Il processo $e^+e^- \to \mu^+\mu^-$ in approssimazione di Born nel Modello Standard

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ABSTRACT

I Introduzione

$$\mathcal{L}_{nc} = \sum_{i=e,\mu} \left\{ \frac{g}{4\cos\theta_W} \bar{\ell}_i \gamma^\mu (4\sin^2\theta_W - 1 + \gamma^5) \ell_i Z_\mu - e\bar{\ell}_i \gamma^\mu \ell_i A_\mu \right\} \tag{1}$$

$$\mathcal{L}_{sf} = - \sum_{i=e,\mu} \left\{ y_i \bar{\psi}_{i,L} \Phi \psi_{i,R} + h.c. \right\} \tag{2}$$

II L'interazione con il campo scalare

$$\mathcal{L}_s = (D_\mu \Phi)^\dagger D^\mu \Phi + \frac{\lambda}{4!} \left(|\Phi|^2 - F^2 \right)^2, \qquad \lambda > 0 \tag{3}$$

$$\Phi = \begin{pmatrix} 0 \\ F \end{pmatrix} + \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} = \begin{pmatrix} 0 \\ F \end{pmatrix} + \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2}\phi_1 \\ H + i\phi^0 \end{pmatrix} \tag{4}$$

$$y_j = \frac{1}{\sqrt{2}} g \frac{m_j}{M_W} \tag{5}$$

$$\begin{aligned} \mathcal{L}_{sf} \Longrightarrow & \frac{y_j}{\sqrt{2}} \begin{pmatrix} \bar{\nu}_j & \bar{\ell}_j \end{pmatrix} \left(\frac{1+\gamma^5}{2} \right) \begin{pmatrix} - \\ \sqrt{2}F + H + i\phi^0 \end{pmatrix} \left(\frac{1+\gamma^5}{2} \right) \ell_j + h.c. \\ = & y_j \bar{\ell}_j F \ell_j + \frac{y_j}{\sqrt{2}} \bar{\ell}_j H \ell_j + i \frac{y_j}{\sqrt{2}} \bar{\ell}_j \gamma^5 \phi^0 \ell_j \end{aligned} \tag{6}$$

$$\frac{m_\mu}{M_W} \simeq 10^{-3} \tag{7}$$

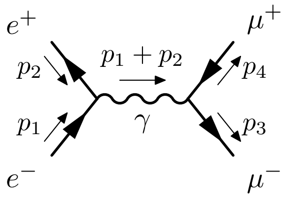
III L'invarianza di gauge dell'elemento di matrice S

$$|\mathcal{M}|^2 = |\mathcal{M}_1 + \mathcal{M}_2 + \mathcal{M}_3 + \mathcal{M}_4|^2 \tag{8}$$

$$iG_F^{\mu\nu}(p) = \frac{-i}{p^2 - M_Z^2} \left\{ g^{\mu\nu} - (1-\xi^2) \frac{p^\mu p^\nu}{p^2 - \xi^2 M_Z^2} \right\}; \qquad i\Delta_F(p) = \frac{i}{p^2 - \xi^2 M_Z^2} \tag{9}$$

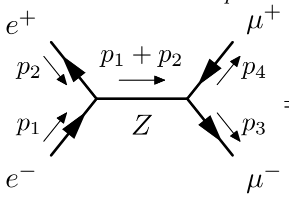
$$\mathcal{M}_1 + \mathcal{M}_2 \Longrightarrow - \left(\frac{g}{4c} \right)^2 \frac{i(1-\xi^2)}{s^2 - M_Z^2} \frac{(p_1+p_2)^\mu (p_3+p_4)^\nu}{s - \xi^2 M_Z^2} \bar{v}(p_1) \gamma_\mu (V + \gamma^5) u(p_2) \bar{u}(p_3) \gamma_\nu (V + \gamma^5) v(p_4) \tag{10}$$

IV La sezione d'urto differenziale



$$\Longrightarrow \mathcal{M}_1 = (-ie)^2\, \bar{v}(p_1)\gamma^\mu u(p_2) \frac{-ig_{\mu\nu}}{(p_1+p_2)^2}\, \bar{u}(p_3)\gamma^\nu v(p_4)$$

$$\frac{1}{4}\sum_{Spin} |\mathcal{M}_1|^2 = \frac{2e^4}{s^2}\{(u - m_e^2 - m_\mu^2)^2 + (t - m_e^2 - m_\mu^2)^2 + 2(m_e^2 + m_\mu^2)s\} \tag{11}$$



$$\Longrightarrow \mathcal{M}_2 = \left(\frac{ig}{4c}\right)^2 \bar{v}(p_1)\gamma^\mu(V+\gamma^5)u(p_2)\frac{-ig_{\mu\nu}}{(p_1+p_2)^2 - M_Z^2}\bar{u}(p_3)\gamma^\nu(V+\gamma^5)v(p_4)$$

$$\begin{aligned}\frac{1}{4}\sum_{Spin}|\mathcal{M}_2|^2 = 2\left(\frac{g}{4c}\right)^4 \frac{1}{(s-M_Z^2)^2}\{&[(V^2+1)^2 + 4V^2](u - m_e^2 - m_\mu^2)^2 \\ &+[(V^2+1)^2 - 4V^2](t - m_e^2 - m_\mu^2)^2 \\ &+2(V^4-1)[(m_\mu^2 + m_e^2)s - 4m_\mu^2 m_e^2] \\ &+8m_\mu^2 m_e^2 (V^2-1)^2\}\end{aligned} \tag{12}$$

$$\begin{aligned}\frac{1}{4}\sum_{Spin}\mathcal{M}_1\mathcal{M}_2^\dagger = 2e^2\left(\frac{g}{4c}\right)^2 \frac{1}{s(s-M_Z^2)}\{&(V^2+1)(u - m_e^2 - m_\mu^2)^2 \\ &+(V^2-1)(t - m_e^2 - m_\mu^2)^2 \\ &+2V^2(m_e^2 + m_\mu^2)s\}\end{aligned} \tag{13}$$

$$\begin{aligned}\frac{d\sigma}{dt} &= (2\pi)^{4-6}\cdot F \cdot \int \mathrm{d}\underset{2\to 2}{PS} \cdot \frac{1}{4}\sum_{Spin}|\mathcal{M}|^2 \\ &= \frac{1}{(2\pi)^2}\cdot\frac{1}{2s^{\frac{1}{2}}(s-4m_e^2)^{\frac{1}{2}}}\cdot\frac{\pi}{2s^{\frac{1}{2}}(s-4m_e^2)^{\frac{1}{2}}}\cdot\frac{1}{4}\sum_{Spin}\left\{|\mathcal{M}_1|^2 + |\mathcal{M}_2|^2 + 2\mathcal{M}_1\mathcal{M}_2^\dagger\right\}\end{aligned} \tag{14}$$

$$\frac{d\sigma}{d\cos\theta} = \frac{(s-4m_e^2)^{\frac{1}{2}}(s-4m_\mu^2)^{\frac{1}{2}}}{2}\frac{d\sigma}{dt} \tag{15}$$

$$\begin{aligned}\frac{d\sigma}{d\cos\theta} = \frac{1}{32\pi s}\left(\frac{s-4m_\mu^2}{s-4m_e^2}\right)^{\frac{1}{2}}&\left\{\frac{e^4}{s^2}\Big\{s^2 + (s-4m_\mu^2)(s-4m_e^2)\cos^2\theta + 4(m_e^2+m_\mu^2)s\Big\}\right. \\ &+\left(\frac{g}{4c}\right)^4\frac{1}{(s-M_Z^2)^2}\Big\{(V^2+1)^2\left[s^2 + (s-4m_e^2)(s-4m_\mu^2)\cos^2\theta\right] + 4V^2 s(s-4m_e^2)^{\frac{1}{2}}(s-4m_\mu^2)^{\frac{1}{2}}\cos\theta \\ &+2(V^4-1)\left[(m_\mu^2+m_e^2)s - 4m_\mu^2 m_e^2\right] + 8m_\mu^2 m_e^2(V^2-1)^2\Big\} \\ &+2e^2\left(\frac{g}{4c}\right)^2\frac{1}{s(s-M_Z^2)}\Big\{V^2\left[s^2 + (s-4m_e^2)(s-4m_\mu^2)\cos^2\theta\right] + 2s(s-4m_e^2)^{\frac{1}{2}}(s-4m_\mu^2)^{\frac{1}{2}}\cos\theta \\ &\left.+4V^2(m_e^2+m_\mu^2)\Big\}\right\}\end{aligned} \tag{16}$$

Riferimenti bibliografici

[1] George Sterman. *An Introduction to Quantum Field Theory*. Cambridge University Press, 1993.

[2] Martinus Veltman. *Diagrammatica: The Path to Feynman Diagrams*. Cambridge University Press, 1994.

[3] *PDG: Particle Listings*. `http://pdg.lbl.gov/2019/listings/contents_listings.html`. [ultima consultazione 11/04/2020].