Il processo $e^+e^- \to \mu^+\mu^-$ in approssimazione di Born nel Modello Standard

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ABSTRACT

I Introduzione

$$\mathcal{L}_{nc} = \sum_{i=e,\mu} \left\{ \frac{g}{4\cos\theta_W} \bar{\ell}_i \gamma^\mu (4\sin^2\theta_W - 1 + \gamma^5)\ell_i Z_\mu - e\bar{\ell}_i\gamma^\mu \ell_i A_\mu \right\} \tag{1}$$

$$\mathcal{L}_{sf} = -\sum_{i=e,\mu} \left\{ y_i \bar{\psi}_{i,L} \Phi \psi_{i,R} + h.c. \right\} \tag{2}$$

II L'interazione con il campo scalare

$$\mathcal{L}_s = (D_\mu\Phi)^\dagger D^\mu\Phi + \frac{\lambda}{4!}\left(|\Phi|^2 - F^2\right)^2, \qquad \lambda > 0 \tag{3}$$

$$\Phi = \begin{pmatrix} 0 \\ F \end{pmatrix} + \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} = \begin{pmatrix} 0 \\ F \end{pmatrix} + \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2}\phi_1 \\ H + i\phi^0 \end{pmatrix} \tag{4}$$

$$y_j = \frac{1}{\sqrt{2}} g \frac{m_j}{M_W} \tag{5}$$

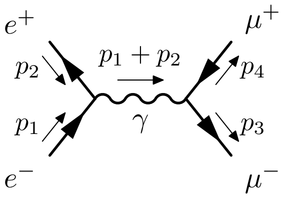
$$\begin{aligned} \mathcal{L}_{sf} \Longrightarrow & \frac{y_j}{\sqrt{2}} \begin{pmatrix} \bar{\nu}_j & \bar{\ell}_j \end{pmatrix} \left(\frac{1+\gamma^5}{2}\right) \begin{pmatrix} - \\ \sqrt{2}F + H + i\phi^0 \end{pmatrix} \left(\frac{1+\gamma^5}{2}\right) \ell_j + h.c. \\ = & y_j \bar{\ell}_j F \ell_j + \frac{y_j}{\sqrt{2}} \bar{\ell}_j H \ell_j + i\frac{y_j}{\sqrt{2}} \bar{\ell}_j \gamma^5 \phi^0 \ell_j \end{aligned} \tag{6}$$

$$\frac{m_\mu}{M_W} \simeq 10^{-3} \tag{7}$$

III L'invarianza di gauge dell'elemento di matrice S

IV La sezione d'urto differenziale

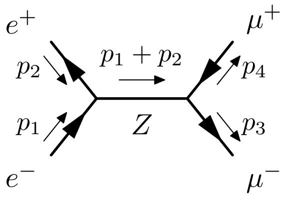
$$|\mathcal{M}|^2 = |\mathcal{M}_1 + \mathcal{M}_2 + \mathcal{M}_3 + \mathcal{M}_4|^2 \tag{8}$$



$$\Longrightarrow \mathcal{M}_1 = (-ie)^2\, \bar{v}(p_1)\gamma^\mu u(p_2) \frac{-ig_{\mu\nu}}{(p_1+p_2)^2} \bar{u}(p_3)\gamma^\nu v(p_4)$$

$$\begin{aligned} \frac{1}{4}\sum_{Spin} |\mathcal{M}_1|^2 = \frac{1}{4} e^4 \frac{1}{(p_1+p_2)^2} & \mathrm{tr}\{(\not{p}_2 + m_e)\gamma^\nu(\not{p}_1 - m_e)\gamma^\mu\} \\ & \times \mathrm{tr}\{(\not{p}_4 - m_\mu)\gamma_\nu(\not{p}_3 + m_\mu)\gamma_\mu\} \end{aligned} \tag{9}$$

$$\frac{1}{4}\sum_{Spin}|\mathcal{M}_1|^2 = \frac{2e^4}{s^2}\{(u-m_e^2-m_\mu^2)^2+(t-m_e^2-m_\mu^2)^2+2(m_e^2+m_\mu^2)s\} \tag{10}$$



$$\Longrightarrow \mathcal{M}_2 = \left(\frac{ig}{4c}\right)^2 \bar{v}(p_1)\gamma^\mu(V+\gamma^5)u(p_2)\frac{-ig_{\mu\nu}}{(p_1+p_2)^2-M_Z^2}\bar{u}(p_3)\gamma^\nu(V+\gamma^5)v(p_4)$$

$$\begin{aligned}\frac{1}{4}\sum_{Spin}|\mathcal{M}_2|^2 = 2\left(\frac{g}{4c}\right)^4\frac{1}{(s-M_Z^2)^2}\{&[(V^2+1)^2+4V^2](u-m_e^2-m_\mu^2)^2\\ &+[(V^2+1)^2-4V^2](t-m_e^2-m_\mu^2)^2\\ &+2(V^4-1)[(m_\mu^2+m_e^2)s-4m_\mu^2m_e^2]\\ &+8m_\mu^2m_e^2(V^2-1)^2\}\end{aligned} \tag{11}$$

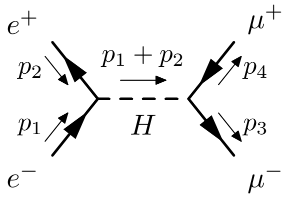
$$\begin{aligned}\frac{1}{4}\sum_{Spin}\mathcal{M}_1\mathcal{M}_2^\dagger = 2e^2\left(\frac{g}{4c}\right)^2\frac{1}{s(s-M_Z^2)}\{&(V^2+1)(u-m_e^2-m_\mu^2)^2\\ &+(V^2-1)(t-m_e^2-m_\mu^2)^2\\ &+2V^2(m_e^2+m_\mu^2)s\}\end{aligned} \tag{12}$$

$$\begin{aligned}\frac{d\sigma}{dt} &= (2\pi)^{4-6}\cdot F\cdot\int \mathrm{d}\underset{2\to2}{PS}\cdot\frac{1}{4}\sum_{Spin}|\mathcal{M}|^2\\ &= \frac{1}{(2\pi)^2}\cdot\frac{1}{2s^{\frac{1}{2}}(s-4m_e^2)^{\frac{1}{2}}}\cdot\frac{\pi}{2s^{\frac{1}{2}}(s-4m_e^2)^{\frac{1}{2}}}\cdot\frac{1}{4}\sum_{Spin}\left\{|\mathcal{M}_1|^2+|\mathcal{M}_2|^2+2\mathcal{M}_1\mathcal{M}_2^\dagger\right\}\end{aligned} \tag{13}$$

$$\frac{d\sigma}{d\cos\theta} = \frac{(s-4m_e^2)^{\frac{1}{2}}(s-4m_\mu^2)^{\frac{1}{2}}}{2}\frac{d\sigma}{dt} \tag{14}$$

$$\begin{aligned}\frac{d\sigma}{d\cos\theta} = \frac{1}{32\pi s}\left(\frac{s-4m_\mu^2}{s-4m_e^2}\right)^{\frac{1}{2}}\Bigg\{&\frac{e^4}{s^2}\left\{s^2+(s-4m_\mu^2)(s-4m_e^2)\cos^2\theta+4(m_e^2+m_\mu^2)s\right\}\\ +\left(\frac{g}{4c}\right)^4&\frac{1}{(s-M_Z^2)^2}\left\{(V^2+1)^2\left[s^2+(s-4m_e^2)(s-4m_\mu^2)\cos^2\theta\right]+4V^2s(s-4m_e^2)^{\frac{1}{2}}(s-4m_\mu^2)^{\frac{1}{2}}\cos\theta\right.\\ &\left.+2(V^4-1)\left[(m_\mu^2+m_e^2)s-4m_\mu^2m_e^2\right]+8m_\mu^2m_e^2(V^2-1)^2\right\}\\ +e^2\left(\frac{g}{4c}\right)^2&\frac{1}{s(s-M_Z^2)}\left\{V^2\left[s^2+(s-4m_e^2)(s-4m_\mu^2)\cos^2\theta\right]+2s(s-4m_e^2)^{\frac{1}{2}}(s-4m_\mu^2)^{\frac{1}{2}}\cos\theta\right.\\ &\left.+4V^2(m_e^2+m_\mu^2)\right\}\Bigg\}\end{aligned} \tag{15}$$

V L'ampiezza del diagramma con linea interna di Higgs



$$\Longrightarrow \mathcal{M}_H = \left(\frac{ig}{2}\right)^2 \frac{m_e m_\mu}{M_W^2} \bar{v}(p_1) u(p_2) \frac{-i}{(p_1+p_2)^2 - M_H^2} \bar{u}(p_3) v(p_4)$$

$$\frac{1}{4}\sum_{Spin} |\mathcal{M}_H|^2 = \left(\frac{ig}{2}\right)^4 \left(\frac{m_e m_\mu}{M_W^2}\right)^2 \frac{1}{(s-M_H^2)^2} \{(s-2m_e^2)(s-2m_\mu^2) \\ -2(m_e^2+m_\mu^2)s + 8m_e^2 m_\mu^2\} \tag{16}$$

Riferimenti bibliografici

[1] George Sterman. *An Introduction to Quantum Field Theory*. Cambridge University Press, 1993.

[2] Martinus Veltman. *Diagrammatica: The Path to Feynman Diagrams*. Cambridge University Press, 1994.

[3] *PDG: Particle Listings*. `http://pdg.lbl.gov/2019/listings/contents_listings.html`. [ultima consultazione 11/04/2020].