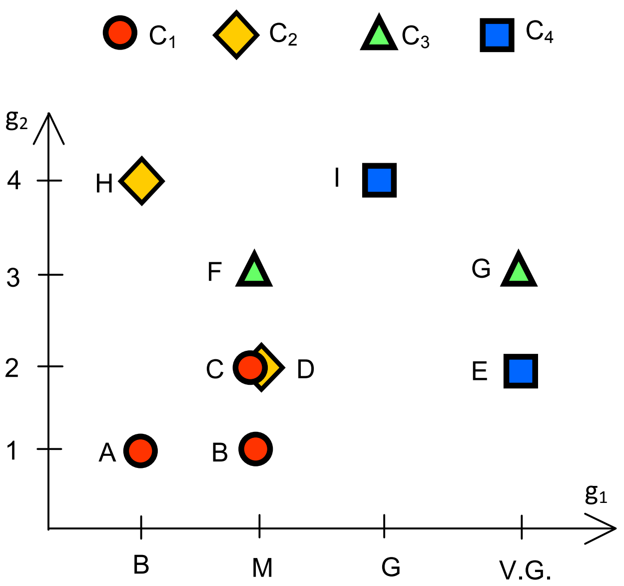
INTELLIGENT DECISION SUPPORT SYSTEMS – EXERCISES VII – DOMINANCE-BASED ROUGH SET APPROACH

I. Indicate the truth (T) or falsity (F) for the below statements.

a) DRSA handles inconsistencies with respect to the indiscernibility relation	
b) The upper approximation of a given class union contains all objects that possibly belong to this union	T
c) The upper approximation of any class union is always a proper superset of its lower approximation	F
d) Possible rules are induced from upper approximations of classes	
e) For certain decision rules, the certainty factor is always equal to 1	T
f) The DOMLEM algorithm finds a local covering of a given set	
g) Each decision rule coupled with DRSA must use all criteria in its decision part	
h) The core is defined as the intersection of all reducts	
i) The quality of approximation of classification for a reduct is always equal to one	
j) When considering two subsets of condition attributes *P*, *P*', such that *P*' ⊂ *P*, the quality of approximation of classification for *P*' is always not higher than for *P*	
k) The standard classification algorithm coupled with DRSA may return imprecise assignment	

II. Consider a decision table composed of 2 gain-type criteria g_1 and g_2, and 9 objects A-I. The relevant data is provided in the below figure, where C_1 is the least preferred class, and C_4 is the most preferred class. First, compute the dominance cones specified below. Then, compute the lower and upper approximations of all class unions. Finally, compute the quality of approximation of classification, and find the reduct.



$$\gamma_P(\mathbf{Cl}) = \frac{\left|U - \bigcup_{t\in\{2,\dots,m\}} Bn_P\left(Cl_t^{\ge}\right)\right|}{|U|} =$$

$$RED_{\mathbf{Cl}}(P) =$$

$$CORE_{\mathbf{Cl}}(P) = \bigcap RED_{\mathbf{Cl}}(P)$$

$D_P^+(x) = \{y \in U : y D_P x\}$

$D_P^-(x) = \{y \in U : x D_P y\}$

$D_P^+(F) = \{F, G, I\}$

$D_P^-(F) = \{F, A, B, C, D\}$

$D_P^+(H) = \{H, I\}$

$D_P^-(H) = \{H, A\}$

$D_P^+(D) =$ {D,C,F,I,G,E}

$D_P^-(D) =$ {D,C,B,A}

$D_P^+(E) =$ {E, G}

$D_P^-(E) =$ {E, C, D, B, A}

$\underline{P}\left(Cl_t^{\ge}\right) = U - \overline{P}\left(Cl_{t-1}^{\le}\right)$

$\underline{P}\left(Cl_3^{\ge}\right) = U - \overline{P}\left(Cl_2^{\le}\right) = \{E, F, G, I\}$

$\underline{P}\left(Cl_t^{\le}\right) = \left\{x \in U : D_P^-(x) \subseteq Cl_t^{\le}\right\}$

$\underline{P}(C_1^{\le}) = \{A, B\}$

$\overline{P}(C_1^{\le}) = \{A, B, C, D\}$

$\underline{P}(C_2^{\le}) =$ {A, B, C, D, H}

$\overline{P}(C_2^{\le}) =$ {A, B, C, D, H}

$\underline{P}(C_3^{\le}) =$ {A,B,C,D,H,F}

$\overline{P}(C_3^{\le}) =$ {A,B,C,D,F,H,G,E}

$\underline{P}\left(Cl_t^{\ge}\right) = \left\{x \in U : D_P^+(x) \subseteq Cl_t^{\ge}\right\}$

$\underline{P}(C_4^{\ge}) = \{I\}$

$\overline{P}(C_4^{\ge}) = \{I, E, G\}$

$\underline{P}(C_3^{\ge}) =$ {F,G,I,E}

$\overline{P}(C_3^{\ge}) =$ {F,I,G,E}

$\underline{P}(C_2^{\ge}) =$ {H,F,I,G,E}

$\overline{P}(C_2^{\ge}) =$ {C,D,F,H,I,G,E}

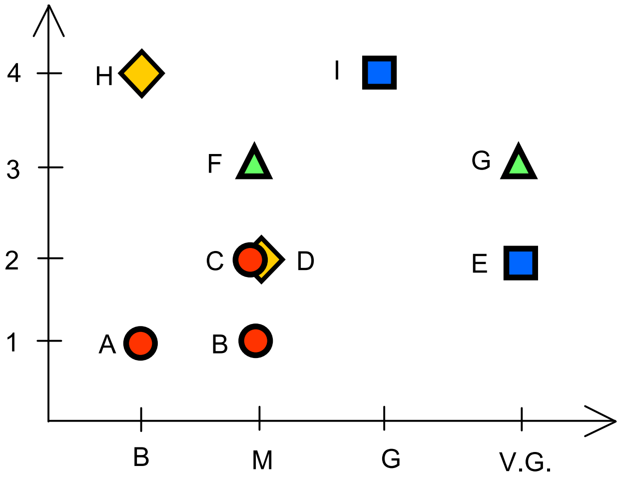
$Bn_P\left(Cl_t^{\ge}\right) = Bn_P\left(Cl_{t-1}^{\le}\right)$

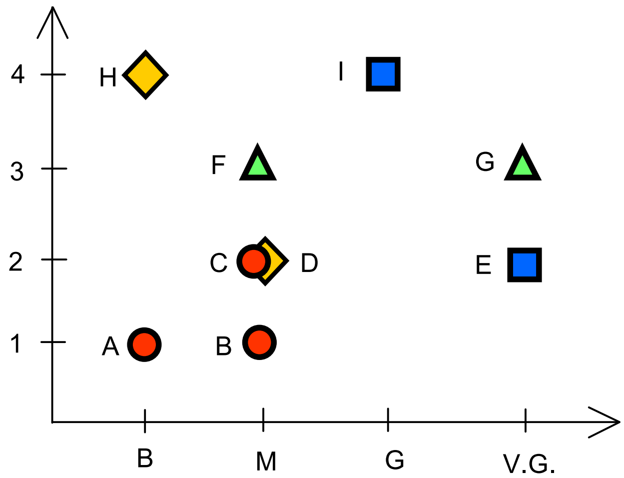
$Bn_P\left(Cl_2^{\ge}\right) = Bn_P\left(Cl_1^{\le}\right) = \{C, D\}$

III. Mark (graphically) the lower and upper approximations for the selected class unions given below.

$$\underline{P}(C_1^{\le}) = \{A,B\}$$
$$\overline{P}(C_1^{\le}) = \{A,B,C,D\}$$
$$\underline{P}(C_2^{\le}) = \{A,B,C,D,H\}$$
$$\overline{P}(C_2^{\le}) = \{A,B,C,D,H\}$$

$$\underline{P}(C_4^{\ge}) = \{I\}$$
$$\overline{P}(C_4^{\ge}) = \{I,E,G\}$$
$$\underline{P}(C_3^{\ge}) = \{I,E,G,F\}$$
$$\overline{P}(C_3^{\ge}) = \{I,E,G,F\}$$





IV. Induce a minimal set of minimal certain decision rules for upward $Cl^{\ge}$ and downward $Cl^{\le}$ class unions. The rules should cover the objects from the lower approximations computed in the previous tasks.

$Cl_1^{\le}$ {A,B}

$t_1=(g_1(x)\le M)$ {A, B, C, D, F, H} 0.33 2

$t_2=(g_1(x)\le B)$ {A,H} 0.5 1

$t_3=(g_2(x)\le 1)$ {A,B} 1.0 2

if $g_2(x) \le 1$ then $x\in Cl_1^{\le}$

$C_2^{\ge}$ {E,F,G,H,I}

$t_1=(g_1(x)\ge B)$ {A,B,C,D,E,F,G,H, I} 0.55 5

$t_2=(g_1(x)\ge M)$ {B,C,D,E,F,G,I} 0.58 4

$t_3=(g_1(x)\ge G)$ {I,E,G} 1.0 3

$t_4=(g_1(x)\ge V.G)$ {E,G} 1.0 2

$t_5=(g_2(x) \ge 2)$ {C,D,E,F,G,H,I} 0.55 5

$t_9=(g_2(x) \ge 3)$ {F,G,H,I} 1.0 4

$t_7=(g_2(x) \ge 4)$ {H,I} 1.0 2

if $g_2(x) \ge 3$ then $x\in C_2^{\ge}$

$t_1=(g_1(x)\ge V.G)$ {E} 1.0 1

$t_2=(g_2(x)\ge 2)$ { C,D,E} 0.33 1

if $g_1(x) \ge V.G$ then $x\in C_2^{\ge}$

$Cl_2^{\le}$ {A,B,C,D,H}

$t_1=(g_1(x)\le M)$ {A,B,C,D,F,H} 0.83 5

$t_2=(g_1(x)\le B)$ {A,H} 1.0 2

$t_3=(g_2(x)\le 4)$ {A,B,C,D,E,F,G,H} 0.55 5

$t_4=(g_2(x)\le 2)$ {A,B,C,D,E} 0.8 4

$t_5=(g_2(x)\le 1)$ {A,B} 1.0 2

a) if $g_1(x) \le B$ then $x\in Cl_2^{\le}$ (covers {A,H}, still to be covered {B,C,D})

$t_1=(g_1(x)\le M)$ {B, C, D, F} 0.75 3 (nie wypisuję {A,H})

$t_2=(g_2(x)\le 2)$ {B,C,D,E} 0.75 3

$t_3=(g_2(x)\le 1)$ {B} 1.0 1

b) if $g_2(x) \le 1$ then $x\in Cl_2^{\le}$ (covers {B}, still to be covered {C,D})

$t_1=(g_1(x)\le M)$ {C, D, F} 0.67 2

$t_2=(g_2(x)\le 2)$ {C,D,E} 0.67 2

c) if $g_1(x) \le M$ and $g_2(x) \le 2$ then $x\in Cl_2^{\le}$ (covers {A,B,C,D})

rule b) is redundant

$C_3^{\le}$ {A,B,C,D,H,F}

$C_3^{\ge}$ {E,F,G,I}

$C_4^{\ge}$ {I}

V. Mark (graphically) the below provided decision rules.

R1) if $g_2(x) \le 1$ then $x \in Cl_1^{\le}$

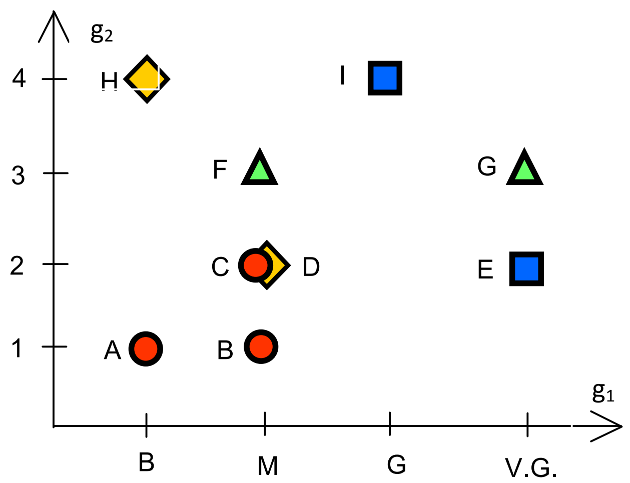
R2) if $g_1(x) \le B$ then $x \in Cl_2^{\le}$

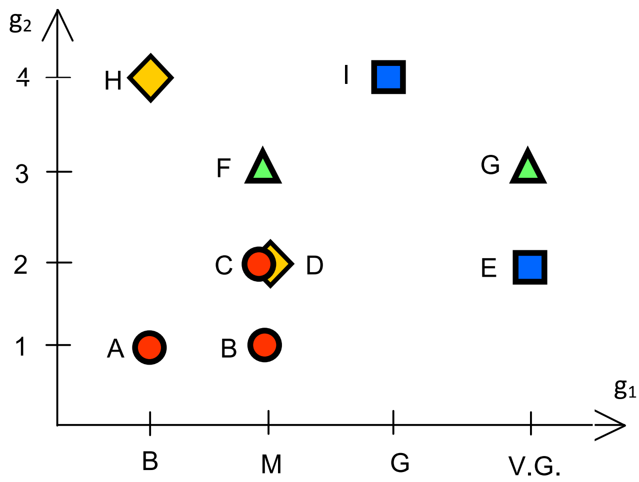
R3) if $g_1(x) \le M$ and $g_2(x) \le 2$ then $x \in Cl_2^{\le}$

R4) if $g_2(x) \ge 3$ then $x \in C_2^{\ge}$

R5) if $g_1(x) \ge V.G$ then $x \in C_2^{\ge}$

R6) if $g_1(x) \ge G$ and $g_2(x) \ge 4$ then $x \in C_4^{\ge}$





VI. For the below-provided decision table with 3 gain-type criteria (K1, K2, K3) and 7 objects, compute the lower and upper approximations of class unions (C_1 is the least preferred class, C_3 is the most preferred class).

	K1	K2	K3	Cl
A	3	2	3	C_3
B	**2**	**3**	**2**	**C_2**
C	1	1	2	C_1
D	**2**	**3**	**2**	**C_3**
E	2	3	1	C_2
F	1	2	1	C_1
G	3	1	3	C_3

$\underline{P}(C_1^{\le}) =$ {C,F}

$\overline{P}(C_1^{\le}) =$ {C,F}

$\underline{P}(C_2^{\le}) =$ {C,F,E}

$\overline{P}(C_2^{\le}) =$ {C,F,E,B,D}

$\underline{P}(C_3^{\ge}) =$ {A,G}

$\overline{P}(C_3^{\ge}) =$ {A,G,B,D}

$\underline{P}(C_2^{\ge}) =$ {A,B,D,E,G}

$\overline{P}(C_2^{\ge}) =$ {A,B,D,E,G}

Induce certain decision rules for the following two unions: $C_2^{\le}$ and $C_2^{\ge}$.

if K1 <= 1 then x in C<=2
if K3 <= 1 then x in C<=2

Determine the reducts and the core.

Candidates for being reducts: {K1}, {K2}, {K3}, {K1,K2}, {K1,K3}, {K2,K3}, {K1,K2,K3}.

Core:

VII. Consider the below set of decision rules. Use it to classify non-reference objects O1-O4 using a standard classification algorithm.

Rules	Objects	Activated rules (decision parts)	Recommended class(es)
if $A \ge 3$ and $C \ge 2$ then $\ge C_5$	O1: A=2, B=2, C=2		
if $A \ge 2$ and $C \ge 2$ then $\ge C_3$	O2: A=2, B=1, C=1		
if $B \le 2$ and $C \le 2$ then $\le C_4$	O3: A=2, B=3, C=3		
if $A \le 1$ and $C \le 1$ then $\le C_2$	O4: A=3, B=2, C=2		
if $B \le 1$ then $\le C_1$			