When does a sampled cosinuoid equal 0?

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August 11, 2014

For continuous time t, trivially:

$$y = \cos t = \pm 1, \quad t \in [0, \pm \pi, \pm 2\pi, \dots, \pi m, \quad m \in \mathbb{Z}$$

Likewise for continous time, a cosinusoid with angular frequency ω and time t:

$$y = \cos \omega t = \pm 1, \quad t \in [0, \frac{\pi}{\omega}, \frac{2\pi}{\omega}, \dots, \frac{\pi m}{w}, \quad m \in \mathbb{Z}]$$

By inspection, for continuous time frequency f:

$$y = \cos 2\pi f t = \pm 1, \quad t \in 0, \frac{\pi}{2\pi f}, \frac{2\pi}{2\pi f}, \dots, \frac{\pi m}{2\pi f}, \quad m \in \mathbb{Z}$$

The same result is realized in discrete time with sampling period T_s at time samples n:

$$y = \cos 2\pi f t = \pm 1, \quad n = \text{nint} \left[\frac{\pi}{2\pi f T_s} \right] m, \quad m \in \mathbb{Z}$$

where $nint[\cdot]$ means round to the nearest integer.

That is, the output $y = \pm 1$ in discrete time every n samples.

To account for a phase shift ϕ , a similar argument leads to the final desired result: