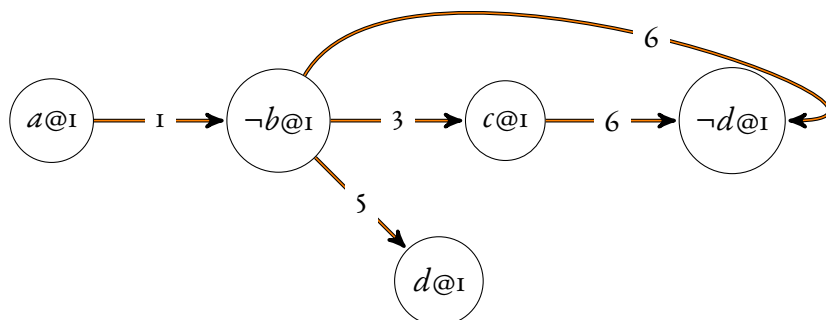
$$S_3 = \text{False}$$



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**QUESTION 3**

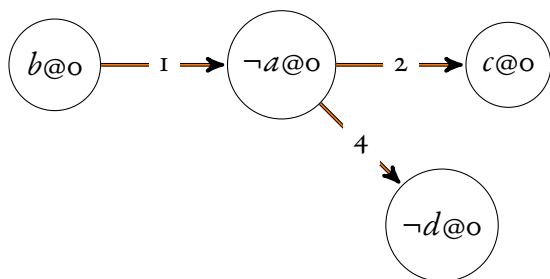
Apply the CDCL procedure to the CNF formula of Problem 1. Assume that procedure CHOOSENEXTASSIGNMENT always assigns true to the first unsigned variable in alphabetic order. Show all the implication graphs produced by the execution of the algorithm and detail the derivation of the conflict-learned clauses.

SOLUTION:



We add  $\gamma_7 = b$

$$(\neg a \vee \neg b)_1 \wedge (a \vee c)_2 \wedge (b \vee c)_3 \wedge (a \vee \neg b \vee \neg d)_4 \wedge (b \vee d)_5 \wedge (b \vee \neg c \vee \neg d)_6 \wedge (b)$$

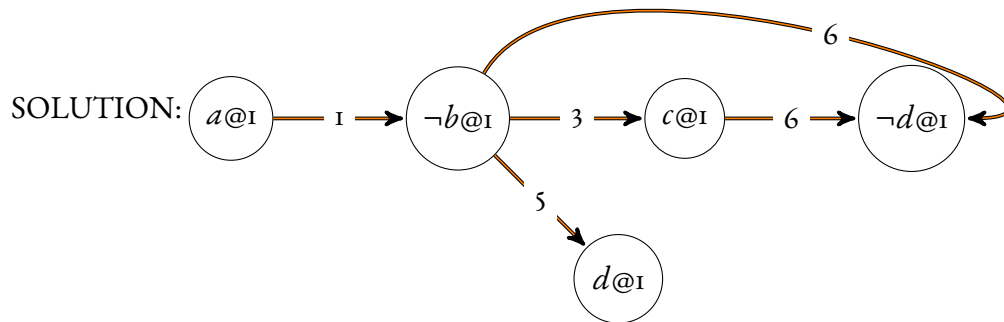


Satisfying assignments of  $\{b, \neg a, c, \neg d\}$

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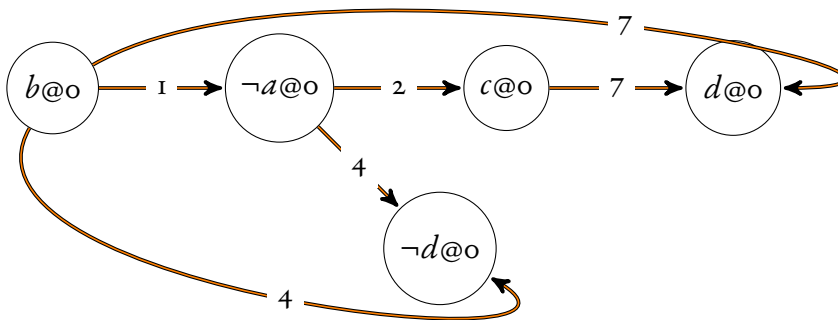
## QUESTION 4

Apply the CDCL procedure to the CNF formula of Problem 2. Assume that procedure CHOOSENEXTASSIGNMENT always assigns true to the first unsigned variable in alphabetic order. Show all the implication graphs produced by the execution of the algorithm and detail the derivation of the conflict-learned clauses. Finally, draw the resolution proof of unsatisfiability derived from the CDCL run.



We add  $\gamma_8 = b$

$$(\neg a \vee \neg b)_1 \wedge (a \vee c)_2 \wedge (b \vee c)_3 \wedge (a \vee \neg b \vee \neg d)_4 \wedge (b \vee d)_5 \wedge (b \vee \neg c \vee \neg d)_6 \wedge (\neg b \vee \neg c \vee d)_7 \wedge (b).$$



Conflict at level 0 means it is unsatisfiable

