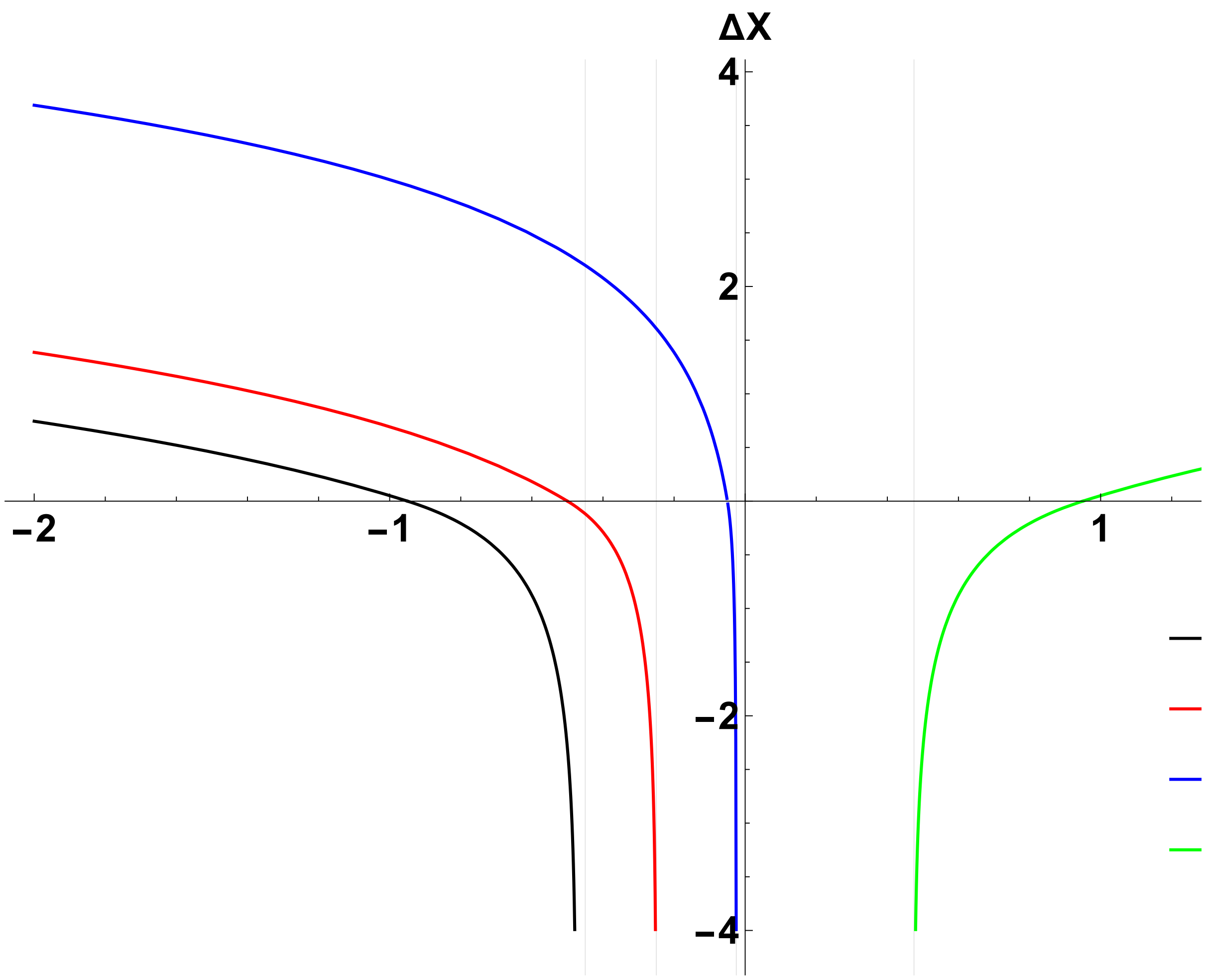
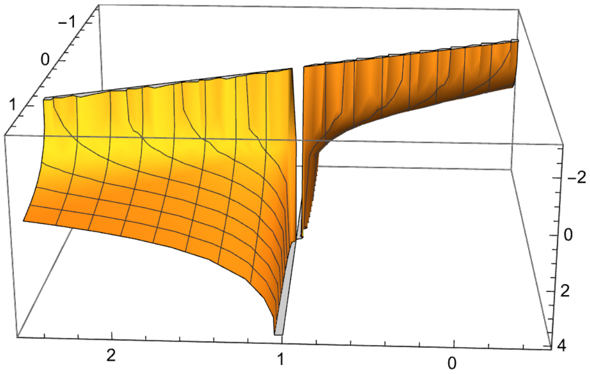
In[ ]:=

```
Plot[
 Evaluate[Sign[r] * Log[1 - Abs[r]] /. r → (s - 1) / x - 1 /. s → {0.05, 0.5, 0.95, 1.95}],
 {x, -2, 2}, AxesLabel → {"χi", "ΔX"},
 PlotStyle → {Black, Red, Blue, Green}, LabelStyle → {20, Bold}, GridLines →
  {{{(0.1 - 1) / 2, {Dashed, Black, Thick}}, {(0.5 - 1) / 2, {Red, Dashed, Thick}},
    {(0.95 - 1) / 2, {Blue, Dashed, Thick}}, {(1.95 - 1) / 2, {Green, Dashed, Thick}}},
   None}, PlotLegends → Placed[{"Si=0.05", "Si=0.5", "Si=0.95", "Si=1.95"},
   Scaled[{0.85, 0.25}]], ImageSize → {800, 600}]
```

Out[ ]=



In[ ]:=
```
Plot3D[Sign[r] * Log[1 - Abs[r]] /. r → (s - 1) / x - 1, {x, -1.5, 1.5}, {s, -0.5, 2.5}]
```
Out[ ]=



In[ ]:=
```
L = 100;
tabS = {-1.5, -0.5, 0.25, 0.5, 0.75, 1.05};
sol1neg = Table[NDSolve[{
     D[n[x, t], t] == D[n[x, t], {x, 2}] - n[x, t] + UnitStep[(Sign[S] * n[x, t] - S)],
     n[x, 0] == 1.25 * UnitStep[-x], (D[n[x, t], x] /. x → -L / 2) == 0,
     (D[n[x, t], x] /. x → L / 2) == 0}, {n}, {x, -L / 2, L / 2}, {t, 0, 5}, AccuracyGoal → 2,
    Method → {"PDEDiscretization" → {"MethodOfLines", "TemporalVariable" → t,
        "SpatialDiscretization" → {"TensorProductGrid", "MinPoints" → 2^8},
        "DifferentiateBoundaryConditions" → True}, "TimeIntegration" → Automatic},
    MaxSteps → 1000, MaxStepSize → Automatic, StartingStepSize → Automatic], {S,
    Take[tabS, 2]}];
sol1pos = Table[NDSolve[{
     D[n[x, t], t] == D[n[x, t], {x, 2}] - n[x, t] + UnitStep[(Sign[S] * n[x, t] - S)],
     n[x, 0] == 1.25 * UnitStep[-x], (D[n[x, t], x] /. x → -L / 2) == 0,
     (D[n[x, t], x] /. x → L / 2) == 0}, {n}, {x, -L / 2, L / 2}, {t, 0, 50}, AccuracyGoal → 3,
    Method → {"PDEDiscretization" → {"MethodOfLines", "TemporalVariable" → t,
        "SpatialDiscretization" → {"TensorProductGrid", "MinPoints" → 2^8},
        "DifferentiateBoundaryConditions" → True}, "TimeIntegration" → Automatic},
    MaxSteps → Automatic, MaxStepSize → Automatic, StartingStepSize → Automatic], {S,
    Take[tabS, -4]}];
```

NDSolve: Using maximum number of grid points 10000 allowed by the MaxPoints or MinStepSize options for independent variable x.

NDSolve: Using maximum number of grid points 10000 allowed by the MaxPoints or MinStepSize options for independent variable x.

NDSolve: Maximum number of 1000 steps reached at the point t == 2.7061057325078015`.

NDSolve: Using maximum number of grid points 10000 allowed by the MaxPoints or MinStepSize options for independent variable x.

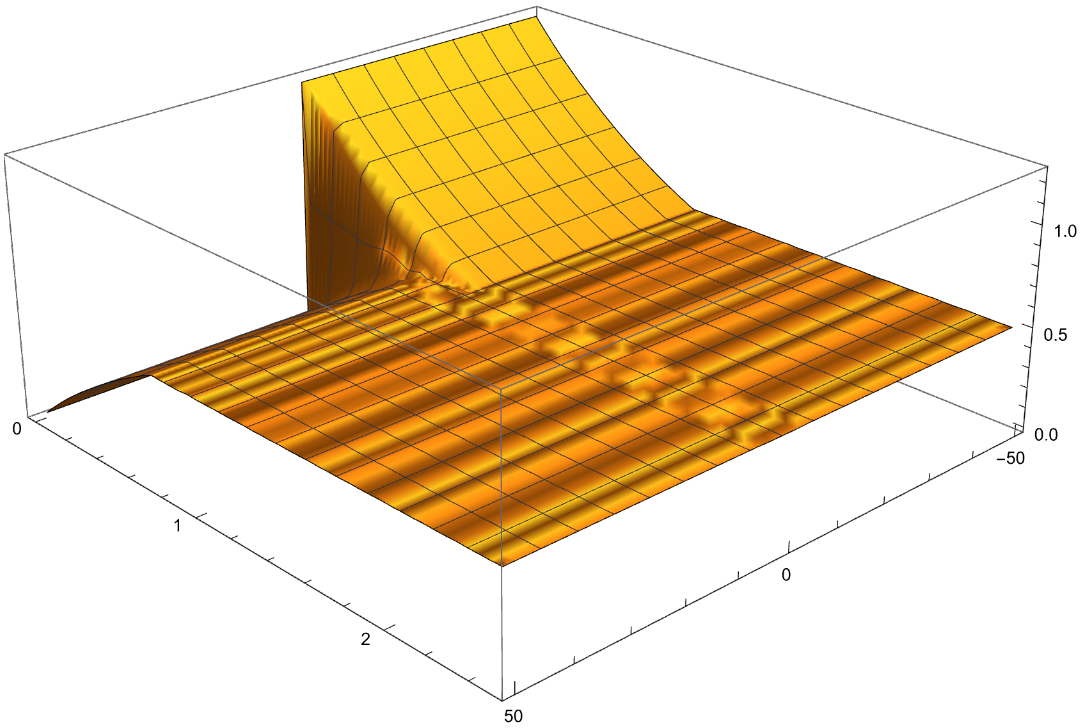
NDSolve: Using maximum number of grid points 10000 allowed by the MaxPoints or MinStepSize options for independent variable x.

NDSolve: Using maximum number of grid points 10000 allowed by the MaxPoints or MinStepSize options for independent variable x.

General: Further output of NDSolve::mxsst will be suppressed during this calculation.

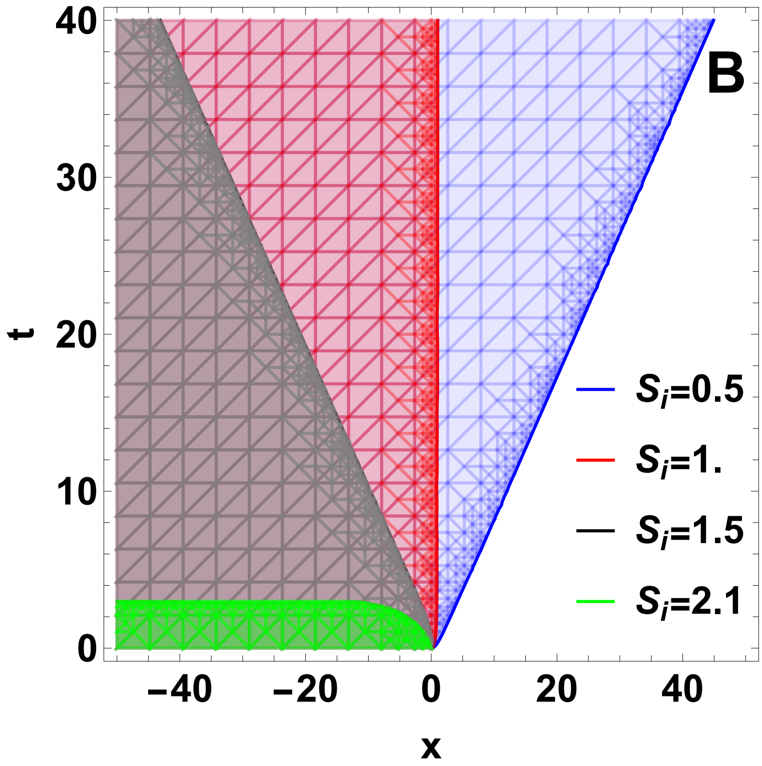
In[ ]:=
```
Plot3D[n[x, t] /. sol1neg[[2]], {x, -L / 2, L / 2},
 {t, 0, 2.5}, PlotRange → All, PlotPoints → 50]
```

Out[ ]=



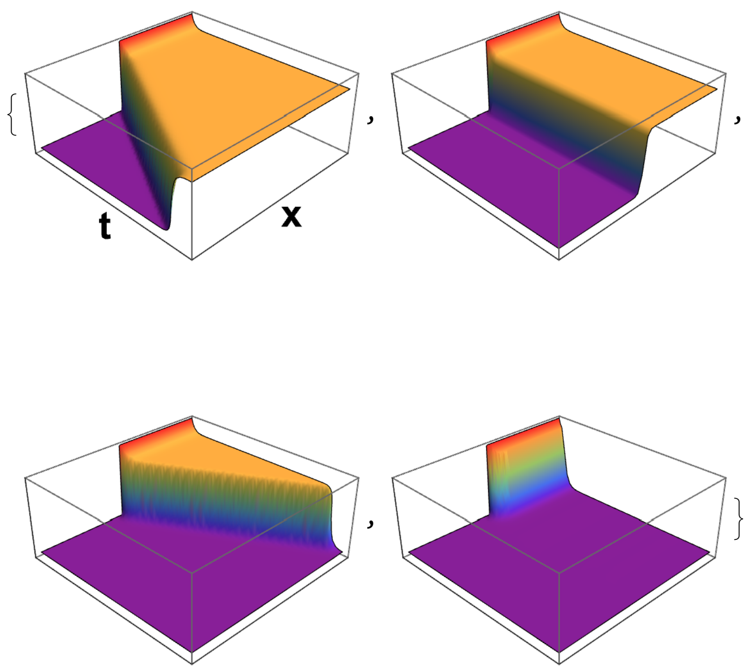
In[ ]:=
```
Show[ContourPlot[Evaluate[Table[n[x, t] == tabS[[i]] /. sol[[i]], {i, 1, 4}]],
  {x, -L / 2, L / 2}, {t, 0, 40}, MaxRecursion → 4, PerformanceGoal → "Quality",
  ContourStyle → {Blue, Red, Black, Green}, FrameLabel → {"x", "t"},
  LabelStyle → {20, Bold}, ImageSize → {400, 400}, PlotLegends →
   Placed[Table[StringForm["Sᵢ=``", 2 * tabS[[i]]], {i, 1, 4}], Scaled[{0.85, 0.25}]],
  (*Epilog→Inset[Plot[n[x,500]/.sol[[2]],{x,-L/2,L/2},AxesLabel→{"x","ψ(x,+∞)"},
     Ticks→{{-40,0,40},{0,1}},PlotStyle→Red,Filling→{1→Bottom},LabelStyle→
      {20,Bold},ImageSize→{120,100}],Scaled[{0.72,0.85}]],*)ContourLabels → None],
 RegionPlot[Evaluate[Table[n[x, t] > tabS[[i]] /. sol[[i]], {i, 1, 4}]],
  {x, -L / 2, L / 2}, {t, 0, 40}, MaxRecursion → 4,
  PerformanceGoal → "Quality", BoundaryStyle → None,
  PlotStyle → {Directive[Blue, Opacity[0.1]], Directive[Red, Opacity[0.2]],
    Directive[Gray, Opacity[0.5]], Directive[Green, Opacity[0.4]]}],
 Epilog → Inset[Style["B", 30, Bold], Scaled[{0.95, 0.9}]]]
```

Out[ ]=

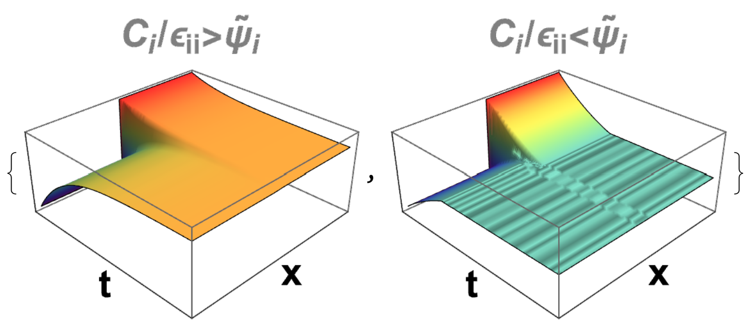


In[ ]:=
```
tab1pos = Table[Plot3D[n[x, t] /. sol1pos⟦k⟧, {x, -L / 2, L / 2},
    {t, 0, 50}, PlotRange → {Automatic, Automatic, {-0.1, 1.25}},
    (*Epilog→Inset[Style[StringForm["S_i=``",2tabS⟦k+2⟧],20,Bold,Gray],{0.5,0.9}],*)
    AxesLabel → {If[k > 1, None, "x"], If[k > 1, None, "t"], (*"ψ(x,t)"*)None},
    PlotPoints → 50, AxesEdge → {{1, -1}, {1, -1}, {1, -1}},
    ViewPoint → Evaluate[{r * Cos[ϕ] * Sin[θ], r * Sin[ϕ] Sin[θ], r * Cos[θ]} /.
        {r → 3, ϕ → 2 π / 8, θ → 4 π / 12}],
    ColorFunction → "Rainbow", LabelStyle → {20, Bold}, Mesh → None,
    Ticks → (*{Automatic,Automatic,{0,1,2}}*)None], {k, 1, 4}]
tab1neg = Table[Plot3D[n[x, t] /. sol1neg⟦k⟧, {x, -L / 2, L / 2}, {t, 0, If[k == 1, 5, 2.5]},
    PlotRange → {Automatic, Automatic, {-0.1, 1.25}}, Mesh → None, Epilog →
     Inset[Style[If[k == 1, "C_i/ϵ_ii>ψ̃_i", "C_i/ϵ_ii<ψ̃_i"], 20, Bold, Gray], {0.5, 0.9}],
    AxesLabel → {"x", "t", (*"ψ(x,t)"*)None}, PlotPoints → 50,
    AxesEdge → {{1, -1}, {1, -1}, {1, -1}},
    ViewPoint → Evaluate[{r * Cos[ϕ] * Sin[θ], r * Sin[ϕ] Sin[θ], r * Cos[θ]} /.
        {r → 3, ϕ → 2 π / 8, θ → 4 π / 12}], ColorFunction → "Rainbow",
    LabelStyle → {20, Bold}, Ticks → (*{Automatic,Automatic,{0,1,2}}*)None], {k, 1, 2}]
```

Out[ ]=

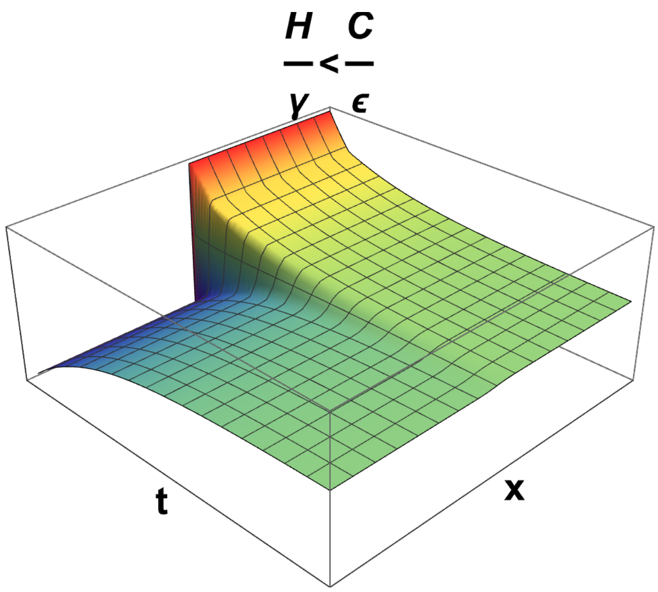


Out[ ]=

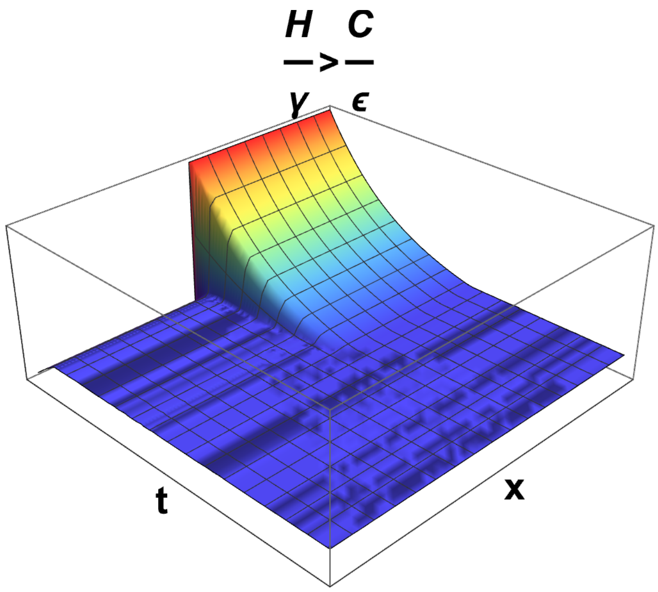


In[ ]:=
```
Plot3D[n[x, t] /. sol[[1]], {x, -L / 2, L / 2},
 {t, 0, 4}, PlotRange → {Automatic, Automatic, {-0.1, 2}},
 Epilog → Inset[Style["H/γ < C/ϵ", 20, Bold], {0.5, 1.0}],
 AxesLabel → {"x", "t", (*"ψ(x,t)"*)None}, PlotPoints → 50,
 AxesEdge → {{1, -1}, {1, -1}, {1, -1}}, ViewPoint → Evaluate[
   {r * Cos[ϕ] * Sin[θ], r * Sin[ϕ] Sin[θ], r * Cos[θ]} /. {r → 3, ϕ → 2 π / 8, θ → 4 π / 12}],
 ColorFunction → "Rainbow", LabelStyle → {20, Bold},
 Ticks → (*{Automatic,Automatic,{0,1,2}}*)None, ImageSize → {400, 300},
 ImagePadding → {{Automatic, Automatic}, {Automatic, 50}}]
Plot3D[n[x, t] /. sol[[2]], {x, -L / 2, L / 2},
 {t, 0, 4}, PlotRange → {Automatic, Automatic, {-0.1, 2}},
 Epilog → Inset[Style["H/γ > C/ϵ", 20, Bold], {0.5, 1.0}],
 AxesLabel → {"x", "t", (*"ψ(x,t)"*)None}, PlotPoints → 50,
 AxesEdge → {{1, -1}, {1, -1}, {1, -1}}, ViewPoint → Evaluate[
   {r * Cos[ϕ] * Sin[θ], r * Sin[ϕ] Sin[θ], r * Cos[θ]} /. {r → 3, ϕ → 2 π / 8, θ → 4 π / 12}],
 ColorFunction → "Rainbow", LabelStyle → {20, Bold},
 Ticks → (*{Automatic,Automatic,{0,1,2}}*)None, ImageSize → {400, 300},
 ImagePadding → {{Automatic, Automatic}, {Automatic, 50}}]
```

Out[ ]=



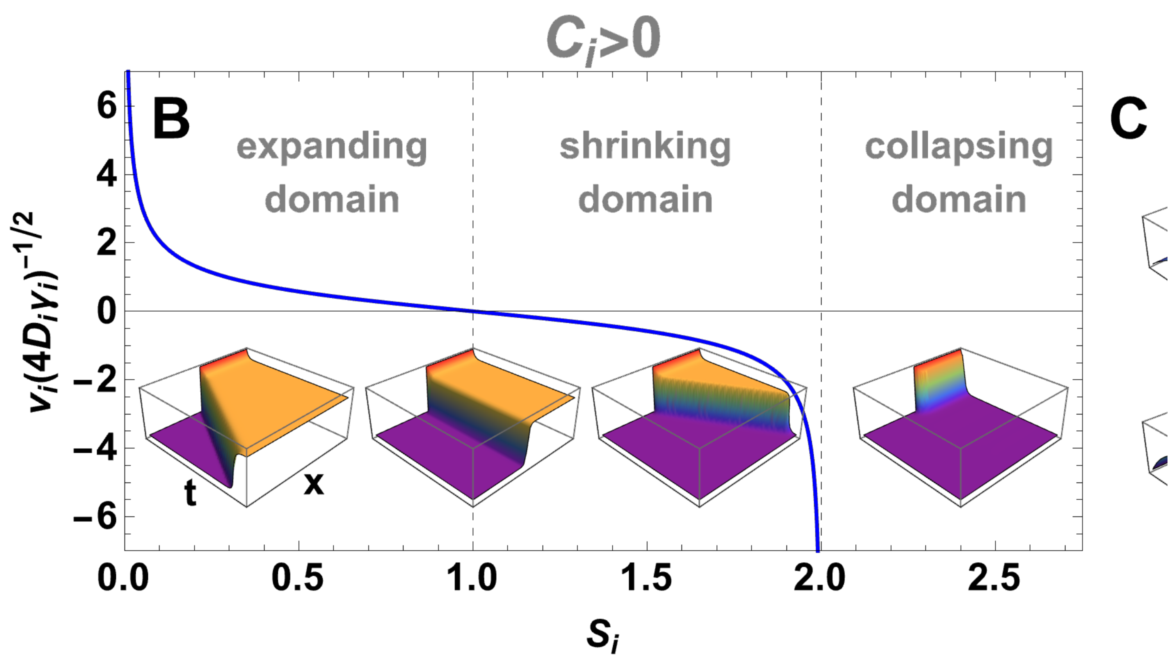
Out[ ]=



In[ ]:=

```
plotV1d = GraphicsRow[
  {Plot[- (S - 1) / Sqrt[(1 - (S - 1) ^2)], {S, 0, 2}, FrameLabel → {"Si", "vi (4Diγi) -1/2"},
    PlotRange → {{0, 2.75}, {-7, 7}}, LabelStyle → {20, Bold}, PlotStyle → {Thick, Blue},
    GridLines → {{1, 2}, {}}, GridLinesStyle → Directive[{Dashed, Thick, Black}],
    PlotRangeClipping → False, Frame → True, AspectRatio → 0.5, Epilog → {
      (*Inset[Graphics[{Dashed,Line[{{0,0},{0,2}}]}],{1.0,4}],*)
      Inset[Style["expanding\ndomain", Bold, 20, Gray], {0.6, 4}],
      Inset[Style["shrinking\ndomain", Bold, 20, Gray], {1.5, 4}],
      Inset[Style["collapsing\ndomain", Bold, 20, Gray], {2.4, 4}],
      (*Inset[Style["C",30,Bold],Scaled[{1,1.03}]],*)
      Inset[Style["B", 30, Bold], Scaled[{0.05, 0.9}]],
      Inset[Style["Ci>0", 30, Bold, Gray], Scaled[{0.5, 1.07}]],
      Inset[tab1pos[[1]], {0.35, -3}, Automatic, {6.5, 6.5}],
      Inset[tab1pos[[2]], {1.0, -3}, Automatic, {6.5, 6.5}],
      Inset[tab1pos[[3]], {1.65, -3}, Automatic, {6.5, 6.5}],
      Inset[tab1pos[[4]], {2.4, -3}, Automatic, {6.5, 6.5}],
      Inset[tab1neg[[1]], {3.23, -4}, Automatic, {6.5, 6.5}],
      Inset[tab1neg[[2]], {3.23, 2}, Automatic, {6.5, 6.5}],
      Inset[Style["Ci<0", 30, Bold, Gray], Scaled[{1.15, 1.07}]],
      Inset[Style["C", 30, Bold, Black], Scaled[{1.05, 0.9}]] (*,
      Arrow[{{0.5,-7},{0.33,-5}}],Arrow[{{1.0,-7},{1.0,-5}}],
      Arrow[{{1.5,-7},{1.67,-5}}],Arrow[{{2.1,-7},{2.4,-5}}]*)},
    ImagePadding → {{60, 145}, {Automatic, 30}}] (*,Plot[,{x,-1,1},AspectRatio→2]*)},
  ImageSize → {750, Automatic}, ImagePadding → {{0, 0}, {0, 0}}]
(*Plot[,{x,-1,1},Axes→False,Epilog→Inset[plotV1d],ImageSize→{900,Automatic}]*)
```

Out[ ]=



In[ ]:=
```
G[x_, t_] := Exp[-0.55 t - x^2 / t / 0.2] / Sqrt[2 π * t];
X[t_] := Exp[-2 t] (0.65) + 0.1 * t - 0.5;
t1 = 4; t2 = 9;
int1plot = Show[
  DensityPlot[G[x - X[t1], t1 - t], {t, 0, 10},
   {x, -1, 1}, FrameLabel → None, LabelStyle → {20, Bold}, FrameTicks →
    {{{{X[0], "Xi(0)"}, {X[t1], "Xi(t1)"}, {X[t2], "Xi(t2)"}, {0.749, "x"}}, None},
     {{0, {t1, "t1"}, {t2, "t2"}, {9.99, "t'"}}, None}},
   PlotRange → {All, {-1, 0.75}, {0, 2.0}}, PlotRangePadding → None,
   RegionFunction → Function[{x, y, z}, x < t1 && y < X[x]],
   ColorFunction → Function[{z}, If[z < 0.5, ColorData[{"SunsetColors", "Reverse"}][
        z / 0.5], ColorData[{"SunsetColors", "Reverse"}][1]]],
   PlotLegends → Placed[LineLegend[{Red, {Black, Dashed}},
      {Style["Xi(t')", Red], "vit'+X̃i"}], {0.2, 0.7}], PlotRangeClipping → True,
   Epilog → {
     {Red, Thick, Dashed, Line[{{t1, -1}, {t1, X[t1]}}]},
     {Red, Thick, Dashed, Line[{{t2, -1}, {t2, X[t2]}}]},
     Inset[Style["Gi(x-Xi(t1),t1-t')", 20, Bold, Orange], {2.5, -0.5}],
     Inset[Style["Gi(x-Xi(t2),t2-t')", 20, Bold, Orange], {7.5, -0.2}],
     Inset[Style["A", Bold, 30], Scaled[{0.04, 0.8}]]}
  ],
  DensityPlot[G[x - X[t2], t2 - t],
   {t, 0, 10}, {x, -1, 1}, PlotRange → {All, All, {0, 2}},
   RegionFunction → Function[{x, y, z}, t1 < x < t2 && y < X[x]],
   ColorFunction →
    Function[{z}, If[z < 0.5, ColorData[{"SunsetColors", "Reverse"}][z / 0.5],
      ColorData[{"SunsetColors", "Reverse"}][1]]]],
  DensityPlot[G[x - X[t2], t2 - t],
   {t, 0, 10}, {x, -1., 1}, PlotRange → {All, All, {0, 2}},
   RegionFunction → Function[{x, y, z}, t1 < x && 1 > y > X[x]],
   ColorFunction →
    Function[{z}, If[z < 0.5, ColorData[{"PigeonTones", "Reverse"}][z / 0.5],
      ColorData[{"PigeonTones", "Reverse"}][1]]]],
  DensityPlot[G[x - X[t1], t1 - t],
   {t, 0, 10}, {x, -1., 1}, PlotRange → {All, All, {0, 2}},
   RegionFunction → Function[{x, y, z}, x < t1 && 1 > y > X[x]],
   ColorFunction →
    Function[{z}, If[z < 0.5, ColorData[{"PigeonTones", "Reverse"}][z / 0.5],
      ColorData[{"PigeonTones", "Reverse"}][1]]]],
  Plot[{X[t], 0.1 t - 0.5}, {t, 0, 10}, PlotStyle → {{Red, Thick}, {Black, Dashed}}],
  PlotRange → {Automatic, {-0.75, 0.75}},
  ImagePadding → {{Automatic, Automatic}, {Automatic, Automatic}},
  AspectRatio → 0.2, ImageSize → {690, Automatic}]
```

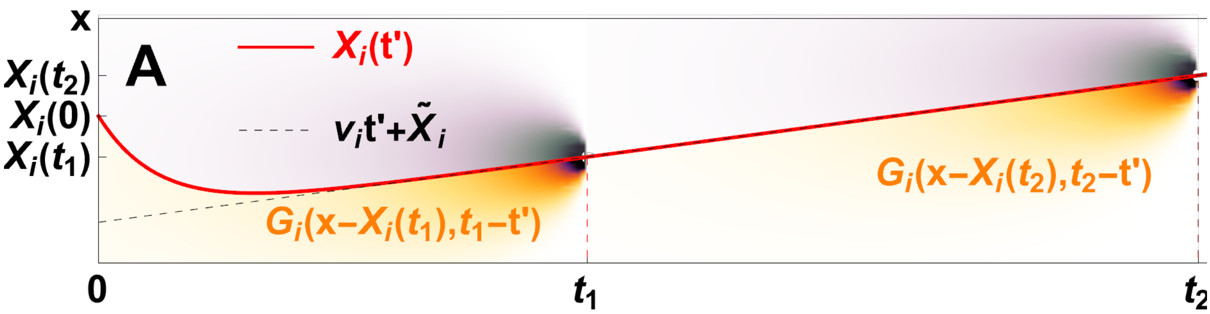
General: Exp[-6355.56] is too small to represent as a normalized machine number; precision may be lost.

General: Exp[-6403.29] is too small to represent as a normalized machine number; precision may be lost.

General: Exp[-57610.3] is too small to represent as a normalized machine number; precision may be lost.

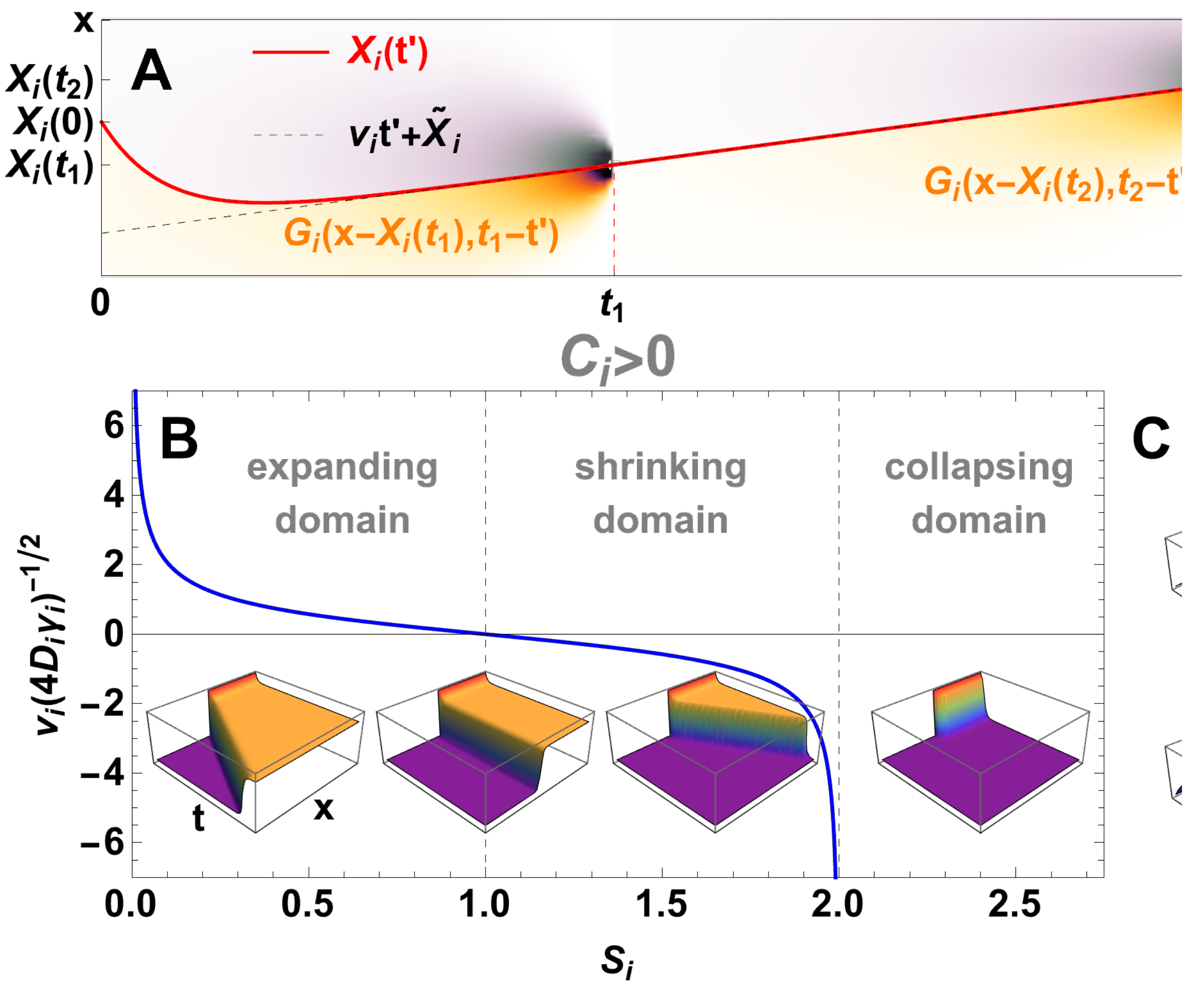
General: Further output of General::munfl will be suppressed during this calculation.

Out[ ]=



In[ ]:=
```
GraphicsColumn[{int1plot, plotV1d}, ImageSize → {750, Automatic},
 Spacings → {-20, -9}, ImagePadding → {{Automatic, Automatic}, {0, 0}}]
```

Out[ ]=

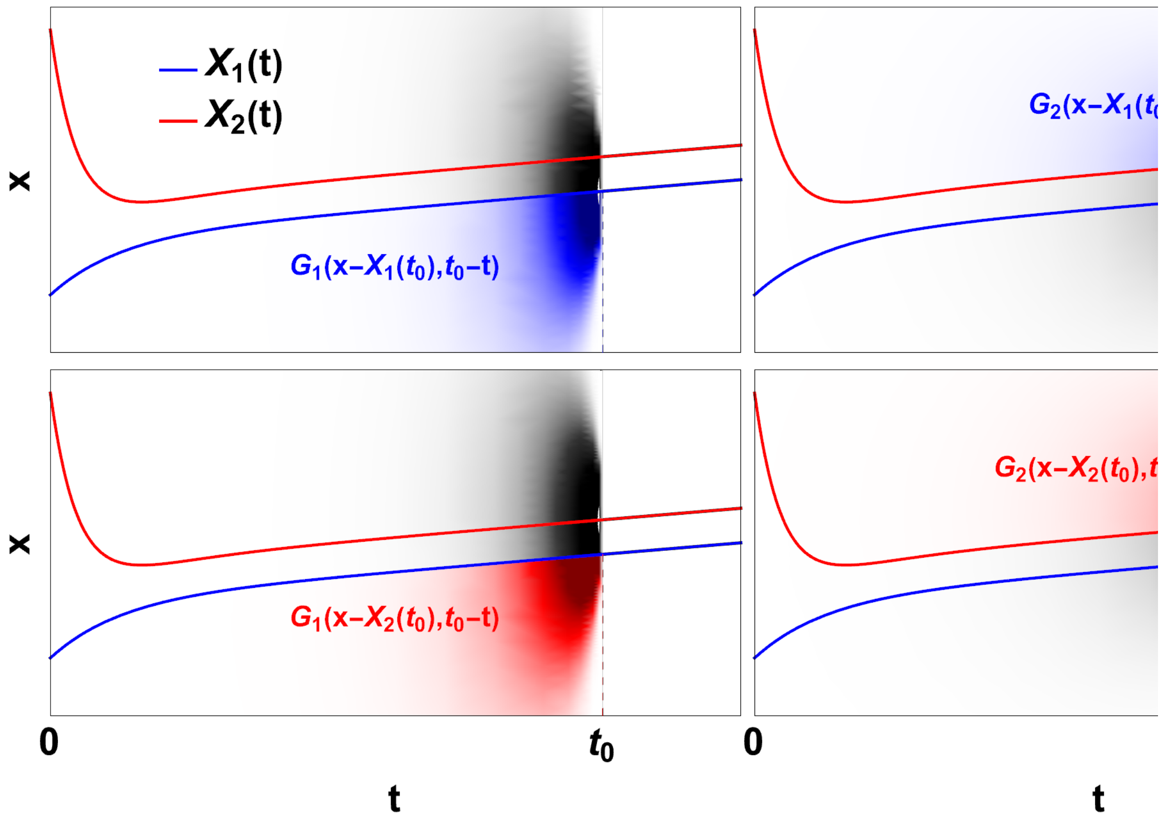


In[ ]:=
```
G1[x_, t_] := Exp[-2.5 t - x^2 / t / 4.2] / Sqrt[2 π * t];
G2[x_, t_] := Exp[-1.5 t - x^2 / t / 1] / Sqrt[2 π * t];
X1[t_] := Exp[-2 t] (-0.5) + 0.1 * t - 0.5;
X2[t_] := X1[t] + 2 Exp[-5 t] + 0.3;
t0 = 4;
GraphicsGrid[{{
   Show[
    DensityPlot[G1[x - X1[t0], t0 - t], {t, 0, 5}, {x, -1.5, 1.5},
     PlotRange → {{0, 5}, {-1.5, 1.5}, {0, 2}}, RegionFunction →
```

```
      Function[{x, y, z}, x < t0 && y < X1[x]], ColorFunction → Function[{z}, If[z < 0.3,
        Blend[{White, Blue, RGBColor[0, 0, 0.5]}, z / 0.3], RGBColor[0, 0, 0.5]]],
     FrameLabel → {None, "x"},
     FrameTicks → {{None, None}, {None, None}}, LabelStyle → {20, Bold},
     Epilog → {{Dashed, Blue, Line[{{t0, -1.5}, {t0, X1[t0]}}]},
       Inset[Style["G₁(x-X₁(t₀),t₀-t)", Blue, Bold, 15], {2.5, -0.75}],
       Inset[LineLegend[{Blue, Red}, {Style["X₁(t)", 20, Bold],
          Style["X₂(t)", 20, Bold]}], Scaled[{0.25, 0.75}]]},
     ImageSize → {400, 400}, AspectRatio → 1 / 2
    ],
    DensityPlot[G1[x - X1[t0], t0 - t], {t, 0, 5},
     {x, -1.5, 1.5}, PlotRange → {{0, 5}, {-1.5, 1.5}, {0, 2}},
     RegionFunction → Function[{x, y, z}, X1[x] < y], ColorFunction → Function[{z},
       If[z < 0.3, Blend[{White, RGBColor[0.2, 0.2, 0.2], Black}, z / 0.3], Black]]
    ],
    Plot[{X1[t], X2[t]}, {t, 0, 5}, PlotStyle → {Blue, Red}]
   ],
   Show[
    DensityPlot[G2[x - X1[t0], t0 - t], {t, 0, 5}, {x, -1.5, 1.5},
     PlotRange → {{0, 5}, {-1.5, 1.5}, {0, 2}}, RegionFunction →
      Function[{x, y, z}, x < t0 && y > X1[x]], ColorFunction → Function[{z}, If[z < 0.3,
        Blend[{White, Blue, RGBColor[0, 0, 0.5]}, z / 0.3], RGBColor[0, 0, 0.5]]],
     FrameTicks → {{None, {{X1[t0], "X₁(t₀)"}, {X2[t0], "X₂(t₀)"}}}, {None, None}},
     LabelStyle → {20, Bold},
     Epilog → {{Dashed, Blue, Line[{{t0, X2[t0]}, {t0, 1.5}}]},
       Inset[Style["G₂(x-X₁(t₀),t₀-t)", Blue, Bold, 15], {2.75, 0.65}]},
     ImageSize → {400, 400}, AspectRatio → 1 / 2
    ],
    DensityPlot[G2[x - X1[t0], t0 - t], {t, 0, 5},
     {x, -1.5, 1.5}, PlotRange → {{0, 5}, {-1.5, 1.5}, {0, 2}},
     RegionFunction → Function[{x, y, z}, x < t0 && X2[x] > y], ColorFunction → Function[
       {z}, If[z < 0.3, Blend[{White, RGBColor[0.2, 0.2, 0.2], Black}, z / 0.3], Black]]
    ],
    Plot[{X1[t], X2[t]}, {t, 0, 5}, PlotStyle → {Blue, Red}]
   ]
  }, {
   Show[
    DensityPlot[G1[x - X2[t0], t0 - t], {t, 0, 5}, {x, -1.5, 1.5},
     PlotRange → {{0, 5}, {-1.5, 1.5}, {0, 2}}, RegionFunction →
      Function[{x, y, z}, x < t0 && y ≤ X1[x]], ColorFunction → Function[{z}, If[z < 0.3,
        Blend[{White, Red, RGBColor[0.5, 0, 0]}, z / 0.3], RGBColor[0.5, 0, 0]]],
     FrameLabel → {"t", "x"},
     FrameTicks → {{None, None}, {{0, {t0, "t₀"}}, None}}, LabelStyle → {20, Bold},
     Epilog → {{Dashed, Red, Line[{{t0, X1[t0]}, {t0, -1.5}}]},
       Inset[Style["G₁(x-X₂(t₀),t₀-t)", Red, Bold, 15], {2.5, -0.65}]},
     ImageSize → {400, 400}, AspectRatio → 1 / 2
    ],
    DensityPlot[G1[x - X2[t0], t0 - t], {t, 0, 5},
     {x, -1.5, 1.5}, PlotRange → {{0, 5}, {-1.5, 1.5}, {0, 2}},
     RegionFunction → Function[{x, y, z}, x < t0 && X1[x] < y], ColorFunction → Function[
```

```
      {z}, If[z < 0.3, Blend[{White, RGBColor[0.2, 0.2, 0.2], Black}, z / 0.3], Black]]
    ],
   Plot[{X1[t], X2[t]}, {t, 0, 5}, PlotStyle → {Blue, Red}]
  ],
  Show[
   DensityPlot[G2[x - X2[t0], t0 - t], {t, 0, 5}, {x, -1.5, 1.5},
    PlotRange → {{0, 5}, {-1.5, 1.5}, {0, 2}}, RegionFunction →
     Function[{x, y, z}, x < t0 && y > X2[x]], ColorFunction → Function[{z}, If[z < 0.3,
       Blend[{White, Red, RGBColor[0.5, 0, 0]}, z / 0.3], RGBColor[0.5, 0, 0]]],
    FrameLabel → {"t", None},
    FrameTicks → {{None, {{X1[t0], "X₁(t₀)"}, {X2[t0], "X₂(t₀)"}}},
      {{0, {t0, "t₀"}}, None}}, LabelStyle → {20, Bold},
    Epilog → {{Dashed, Red, Line[{{t0, X2[t0]}, {t0, 1.5}}]},
      Inset[Style["G₂(x-X₂(t₀),t₀-t)", Red, Bold, 15], {2.5, 0.65}]},
    ImageSize → {400, 400}, AspectRatio → 1 / 2
   ],
   DensityPlot[G2[x - X2[t0], t0 - t], {t, 0, 5},
    {x, -1.5, 1.5}, PlotRange → {{0, 5}, {-1.5, 1.5}, {0, 2}},
    RegionFunction → Function[{x, y, z}, x < t0 && X2[x] > y], ColorFunction → Function[
      {z}, If[z < 0.3, Blend[{White, RGBColor[0.2, 0.2, 0.2], Black}, z / 0.3], Black]]
   ],
   Plot[{X1[t], X2[t]}, {t, 0, 5}, PlotStyle → {Blue, Red}]
  ]
 }}, ImageSize → {850, 500}, Spacings → {-70, -230},
ImagePadding → {{Automatic, Automatic}, {150, 150}}]
```

Out[ ]=



In[ ]:=
```
G1[x_, t_] := Exp[-2.5 t - x^2 / t / 4.2] / Sqrt[2 π * t];
G2[x_, t_] := Exp[-1.5 t - x^2 / t / 1] / Sqrt[2 π * t];
X1[t_] := Exp[-2 t] (-0.5) + 0.75 * t - 0.5;
X2[t_] := -0.5 t + 2 Exp[-5 t] + 0.2;
t0 = 4;
GraphicsGrid[{{
   Show[
    DensityPlot[G2[x - X2[t0], t0 - t] + G1[x - X1[t0], t0 - t], {t, 0, 5},
     {x, -2.5, 3.5}, PlotRange → {{0, 5}, {-2.5, 3.5}, {0, 2}}, RegionFunction →
      Function[{x, y, z}, x < t0 && y < X1[x]], ColorFunction → Function[{z}, If[z < 0.3,
        Blend[{White, Blue, RGBColor[0.0, 0, 0.5]}, z / 0.3], RGBColor[0.0, 0, 0.5]]],
     FrameLabel → {"t", "x"},
     FrameTicks → {{None, (*{{X1[t0],"X₁(t₀)"},{X2[t0],"X₂(t₀)"}}*)None},
       {{0, {t0, "t₀"}}, None}}, LabelStyle → {20, Bold},
     Epilog → {{Dashed, Blue, Line[{{t0, X1[t0]}, {t0, -2.5}}]},
       Inset[Style["G₂(x-X₂(t₀),t₀-t)", Red, Bold, 15], {2.5, -1.9}],
       Inset[Style["G₁(x-X₁(t₀),t₀-t)", Blue, Bold, 15], {2.5, 2.75}]},
     (*ImageSize→{400,400},*)AspectRatio → 1 / 2
    ],
    DensityPlot[G2[x - X2[t0], t0 - t] + G1[x - X1[t0], t0 - t],
     {t, 0, 5}, {x, -2.5, 3.5}, PlotRange → {{0, 5}, {-1.5, 3.5}, {0, 2}},
     RegionFunction → Function[{x, y, z}, x < t0 && X1[x] < y], ColorFunction → Function[
       {z}, If[z < 0.3, Blend[{White, RGBColor[0.2, 0.2, 0.2], Black}, z / 0.3], Black]]
    ],
    Plot[{X1[t], X2[t]}, {t, 0, 5}, PlotStyle → {Blue, Red}]
   ],
   Show[
    DensityPlot[G2[x - X2[t0], t0 - t] + G1[x - X1[t0], t0 - t], {t, 0, 5},
     {x, -2.5, 3.5}, PlotRange → {{0, 5}, {-2.5, 3.5}, {0, 2}}, RegionFunction →
      Function[{x, y, z}, x < t0 && y > X2[x]], ColorFunction → Function[{z}, If[z < 0.3,
        Blend[{White, Red, RGBColor[0.5, 0, 0]}, z / 0.3], RGBColor[0.5, 0, 0]]],
     FrameLabel → {"t", None},
     FrameTicks → {{None, {{X1[t0], "X₁(t₀)"}, {X2[t0], "X₂(t₀)"}}},
       {{0, {t0, "t₀"}}, None}}, LabelStyle → {20, Bold},
     Epilog → {{Dashed, Red, Line[{{t0, X2[t0]}, {t0, 3.5}}]},
       Inset[Style["G₂(x-X₂(t₀),t₀-t)", Red, Bold, 15], {2.5, -1.9}],
       Inset[Style["G₁(x-X₁(t₀),t₀-t)", Blue, Bold, 15], {2.5, 2.75}]},
     (*ImageSize→{400,400},*)AspectRatio → 1 / 2
    ],
    DensityPlot[G2[x - X2[t0], t0 - t] + G1[x - X1[t0], t0 - t],
     {t, 0, 5}, {x, -2.5, 1.5}, PlotRange → {{0, 5}, {-1.5, 1.5}, {0, 2}},
     RegionFunction → Function[{x, y, z}, x < t0 && X2[x] > y], ColorFunction → Function[
       {z}, If[z < 0.3, Blend[{White, RGBColor[0.2, 0.2, 0.2], Black}, z / 0.3], Black]]
    ],
    Plot[{X1[t], X2[t]}, {t, 0, 5}, PlotStyle → {Blue, Red}]]
  }}, ImageSize → {750, 250}, Spacings → {-50, Automatic},
 ImagePadding → {{175, 190}, {Automatic, Automatic}}]
```

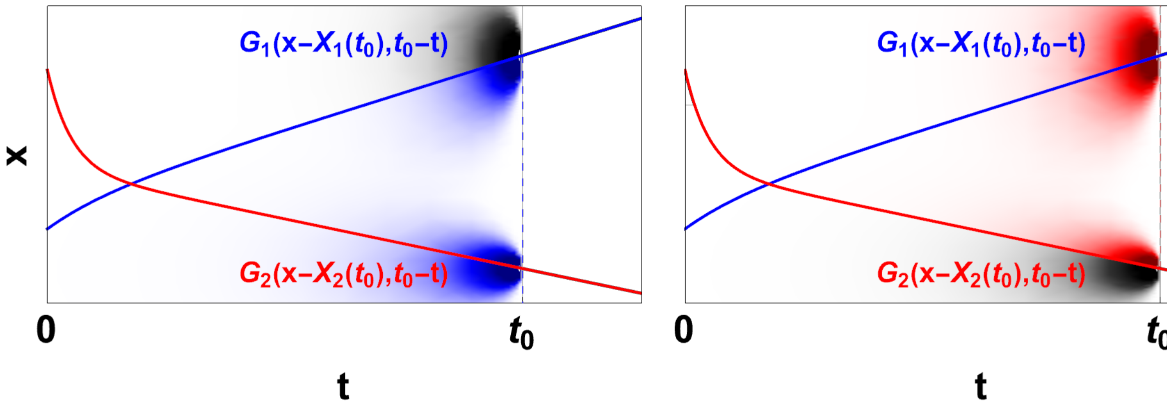
General: Exp[-1358.03] is too small to represent as a normalized machine number; precision may be lost.

General: Exp[-1375.97] is too small to represent as a normalized machine number; precision may be lost.

General: Exp[−4.83095 × 10^6] is too small to represent as a normalized machine number; precision may be lost.

General: Further output of General::munfl will be suppressed during this calculation.

Out[ ]=



In[ ]:=
```
G1[x_, t_] := Exp[-2.5 t - x^2 / t / 4.2] / Sqrt[2 π * t];
G2[x_, t_] := Exp[-1.5 t - x^2 / t / 1] / Sqrt[2 π * t];
X1[t_] := Exp[-3 t] (-0.4) - 0.25 * t - 0.4;
X2[t_] := 0.4 t + Exp[-4 t] + 1.0;
t0 = 4;
GraphicsGrid[{{
   Show[
    DensityPlot[G2[x - X2[t0], t0 - t] + G1[x - X1[t0], t0 - t], {t, 0, 5},
     {x, -2.5, 3.5}, PlotRange → {{0, 5}, {-2.5, 3.5}, {0, 2}}, RegionFunction →
      Function[{x, y, z}, x < t0 && y < X1[x]], ColorFunction → Function[{z}, If[z < 0.3,
        Blend[{White, Blue, RGBColor[0.0, 0, 0.5]}, z / 0.3], RGBColor[0.0, 0, 0.5]]],
     FrameLabel → {"t", "x"},
     FrameTicks → {{None, (*{{X1[t0],"X₁(t₀)"},{X2[t0],"X₂(t₀)"}}*)None},
       {{0, {t0, "t₀"}}, None}}, LabelStyle → {20, Bold},
     Epilog → {{Dashed, Blue, Line[{{t0, X1[t0]}, {t0, -2.5}}]},
       Inset[Style["G₂(x-X₂(t₀),t₀-t)", Red, Bold, 15], {2.5, 2.75}],
       Inset[Style["G₁(x-X₁(t₀),t₀-t)", Blue, Bold, 15], {2.5, -1.9}]},
     (*ImageSize→{400,400},*)AspectRatio → 1 / 2
    ],
    DensityPlot[G2[x - X2[t0], t0 - t] + G1[x - X1[t0], t0 - t],
     {t, 0, 5}, {x, -2.5, 3.5}, PlotRange → {{0, 5}, {-1.5, 3.5}, {0, 2}},
     RegionFunction → Function[{x, y, z}, x < t0 && X1[x] < y], ColorFunction → Function[
       {z}, If[z < 0.3, Blend[{White, RGBColor[0.2, 0.2, 0.2], Black}, z / 0.3], Black]]
    ],
    Plot[{X1[t], X2[t]}, {t, 0, 5}, PlotStyle → {Blue, Red}]
   ],
   Show[
    DensityPlot[G2[x - X2[t0], t0 - t] + G1[x - X1[t0], t0 - t], {t, 0, 5},
     {x, -2.5, 3.5}, PlotRange → {{0, 5}, {-2.5, 3.5}, {0, 2}}, RegionFunction →
      Function[{x, y, z}, x < t0 && y > X2[x]], ColorFunction → Function[{z}, If[z < 0.3,
        Blend[{White, Red, RGBColor[0.5, 0, 0]}, z / 0.3], RGBColor[0.5, 0, 0]]],
     FrameLabel → {"t", None},
     FrameTicks → {{None, {{X1[t0], "X₁(t₀)"}, {X2[t0], "X₂(t₀)"}}},
       {{0, {t0, "t₀"}}, None}}, LabelStyle → {20, Bold},
     Epilog → {{Dashed, Red, Line[{{t0, X2[t0]}, {t0, 3.5}}]},
       Inset[Style["G₂(x-X₂(t₀),t₀-t)", Red, Bold, 15], {2.5, 2.75}],
       Inset[Style["G₁(x-X₁(t₀),t₀-t)", Blue, Bold, 15], {2.5, -1.9}]},
     (*ImageSize→{400,400},*)AspectRatio → 1 / 2
    ],
    DensityPlot[G2[x - X2[t0], t0 - t] + G1[x - X1[t0], t0 - t],
     {t, 0, 5}, {x, -2.5, 3.5}, PlotRange → {{0, 5}, {-1.5, 3.5}, {0, 2}},
     RegionFunction → Function[{x, y, z}, x < t0 && X2[x] > y], ColorFunction → Function[
       {z}, If[z < 0.3, Blend[{White, RGBColor[0.2, 0.2, 0.2], Black}, z / 0.3], Black]]
    ],
    Plot[{X1[t], X2[t]}, {t, 0, 5}, PlotStyle → {Blue, Red}]]
  }}, ImageSize → {750, 250}, Spacings → {-50, Automatic},
 ImagePadding → {{175, 190}, {Automatic, Automatic}}]
```

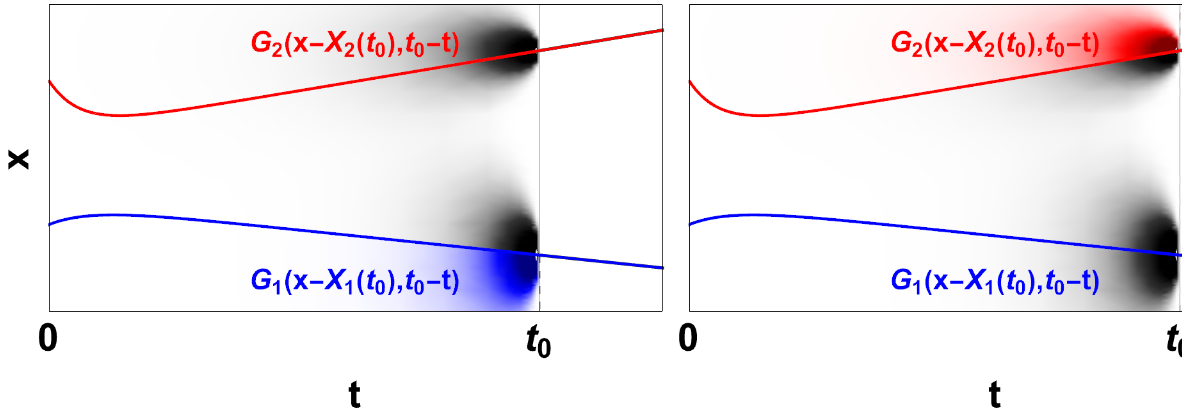
General: Exp[-1189.31] is too small to represent as a normalized machine number; precision may be lost.

General: Exp[-4.1756×10^{6}] is too small to represent as a normalized machine number; precision may be lost.

General: Exp[-1205.33] is too small to represent as a normalized machine number; precision may be lost.

General: Further output of General::munfl will be suppressed during this calculation.

Out[ ]=



In[ ]:=

```
tmax = 36000;
L = 500;
(*non-stable, overlapping*)
ϵ11 = 1.0;
ϵ22 = 1.0;
ϵ12 = -0.2;
ϵ21 = -1.0;
d1 = 1;
d2 = 1;
γ1 = 0.0005;
γ2 = 0.0004;
h1 = 0.01;
h2 = 0.02;
c1 = 3.5;
c2 = 6.0;
x1 = -3 L / 8;
x2 = 3 L / 8;
S1 = 2 c1 * γ1 / h1 / ϵ11
S2 = 2 c2 * γ2 / h2 / ϵ22
sol2 = NDSolve[{
    D[n1[x, t], t] == d1 * D[n1[x, t], {x, 2}] -
      γ1 * n1[x, t] + h1 * UnitStep[ϵ11 * n1[x, t] + ϵ12 * n2[x, t] - c1],
    D[n2[x, t], t] == d2 * D[n2[x, t], {x, 2}] -
      γ2 * n2[x, t] + h2 * UnitStep[ϵ22 * n2[x, t] + ϵ21 * n1[x, t] - c2],
    n1[x, 0] == 1.1 c1 / ϵ11 * UnitStep[x1 - x], n2[x, 0] == 1.1 c2 / ϵ22 * UnitStep[x - x2],
    (D[n1[x, t], x] /. x → -L / 2) == 0, (D[n1[x, t], x] /. x → L / 2) == 0,
    (D[n2[x, t], x] /. x → -L / 2) == 0, (D[n2[x, t], x] /. x → L / 2) == 0},
   {n1, n2}, {x, -L / 2, L / 2}, {t, 0, tmax}, AccuracyGoal → 3,
   Method → {"PDEDiscretization" → {"MethodOfLines", "TemporalVariable" → t,
       "SpatialDiscretization" → {"TensorProductGrid", "MinPoints" → 2^11},
       "DifferentiateBoundaryConditions" → True}, "TimeIntegration" → "BDF"},
   MaxStepSize → Automatic, StartingStepSize → Automatic];
Plot3D[Evaluate[{n1[x, t], n2[x, t]} /. sol2],
```

```
 {x, -L / 2, L / 2}, {t, 0, tmax}, PlotRange → All]
v1 = Sqrt[4 d1 * γ1] * Abs[S1 - 1] / Sqrt[1 - (S1 - 1) ^2];
v2 = Sqrt[4 d2 * γ2] * Abs[S2 - 1] / Sqrt[1 - (S2 - 1) ^2];
T0 = (x2 - x1) / (Abs[v2] + Abs[v1]);
If[S1 > 1, v1 = -v1];
If[S2 < 1, v2 = -v2];
If[v1 > 0 && v2 < 0, X0 = (Abs[v1] * x2 + Abs[v2] * x1) / (Abs[v1] + Abs[v2])];
(*ContourPlot[{c1==ϵ11*h1/2/γ1*(-v/Sqrt[4d1*γ1+v^2]+1)+ϵ12*h2/2/γ2*(1-Sign[Δc]+
        Exp[v*Δc/4/d2-Abs[Δc]*Sqrt[4d2*γ2+v^2]/2/d2]*(v/Sqrt[4d2*γ2+v^2]+Sign[Δc])),
  c2==ϵ21*h1/2/γ1*(1-Sign[Δc]-Exp[-v*Δc/4/d1-Abs[Δc]*Sqrt[4d1*γ1+v^2]/2/d1]*
          (v/Sqrt[4d1*γ1+v^2]-Sign[Δc]))+ϵ22*h2/2/γ2*(v/Sqrt[4d2*γ2+v^2]+1)},
 {v,-200,200},{Δc,-200,200},FrameLabel→{"v","Δc"}]*)
solV = FindRoot[{c1 == ϵ11 * h1 / 2 / γ1 * (-v / Sqrt[4 d1 * γ1 + v^2] + 1) + ϵ12 * h2 / 2 / γ2 *
        (1 - Sign[Δc] + Exp[v * Δc / 4 / d2 - Abs[Δc] * Sqrt[4 d2 * γ2 + v^2] / 2 / d2] *
           (v / Sqrt[4 d2 * γ2 + v^2] + Sign[Δc])),
    c2 == ϵ21 * h1 / 2 / γ1 * (1 - Sign[Δc] - Exp[-v * Δc / 4 / d1 - Abs[Δc] *
              Sqrt[4 d1 * γ1 + v^2] / 2 / d1] * (v / Sqrt[4 d1 * γ1 + v^2] - Sign[Δc])) +
      ϵ22 * h2 / 2 / γ2 * (v / Sqrt[4 d2 * γ2 + v^2] + 1)}, {{v, 0(*RandomReal[
      {-0.5,0.5}]*)}, {Δc, (*RandomReal[{-0.5,0.5}]*)-20}}]
empX1 = FindRoot[ϵ11 * n1[x1, tmax] + ϵ12 * n2[x1, tmax] == c1 /. sol2,
     {x1, X0}, AccuracyGoal → 3][[1, 2]];
empX2 = FindRoot[ϵ21 * n1[x2, tmax] + ϵ22 * n2[x2, tmax] == c2 /. sol2,
     {x2, X0}, AccuracyGoal → 3][[1, 2]];
center = (empX2 + empX1) / 2;
```

Out[∘]=

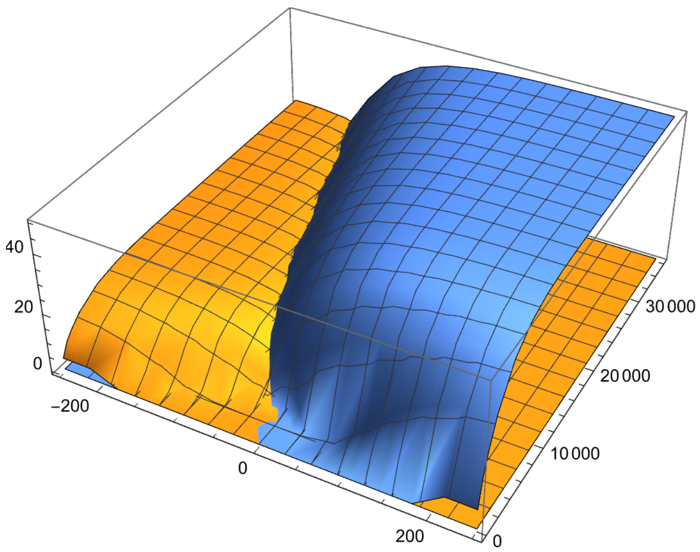
0.35

Out[∘]=

0.24

NDSolve: Using maximum number of grid points 10000 allowed by the MaxPoints or MinStepSize options for independent variable x.

Out[∘]=



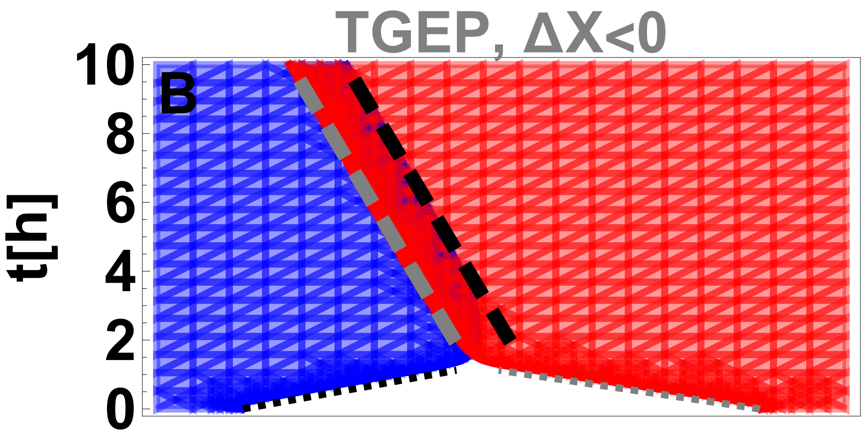
Out[∘]=

{v → -0.00412787, Δc → -39.793}

In[ ]:=

```
plotTGEP1 = Show[{ContourPlot[Evaluate[{∊11 * n1[x, t] + ∊12 * n2[x, t] == c1 /. sol2,
        ∊21 * n1[x, t] + ∊22 * n2[x, t] == c2 /. sol2} /. t → 3600 * τ], {x, -L / 2, L / 2}, {τ,
      0, tmax / 3600}, ContourStyle → {{Blue, Thickness[0.01]}, {Red, Thickness[0.01]}},
     LabelStyle → {Bold, 30}, ContourLabels → None,
     FrameLabel → {None (*Style["x[μm]",Opacity[0]]*), "t[h]"},
     FrameTicks → {{Automatic, None},
       {(*Table[{i,Style["a",Opacity[0]]},{i,-250,250,20}]*)None, None}},
     PlotRangeClipping → False, ImagePadding → {{Automatic, Automatic}, {Automatic, 50}},
     MaxRecursion → 5, PerformanceGoal → "Quality", AspectRatio → 1 / 2, Epilog → {
       (*Inset[LineLegend[{Directive[{Blue,Thickness[0.01]}],
          Directive[{Red,Thickness[0.01]}],Directive[{Black,Dashed,Thickness[0.01]}],
          Directive[{Gray,Dashed,Thickness[0.01]}],
          Directive[{Black,Dashing[{0.05,0.025}],Thickness[0.02]}],
          Directive[{Gray,Dashing[{0.05,0.025}],Thickness[0.02]}]},
         {Style["X1(t)",30,Bold],Style["X2(t)",30,Bold],
          Style["X1(0)+v1t",30,Bold],Style["X2(0)+v2t",30,Bold],
          Style["vt-ΔX/2+c0",30,Bold],Style["vt+ΔX/2+c0",30,Bold]},
         LegendLayout→{"Column",2}],Scaled[{0.5,0.5}]],*)
       Inset[Style["B", 30, Bold], Scaled[{0.05, .9}]],
       Inset[Style["TGEP,  ΔX<0", Bold, 30, Gray], Scaled[{0.5, 1.07}]]}
    ],
    RegionPlot[Evaluate[{∊11 * n1[x, t] + ∊12 * n2[x, t] > c1 /. sol2,
        ∊21 * n1[x, t] + ∊22 * n2[x, t] > c2 /. sol2} /. t → 3600 * τ],
     {x, -L / 2, L / 2}, {τ, 0, tmax / 3600}, BoundaryStyle → None,
     PlotStyle → {Directive[Blue, Opacity[0.4]], Directive[Red, Opacity[0.4]]},
     MaxRecursion → 5, PerformanceGoal → "Quality"],
    ParametricPlot[Evaluate[{{HeavisideTheta[T0 / 3600 - t] * (x1 + v1 * 3600 * t) +
          HeavisideTheta[t - T0 / 3600] * (center + v * (3600 * t - tmax) - Δc / 2), t},
        {HeavisideTheta[T0 / 3600 - t] * (x2 + v2 * 3600 * t) + HeavisideTheta[t - T0 / 3600] *
           (center + v * (3600 * t - tmax) + Δc / 2), t}} /. solV],
     {t, 0, T0 / 3600 - 0.1}, (*RegionFunction→Function[{x,y,u,v},!((T0-100)<u)],*)
     PlotStyle → {{Black, Thickness[0.01], Dashed}, {Gray, Thickness[0.01], Dashed}}],
    ParametricPlot[Evaluate[{{center + v * (3600 * t - tmax) - Δc / 2, t},
        {center + v * (3600 * t - tmax) + Δc / 2, t}} /. solV],
     {t, 7000 / 3600, tmax / 3600}, (*,PlotRange→{Automatic,{0,tmax}},*)
     PlotStyle → {{Black, Thickness[0.02], Dashing[{0.05, 0.025}]},
       {Gray, Thickness[0.02], Dashing[{0.05, 0.025}]}}]}, ImageSize → {500, 250}]
```

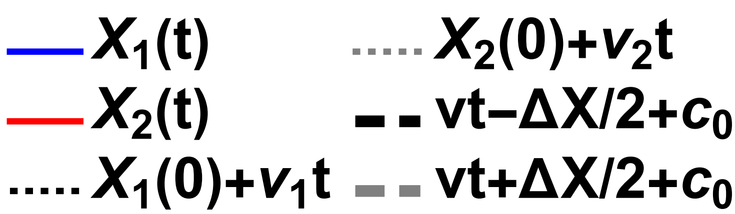
Out[ ]=



In[ ]:=

```
legend = Plot[, {x, -L / 2, L / 2}, Axes → False, PlotRangeClipping → False,
  Epilog → Inset[LineLegend[{Directive[{Blue, Thickness[0.01]}],
      Directive[{Red, Thickness[0.01]}], Directive[{Black, Dashed, Thickness[0.01]}],
      Directive[{Gray, Dashed, Thickness[0.01]}],
      Directive[{Black, Dashing[{0.05, 0.025}], Thickness[0.02]}],
      Directive[{Gray, Dashing[{0.05, 0.025}], Thickness[0.02]}]},
     {Style["X1(t)", 30, Bold], Style["X2(t)", 30, Bold], Style["X1(0)+v1t", 30, Bold],
      Style["X2(0)+v2t", 30, Bold], Style["vt-ΔX/2+c0", 30, Bold],
      Style["vt+ΔX/2+c0", 30, Bold]}, LegendLayout → {"Column", 2}],
    Scaled[{0.5, 0.5}]], ImageSize → {500, 250}]
```

Out[ ]=



In[ ]:=
```
tmax = 36 000;
L = 500;
(*non-stable, depleted zone*)
ϵ11 = 1.0;
ϵ22 = 1.0;
ϵ12 = -0.45;
ϵ21 = -2.1;
d1 = 1;
d2 = 1;
γ1 = 0.0005;
γ2 = 0.0004;
h1 = 0.01;
h2 = 0.02;
c1 = 4.0;
c2 = 6.5;
S1 = 2 c1 * γ1 / h1 / ϵ11
S2 = 2 c2 * γ2 / h2 / ϵ22
sol2 = NDSolve[{
    D[n1[x, t], t] == d1 * D[n1[x, t], {x, 2}] -
      γ1 * n1[x, t] + h1 * UnitStep[ϵ11 * n1[x, t] + ϵ12 * n2[x, t] - c1],
    D[n2[x, t], t] == d2 * D[n2[x, t], {x, 2}] -
      γ2 * n2[x, t] + h2 * UnitStep[ϵ22 * n2[x, t] + ϵ21 * n1[x, t] - c2],
    n1[x, 0] == 1.1 c1 / ϵ11 * UnitStep[x1 - x], n2[x, 0] == 1.1 c2 / ϵ22 * UnitStep[x - x2],
    (D[n1[x, t], x] /. x → -L / 2) == 0, (D[n1[x, t], x] /. x → L / 2) == 0,
    (D[n2[x, t], x] /. x → -L / 2) == 0, (D[n2[x, t], x] /. x → L / 2) == 0},
   {n1, n2}, {x, -L / 2, L / 2}, {t, 0, tmax}, AccuracyGoal → 3,
   Method → {"PDEDiscretization" → {"MethodOfLines", "TemporalVariable" → t,
       "SpatialDiscretization" → {"TensorProductGrid", "MinPoints" → 2^11},
       "DifferentiateBoundaryConditions" → True}, "TimeIntegration" → "BDF"},
   MaxStepSize → Automatic, StartingStepSize → Automatic];
Plot3D[Evaluate[{n1[x, t], n2[x, t]} /. sol2],
 {x, -L / 2, L / 2}, {t, 0, tmax}, PlotRange → All]
v1 = Sqrt[4 d1 * γ1] * Abs[S1 - 1] / Sqrt[1 - (S1 - 1) ^2];
v2 = Sqrt[4 d2 * γ2] * Abs[S2 - 1] / Sqrt[1 - (S2 - 1) ^2];
T0 = (x2 - x1) / (Abs[v2] + Abs[v1]);
If[S1 > 1, v1 = -v1];
If[S2 < 1, v2 = -v2];
If[v1 > 0 && v2 < 0, X0 = (Abs[v1] * x2 + Abs[v2] * x1) / (Abs[v1] + Abs[v2])];
solV = FindRoot[{c1 == ϵ11 * h1 / 2 / γ1 * (-v / Sqrt[4 d1 * γ1 + v^2] + 1) + ϵ12 * h2 / 2 / γ2 *
      (1 - Sign[Δc] + Exp[v * Δc / 4 / d2 - Abs[Δc] * Sqrt[4 d2 * γ2 + v^2] / 2 / d2] *
         (v / Sqrt[4 d2 * γ2 + v^2] + Sign[Δc])),
   c2 == ϵ21 * h1 / 2 / γ1 * (1 - Sign[Δc] - Exp[-v * Δc / 4 / d1 - Abs[Δc] *
             Sqrt[4 d1 * γ1 + v^2] / 2 / d1] * (v / Sqrt[4 d1 * γ1 + v^2] - Sign[Δc])) +
     ϵ22 * h2 / 2 / γ2 * (v / Sqrt[4 d2 * γ2 + v^2] + 1)}, {{v, 0(*RandomReal[
     {-0.5,0.5}]*)}, {Δc, (*RandomReal[{-0.5,0.5}]*)-20}}]
empX1 = FindRoot[ϵ11 * n1[x1, tmax] + ϵ12 * n2[x1, tmax] == c1 /. sol2,
    {x1, -200}, AccuracyGoal → 3][[1, 2]];
empX2 = FindRoot[ϵ21 * n1[x2, tmax] + ϵ22 * n2[x2, tmax] == c2 /. sol2,
    {x2, -200}, AccuracyGoal → 3][[1, 2]];
center = (empX2 + empX1) / 2;
```

Out[ ]=

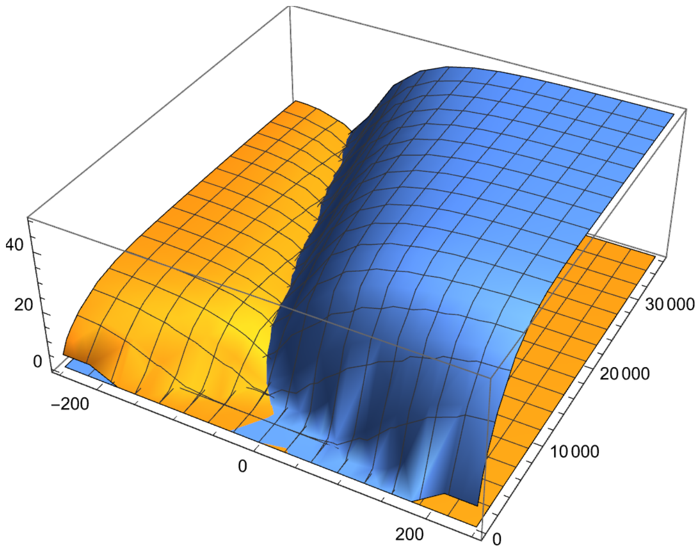
0.4

Out[ ]=

0.26

NDSolve: Using maximum number of grid points 10000 allowed by the MaxPoints or MinStepSize options for independent variable x.

Out[ ]=

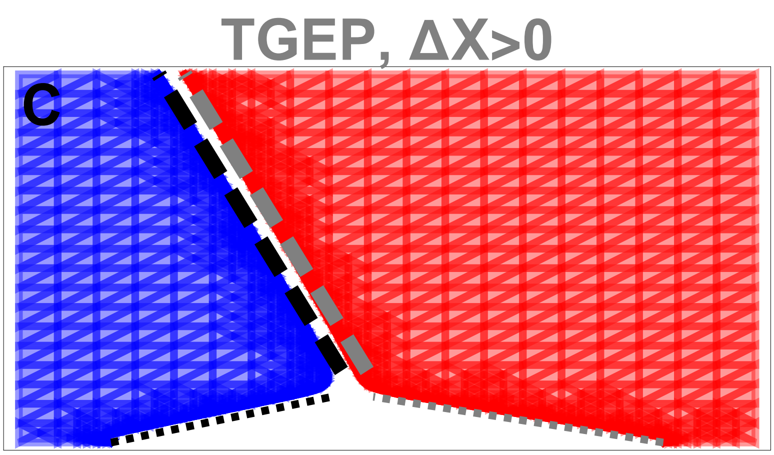


Out[ ]=

{v → -0.00427539, Δc → 17.421}

In[ ]:=
```
plotTGEP2 = Show[{ContourPlot[Evaluate[{ϵ11 * n1[x, t] + ϵ12 * n2[x, t] == c1 /. sol2,
      ϵ21 * n1[x, t] + ϵ22 * n2[x, t] == c2 /. sol2} /. t → 3600 * τ], {x, -L / 2, L / 2}, {τ,
    0, tmax / 3600}, ContourStyle → {{Blue, Thickness[0.01]}, {Red, Thickness[0.01]}},
   ContourLabels → None, LabelStyle → {30, Bold},
   FrameLabel → {(*"x[μm]"*)None, None(*,"t[h]"*)}, FrameTicks →
    {{(*Table[{i,Style["a",Opacity[0]]},{i,0,10,0.5}]*)None, None}, {None, None}},
   PlotRangeClipping → False, ImagePadding → {{Automatic, Automatic}, {Automatic, 50}},
   MaxRecursion → 5, PerformanceGoal → "Quality", AspectRatio → 1 / 2, Epilog → {
     (*Inset[LineLegend[{Directive[{Blue,Thickness[0.01]}],
        Directive[{Red,Thickness[0.01]}],Directive[{Black,Dashed,Thickness[0.01]}],
        Directive[{Gray,Dashed,Thickness[0.01]}],
        Directive[{Black,Dashing[{0.05,0.025}],Thickness[0.02]}],
        Directive[{Gray,Dashing[{0.05,0.025}],Thickness[0.02]}]},
       {Style["X1(t)",20,Bold],Style["X2(t)",20,Bold],Style["X1(0)+v1t",20,Bold],
        Style["X2(0)+v2t",20,Bold],Style["vt-ΔX/2+c0",20,Bold],
        Style["vt+ΔX/2+c0",20,Bold]}],Scaled[{0.75,0.75}]],*)
     Inset[Style["C", 30, Bold], Scaled[{0.05, .9}]],
     Inset[Style["TGEP,  ΔX>0", 30, Bold, Gray], Scaled[{0.5, 1.07}]]}
  ],
  RegionPlot[Evaluate[{ϵ11 * n1[x, t] + ϵ12 * n2[x, t] > c1 /. sol2,
      ϵ21 * n1[x, t] + ϵ22 * n2[x, t] > c2 /. sol2} /. t → 3600 * τ],
   {x, -L / 2, L / 2}, {τ, 0, tmax / 3600}, BoundaryStyle → None,
   PlotStyle → {Directive[Blue, Opacity[0.4]], Directive[Red, Opacity[0.4]]},
   MaxRecursion → 5, PerformanceGoal → "Quality"],
  ParametricPlot[Evaluate[{{HeavisideTheta[T0 / 3600 - t] * (x1 + v1 * 3600 * t) +
        HeavisideTheta[t - T0 / 3600] * (center + v * (3600 * t - tmax) - Δc / 2), t},
      {HeavisideTheta[T0 / 3600 - t] * (x2 + v2 * 3600 * t) + HeavisideTheta[t - T0 / 3600] *
         (center + v * (3600 * t - tmax) + Δc / 2), t}} /. solV],
   {t, 0, T0 / 3600 - 0.1}, (*RegionFunction→Function[{x,y,u,v},!((T0-100)<u)],*)
   PlotStyle → {{Black, Thickness[0.01], Dashed}, {Gray, Thickness[0.01], Dashed}}],
  ParametricPlot[Evaluate[{{center + v * (3600 * t - tmax) - Δc / 2, t},
      {center + v * (3600 * t - tmax) + Δc / 2, t}} /. solV],
   {t, 7000 / 3600, tmax / 3600}, (*,PlotRange→{Automatic,{0,tmax}},*)
   PlotStyle → {{Black, Thickness[0.02], Dashing[{0.05, 0.025}]},
     {Gray, Thickness[0.02], Dashing[{0.05, 0.025}]}}]}, ImageSize → {500, 250}]
```

Out[ ]=



In[ ]:=
```
tmax = 36000;
L = 500;
(*exemplary params for non-stable dynamics*)
ϵ11 = 1.0;
ϵ22 = 1.0;
d1 = 1;
d2 = 1;
γ1 = 0.0005;
γ2 = 0.0004;
h1 = 0.01;
h2 = 0.02;
c1 = 4.5;
c2 = 7.5;
λ1 = Sqrt[d1 / γ1]
λ2 = Sqrt[d2 / γ2]
S1 = 2 c1 * γ1 / h1 / ϵ11
S2 = 2 c2 * γ2 / h2 / ϵ22
R1 = 0.5;
ϵ12 = (2 c1 - ϵ11 * h1 / γ1) / (R1 + 1) / (h2 / γ2)
R2 = Sign[R1] * Abs[1 - (1 - Abs[R1]) ^ (λ2 / λ1)]
ϵ21 = (2 c2 - ϵ22 * h2 / γ2) / (R2 + 1) / (h1 / γ1)
Sign[R1] * λ2 * Log[1 - Abs[R1]]
sol2 = NDSolve[{
    D[n1[x, t], t] == d1 * D[n1[x, t], {x, 2}] -
      γ1 * n1[x, t] + h1 * UnitStep[ϵ11 * n1[x, t] + ϵ12 * n2[x, t] - c1],
    D[n2[x, t], t] == d2 * D[n2[x, t], {x, 2}] -
      γ2 * n2[x, t] + h2 * UnitStep[ϵ22 * n2[x, t] + ϵ21 * n1[x, t] - c2],
    n1[x, 0] == 1.1 c1 / ϵ11 * UnitStep[x1 - x], n2[x, 0] == 1.1 c2 / ϵ22 * UnitStep[x - x2],
    (D[n1[x, t], x] /. x → -L / 2) == 0, (D[n1[x, t], x] /. x → L / 2) == 0,
    (D[n2[x, t], x] /. x → -L / 2) == 0, (D[n2[x, t], x] /. x → L / 2) == 0},
   {n1, n2}, {x, -L / 2, L / 2}, {t, 0, tmax}, AccuracyGoal → 3,
   Method → {"PDEDiscretization" → {"MethodOfLines", "TemporalVariable" → t,
       "SpatialDiscretization" → {"TensorProductGrid", "MinPoints" → 2^11},
       "DifferentiateBoundaryConditions" → True}, "TimeIntegration" → "BDF"},
   MaxStepSize → Automatic, StartingStepSize → Automatic];
Plot3D[Evaluate[{n1[x, t], n2[x, t]} /. sol2],
 {x, -L / 2, L / 2}, {t, 0, tmax}, PlotRange → All]
v1 = Sqrt[4 d1 * γ1] * Abs[S1 - 1] / Sqrt[1 - (S1 - 1) ^2];
v2 = Sqrt[4 d2 * γ2] * Abs[S2 - 1] / Sqrt[1 - (S2 - 1) ^2];
T0 = (x2 - x1) / (Abs[v2] + Abs[v1]);
If[S1 > 1, v1 = -v1];
If[S2 < 1, v2 = -v2];
If[v1 > 0 && v2 < 0, X0 = (Abs[v1] * x2 + Abs[v2] * x1) / (Abs[v1] + Abs[v2])];
solV = FindRoot[{c1 == ϵ11 * h1 / 2 / γ1 * (-v / Sqrt[4 d1 * γ1 + v^2] + 1) + ϵ12 * h2 / 2 / γ2 *
      (1 - Sign[Δc] + Exp[v * Δc / 4 / d2 - Abs[Δc] * Sqrt[4 d2 * γ2 + v^2] / 2 / d2] *
          (v / Sqrt[4 d2 * γ2 + v^2] + Sign[Δc])),
    c2 == ϵ21 * h1 / 2 / γ1 * (1 - Sign[Δc] - Exp[-v * Δc / 4 / d1 - Abs[Δc] *
             Sqrt[4 d1 * γ1 + v^2] / 2 / d1] * (v / Sqrt[4 d1 * γ1 + v^2] - Sign[Δc])) +
      ϵ22 * h2 / 2 / γ2 * (v / Sqrt[4 d2 * γ2 + v^2] + 1)}, {{v, 0(*RandomReal[
```

```
     {-0.5,0.5}]*)}, {Δc, (*RandomReal[{-0.5,0.5}]*)-20}}]
empX1 = FindRoot[ϵ11 * n1[x1, tmax] + ϵ12 * n2[x1, tmax] == c1 /. sol2,
    {x1, X0}, AccuracyGoal → 3][[1, 2]];
empX2 = FindRoot[ϵ21 * n1[x2, tmax] + ϵ22 * n2[x2, tmax] == c2 /. sol2,
    {x2, X0}, AccuracyGoal → 3][[1, 2]];
center = (empX2 + empX1) / 2;
```

Out[ ]=

44.7214

Out[ ]=

50.

Out[ ]=

0.45

Out[ ]=

0.3

Out[ ]=

-0.146667

Out[ ]=

0.539279

Out[ ]=

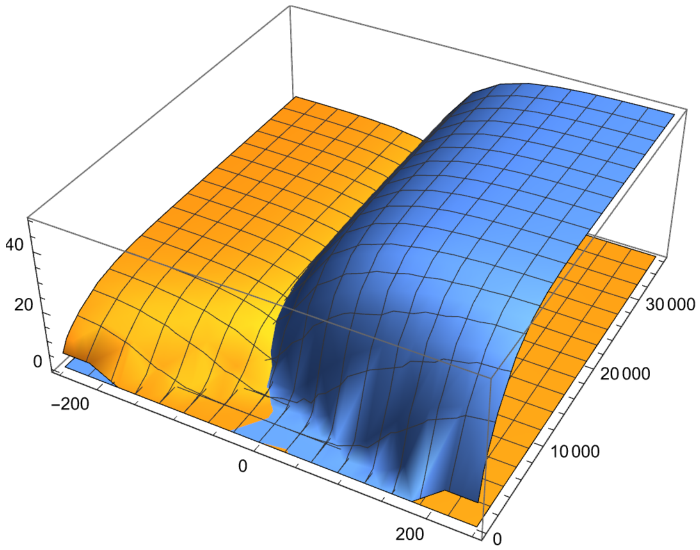
-1.1369

Out[ ]=

-34.6574

NDSolve: Using maximum number of grid points 10000 allowed by the MaxPoints or MinStepSize options for independent variable x.

Out[ ]=

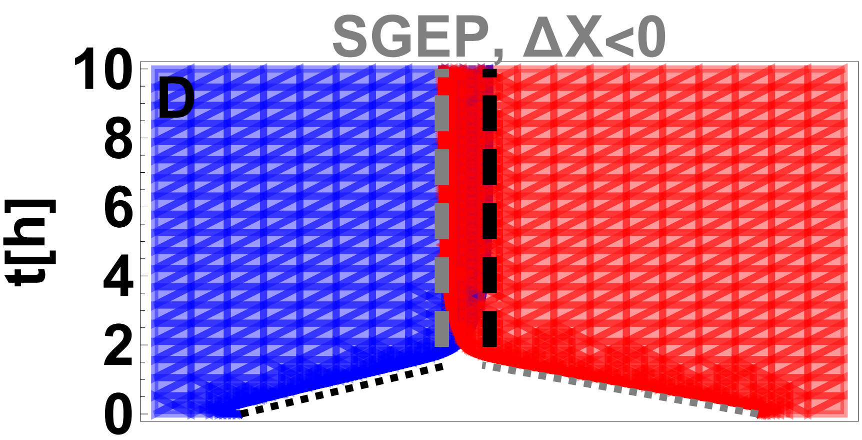


Out[ ]=

$\{v \to 7.65782 \times 10^{-19}, \Delta c \to -34.6574\}$

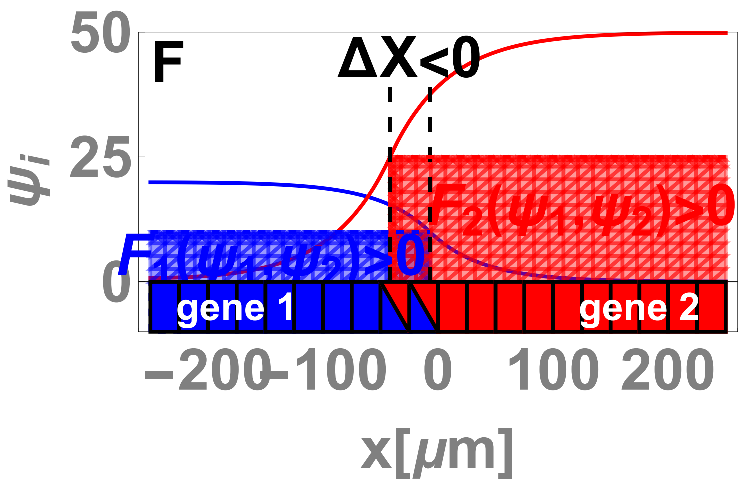
In[ ]:=
```
plotSGEP1 = Show[{ContourPlot[Evaluate[{ϵ11 * n1[x, t] + ϵ12 * n2[x, t] == c1 /. sol2,
      ϵ21 * n1[x, t] + ϵ22 * n2[x, t] == c2 /. sol2} /. t → 3600 * τ],
    {x, -L / 2, L / 2}, {τ, 0, tmax / 3600}, ContourStyle →
     {{Blue, Thickness[0.01]}, {Red, Thickness[0.01]}}, LabelStyle → {Bold, 30},
    ContourLabels → None, FrameLabel → {None(*"x[μm]"*), "t[h]"},
    FrameTicks → {{Automatic, None}, {None, None}}, PlotRangeClipping → False,
    ImagePadding → {{Automatic, Automatic}, {Automatic, 50}}, MaxRecursion → 5,
    PerformanceGoal → "Quality", AspectRatio → 1 / 2, Epilog → {
      (*Inset[LineLegend[{Directive[{Blue,Thickness[0.01]}],
         Directive[{Red,Thickness[0.01]}],Directive[{Black,Dashed,Thickness[0.01]}],
         Directive[{Gray,Dashed,Thickness[0.01]}],
         Directive[{Black,Dashing[{0.05,0.025}],Thickness[0.02]}],
         Directive[{Gray,Dashing[{0.05,0.025}],Thickness[0.02]}]},
        {Style["X1(t)",20,Bold],Style["X2(t)",20,Bold],Style["X1(0)+v1t",20,Bold],
         Style["X2(0)+v2t",20,Bold],Style["vt-ΔX/2+c0",20,Bold],
         Style["vt+ΔX/2+c0",20,Bold]}],Scaled[{0.75,0.75}]],*)
      Inset[Style["D", 30, Bold], Scaled[{0.05, .9}]],
      Inset[Style["SGEP,  ΔX<0", Bold, 30, Gray], Scaled[{0.5, 1.07}]]}
   ],
   RegionPlot[Evaluate[{ϵ11 * n1[x, t] + ϵ12 * n2[x, t] > c1 /. sol2,
      ϵ21 * n1[x, t] + ϵ22 * n2[x, t] > c2 /. sol2} /. t → 3600 * τ],
    {x, -L / 2, L / 2}, {τ, 0, tmax / 3600}, BoundaryStyle → None,
    PlotStyle → {Directive[Blue, Opacity[0.4]], Directive[Red, Opacity[0.4]]},
    MaxRecursion → 5, PerformanceGoal → "Quality"],
   ParametricPlot[Evaluate[{{HeavisideTheta[T0 / 3600 - t] * (x1 + v1 * 3600 * t) +
         HeavisideTheta[t - T0 / 3600] * (center + v * (3600 * t - tmax) - Δc / 2), t},
      {HeavisideTheta[T0 / 3600 - t] * (x2 + v2 * 3600 * t) + HeavisideTheta[t - T0 / 3600] *
          (center + v * (3600 * t - tmax) + Δc / 2), t}} /. solV],
    {t, 0, T0 / 3600 - 0.1}, (*RegionFunction→Function[{x,y,u,v},!((T0-100)<u)],*)
    PlotStyle → {{Black, Thickness[0.01], Dashed}, {Gray, Thickness[0.01], Dashed}}],
   ParametricPlot[Evaluate[{{center + v * (3600 * t - tmax) - Δc / 2, t},
      {center + v * (3600 * t - tmax) + Δc / 2, t}} /. solV],
    {t, 7000 / 3600, tmax / 3600}, (*,PlotRange→{Automatic,{0,tmax}},*)
    PlotStyle → {{Black, Thickness[0.02], Dashing[{0.05, 0.025}]},
      {Gray, Thickness[0.02], Dashing[{0.05, 0.025}]}}]}, ImageSize → {500, 250}]
```

Out[ ]=



In[ ]:=
```
profile1 =
 Show[Plot[Evaluate[{n1[x, tmax], n2[x, tmax] (*n1[empX1,tmax]*UnitStep[empX1-x],
     n2[empX2,tmax]UnitStep[x-empX2]*)} /. sol2], {x, -L / 2, L / 2},
   FrameLabel → {"x[μm]", "ψi"}, PlotRange → {Automatic, {-10, 50}},
   LabelStyle → {Bold, 30, Gray},
   PlotStyle → {{Thick, Blue}, {Red, Thick}, {Blue, Dashed}, {Red, Dashed}},
   Frame → True, FrameTicks → {{{0, 25, 50}, None}, {Automatic, None}}, AspectRatio → 0.5,
   ImageSize → {500, 250}, AxesOrigin → {-250, 0}, PlotRangeClipping → False,
   ImagePadding → {{Automatic, Automatic}, {Automatic, Automatic}},
   Epilog → {Inset[Style["F1(ψ1,ψ2)>0", Bold, 30, Blue], {-145, 5}],
     Inset[Style["F2(ψ1,ψ2)>0", Bold, 30, Red], {125, 15}],
     Inset[Style["ΔX<0", 30, Bold], {(empX1 + empX2) / 2, 45}],
     Inset[Style["F", 30, Bold], Scaled[{0.05, .9}]],
     {Dashing[0.025], Thick, Line[{{empX1, 0}, {empX1, 42}}]},
     {Dashing[0.025], Thick, Line[{{empX2, 0}, {empX2, 42}}]}, {Dashed, Thick, Blue,
      Line[{{-L / 2, n1[empX1, tmax]}, {empX1, n1[empX1, tmax]}} /. sol2]}, {Dashed,
      Thick, Red, Line[{{L / 2, n2[empX2, tmax]}, {empX2, n2[empX2, tmax]}} /. sol2]},
     y0 = -10; y1 = 0;
     Table[{Blue, EdgeForm[Thick],
       Rectangle[{-250 + i * 25, y0}, {-250 + (i + 1) * 25, y1}]}, {i, 0, 7}],
     Table[{Blue, EdgeForm[Thick], Triangle[
         {{-250 + i * 25, y0}, {-250 + (i + 1) * 25, y0}, {-250 + i * 25, y1}}]}, {i, 8, 9}],
     Table[{Red, EdgeForm[Thick], Triangle[{{-250 + i * 25, y1},
          {-250 + (i + 1) * 25, y1}, {-250 + (i + 1) * 25, y0}}]}, {i, 8, 9}],
     Table[{Red, EdgeForm[Thick],
       Rectangle[{-250 + i * 25, y0}, {-250 + (i + 1) * 25, y1}]}, {i, 10, 19}],
     Inset[Style["gene 1", White, Bold, 20], {-175 , (y0 + y1) / 2}],
     Inset[Style["gene 2", White, Bold, 20], {175 , (y0 + y1) / 2}]
    }
  ],
  RegionPlot[
   Evaluate[{ψ < n1[empX1, tmax] && x < empX1, ψ < n2[empX2, tmax] && x > empX2} /. sol2],
   {x, -L / 2, L / 2}, {ψ, 0, 100}, BoundaryStyle → None,
   PlotStyle → {{Directive[Blue, Opacity[0.4]]}, {Directive[Red, Opacity[0.4]]}},
   PlotPoints → 40]]
```

Out[ ]=



In[ ]:=
```
tmax = 36 000;
L = 500;
```

```
(*exemplary params for non-stable dynamics*)
ϵ11 = 1.0;
ϵ22 = 1.0;
d1 = 1;
d2 = 1;
γ1 = 0.0005;
γ2 = 0.0004;
h1 = 0.01;
h2 = 0.02;
c1 = 4.5;
c2 = 7.5;
λ1 = Sqrt[d1 / γ1]
λ2 = Sqrt[d2 / γ2]
S1 = 2 c1 * γ1 / h1 / ϵ11
S2 = 2 c2 * γ2 / h2 / ϵ22
R1 = -0.6;
ϵ12 = (2 c1 - ϵ11 * h1 / γ1) / (R1 + 1) / (h2 / γ2)
R2 = Sign[R1] * Abs[1 - (1 - Abs[R1]) ^ (λ2 / λ1)]
ϵ21 = (2 c2 - ϵ22 * h2 / γ2) / (R2 + 1) / (h1 / γ1)
Sign[R1] * λ2 * Log[1 - Abs[R1]]
sol2 = NDSolve[{
    D[n1[x, t], t] == d1 * D[n1[x, t], {x, 2}] -
      γ1 * n1[x, t] + h1 * UnitStep[ϵ11 * n1[x, t] + ϵ12 * n2[x, t] - c1],
    D[n2[x, t], t] == d2 * D[n2[x, t], {x, 2}] -
      γ2 * n2[x, t] + h2 * UnitStep[ϵ22 * n2[x, t] + ϵ21 * n1[x, t] - c2],
    n1[x, 0] == 1.1 c1 / ϵ11 * UnitStep[x1 - x], n2[x, 0] == 1.1 c2 / ϵ22 * UnitStep[x - x2],
    (D[n1[x, t], x] /. x → -L / 2) == 0, (D[n1[x, t], x] /. x → L / 2) == 0,
    (D[n2[x, t], x] /. x → -L / 2) == 0, (D[n2[x, t], x] /. x → L / 2) == 0},
   {n1, n2}, {x, -L / 2, L / 2}, {t, 0, tmax}, AccuracyGoal → 3,
   Method → {"PDEDiscretization" → {"MethodOfLines", "TemporalVariable" → t,
       "SpatialDiscretization" → {"TensorProductGrid", "MinPoints" → 2^11},
       "DifferentiateBoundaryConditions" → True}, "TimeIntegration" → "BDF"},
   MaxStepSize → Automatic, StartingStepSize → Automatic];
Plot3D[Evaluate[{n1[x, t], n2[x, t]} /. sol2],
 {x, -L / 2, L / 2}, {t, 0, tmax}, PlotRange → All]
v1 = Sqrt[4 d1 * γ1] * Abs[S1 - 1] / Sqrt[1 - (S1 - 1) ^2];
v2 = Sqrt[4 d2 * γ2] * Abs[S2 - 1] / Sqrt[1 - (S2 - 1) ^2];
T0 = (x2 - x1) / (Abs[v2] + Abs[v1]);
If[S1 > 1, v1 = -v1];
If[S2 < 1, v2 = -v2];
If[v1 > 0 && v2 < 0, X0 = (Abs[v1] * x2 + Abs[v2] * x1) / (Abs[v1] + Abs[v2])];
solV = FindRoot[{c1 == ϵ11 * h1 / 2 / γ1 * (-v / Sqrt[4 d1 * γ1 + v^2] + 1) + ϵ12 * h2 / 2 / γ2 *
      (1 - Sign[Δc] + Exp[v * Δc / 4 / d2 - Abs[Δc] * Sqrt[4 d2 * γ2 + v^2] / 2 / d2] *
         (v / Sqrt[4 d2 * γ2 + v^2] + Sign[Δc])),
   c2 == ϵ21 * h1 / 2 / γ1 * (1 - Sign[Δc] - Exp[-v * Δc / 4 / d1 - Abs[Δc] *
             Sqrt[4 d1 * γ1 + v^2] / 2 / d1] * (v / Sqrt[4 d1 * γ1 + v^2] - Sign[Δc])) +
     ϵ22 * h2 / 2 / γ2 * (v / Sqrt[4 d2 * γ2 + v^2] + 1)}, {{v, 0(*RandomReal[
     {-0.5,0.5}]*)}, {Δc, (*RandomReal[{-0.5,0.5}]*)-20}}]
empX1 = FindRoot[ϵ11 * n1[x1, tmax] + ϵ12 * n2[x1, tmax] == c1 /. sol2,
    {x1, X0}, AccuracyGoal → 3][[1, 2]];
```

```
empX2 = FindRoot[ϵ21 * n1[x2, tmax] + ϵ22 * n2[x2, tmax] == c2 /. sol2,
    {x2, X0}, AccuracyGoal → 3] [[1, 2]];
center = (empX2 + empX1) / 2;
```

Out[ ]=

```
44.7214
```

Out[ ]=

```
50.
```

Out[ ]=

```
0.45
```

Out[ ]=

```
0.3
```

Out[ ]=

```
-0.55
```

Out[ ]=

```
-0.641004
```

Out[ ]=

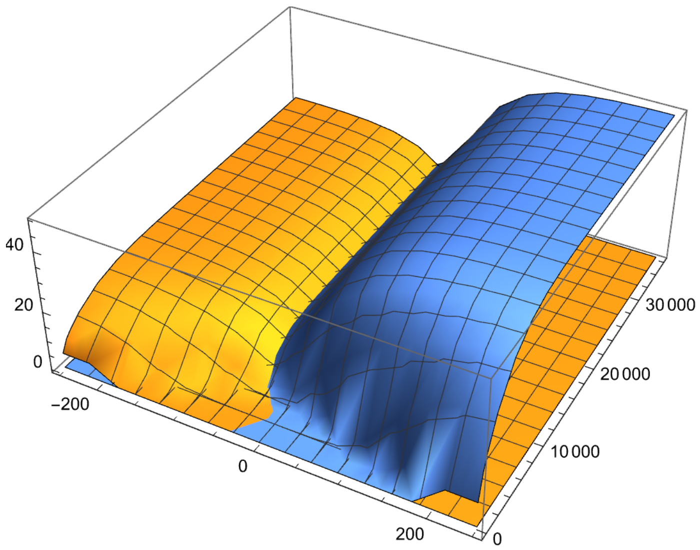
```
-4.87471
```

Out[ ]=

```
45.8145
```

NDSolve: Using maximum number of grid points 10000 allowed by the MaxPoints or MinStepSize options for independent variable x.

Out[ ]=

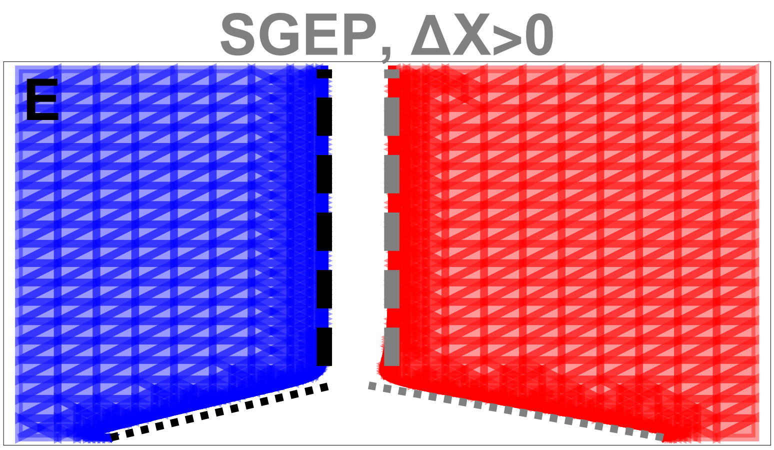


Out[ ]=

$\{v \to -4.08458 \times 10^{-18}, \Delta c \to 45.8145\}$

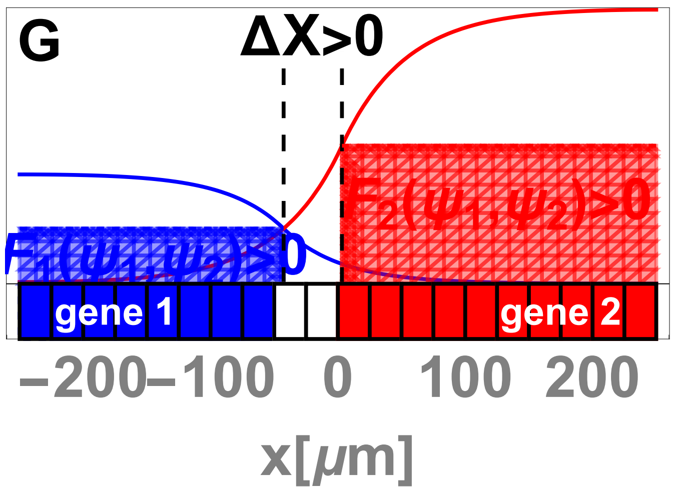
In[ ]:=
```
plotSGEP2 = Show[{ContourPlot[Evaluate[{ϵ11 * n1[x, t] + ϵ12 * n2[x, t] == c1 /. sol2,
      ϵ21 * n1[x, t] + ϵ22 * n2[x, t] == c2 /. sol2} /. t → 3600 * τ],
    {x, -L / 2, L / 2}, {τ, 0, tmax / 3600}, ContourStyle →
     {{Blue, Thickness[0.01]}, {Red, Thickness[0.01]}}, LabelStyle → {Bold, 30},
    ContourLabels → None, FrameLabel → {None(*"x[μm]"*), None}, FrameTicks →
     {{(*Table[{i,Style["a",Opacity[0]]},{i,0,10,0.5}]*)None, None}, {None, None}},
    PlotRangeClipping → False, ImagePadding → {{Automatic, Automatic}, {Automatic, 50}},
    MaxRecursion → 5, PerformanceGoal → "Quality", AspectRatio → 1 / 2, Epilog → {
      (*Inset[LineLegend[{Directive[{Blue,Thickness[0.01]}],
         Directive[{Red,Thickness[0.01]}],Directive[{Black,Dashed,Thickness[0.01]}],
         Directive[{Gray,Dashed,Thickness[0.01]}],
         Directive[{Black,Dashing[{0.05,0.025}],Thickness[0.02]}],
         Directive[{Gray,Dashing[{0.05,0.025}],Thickness[0.02]}]},
        {Style["X1(t)",20,Bold],Style["X2(t)",20,Bold],Style["X1(0)+v1t",20,Bold],
         Style["X2(0)+v2t",20,Bold],Style["vt-ΔX/2+c0",20,Bold],
         Style["vt+ΔX/2+c0",20,Bold]}],Scaled[{0.75,0.75}]],*)
      Inset[Style["E", 30, Bold], Scaled[{0.05, .9}]],
      Inset[Style["SGEP, ΔX>0", Bold, 30, Gray], Scaled[{0.5, 1.07}]]}
   ],
   RegionPlot[Evaluate[{ϵ11 * n1[x, t] + ϵ12 * n2[x, t] > c1 /. sol2,
      ϵ21 * n1[x, t] + ϵ22 * n2[x, t] > c2 /. sol2} /. t → 3600 * τ],
    {x, -L / 2, L / 2}, {τ, 0, tmax / 3600}, BoundaryStyle → None,
    PlotStyle → {Directive[Blue, Opacity[0.4]], Directive[Red, Opacity[0.4]]},
    MaxRecursion → 5, PerformanceGoal → "Quality", ImageSize → {500, 500}],
   ParametricPlot[Evaluate[{{HeavisideTheta[T0 / 3600 - t] * (x1 + v1 * 3600 * t) +
         HeavisideTheta[t - T0 / 3600] * (center + v * (3600 * t - tmax) - Δc / 2), t},
      {HeavisideTheta[T0 / 3600 - t] * (x2 + v2 * 3600 * t) + HeavisideTheta[t - T0 / 3600] *
          (center + v * (3600 * t - tmax) + Δc / 2), t}} /. solV],
    {t, 0, T0 / 3600 - 0.1}, (*RegionFunction→Function[{x,y,u,v},!((T0-100)<u)],*)
    PlotStyle → {{Black, Thickness[0.01], Dashed}, {Gray, Thickness[0.01], Dashed}}],
   ParametricPlot[Evaluate[{{center + v * (3600 * t - tmax) - Δc / 2, t},
      {center + v * (3600 * t - tmax) + Δc / 2, t}} /. solV],
    {t, 7000 / 3600, tmax / 3600}, (*,PlotRange→{Automatic,{0,tmax}},*)
    PlotStyle → {{Black, Thickness[0.02], Dashing[{0.05, 0.025}]},
      {Gray, Thickness[0.02], Dashing[{0.05, 0.025}]}}]}, ImageSize → {500, 250}]
```

Out[ ]=



In[ ]:=
```
profile2 =
 Show[Plot[Evaluate[{n1[x, tmax], n2[x, tmax] (*n1[empX1,tmax]*UnitStep[empX1-x],
      n2[empX2,tmax]UnitStep[x-empX2]*)} /. sol2], {x, -L / 2, L / 2},
   FrameLabel → {"x[μm]", None(*"ψi"*)}, LabelStyle → {Bold, 30, Gray},
   PlotStyle → {{Thick, Blue}, {Red, Thick}, {Blue, Dashed}, {Red, Dashed}},
   Filling → {3 → Bottom, 4 → Bottom}, Frame → True,
   FrameTicks → {{None, None}, {Automatic, None}},
   FillingStyle → {{Directive[Blue, Opacity[0.7]]}, {Directive[Red, Opacity[0.7]]}},
   AspectRatio → 0.5, ImageSize → {500, 250},
   AxesOrigin → {-250, 0}, PlotRangeClipping → False,
   ImagePadding → {{Automatic, Automatic}, {Automatic, Automatic}},
   PlotRange → {Automatic, {-10, 50}},
   Epilog → {Inset[Style["F1(ψ1,ψ2)>0", Bold, 30, Blue], {-145, 5}],
     Inset[Style["F2(ψ1,ψ2)>0", Bold, 30, Red], {125, 15}], Inset[
      Style["ΔX>0", 30, Bold], {(empX1 + empX2) / 2, 45}], Inset[Style["G", 30, Bold],
      Scaled[{0.05, .9}]], {Dashing[0.025], Thick, Line[{{empX1, 0}, {empX1, 42}}]},
     {Dashing[0.025], Thick, Line[{{empX2, 0}, {empX2, 42}}]}, {Dashed, Thick,
      Blue, Line[{{-L / 2, n1[empX1, tmax]}, {empX1, n1[empX1, tmax]}} /. sol2]},
     Dashed, Thick, Red,
     Line[{{L / 2, n2[empX2, tmax]}, {empX2, n2[empX2, tmax]}} /. sol2], y0 = -10;
     y1 = 0;
     Table[{Blue, EdgeForm[Thick],
       Rectangle[{-250 + i * 25, y0}, {-250 + (i + 1) * 25, y1}]}, {i, 0, 7}],
     Table[{White, EdgeForm[Thick],
       Rectangle[{-250 + i * 25, y0}, {-250 + (i + 1) * 25, y1}]}, {i, 8, 9}],
     Table[{Red, EdgeForm[Thick],
       Rectangle[{-250 + i * 25, y0}, {-250 + (i + 1) * 25, y1}]}, {i, 10, 19}],
     Inset[Style["gene 1", White, Bold, 20], {-175 , (y0 + y1) / 2}],
     Inset[Style["gene 2", White, Bold, 20], {175 , (y0 + y1) / 2}]
    }
  ],
  RegionPlot[
   Evaluate[{ψ < n1[empX1, tmax] && x < empX1, ψ < n2[empX2, tmax] && x > empX2} /. sol2],
   {x, -L / 2, L / 2}, {ψ, 0, 100}, BoundaryStyle → None,
   PlotStyle → {{Directive[Blue, Opacity[0.4]]}, {Directive[Red, Opacity[0.4]]}},
   PlotPoints → 40]]
```

Out[ ]=



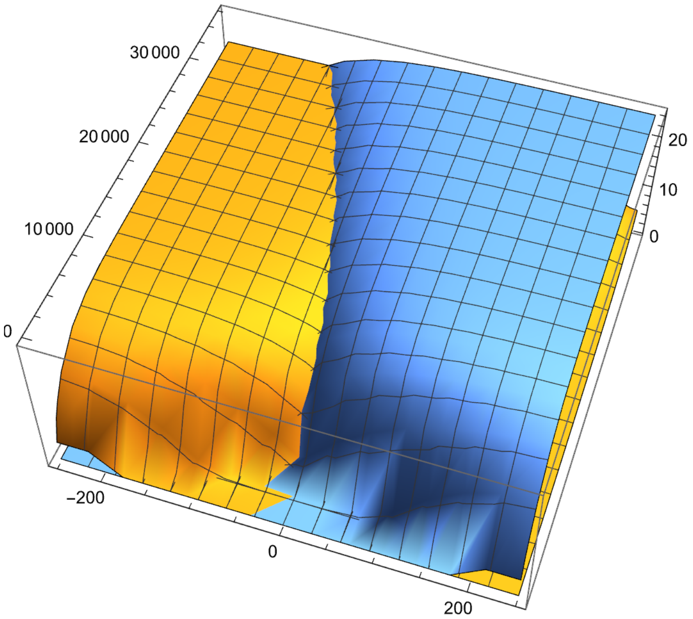
In[ ]:=
```
(*IGEP*)
tmax = 36 000;
L = 500;
(*non-stable, overlapping*)
ϵ11 = 1.0;
ϵ22 = 1.0;
ϵ12 = -0.21;
ϵ21 = -0.37;
d1 = 1;
d2 = 1;
γ1 = 0.0005;
γ2 = 0.0004;
h1 = 0.01;
h2 = 0.01;
c1 = 3.5;
c2 = 3.5;
x1 = -3 L / 8;
x2 = 3 L / 8;
S1 = 2 c1 * γ1 / h1 / ϵ11
S2 = 2 c2 * γ2 / h2 / ϵ22
sol2 = NDSolve[{
    D[n1[x, t], t] == d1 * D[n1[x, t], {x, 2}] -
      γ1 * n1[x, t] + h1 * UnitStep[ϵ11 * n1[x, t] + ϵ12 * n2[x, t] - c1],
    D[n2[x, t], t] == d2 * D[n2[x, t], {x, 2}] -
      γ2 * n2[x, t] + h2 * UnitStep[ϵ22 * n2[x, t] + ϵ21 * n1[x, t] - c2],
    n1[x, 0] == 1.1 c1 / ϵ11 * UnitStep[x1 - x], n2[x, 0] == 1.1 c2 / ϵ22 * UnitStep[x - x2],
    (D[n1[x, t], x] /. x → -L / 2) == 0, (D[n1[x, t], x] /. x → L / 2) == 0,
    (D[n2[x, t], x] /. x → -L / 2) == 0, (D[n2[x, t], x] /. x → L / 2) == 0},
   {n1, n2}, {x, -L / 2, L / 2}, {t, 0, tmax}, AccuracyGoal → 3,
   Method → {"PDEDiscretization" → {"MethodOfLines", "TemporalVariable" → t,
       "SpatialDiscretization" → {"TensorProductGrid", "MinPoints" → 2^11},
       "DifferentiateBoundaryConditions" → True}, "TimeIntegration" → "BDF"},
   MaxStepSize → Automatic, StartingStepSize → Automatic];
Plot3D[Evaluate[{n1[x, t], n2[x, t]} /. sol2],
 {x, -L / 2, L / 2}, {t, 0, tmax}, PlotRange → All]
v1 = Sqrt[4 d1 * γ1] * Abs[S1 - 1] / Sqrt[1 - (S1 - 1) ^2];
v2 = Sqrt[4 d2 * γ2] * Abs[S2 - 1] / Sqrt[1 - (S2 - 1) ^2];
T0 = (x2 - x1) / (Abs[v2] + Abs[v1]);
If[S1 > 1, v1 = -v1];
If[S2 < 1, v2 = -v2];
If[v1 > 0 && v2 < 0, X0 = (Abs[v1] * x2 + Abs[v2] * x1) / (Abs[v1] + Abs[v2])];
```

Out[ ]=
```
0.35
```

Out[ ]=
```
0.28
```

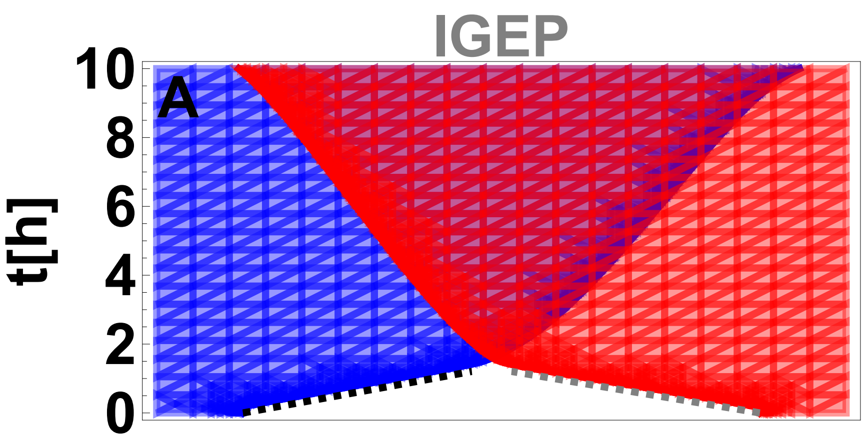
NDSolve: Using maximum number of grid points 10000 allowed by the MaxPoints or MinStepSize options for independent variable x.

Out[ ]=



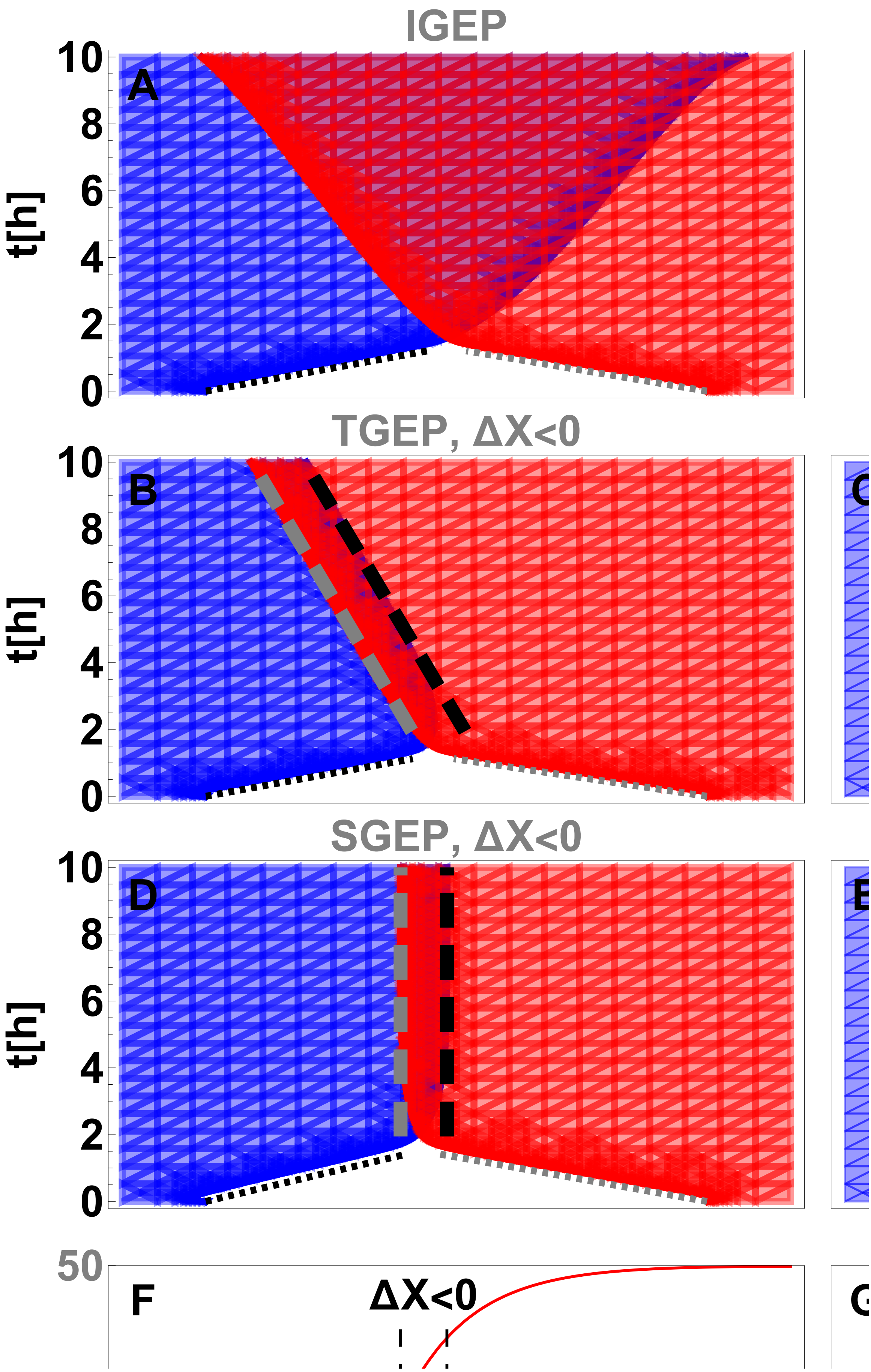
In[ ]:=
```
plotIGEP = Show[{ContourPlot[Evaluate[{ϵ11 * n1[x, t] + ϵ12 * n2[x, t] == c1 /. sol2,
       ϵ21 * n1[x, t] + ϵ22 * n2[x, t] == c2 /. sol2} /. t → 3600 * τ], {x, -L / 2, L / 2}, {τ,
     0, tmax / 3600}, ContourStyle → {{Blue, Thickness[0.01]}, {Red, Thickness[0.01]}},
    LabelStyle → {Bold, 30}, ContourLabels → None, FrameLabel → {None, "t[h]"},
    PlotRangeClipping → False, ImagePadding → {{Automatic, Automatic}, {Automatic, 50}},
    MaxRecursion → 5, PerformanceGoal → "Quality", AspectRatio → 1 / 2,
    FrameTicks → {{True, None}, {None, None}}, Epilog → {
      (*Inset[LineLegend[{Directive[{Blue,Thickness[0.01]}],
         Directive[{Red,Thickness[0.01]}],Directive[{Black,Dashed,Thickness[0.01]}],
         Directive[{Gray,Dashed,Thickness[0.01]}],
         Directive[{Black,Dashing[{0.05,0.025}],Thickness[0.02]}],
         Directive[{Gray,Dashing[{0.05,0.025}],Thickness[0.02]}]},
        {Style["X1(t)",20,Bold],Style["X2(t)",20,Bold],Style["X1(0)+v1t",20,Bold],
         Style["X2(0)+v2t",20,Bold],Style["vt-ΔX/2+c0",20,Bold],
         Style["vt+ΔX/2+c0",20,Bold]}],Scaled[{0.75,0.75}]],*)
      Inset[Style["A", 30, Bold], Scaled[{0.05, .9}]],
      Inset[Style["IGEP", Bold, 30, Gray], Scaled[{0.5, 1.07}]]}
   ],
   RegionPlot[Evaluate[{ϵ11 * n1[x, t] + ϵ12 * n2[x, t] > c1 /. sol2,
       ϵ21 * n1[x, t] + ϵ22 * n2[x, t] > c2 /. sol2} /. t → 3600 * τ],
    {x, -L / 2, L / 2}, {τ, 0, tmax / 3600}, BoundaryStyle → None,
    PlotStyle → {Directive[Blue, Opacity[0.4]], Directive[Red, Opacity[0.4]]},
    MaxRecursion → 5, PerformanceGoal → "Quality"],
   ParametricPlot[{{HeavisideTheta[T0 / 3600 - t] * (x1 + v1 * 3600 * t), t},
     {HeavisideTheta[T0 / 3600 - t] * (x2 + v2 * 3600 * t), t}}, {t, 0, T0 / 3600 - 0.1},
    (*RegionFunction→Function[{x,y,u,v},!((T0-100)<u)],*)PlotStyle →
     {{Black, Thickness[0.01], Dashed}, {Gray, Thickness[0.01], Dashed}}](*,
   ParametricPlot[Evaluate[
     {{center+v*(3600*t-tmax)-Δc/2,t},{center+v*(3600*t-tmax)+Δc/2,t}}/.solV],
    {t,7000/3600,tmax/3600},(*,PlotRange→{Automatic,{0,tmax}},*)
    PlotStyle→{{Black,Thickness[0.02],Dashing[{0.05,0.025}]},
      {Gray,Thickness[0.02],Dashing[{0.05,0.025}]}}]*)}, ImageSize → {500, 250}]
```

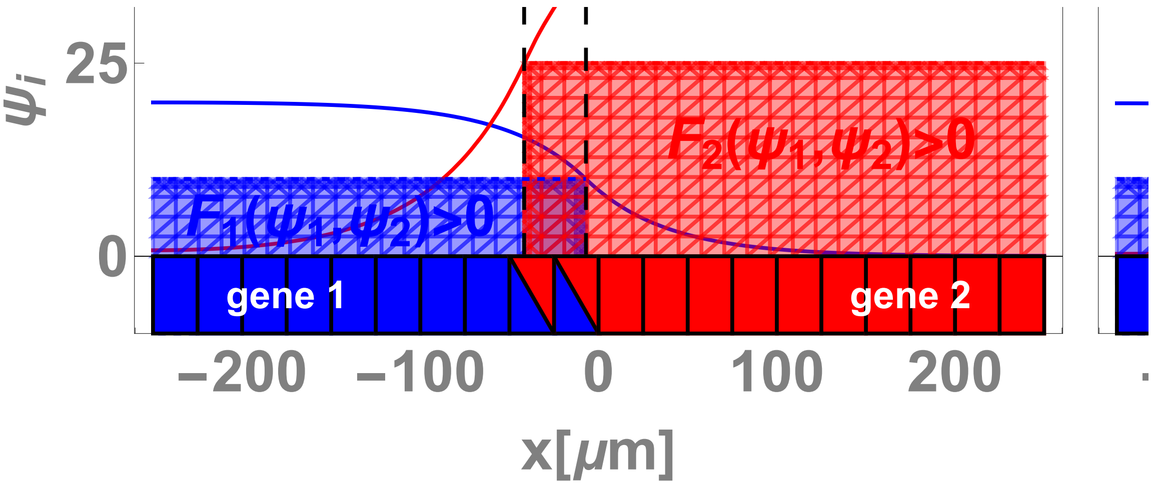
Out[ ]=



In[ ]:=
```
GraphicsGrid[{{plotIGEP, legend}, {plotTGEP1, plotTGEP2},
   {plotSGEP1, plotSGEP2}, {profile1, profile2}}, ImageSize → {1100, Automatic},
  Spacings → {-170, -65}, ImagePadding → {{290, 290}, {Automatic, Automatic}}]
```

Out[ ]=

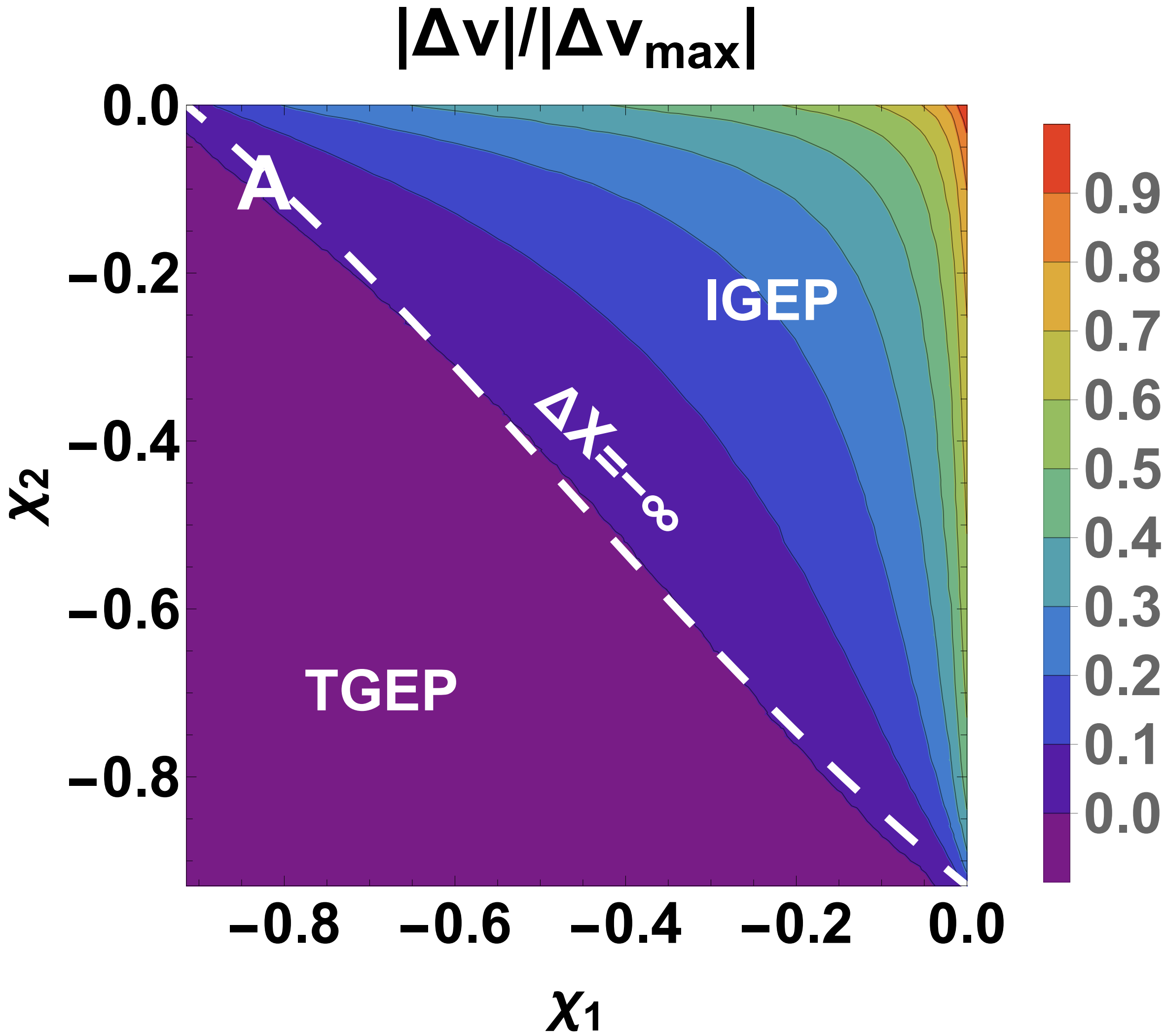

IGEP
A
t[h]
10
8
6
4
2
0
TGEP, ΔX<0
B
C
SGEP, ΔX<0
D
E
50
F
ΔX<0
G



ψ_i
25
0
$F_1(\psi_1,\psi_2)>0$
$F_2(\psi_1,\psi_2)>0$
gene 1
gene 2
−200
−100
0
100
200
x[μm]

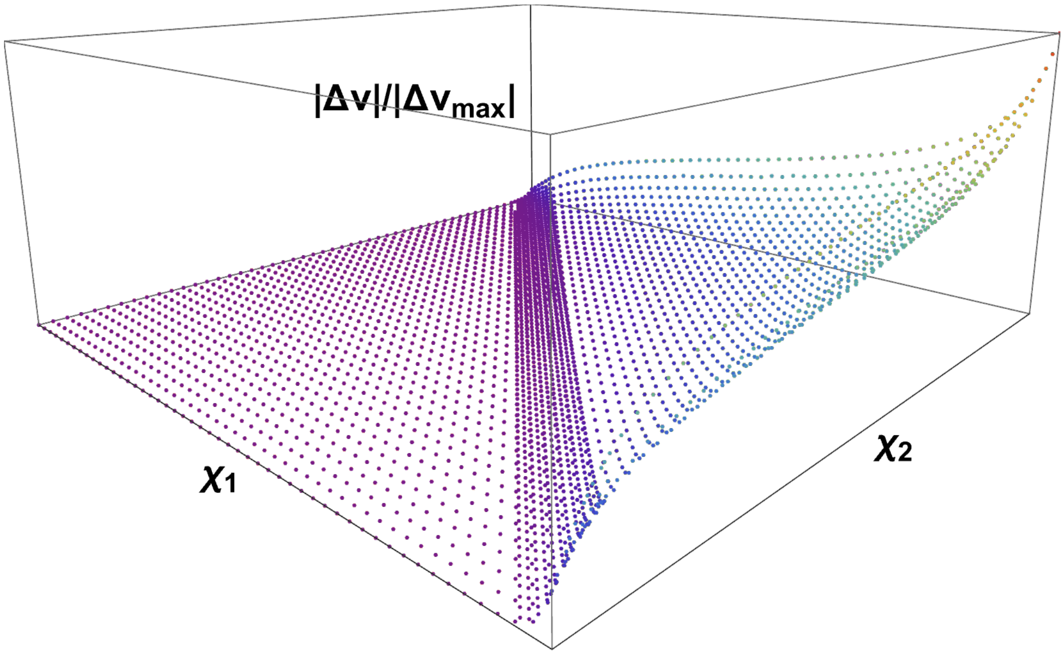
```
In[ ]:= rawfronts =
    Uncompress[Import[NotebookDirectory[] <> "order parameter/fronts2500.dat"][[1, 1]]];
  rawfrontsdiag =
    Uncompress[Import[NotebookDirectory[] <> "order parameter/fronts_diag.dat"][[1, 1]]];
  rawfronts = Join[rawfronts, rawfrontsdiag];
  L = 5000;
  trajlength[data_] := Module[{tab = data, length = Length[data], n = 10},
    While[(L / 2 - Abs[tab[[n, 2]]] ≥ 0.1 L) && (n < length), ++n];
    Return[n]]
  fronts = Table[{rawfronts[[i, 1]], rawfronts[[i, 2]],
      Take[rawfronts[[i, 3]], trajlength[rawfronts[[i, 3]]]],
      Take[rawfronts[[i, 4]], trajlength[rawfronts[[i, 4]]]]}, {i, 1, Length[rawfronts]}];
  dataVfit = Table[{fronts[[i, 1]], fronts[[i, 2]],
      FindFit[Take[fronts[[i, 3]], -Ceiling[Length[fronts[[i, 3]]] / 4]], v * t + c, {v, c}, t,
       AccuracyGoal → 5], FindFit[Take[fronts[[i, 4]], -Ceiling[Length[fronts[[i, 4]]] / 4]],
       v * t + c, {v, c}, t, AccuracyGoal → 5]}, {i, 1, Length[fronts]}];
  deltaVmax = Abs[dataVfit[[1, 4, 1, 2]] - dataVfit[[1, 3, 1, 2]]];
  deltaV =
    Table[{dataVfit[[i, 1]], dataVfit[[i, 2]], Abs[dataVfit[[i, 4, 1, 2]] - dataVfit[[i, 3, 1, 2]]] /
       deltaVmax}, {i, 1, Length[dataVfit]}];
  plotOrderParam =
   Show[ListContourPlot[deltaV, InterpolationOrder → 1, ColorFunction → "Rainbow",
     PlotLegends → BarLegend[Automatic, LegendMarkerSize → 430],
     PlotRange → {Automatic, Automatic, All}, Contours → (*{0,-0.001,-0.005,
       -0.01,-0.015,-0.02}*)Table[If[i == 0, 0.01, 0.1 * i], {i, 0, 10}],
     PlotRangePadding → None, ContourLabels → None, FrameLabel → {"χ1", "χ2"},
     PlotLabel → "|Δv|/|Δvmax|", LabelStyle → {30, Bold},
     (*RegionFunction→Function[{x,y,z},x>-0.4||x<-0.91||y>-0.6||y<-0.91],*)
     Epilog → {Inset[Style["A", 40, Bold, White], Scaled[{0.1, 0.9}]],
       Inset[Rotate[Style["ΔX=-∞", 30, Bold, White], -45 Degree], Scaled[{0.55, 0.55}]],
       Inset[Style["IGEP", 30, Bold, White], Scaled[{0.75, 0.75}]],
       Inset[Style["TGEP", 30, Bold, White], Scaled[{0.25, 0.25}]],
       (*Inset[ListPointPlot3D[deltaV,PlotRange→All,ColorFunction→"Rainbow",
         Ticks→None,AxesLabel→{"χ1","χ2",(*"|Δv|/|Δvmax|"*)"|Δv|/|Δvmax|"},
         LabelStyle→{20,Bold},AxesEdge→{{-1,-1},{1,-1},{-1,1}},
         ViewPoint→{0.95,-1.05,0.25},ImageSize→{400,200}],Scaled[{0.3,0.25}]]*)}
    ],
    Plot[0.5 (S2 - 1 + If[(S1 - 1) / χ1 - 1 > 1, -1, 1] /
          Sqrt[1 + d2 / d1 * γ2 / γ1 * (1 - (1 - S1 + 2 χ1) ^2) / (1 - S1 + 2 χ1) ^2]) /.
      {d1 → 1, d2 → 1, γ1 → 0.0005, γ2 → 0.0004, S1 → 0.05, S2 → 0.1}, {χ1, -1, 0},
     PlotStyle → {White, Dashing[0.05, 0.1], Thickness[0.01]}], ImageSize → {550, 550}]
```

Out[ ]=


|Δv|/|Δvmax|
0.0
−0.2
−0.4
−0.6
−0.8
χ2
−0.8
−0.6
−0.4
−0.2
0.0
χ1
A
IGEP
ΔX=−∞
TGEP
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0

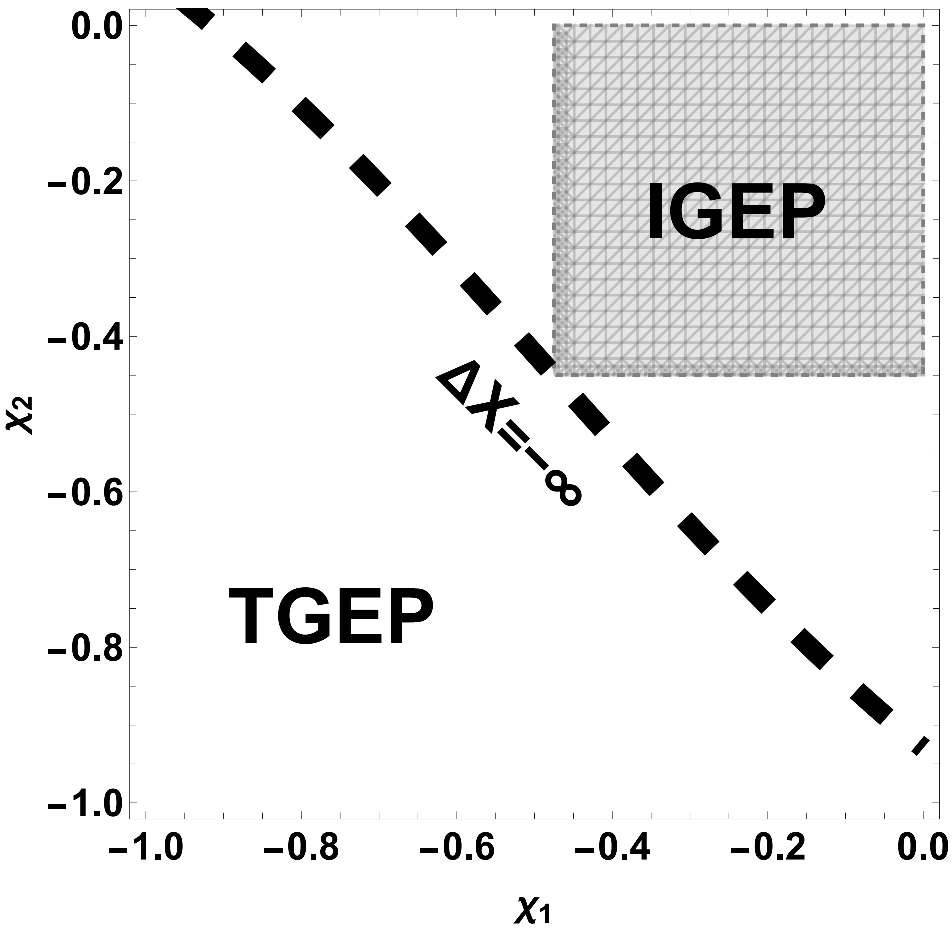
In[ ]:=
```
ListPointPlot3D[deltaV, PlotRange → All, ColorFunction → "Rainbow",
  Ticks → None, AxesLabel → {"χ1", "χ2", (*"|Δv|/|Δvmax|"*)"|Δv|/|Δvmax|"},
  LabelStyle → {20, Bold}, AxesEdge → {{-1, -1}, {1, -1}, {-1, 1}},
  ViewPoint → {0.95, -1.05, 0.25}, ImageSize → {600, 400}]
```

Out[ ]=



In[ ]:=
```
Show[RegionPlot[! ((χ1 < 0 && χ2 < S2 - 1) || ((S1 - 1) / 2 > χ1 && S2 / 2 > χ2 > S2 - 1) ||
      (S1 / 2 > χ1 > (S1 - 1) / 2 && (S2 - 1) / 2 > χ2) || ((S1 - 1) > χ1 && 0 > χ2)) /.
    {S1 → 0.05, S2 → 0.1}, {χ1, -1, 0}, {χ2, -1, 0}, PlotPoints → 50,
   PlotStyle → {Gray, Opacity[0.2]}, BoundaryStyle → {Gray, Thick, Dashed},
   FrameLabel → {"χ₁", "χ₂"}, LabelStyle → {20, Bold}, ImageSize → {500, 500}],
  Plot[0.5 (S2 - 1 + If[(S1 - 1) / χ1 - 1 > 1, -1, 1] /
       Sqrt[1 + d2 / d1 * γ2 / γ1 * (1 - (1 - S1 + 2 χ1) ^2) / (1 - S1 + 2 χ1) ^2]) /.
    {d1 → 1, d2 → 1, γ1 → 0.0005, γ2 → 0.0004, S1 → 0.05, S2 → 0.1}, {χ1, -1, 0},
   PlotStyle → {Thickness[0.025], Dashing[0.05, 0.05], Black}],
  Epilog → {Inset[Style["TGEP", Bold, 40], Scaled[{0.25, 0.25}]],
    Inset[Style["IGEP", Bold, 40], Scaled[{0.75, 0.75}]],
    Inset[Rotate[Style["ΔX=-∞", 30, Bold, Black], -45 Degree], Scaled[{0.48, 0.48}]]}]
```

Out[ ]=

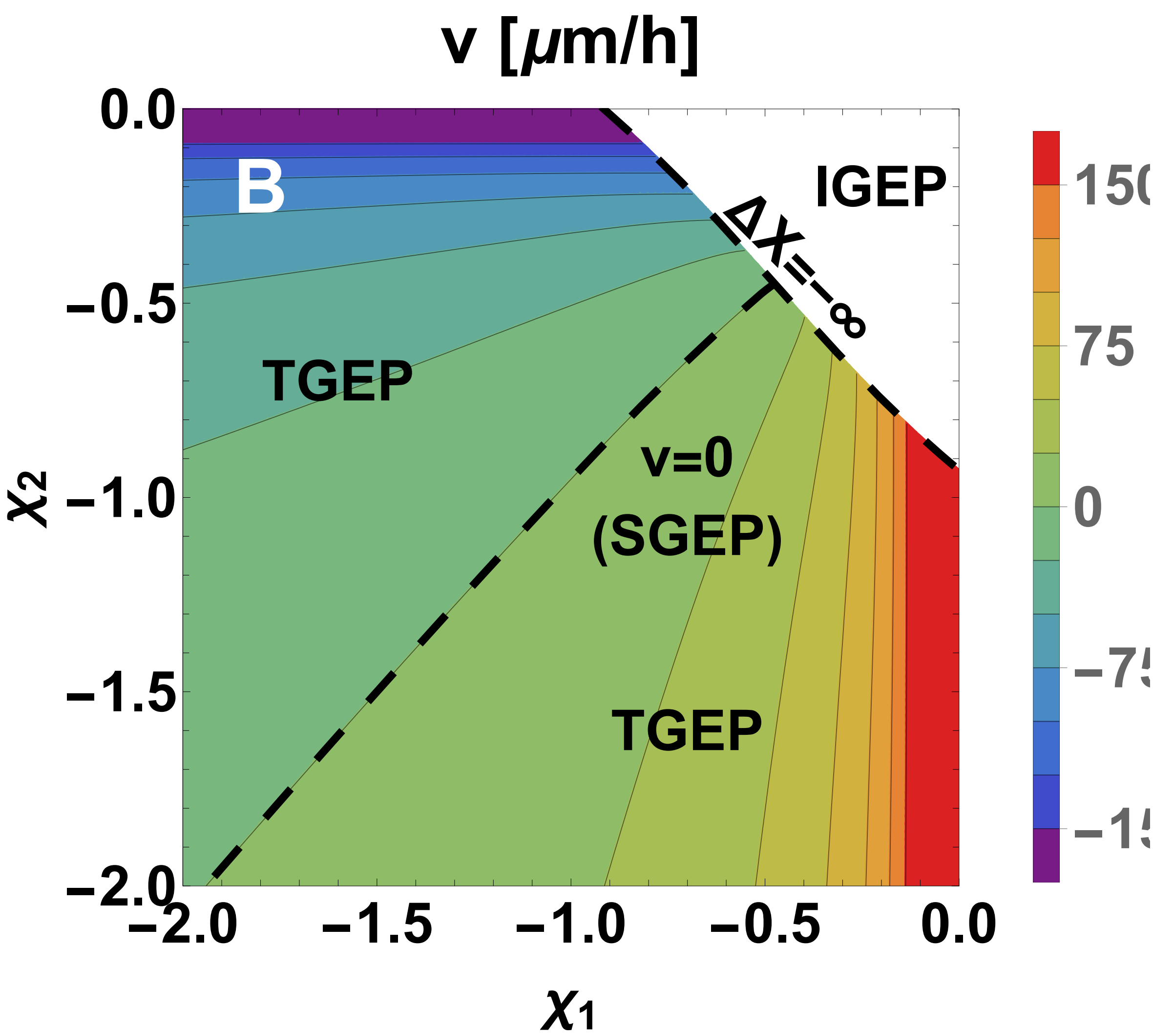


In[ ]:=
```
d1 = 1; d2 = 1; γ1 = 0.0005; γ2 = 0.0004; S1 = 0.05; S2 = 0.1;
r = d2 * γ2 / (d1 * γ1);
(*v→v/Sqrt[4d1γ1], Δc→Δc*Sqrt[4d1γ1]*)
plotχ = Flatten[ParallelTable[{χ1, χ2,
     Quiet[FindRoot[{S1 == (-v / Sqrt[1 + v^2] + 1) + χ1 (1 - Sign[Δc] + Exp[v * Δc / 2 / d2 *
                (1 - Sign[Δc] * Sqrt[r + v^2] / v)] * (v / Sqrt[r + v^2] + Sign[Δc])),
         S2 == χ2 (1 - Sign[Δc] - Exp[-v * Δc / 2 / d1 * (1 + Sign[Δc] * Sqrt[1 + v^2] / v)] *
               (v / Sqrt[1 + v^2] - Sign[Δc])) + (v / Sqrt[r + v^2] + 1)},
        {{v, RandomReal[{-10, 10}]}, {Δc, 0(*RandomReal[{-100,100}]*)}},
        PrecisionGoal → 4, MaxIterations → 200]]},
    {χ1, -2, 0, 0.02}, {χ2, -2, 0, 0.02}], 1];
dataV = Table[{plotχ[[i, 1]], plotχ[[i, 2]], plotχ[[i, 3, 1, 2]] * Sqrt[4 * d1 * γ1] * 3600},
   {i, 1, Length[plotχ]}];
dataΔX = Table[{plotχ[[i, 1]], plotχ[[i, 2]], plotχ[[i, 3, 2, 2]] / Sqrt[4 * d1 * γ1]},
   {i, 1, Length[plotχ]}];
```

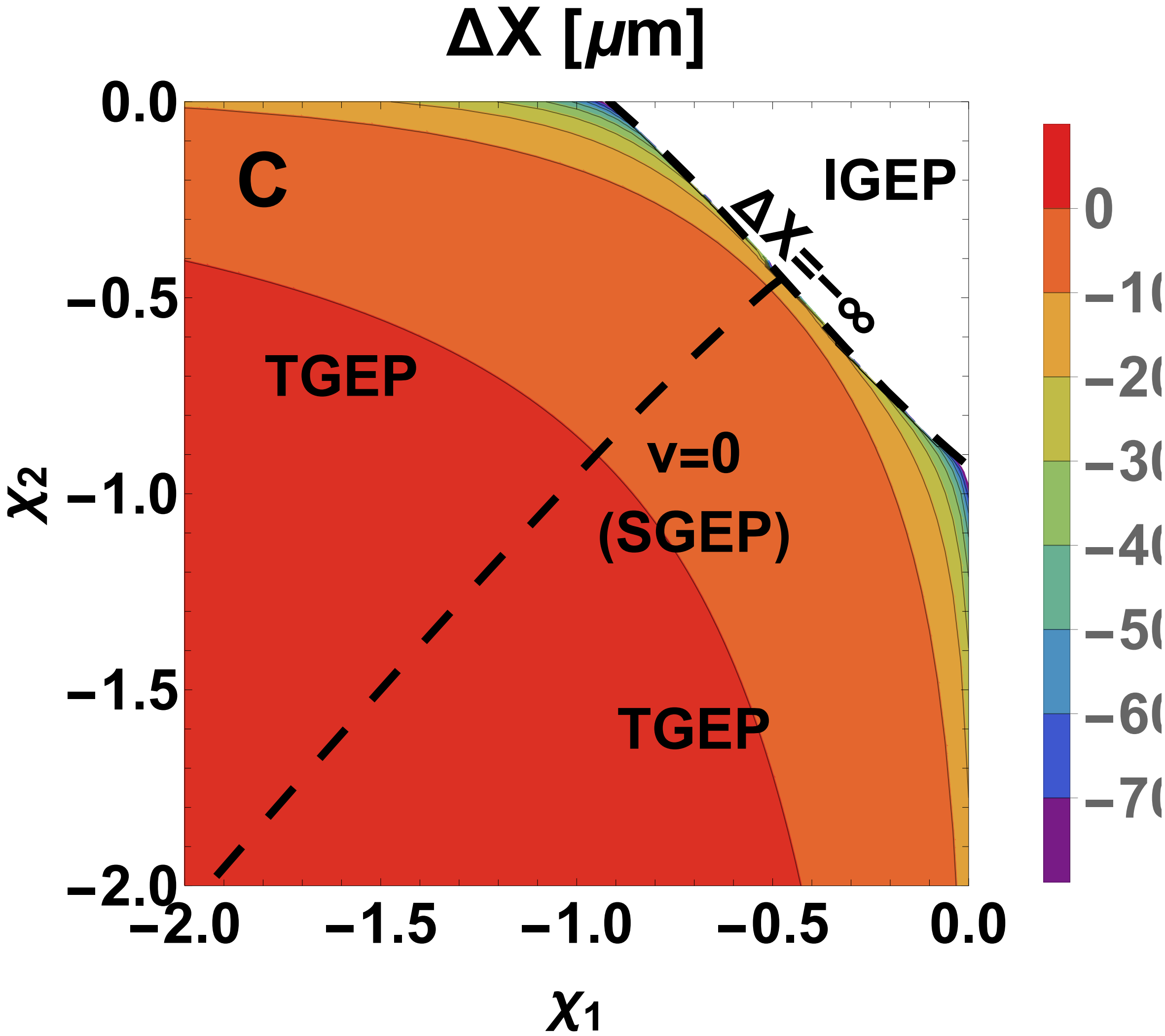
In[ ]:=
```
plotV = Show[
  ListContourPlot[dataV, PlotRange → {{-2, 0}, {-2, 0}, {-200, 200}}, Axes → True,
   InterpolationOrder → 1, PlotLegends → BarLegend[Automatic, LegendMarkerSize → 430],
   Contours → Table[i, {i, -150, 150, 25}], ColorFunction → "Rainbow",
   ClippingStyle → {ColorData["Rainbow"][0], ColorData["Rainbow"][1]}(*Function[{z},
    If[z>ϵ,If[z<1-ϵ,ColorData["Rainbow"][(z-ϵ)/(1-2ϵ)],ColorData["Rainbow"][1]],
      ColorData["Rainbow"][0]]/.ϵ→0.2]*), ColorFunctionScaling → True,
   PlotRangePadding → None, ContourLabels → False, FrameLabel → {"χ₁", "χ₂"},
   PlotLabel → "v  [μm/h]", LabelStyle → {30, Bold}, PlotRangePadding → None,
   RegionFunction → Function[{x, y, z}, y < 0.5 (S2 - 1 + If[(S1 - 1) / x - 1 > 1, -1, 1] /
          Sqrt[1 + d2 / d1 * γ2 / γ1 * (1 - (1 - S1 + 2 x)^2) / (1 - S1 + 2 x)^2])],
   Epilog → {Inset[Style["v=0\n(SGEP)", Bold, 30], Scaled[{0.65, 0.5}]],
     Inset[Rotate[Style["ΔX=-∞", Black, Bold, 30], -45 Degree], Scaled[{0.8, 0.8}]]
     (*,Inset[Style["(S₁-1)/2<χ₁\n(S₂-1)/2<χ₂",Bold,20,Gray],Scaled[{0.89,0.89}]]*),
     Inset[Style["TGEP", 30, Bold], Scaled[{0.2, 0.65}]], Inset[Style["TGEP", 30, Bold],
      Scaled[{0.65, 0.2}]], Inset[Style["IGEP", 30, Bold], Scaled[{0.9, 0.9}]],
     Inset[Style["B", 40, White, Bold], Scaled[{0.1, 0.9}]]}],
  (*RegionPlot[!((χ1<0&&χ2<S2-1)||((S1-1)/2>χ1&&S2/2>χ2>S2-1)||
     (S1/2>χ1>(S1-1)/2&&(S2-1)/2>χ2)||((S1-1)>χ1&&0>χ2)),{χ1,-2,0},{χ2,-2,0},
   PlotPoints→50,PlotStyle→{Gray,Opacity[0.2]},BoundaryStyle→{Gray,Dashed}],*)
  Plot[{(S2 - 1) / (1 + Sign[R] (1 - (1 - Abs[R])^Sqrt[d2 * γ1 / d1 / γ2])) /.
     {R → (S1 - 1) / χ1 - 1}, 0.5 (S2 - 1 + If[(S1 - 1) / χ1 - 1 > 1, -1, 1] /
        Sqrt[1 + d2 / d1 * γ2 / γ1 * (1 - (1 - S1 + 2 χ1)^2) / (1 - S1 + 2 χ1)^2])},
   {χ1, -5, 0}, PlotStyle → {{Black, Dashing[0.05, 0.1], Thickness[0.01]},
     {Black, Dashing[0.05, 0.1], Thickness[0.01]}}], ImageSize → {550, 550}]
plotDeltaX = Show[
  ListContourPlot[dataΔX, PlotRange → {{-2, 0}, {-2, 0}, {-800, 100}},
   ColorFunction → "Rainbow", InterpolationOrder → 1,
   PlotLegends → Placed[BarLegend[Automatic, LegendMarkerSize → 430], Automatic],
   ContourLabels → False, FrameLabel → {"χ₁", "χ₂"}, PlotLabel → "ΔX  [μm]",
```

```
LabelStyle → {30, Bold}, PlotRangePadding → None,
RegionFunction → Function[{x, y, z}, y < 0.5 (S2 - 1 + If[(S1 - 1) / x - 1 > 1, -1, 1] /
      Sqrt[1 + d2 / d1 * γ2 / γ1 * (1 - (1 - S1 + 2 x) ^2) / (1 - S1 + 2 x) ^2])],
Epilog → {Inset[Style["v=0\n(SGEP)", Bold, 30], Scaled[{0.65, 0.5}]],
   Inset[Rotate[Style["ΔX=-∞", Black, Bold, 30], -45 Degree], Scaled[{0.8, 0.8}]]
   (*,Inset[Style["(S1-1)/2<χ1\n(S2-1)/2<χ2",Bold,20,Gray],Scaled[{0.89,0.89}]]*),
   Inset[Style["TGEP", 30, Bold], Scaled[{0.2, 0.65}]],
   Inset[Style["TGEP", 30, Bold], Scaled[{0.65, 0.2}]], Inset[Style["IGEP", 30, Bold],
    Scaled[{0.9, 0.9}]], Inset[Style["C", 40, Bold], Scaled[{0.1, 0.9}]]}],
(*RegionPlot[!((χ1<0&&χ2<S2-1)||((S1-1)/2>χ1&&S2/2>χ2>S2-1)||
    (S1/2>χ1>(S1-1)/2&&(S2-1)/2>χ2)||((S1-1)>χ1&&0>χ2)),{χ1,-2,0},{χ2,-2,0},
 PlotPoints→50,PlotStyle→{Gray,Opacity[0.2]},BoundaryStyle→{Gray,Dashed}],*)
Plot[{(S2 - 1) / (1 + Sign[R] (1 - (1 - Abs[R]) ^Sqrt[d2 * γ1 / d1 / γ2])) /.
   {R → (S1 - 1) / χ1 - 1}, 0.5 (S2 - 1 + If[(S1 - 1) / χ1 - 1 > 1, -1, 1] /
     Sqrt[1 + d2 / d1 * γ2 / γ1 * (1 - (1 - S1 + 2 χ1) ^2) / (1 - S1 + 2 χ1) ^2])},
 {χ1, -5, 0}, PlotStyle → {{Black, Dashing[0.05, 0.1], Thickness[0.01]},
   {Black, Dashing[0.05, 0.1], Thickness[0.01]}}], ImageSize → {550, 550}]
```

Out[ ]=



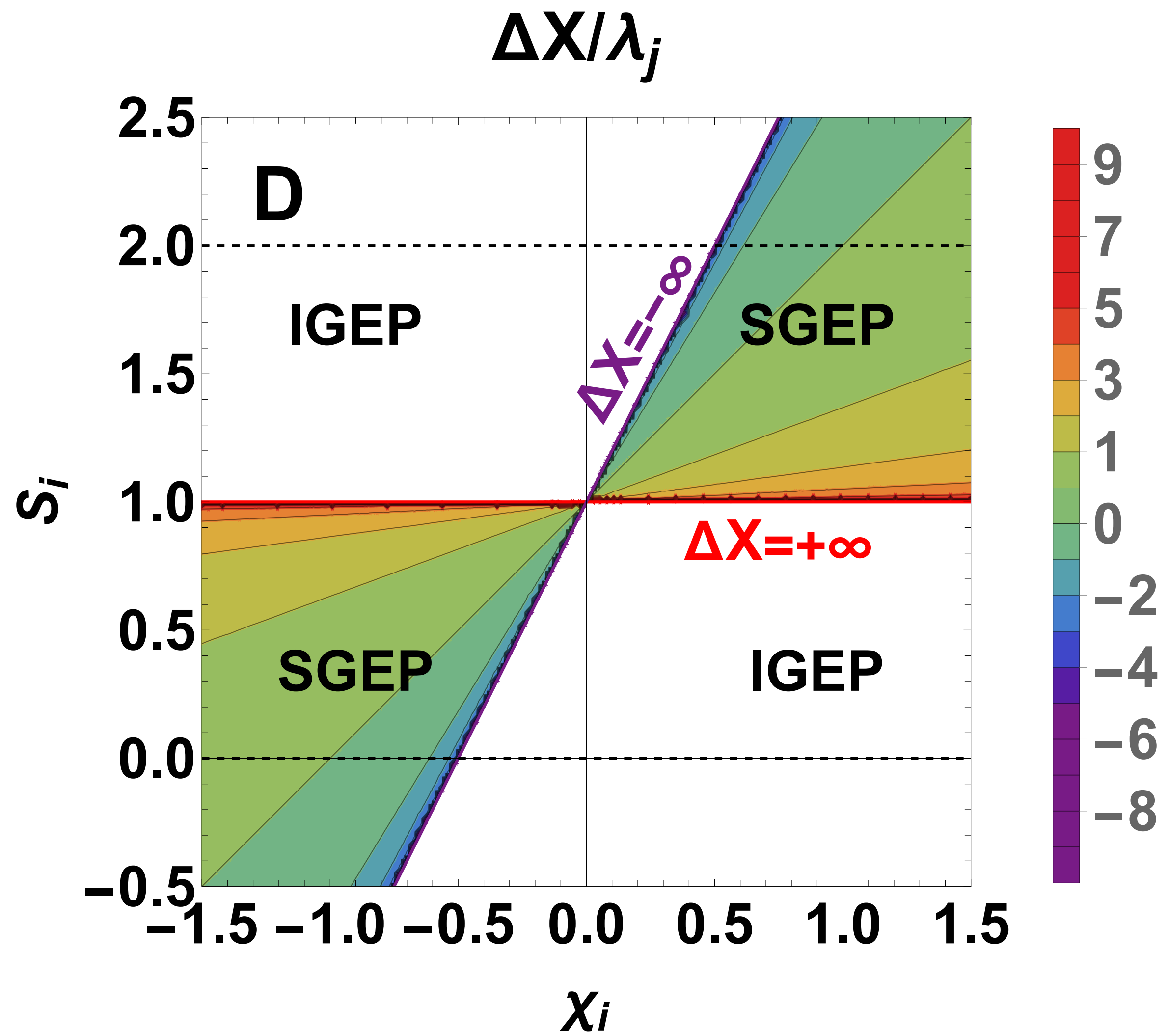
Out[ ]=



ΔX [μm]
C
IGEP
ΔX=−∞
TGEP
v=0
(SGEP)
TGEP
χ_2
χ_1
0.0
−0.5
−1.0
−1.5
−2.0
−2.0
−1.5
−1.0
−0.5
0.0
0
−1(
−2(
−3(
−4(
−5(
−6(
−7(

In[ ]:=
```
plotSGEP = Show[ContourPlot[Sign[R] * Log[1 - Abs[R]] /. R → (s - 1) / x - 1, {x, -1.5, 1.5},
    {s, -0.5, 2.5}, PlotRange → {Automatic, Automatic, {-10, 10}}, Contours →
     Table[If[i == 0, 0, i - 10], {i, 0, 20, 1}], (*PlotPoints→100,*) ColorFunction →
     Function[{z}, If[z > ϵ, If[z < 1 - ϵ, ColorData["Rainbow"][(z - ϵ) / (1 - 2 ϵ)],
         ColorData["Rainbow"][1]], ColorData["Rainbow"][0]] /. ϵ → 0.25],
    ColorFunctionScaling → True, Axes → True, FrameLabel → {"χi", "Si"},
    PlotLabel → "ΔX/λj ", LabelStyle → {Bold, 30},
    PlotLegends → BarLegend[Automatic, LegendMarkerSize → 430],
    PlotRangePadding → None, ContourLabels → None,
    Epilog → {Inset[Style["ΔX=+∞", Red, 30, Bold], Scaled[{0.75, 0.45}]],
      (*Inset[Style["ΔX=0",Gray,30,Bold],Scaled[{0.68,0.6}]],*)
      Inset[Rotate[Style["ΔX=-∞", ColorData["Rainbow"][0], 30, Bold], 60 Degree],
       Scaled[{0.56, 0.72}]], Inset[Style["IGEP", 30, Bold], Scaled[{0.8, 0.28}]],
      Inset[Style["IGEP", 30, Bold], Scaled[{0.2, 0.73}]],
      Inset[Style["SGEP", 30, Bold], Scaled[{0.2, 0.28}]],
      Inset[Style["SGEP", 30, Bold], Scaled[{0.8, 0.73}]],
      Inset[Style["D", Bold, 40], Scaled[{0.1, 0.9}]]}
   ],
   Plot[{1, (*x+1,*) 2 x + 1}, {x, -1.5, 1.5}, PlotRange → {Automatic, {-0.5, 2.5}},
    PlotStyle → {{Red}, (*{Gray},*) {ColorData["Rainbow"][0]}}],
   Plot[{0, 2}, {x, -1.5, 1.5}, PlotRange → {Automatic, {-0.5, 2.5}},
    PlotStyle → {{Dashed, Black}, {Dashed, Black}} (*,Filling→{1→Bottom,2→Top}*)],
   ImageSize → {550, 550}, ImageMargins → Automatic]
```

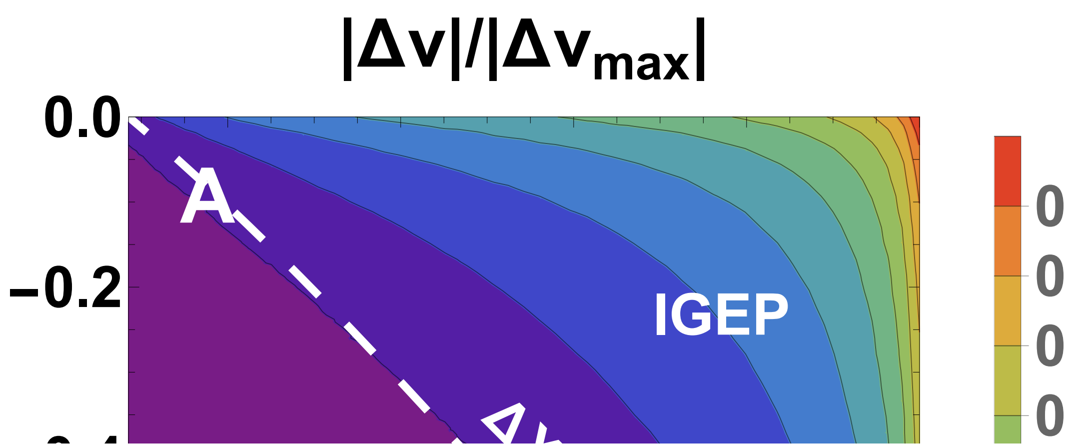
Out[ ]=

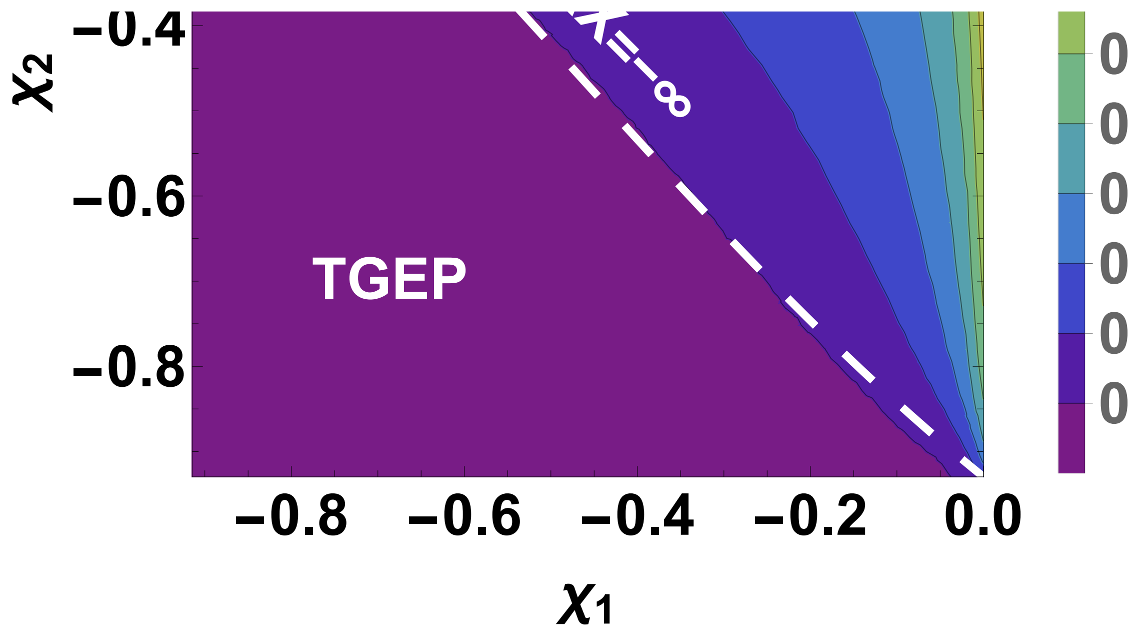


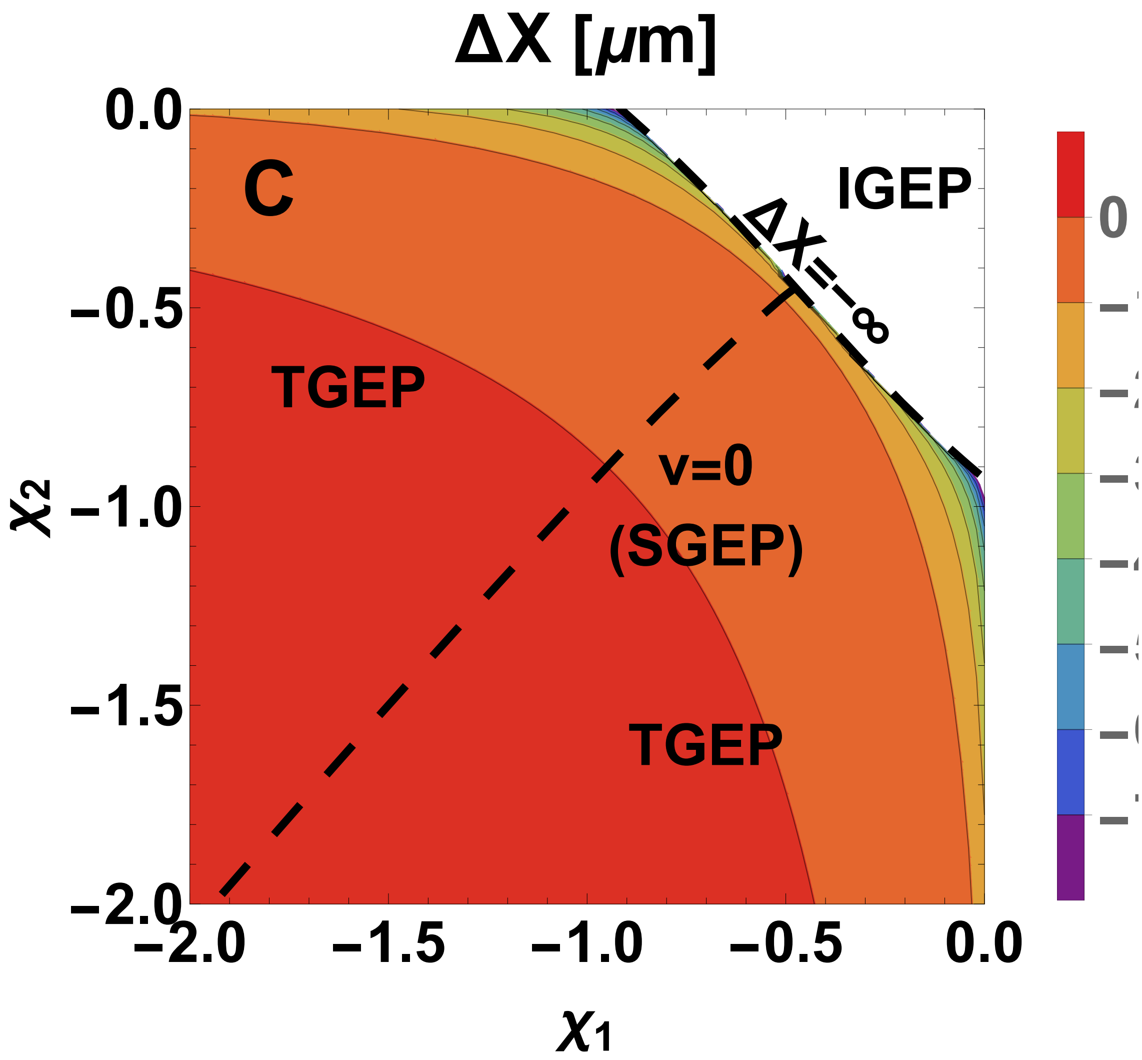
In[ ]:=

```
GraphicsGrid[{{plotOrderParam, plotV}, {plotDeltaX, plotSGEP}},
  ImageSize → {1350, 1200}, Spacings → {-50, -25}]
Export[NotebookDirectory[] <> "fig3all.png",
   GraphicsGrid[{{plotOrderParam, plotV}, {plotDeltaX, plotSGEP}},
    ImageSize → {1350, 1200}, Spacings → {-50, -25}]];
```

Out[ ]=








```
In[ ]:= Export[NotebookDirectory[] <> "data/05_07.dat", Compress[plotχ]];
```

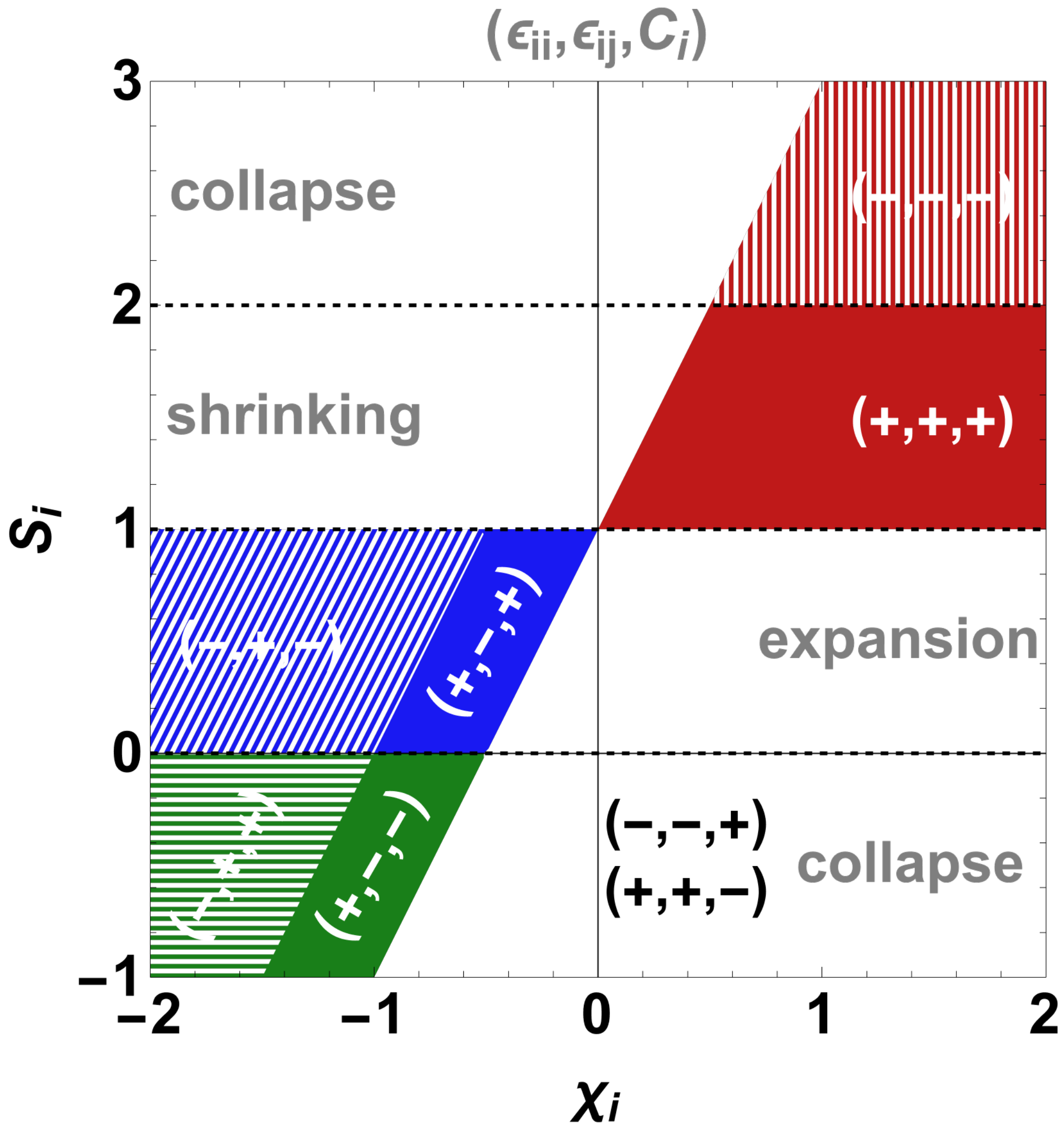
In[ ]:=
```
test = Uncompress[Import[NotebookDirectory[] <> "data/05_07.dat"][[1, 1]]];
```

In[ ]:=
```
test[[2]]
```

Out[ ]=
```
{-4., -3.99, {v → 0.00110051, △c → 109.314}}
```

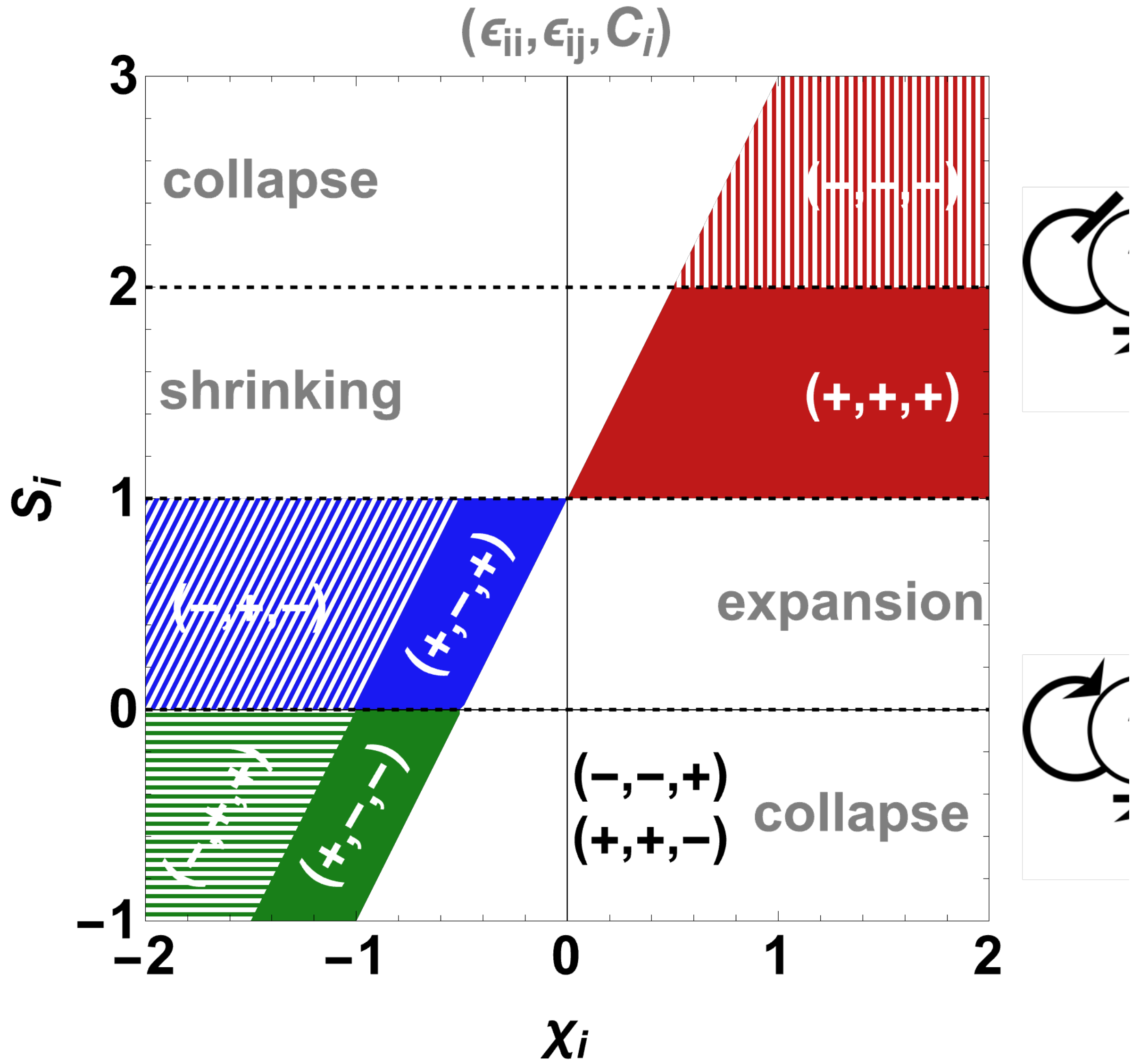
In[ ]:=
```
plotregion = Show[RegionPlot[{
     2 x + 1 > s && s > 1,
     s < 0 && s > 2 x + 1,
     0 < s < 1 && s > 2 x + 1,
     1 > s > 0 && s > 2 + 2 x,
     2 x + 1 > s && s > 2,
     s < 0 && s > 2 + 2 x
    }, {x, -2, 2}, {s, -1, 3}, PlotPoints → 50,
    PlotStyle → {Directive[RGBColor[0.75, 0.1, 0.1], Opacity[1]],
      Directive[RGBColor[0.1, 0.5, 0.1], Opacity[1]], Directive[
       RGBColor[0.1, 0.1, 0.95], Opacity[1]], {HatchFilling[63.26 Degree, 2], White},
      {HatchFilling[90 Degree, 2], White}, {HatchFilling[0 Degree, 2], White}},
    BoundaryStyle → None, FrameLabel → {"χi", "Si"}, LabelStyle → {30, Bold},
    Axes → True, PlotRangeClipping → False, PlotRangePadding → None,
    ImagePadding → {{Automatic, Automatic}, {Automatic, 50}}],
   Plot[{0, 1, 2}, {x, -2, 2}, PlotStyle → Table[{Dashed, Thick, Black}, 3]],
   (*Plot[Table[2x-δ,{δ,-1,2,0.1}],{x,0.5,2},
    PlotRange→{Automatic,{2,3}},PlotStyle→{Orange,Dashed}],
   Plot[Table[2x-δ,{δ,-5,-2,0.1}],{x,-3,-0.5},
    PlotRange→{Automatic,{0,1}},PlotStyle→{White,Dashed}],
   Plot[Table[2x-δ,{δ,-5,-2,0.1}],{x,-3,-0.5},
    PlotRange→{Automatic,{-1,0}},PlotStyle→{RGBColor[0.15,0.15,0.15],Dashed},
    RegionFunction→Function[{x,y},x>-2]],*)
   ImageSize → {600, 600}, Epilog → {
     Inset[Style["(ϵii,ϵij,Ci)", 30, Gray, Bold], Scaled[{0.5, 1.05}]],
     Inset[Style["(+,+,+)", White, 30, Bold], {1.5, 1.5}],
     Inset[Rotate[Style["(+,-,+)", White, 30, Bold], 60 Degree], {-0.5, 0.5}],
     Inset[Rotate[Style["(+,-,-)", White, 30, Bold], 60 Degree], {-1, -0.5}],
     Inset[Rotate[Style["(-,+,+)", White, 30, Bold], 60 Degree], {-1.65, -0.5}],
     Inset[Style["(-,+,-)", White, 30, Bold], {-1.5, 0.5}],
     Inset[Style["(-,-,-)", White, 30, Bold], {1.5, 2.5}],
     Inset[Style["(-,-,+)", Black, 30, Bold], {0.4, -0.3}],
     Inset[Style["(+,+,-)", Black, 30, Bold], {0.4, -0.6}],
     Inset[Style["collapse", 30, Bold, Gray], {-1.4, 2.5}],
     Inset[Style["shrinking", 30, Bold, Gray], {-1.35, 1.5}],
     Inset[Style["expansion", 30, Bold, Gray], {1.35, 0.5}],
     Inset[Style["collapse", 30, Bold, Gray], {1.4, -0.5}]}
   (*,Prolog→Inset[Image[
      Import[NotebookDirectory[]<>"halfmotif.png"],ImageSize→250],{-0.75,2.25}]*)
  ]
```

Out[ ]=



In[ ]:=
```
motifs = Plot[, {x, -1, 1}, AspectRatio → 1, Axes → False, ImageSize → {600, 600}];
GraphicsRow[{plotregion, motifs},
 ImageSize → {1100, Automatic}, Spacings → {-60, Automatic}, Epilog → {
   Inset[Image[Import[NotebookDirectory[] <> "interaction_charts/minus_minus.png"],
     ImageSize → 240], Scaled[{0.625, 0.7}]],
   Inset[Image[Import[NotebookDirectory[] <> "interaction_charts/minus_plus.png"],
     ImageSize → 240], Scaled[{0.875, 0.7}]],
   Inset[Image[Import[NotebookDirectory[] <> "interaction_charts/plus_minus.png"],
     ImageSize → 240], Scaled[{0.625, 0.3}]],
   Inset[Image[Import[NotebookDirectory[] <> "interaction_charts/plus_plus.png"],
     ImageSize → 240], Scaled[{0.875, 0.3}]],
   Inset[Style["(-,-,±)", 40, Bold], Scaled[{0.625, 0.85}]],
   Inset[Style["(-,+,±)", 40, Bold], Scaled[{0.875, 0.85}]],
   Inset[Style["(+,-,±)", 40, Bold], Scaled[{0.625, 0.45}]],
   Inset[Style["(+,+,±)", 40, Bold], Scaled[{0.875, 0.45}]]
  }]
```

Out[ ]=



$(\epsilon_{ii}, \epsilon_{ij}, C_i)$
collapse
shrinking
expansion
collapse
(+,+,+)
(−,−,−)
(−,+,−)
(+,−,+)
(+,−,−)
(−,+,+)
(−,−,+)
(+,+,−)
S_i
χ_i
3
2
1
0
−1
−2
−1
0
1
2