INTELLIGENT DECISION SUPPORT SYSTEMS – EXERCISES IX – DATA ENVELOPMENT ANALYSIS

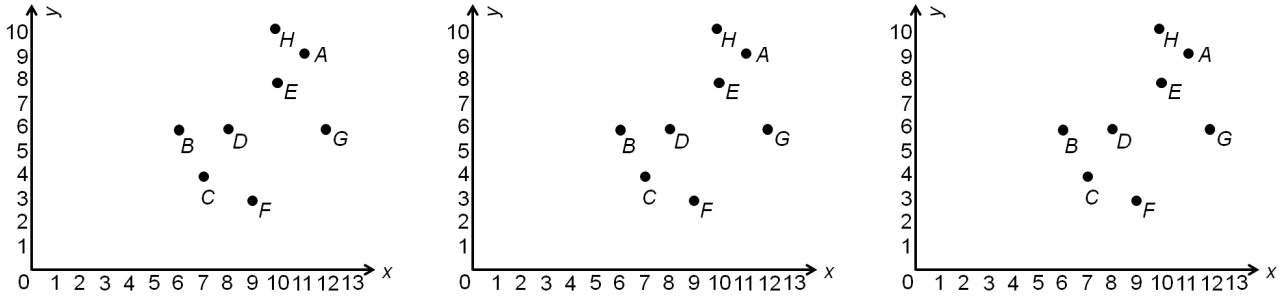
I. Indicate the truth (T) or falsity (F) for the below statements.

a) For all problems, there can be only one efficient unit in the CCR model
b) The CCR model admits convex combinations of existing units
c) The BCC model assumes constat returns to scale
d) The efficiency score of one denotes an efficient unit
e) The input- and output-oriented efficiencies in the CCR model are the same
f) All units efficient in the BCC model are also efficient in the CCR model

II. For the problem involving eight Decision Making Units (see the table given below), draw an efficient frontier while assuming that:

- x is an input and y is an output for the CCR or BCC model; for units D and F compute the efficiency in the input- and output-oriented CCR and BCC model;
- x and y are outputs, and input is the same for all units; show, graphically, how to compute the efficiency for units A, D, and E;
- x and y are inputs, and output is the same for all units for the CCR model; show, graphically, how to compute the efficiency for units B, D, and G.

DMU	A	B	C	D	E	F	G	H
x	11	6	7	8	10	9	12	10
y	9	6	4	6	8	3	6	10



III. Formulate the problem that verifies the efficiency of DMU I according to the input-oriented CCR model. Consider both perspectives: efficiency- and combination-oriented.

DMU	Input 1	Input 2	Output 1	Output 2	Output 3
I	5	14	**9**	**4**	**16**
II	8	15	5	7	10
III	7	12	4	9	13

$\max \mathbf{9}\mu_1 + \mathbf{4}\mu_2 + \mathbf{16}\mu_3$

…

$\mathbf{9}\mu_1 + \mathbf{4}\mu_2 + \mathbf{16}\mu_3 \le 5v_1 + 14v_2$

$5\mu_1 + 7\mu_2 + 10\mu_3 \le 8v_1 + 15v_2$

…

$\mu_1, \mu_2, \mu_3, v_1, v_2 \ge 0$

$\min \theta$

$5\lambda_1 + 8\lambda_2 + 7\lambda_3 \le \ldots$

$14\lambda_1 + 15\lambda_2 + 12\lambda_3 \le \ldots$

$\mathbf{9}\lambda_1 + 5\lambda_2 + 4\lambda_3 \ge \ldots$

$\mathbf{4}\lambda_1 + 7\lambda_2 + 9\lambda_3 \ge \ldots$

$\mathbf{16}\lambda_1 + 10\lambda_2 + 13\lambda_3 \ge \ldots$

$\lambda_1, \lambda_2, \lambda_3 \ge 0$

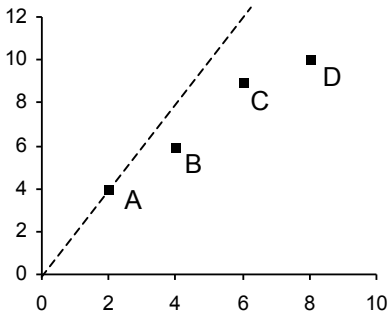
The modifications needed in the BCC model:

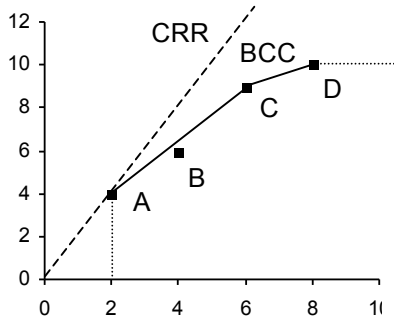
- In the space of efficiencies, objective function: $\max 9\mu_1 + 4\mu_2 + 16\mu_3 + \mathbf{u_0}$,
  and then, in the constraints, e.g.: $5\mu_1 + 7\mu_2 + 10\mu_3 + \mathbf{u_0} \le 8v_1 + 15v_2$ ($\mathbf{u_0}$ free)
- In the space of combinations, we only add: $\lambda_1 + \lambda_2 + \lambda_3 = 1$, which enforces the convex combinations.

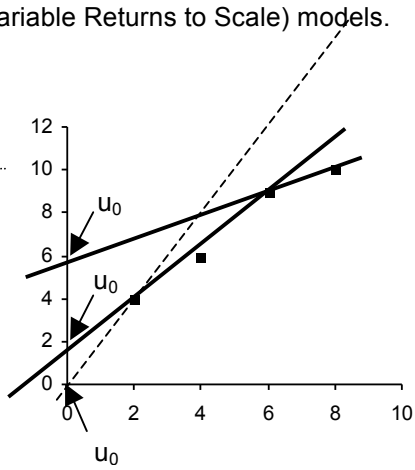
Could you interpret the solutions to the above-formulated problems? That is, indicate the efficiency status (efficient or inefficient), the required changes, and for the combination-oriented perspective, point out the reference set.

IV. Compare the results of the CCR (CRS - Constant Returns to Scale) and BCC (VRS – Variable Returns to Scale) models.

DMU	Input	Output
A	2	4
B	4	6
C	6	9
D	8	10







Which units are efficient in the BCC model? Are they all efficient in the CCR model?

Answer: BCC - CCR -

For example, B is inefficient according to BCC, because a convex combination 50%-50% of units A and C consumes … units of the input and produces … units of the output, which is more than 6 units that are produced by B

- BBC output efficiency for unit B is therefore 1/(6.5/6) = 0.923