When does a sampled cosinuoid equal 0?

Michael Hirsch, MSEE, Ph.D. EE candidate

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For continuous time t, trivially:

$$y = \cos t = \pm 1, \quad t \in 0, \pm\pi, \pm 2\pi, \ldots, \pi m, \quad m \in \mathbb{Z}$$

Likewise for continous time, a cosinusoid with angular frequency ω and time t:

$$y = \cos \omega t = \pm 1, \quad t \in 0, \frac{\pi}{\omega}, \frac{2\pi}{\omega}, \ldots, \frac{\pi m}{w}, \quad m \in \mathbb{Z}$$

By inspection, for continuous time frequency f:

$$y = \cos 2\pi f t = \pm 1, \quad t \in 0, \frac{\pi}{2\pi f}, \frac{2\pi}{2\pi f}, \ldots, \frac{\pi m}{2\pi f}, \quad m \in \mathbb{Z}$$

The same result is realized in discrete time with sampling period T_s at time samples n:

$$y = \cos 2\pi f t = \pm 1, \quad n = \text{round}\left[\frac{\pi}{2\pi f T_s}\right] m, \quad m \in \mathbb{Z}$$

where round$[\cdot]$ means round to the nearest integer. That is, the discrete time output $y = \pm 1$ at the sample indices in the vector n. To account for a phase shift ϕ, a similar argument leads to the final desired result:

$$n = \text{round}\left[\frac{\pi}{2\pi f T_s} + \left|\frac{\phi}{2\pi f T_s}\right|\right] \tag{1}$$

In Matlab, (1) is implemented via the code:

```
sampinterval = pi./(2*pi*f*Ts);
sampoffset = abs(phase./(2*pi*f*Ts)) + 1
n = round(sampinterval:sampoffset:ns)
```

where the `+1` accounts for the one-based Matlab indexing, and `ns` is the total number of samples.