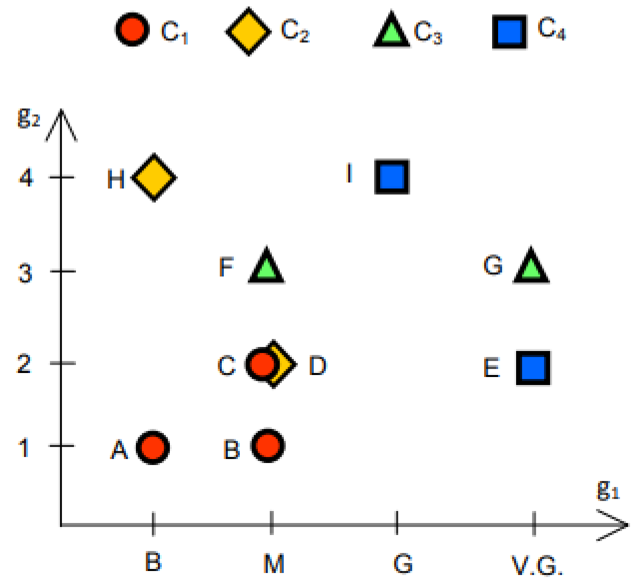
Egzamin 2

Wykład 7 – DOMINANCE-BASED ROUGH SET APPROACH

I. Indicate the truth (T) or falsity (F) for the below statements.

a) DRSA handles inconsistencies with respect to the indiscernibility relation	T
b) The upper approximation of a given class union contains all objects that possibly belong to this union	T
c) The upper approximation of any class union is always a proper superset of its lower approximation	F
d) Possible rules are induced from upper approximations of classes	T
e) For certain decision rules, the certainty factor is always equal to 1	T
f) The DOMLEM algorithm finds a local covering of a given set	T
g) Each decision rule coupled with DRSA must use all criteria in its decision part	F
h) The core is defined as the intersection of all reducts	T
i) The quality of approximation of classification for a reduct is always equal to one	F
j) When considering two subsets of condition attributes P, P', such that P' $\subset$ P, the quality of approximation of classification for P' is always not higher than for P	T
k) The standard classification algorithm coupled with DRSA may return imprecise assignment	T

II. Consider a decision table composed of 2 gain-type criteria g1 and g2, and 9 objects A-I. The relevant data is provided in the below figure, where C1 is the least preferred class, and C4 is the most preferred class. First, compute the dominance cones specified below. Then, compute the lower and upper approximations of all class unions. Finally, compute the quality of approximation of classification, and find the reduct.



$\gamma_p(Cl)$ = (9 - (2+2))/9 → 5/9

$RED_{Cl}(P)$ = {g_1, g_2}

$CORE_{Cl}(P) = \cap\ RED_{Cl}(P)$ = {g_1, g_2}

$D_P^+(x) = \{y \in U: yD_p x\}$

$D_P^-(x) = \{y \in U: yD_p y\}$

$D_P^+(F) = \{F, G, I\}$

$D_P^-(F) = \{F, A, B, C, D\}$

$D_P^+(H) = \{H, I\}$

$D_P^-(H) = \{H, A\}$

$D_P^+(D) = \{C, D, E, F, G, I\}$ tutaj robimy stożek co jest lepsze od naszego pkt

$D_P^-(D) = \{A, B, C, D\}$ tutaj robimy stożek co jest gorsze od naszego pkt

$D_P^+(E) = \{E, G\}$

$D_P^-(E) = \{A, B, C, D, E\}$

$\underline{P}Cl_t^{\geq} = U - \overline{P}(Cl^{\leq}_{t-1})$

$\underline{P}Cl_3^{\geq} = U - \overline{P}(Cl^{\leq}_{2}) = \{E, F, G, I\}$

$\underline{P}(Cl_t^{\leq}) = \{x \in U: D_P^{-}(x) \subseteq Cl^{\leq}_t\}$

$\underline{P}(C^{\leq}_{1}) = \{A, B\}$

$\overline{P}(C^{\leq}_{1}) = \{A, B, C, D\}$

$\underline{P}(C^{\leq}_{2}) = \{A, B, C, D, H\}$

$\overline{P}(C^{\leq}_{2}) = \{A, B, C, D, H\}$

brakuje P(C3)

$\underline{P}(C^{\leq}_{3}) = \{A, B, C, D, F, H\}$

$\overline{P}(C^{\leq}_{3}) = \{A, B, C, D, F, H, G, E\}$

$\underline{P}(C^{\leq}_{4}) = \{A, B, C, D, E, F, G, H, I\}$

$\overline{P}(C^{\leq}_{4}) = \{A, B, C, D, E, F, G, H, I\}$

$\underline{P}(Cl_t^{\geq}) = \{x \in U: D_P^{+}(x) \subseteq Cl^{\geq}_t\}$

$\underline{P}(C^{\geq}_{4}) = \{I\}$

$\overline{P}(C_{4}^{\geq}) = \{I, E, G\}$

$\underline{P}(C^{\geq}_{3}) = \{F, I, G, E\}$

$\overline{P}(C_{3}^{\geq}) = \{F, I, G, E\}$ nie zdominowane przez niższe klasy

$\underline{P}(C^{\geq}_{2}) = \{H, F, I, G, E\}$

$\overline{P}(C_{2}^{\geq}) = \{H, F, I, G, E, C, D\}$

$\underline{P}(C^{\geq}_{1}) = \{A, B, C, D, E, F, G, H, I\}$

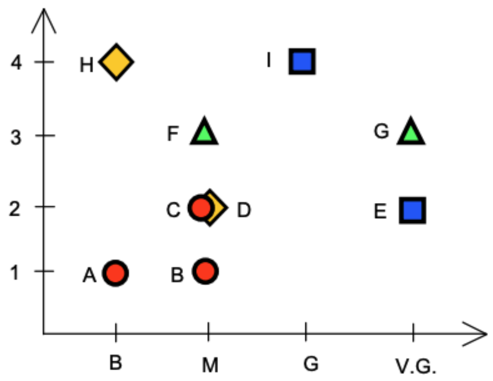
$\overline{P}(C_{1}^{\geq}) = \{A, B, C, D, E, F, G, H, I\}$

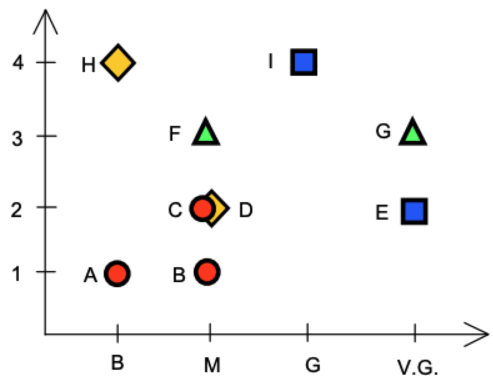
$$Bn_P(Cl_t^{\geq}) \;=\; Bn_P(Cl_{t-1}^{\leq})$$
$$Bn_P(Cl_2^{\geq}) \;=\; Bn_P(Cl_1^{\leq}) \;=\; \{C,\ D\} \rightarrow 2$$
$$Bn_P(Cl_4^{\geq}) \;=\; Bn_P(Cl_3^{\leq}) \;=\; \{E,\ G\} \rightarrow 2$$

III. Mark (graphically) the lower and upper approximations for the selected class unions given below.

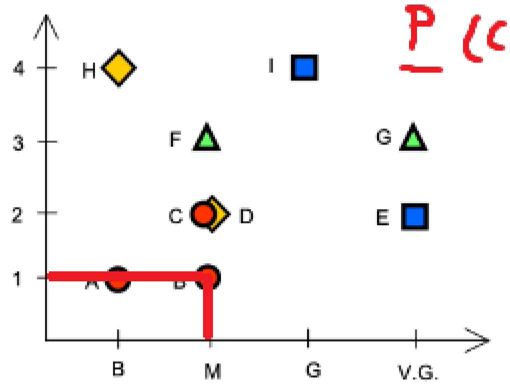
$\underline{P}(C_1^{\leq}) \;=\; \{A,\ B\}$
$\overline{P}(C_1^{\leq}) \;=\; \{A,\ B,\ C,\ D\}$
$\underline{P}(C_2^{\leq}) \;=\; \{A,\ B,\ C,\ D,\ H\}$
$\overline{P}(C_2^{\leq}) \;=\; \{A,\ B,\ C,\ D,\ H\}$

$\underline{P}(C_4^{\geq}) \;=\; \{I\}$
$\overline{P}(C_4^{\geq}) \;=\; \{I,\ E,\ G\}$
$\underline{P}(C_3^{\geq}) \;=\; \{I,\ E,\ G,\ F\}$
$\overline{P}(C_3^{\geq}) \;=\; \{I,\ E,\ G,\ F\}$

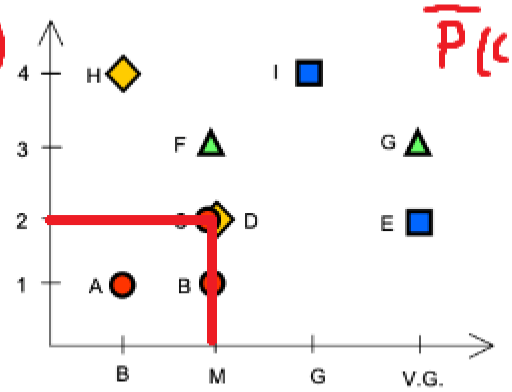




C_1

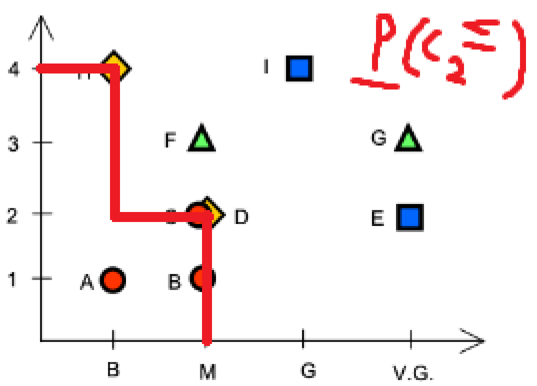


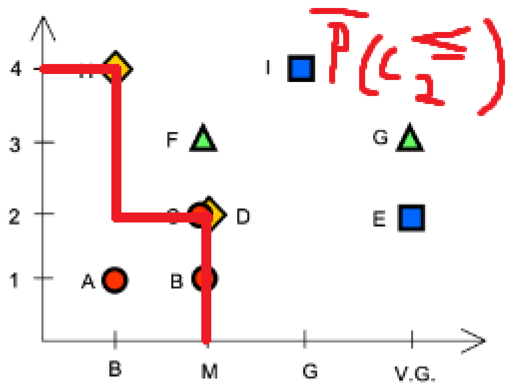
$\underline{P}(c_1^{\leq})$



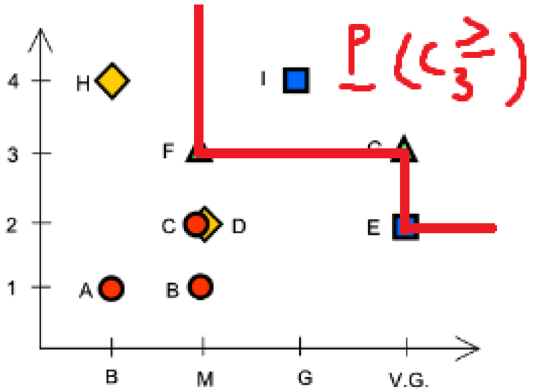
$\overline{P}(c_1^{\leq})$

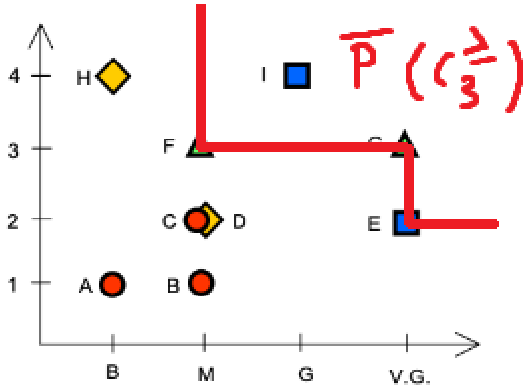
C_2



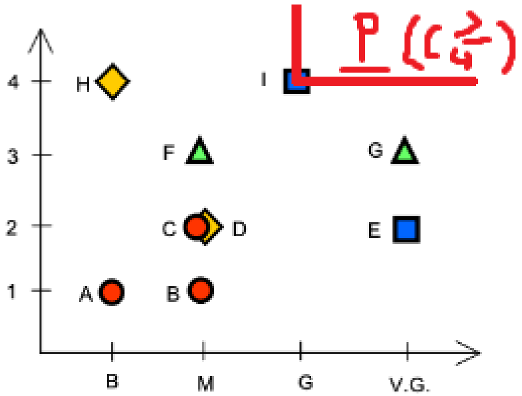


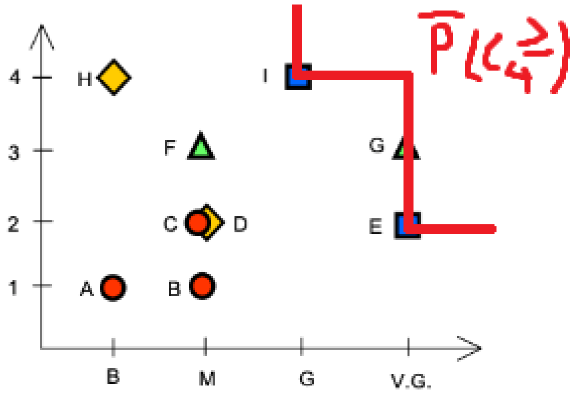
C_3





C_4





IV. Induce a minimal set of minimal certain decision rules for upward Cl≥ and downward Cl≤ class unions. The rules should cover the objects from the lower approximations computed in the previous tasks.

$Cl_1^{\leq}\{A, B\}$

$t_1 = (g_1(x) \leq M)\{A,\ B,\ C,\ D,\ F,\ H\}\quad 0.33\quad 2$

$t_2 = (g_1(x) \leq B)\{A,\ H\}\quad 0.5\quad 1$

$t_3 = (g_2(x) \leq 1)\{A,\ B\}\quad 1.0\quad 2$

if $g_2(x) \leq 1$ $then\ x \in Cl_1^{\leq}$

$C_2^{\geq}\{E, F, G, H, I\}$

$t_1 = (g_1(x) \geq B)\{A, B, C, D, E, F, G, H, I\}$ 0.55 5

$t_2 = (g_1(x) \geq M)\{B, C, D, E, F, G, I\}$ 0.58 4

$t_3 = (g_1(x) \geq G)\{I, E, G\}$ 1.0 3

$t_4 = (g_1(x) \geq V.G)\{E, G\}$ 1.0 2

$t_5 = (g_2(x) \geq 2)\{C, D, E, F, G, H, I\}$ 0.55 5

$t_6 = (g_2(x) \geq 3)\{F, G, H, I\}$ 1.0 4

$t_7 = (g_2(x) \geq 4)\{H, I\}$ 1.0 2

if $g_2(x) \geq 3$ $then\ x \in C_2^{\geq}$

$t_1 = (g_1(x) \geq V.G)\{E\}$ 1.0 1

$t_2 = (g_2(x) \geq 2)\{C, D, E\}$ 0.33 1

$C_3^{\leq}\{A, B, C, D, H, F\}$

$t_1 = (g_1(x) \leq M)\{A, B, C, D, F, H\}$ 1.0 6

$t_2 = (g_1(x) \leq B)\{A, H\}$ 1.0 2

$t_3 = (g_2(x) \leq 1)\{A, B\}$ 1.0 2

$t_4 = (g_2(x) \leq 2)\{A, B, C, D, E\}$ 0.66 4

$t_5 = (g_2(x) \leq 3)\{A, B, C, D, E, F, G\}$ 0.71 5

$t_6 = (g_2(x) \leq 4)\{A, B, C, D, E, F, G, H, I\}$ 0.66 6

if $g_1(x)$ <= M $then\ x \in C_3^{\leq}$ (pokrywa tylko wartości ze zbioru $C_3^{\leq}$ (wszystkie))

$C_3^{\geq}\{E, F, G, I\}$

$t_1 = (g_1(x) \geq B)\{A, B, C, D, E, F, G, H, I\}$ 0.44 4

$t_2 = (g_1(x) \geq M)\{B, C, D, E, F, G, I\}$ 0.57 4

$t_3 = (g_1(x) \geq G)\{I, E, G\}$ 1.0 3

$t_4 = (g_1(x) \geq V.G)\{E, G\}$ 1.0 2

$t_5 = (g_2(x) \geq 2)\{C, D, E, F, G, H, I\}$ 0.57 4

$t_6 = (g_2(x) \geq 3)\{F, G, H, I\}$ 0.75 3

$t_7 = (g_2(x) \geq 4)\{H, I\}$ 0.5 1

if $g_1(x)$ >= G $then\ x \in C_3^{\geq}$ (nie pokrywa wszystkich wartości z $C_3^{\geq}$)

$\{F\}$

$t_1 = (g_1(x) \geq B)\{A, B, C, D, E, F, G, H, I\}$ 0.11 1

$t_2 = (g_1(x) \geq M)\{B, C, D, E, F, G, I\} \quad 0.14 \quad 1$

$t_5 = (g_2(x) \geq 2)\{C, D, E, F, G, H, I\} \quad 0.14 \quad 1$

$t_6 = (g_2(x) \geq 3)\{F, G, H, I\} \quad 0.25 \quad 1$ (pokrywa wartości spoza $C_3^{\geq}$, potrzebny dodatkowy term)

if $g_1(x) >= M$ *and* $g_2(x) >= 3$ *then* $x \in C_3^{\geq}$

$C_4^{\geq}\{I\}$

$t_1 = (g_1(x) \geq B)\{A, B, C, D, E, F, G, H, I\} \quad 0.11 \quad 1$

$t_2 = (g_1(x) \geq M)\{B, C, D, E, F, G, I\} \quad 0.14 \quad 1$

$t_3 = (g_1(x) \geq G)\{I, E, G\} \quad 0.33 \quad 1$

$t_4 = (g_2(x) \geq 1)\{A, B, C, D, E, F, G, H, I\} \quad 0.11 \quad 1$

$t_5 = (g_2(x) \geq 2)\{C, D, E, F, G, H, I\} \quad 0.14 \quad 1$

$t_6 = (g_2(x) \geq 3)\{F, G, H, I\} \quad 0.25 \quad 1$

$t_7 = (g_2(x) \geq 4)\{H, I\} \quad 0.5 \quad 1$(pokrywa wartości spoza $C_4^{\geq}$, potrzebny dodatkowy term)

if $g_1(x) >= G$ *and* $g_2(x) >= 4$ *then* $x \in C_4^{\geq}$

$C_2^{\leq}\{A, B, C, D, H\}$

$t_1 = (g_1(x) \leq M)\{A, B, C, D, E, F H\} \quad 0.83 \quad 5$

$t_2 = (g_1(x) \leq B)\{A, H\} \quad 1.0 \quad 2$

$t_3 = (g_2(x) \leq 4)\{A, B, C, D, E, F, G, H\} \quad 0.55 \quad 5$

$t_4 = (g_2(x) \leq 2)\{A, B, C, D, E\} \quad 0.8 \quad 4$

$t_5 = (g_2(x) \leq 1)\{A, B\} \quad 1.0 \quad 2$

$a)\ if\ g_1(x) \leq B\ then\ x \in Cl_2^{\leq}\ (covers\ \{A, H\},\ still\ to\ be\ covered\ \{B, C, D\})$

$t_1 = (g_1(x) \leq M)\{B, C, D, F\} \quad 0.75 \quad 3\ (nie\ wypisuję\ \{A, H\})\}$

$t_2 = (g_1(x) \leq 2)\{B, C, D, E\} \quad 0.75 \quad 3$

$t_3 = (g_2(x) \leq 1)\{B\} \quad 1.0 \quad 1$

$b)\ if\ g_2(x) \leq 1\ then\ x \in Cl_2^{\leq}\ (covers\ \{B\},\ still\ to\ be\ covered\ \{C, D\})$

$t_1 = (g_1(x) \leq M)\{C, D, F\} \quad 0.67 \quad 2$

$t_2 = (g_2(x) \leq 2)\{C, D, E\} \quad 0.67 \quad 2$

$c)\ if\ g_1(x) \leq M\ and\ g_2(x) \leq 2\ then\ x \in Cl_2^{\leq}\ (covers\ \{A, B, C, D\})$

rule b) is redundant

V. Mark (graphically) the below provided decision rules.

R1) if $g_2(x) \leq 1$ then $x \in Cl_1^{\leq}$

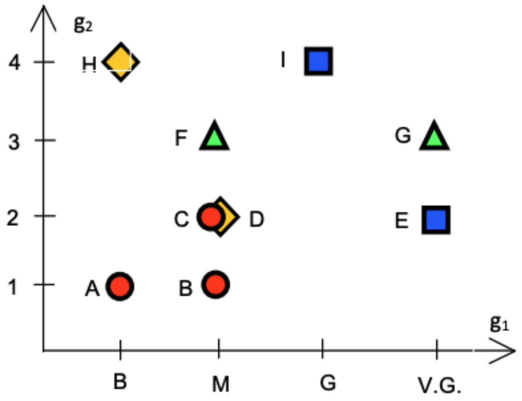
R2) if $g_1(x) \leq B$ then $x \in Cl_2^{\leq}$

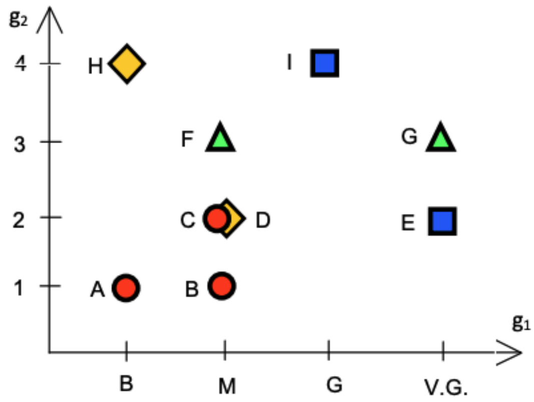
R3) if $g_1(x) \leq M$ and $g_2(x) \leq 2$ then $x \in Cl_2^{\leq}$

R4) if $g_2(x) \geq 3$ then $x \in C_2^{\geq}$

R5) if $g_1(x) \geq V.G$ then $x \in C_2^{\geq}$

R6) if $g_1(x) \geq G$ and $g_2(x) \geq 4$ then $x \in C_4^{\geq}$





R1) if $g_2(x) \leq 1$ then $x \in Cl_1^{\leq}$

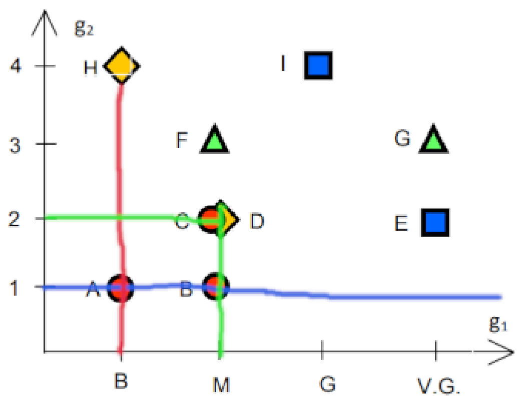
R2) if $g_1(x) \leq B$ then $x \in Cl_2^{\leq}$

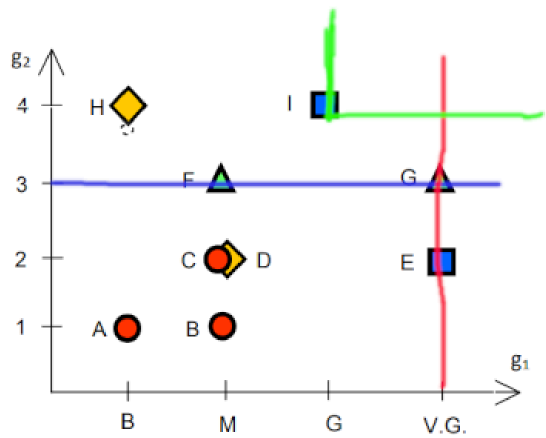
R3) if $g_1(x) \leq M$ and $g_2(x) \leq 2$ then $x \in Cl_2^{\leq}$

R4) if $g_2(x) \geq 3$ then $x \in C_2^{\geq}$

R5) if $g_1(x) \geq V.G$ then $x \in C_2^{\geq}$

R6) if $g_1(x) \geq G$ and $g_2(x) \geq 4$ then $x \in C_4^{\geq}$





VI. For the below-provided decision table with 3 gain-type criteria (K1, K2, K3) and 7 objects, compute the lower and upper approximations of class unions (C1 is the least preferred class, C3 is the most preferred class).

	K1	K2	K3	Cl
A	3	2	3	C_3
B	**2**	**3**	**2**	$\mathbf{C_2}$
C	1	1	2	C_1
D	**2**	**3**	**2**	$\mathbf{C_3}$
E	2	3	1	C_2
F	1	2	1	C_1
G	3	1	3	C_3

$\underline{P}(C_1^{\leq}) =$ {F,C}

$\overline{P}(C_1^{\leq}) =$ {F,C}

$\underline{P}(C_2^{\leq}) =$ {C,F,E}

$\overline{P}(C_2^{\leq}) =$ {C,B,D,E,F}

$\underline{P}(C_3^{\geq}) =$ {A,G}

$\overline{P}(C_3^{\geq}) =$ {A,G,D,B}

$\underline{P}(C_2^{\geq}) =$ {B,A,D,E,G}

$\overline{P}(C_2^{\geq}) =$ {B,A,D,E,G}

Induce certain decision rules for the following two unions: $C_2^{\leq}$ and $C_2^{\geq}$.

Dla $C_2^{\leq}$:

G = {F,C,B,D,E}
B = {F,C,B,D,E}
T(G) = {K1 <= 1, K1 <=2, K2<=1, K2<= 2, K2 <= 3, K3 <= 1, K3 <= 2}

t	|{t} ∩ G |	|{t} |	|{t} ∩ G | / |{t}|
K1 <= 1	2	2	2/2

K1 <=2,	5	5	5/5
K2<=1	1	2	1/2
K2 <= 2	1	4	1/4
K2 <= 3	5	7	5/7
K3 <= 1	2	2	2/2
K3 <= 2	5	5	5/5

Wybieramy regułe o największym pokryciu zbioru G, w naszym przypadku ich 2 (K3 <=2, K1 <= 2)

Najpierw K3 <=2

[T] = [B,C,D,F,E]

G = B - U[T] = [B,C,D,F,E] - [B,C,D,F,E] = Ø (Terminate)

Najpierw K1 <=2

[T] = [B,C,D,F,E]

G = B - U[T] = [B,C,D,F,E] - [B,C,D,F,E] = Ø (Terminate)

Certain Rule for $C_2^{\leq}$ to

T = [K1 <= 2, K3 <= 2]

Dla $C_2^{\leq}$ **:**

$G = \{C, F, E\}$

$B = \{C, F, E\}$

$T(G) = \{K1 \leq 1, K1 \leq 2, K2 \leq 1, K2 \leq 2, K2 \leq 3, K3 \leq 1, K3 \leq 2\}$

$t_1 = (K1 \leq 1)\{C, F\} \quad 1.0 \quad 2$

$t_2 = (K1 \leq 2)\{B, C, D, E, F\} \quad 0.6 \quad 3$

$t_3 = (K2 \leq 1)\{C, G\} \quad 0.5 \quad 1$

$t_4 = (K2 \leq 2)\{A, C, F, G\} \quad 0.5 \quad 2$

$t_5 = (K2 \leq 3)\{A, B, C, D, E, F, G\} \quad 0.42 \quad 3$

$t_6 = (K3 \leq 1)\{E, F\} \quad 1.0 \quad 2$

$t_7 = (K3 \leq 2)\{B, C, D, E, F\} \quad 0.6 \quad 3$

$if\ K1 \leq 1\ then\ x \in C_2^{\leq}\ (nie\ pokrywa\ wszystkich\ wartości\ z\ B\ )$

$G = \{E\}$
$T(G) = \{K1 \leq 2,\ K2 \leq 3,\ K3 \leq 1,\ K3 \leq 2\}$
$t_1 = (K1 \leq 2)\{B, C, D, E, F\} \quad 0.2 \quad 1$

$t_2 = (K2 \leq 3)\{A, B, C, D, E, F, G\} \quad 0.14 \quad 1$

$t_3 = (K3 \leq 1)\{E, F\} \quad 0.5 \quad 1$

$t_4 = (K3 \leq 2)\{B, C, D, E, F\} \quad 0.2 \quad 1$

$if\ K3 \leq 1\ then\ x \in C_2^{\leq}$

Dla $C_2^{\geq}$:

Robimy wszystko to samo, zgodnie z algorytmem DOMLEM i powinno wyjść:

T = [K1>=2, K2>=2, K3>=2]

Końcowa według mnie(Władysław) to:

Certain Rule for $C_2^{\leq}$ to	Certain Rule for $C_2^{\geq}$ to
T = [K1 <= 2, K3 <= 2]	T = [K1>=2, K2>=2, K3>=2]

if K1 <= 1 then C2<=
if K3 <= 1 then C2<=

Dla $C_2^{\geq}$:

$G = \{A, B, D, E, G\}$
$B = \{A\ B, D, E, G\}$
$T(G) = \{K1 \geq 1,\ K1 \geq 2,\ K1 \geq 3,\ K2 \geq 1,\ K2 \geq 2,\ K2 \geq 3,\ K3 \geq 1,\ K3 \geq 2,\ K3 \geq 2\}$

$t_1 = (K1 \geq 1)\{A, B, C, D, E, F, G\} \quad 0.71 \quad 5$

$t_2 = (K1 \geq 2)\{A, B, D, E, G\} \quad 1.0 \quad 5$

$t_3 = (K1 \geq 3)\{A, G\} \quad 1.0 \quad 2$

$t_4 = (K2 \geq 1)\{A, B, C, D, E, F, G\} \quad 0.71 \quad 5$

$t_5 = (K2 \geq 2)\{A, B, D, E, F\} \quad 0.8 \quad 4$

$t_6 = (K2 \geq 3)\{B, D, E\} \quad 1.0 \quad 3$

$t_7 = (K3 \geq 1)\{A, B, C, D, E, F, G\} \quad 0.71 \quad 5$

$t_8 = (K3 \geq 2)\{A, B, C, D, G\} \quad 0.8 \quad 4$

$t_9 = (K3 \geq 3)\{A, G\} \quad 1.0 \quad 2$

$if\ K1 \ \geq 2\ then\ x \in C_2^{\geq}$

Determine the reducts and the core.
Candidates for being reducts: {K1}, {K2}, {K3}, {K1,K2}, {K1,K3}, {K2,K3}, {K1,K2,K3}.
Core:

Tu musimy dla każdego kandydata obliczyć jakość kwalifikacji, jeżeli ta jakość równa się ogólnej jakości to wtedy redukt znaleziony. Ogólną jakość można uzyskać stosunkiem pewnych reguł do wszystkich. W naszym przypadku mamy 5 pewnych reguł i 2 niespójne. Dlatego nasza ogólna jakość kwalifikacji wynosi 5/7.

No i liczymy dla każdego osobno, sprawdzając czy przy jednym kryterium kwalifikacja jest spójna i nie zmienia nic.

C=P=[K1,K2,K3} Ур{Cl} = Ус(Cl)

Ур{Cl} = 5/7

У_K1 = 5/7 - redukt
У_K2 = 1/7
У_K3 = 3/7

У_K1_K2 = 4/7
У_K1_K3 = 5/7 - redukt
У_K2_K3 = 5/7 - redukt

У_K1_K2_K3 = 5/7 - Ale to nie redukt, bo liczba kryteriów nie minimalna

Core(Rdzeń) to przecięcie wszystkich reduktów.

У_**K1**
У_**K1_K3 ->** **Core = [K1,K3]**

У_K2_**K3**

Determine the reducts and the core.
Candidates for being reducts: {K1}, {K2}, {K3}, {K1,K2}, {K1,K3}, {K2,K3}, {K1,K2,K3}.
Core:

$\{K1\}$

$\underline{P}(C_1^{\leq}) = \{C, F\}$

$\overline{P}(C_1^{\leq}) = \{C, F\}$

$\underline{P}(C_2^{\leq}) = \{C, F\}$

$\overline{P}(C_2^{\leq}) = \{B, C, D, E, F\}$

$\underline{P}(C_2^{\geq}) = \{A, B, D, E, G\}$

$\overline{P}(C_2^{\geq}) = \{A, B, D, E, G\}$

$\underline{P}(C_3^{\geq}) = \{A, G\}$

$\overline{P}(C_3^{\geq}) = \{A, B, D, E, G\}$

$Y_p\{K1\} = 4/7$

$\{K2\}$

$\underline{P}(C_1^{\leq}) = \{C\}$

$\overline{P}(C_1^{\leq}) = \{A, C, F, G\}$

$\underline{P}(C_2^{\leq}) = \{\}$

$\overline{P}(C_2^{\leq}) = \{A, B, C, D, E, F, G\}$

$\underline{P}(C_2^{\geq}) = \{B, D, E\}$

$\overline{P}(C_2^{\geq}) = \{A, B, C, D, E, F, G\}$

$\underline{P}(C_3^{\geq}) = \{\}$

$\overline{P}(C_3^{\geq}) = \{A, B, C, D, E, F, G\}$

$Y_p\{K2\} = 0/7$

$\{K3\}$

$\underline{P}(C_1^{\le}) = \{F\}$

$\overline{P}(C_1^{\le}) = \{B, C, D, E, F\}$

$\underline{P}(C_2^{\le}) = \{E, F\}$

$\overline{P}(C_2^{\le}) = \{B, C, D, E, F\}$

$\underline{P}(C_2^{\ge}) = \{A, G\}$

$\overline{P}(C_2^{\ge}) = \{A, B, C, D, E, F, G\}$

$\underline{P}(C_3^{\ge}) = \{A, G\}$

$\overline{P}(C_3^{\ge}) = \{A, B, C, D, G\}$

$Y_p\{K3\} = 4/7$

$\{K1, K2\}$

$\underline{P}(C_1^{\le}) = \{C, F\}$

$\overline{P}(C_1^{\le}) = \{C, F\}$

$\underline{P}(C_2^{\le}) = \{C, F\}$

$\overline{P}(C_2^{\le}) = \{B, C, D, E, F\}$

$\underline{P}(C_2^{\ge}) = \{A, B, D, E, G\}$

$\overline{P}(C_2^{\ge}) = \{A, B, D, E, G\}$

$\underline{P}(C_3^{\ge}) = \{A, G\}$

$\overline{P}(C_3^{\ge}) = \{A, B, D, E, G\}$

$Y_p\{K1, K2\} = 4/7$ *nie jest minimalny poniewaz* $Y_p\{K1\} = 4/7$

$\{K1, K3\}$

$\underline{P}(C_1^{\leq}) = \{C, F\}$

$\overline{P}(C_1^{\leq}) = \{C, F\}$

$\underline{P}(C_2^{\leq}) = \{C, E, F\}$

$\overline{P}(C_2^{\leq}) = \{B, C, D, E, F\}$

$\underline{P}(C_2^{\geq}) = \{A, B, D, E, G\}$

$\overline{P}(C_2^{\geq}) = \{A, B, D, E, G\}$

$\underline{P}(C_3^{\geq}) = \{A, G\}$

$\overline{P}(C_3^{\geq}) = \{A, B, D, G\}$

$Y_p\{K1, K3\} = 5/7 - redukt$

$\{K2, K3\}$

$\underline{P}(C_1^{\leq}) = \{C, F\}$

$\overline{P}(C_1^{\leq}) = \{C, F\}$

$\underline{P}(C_2^{\leq}) = \{C, E, F\}$

$\overline{P}(C_2^{\leq}) = \{B, C, D, E, F\}$

$\underline{P}(C_2^{\geq}) = \{A, B, D, E, G\}$

$\overline{P}(C_2^{\geq}) = \{A, B, D, E, G\}$

$\underline{P}(C_3^{\geq}) = \{A, G\}$

$\overline{P}(C_3^{\geq}) = \{A, B, D, G\}$

$Y_p\{K2, K3\} = 5/7 - redukt$

$\{K1, K2, K3\}$

$\underline{P}(C_1^{\leq}) = \{C,F\}$

$\overline{P}(C_1^{\leq}) = \{C,F\}$

$\underline{P}(C^{\leq}_{2}) = \{C, E, F\}$

$\overline{P}(C^{\leq}_{2}) = \{B, C, D, E, F\}$

$\underline{P}(C^{\geq}_{2}) = \{A, B, D, E, G\}$

$\overline{P}(C^{\geq}_{2}) = \{A, B, D, E, G\}$

$\underline{P}(C^{\geq}_{3}) = \{A, G\}$

$\overline{P}(C^{\geq}_{3}) = \{A, B, D, G\}$

$Y_{p}\{K1\} = 5/7$ $nie\ jest\ minimalny\ poniewaz$ $Y_{p}\{K1, K3\} = 5/7$

Core:

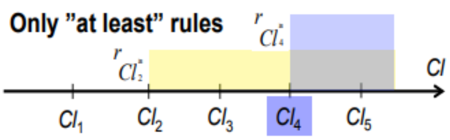
$RED_{Cl}(P) = \{K1, K3\}$

$RED_{Cl}(P) = \{K2, K3\}$

$CORE_{Cl}(P) = \cap\ RED_{Cl}(P)$ = {K3}

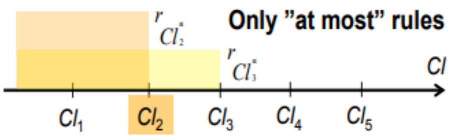
VII. Consider the below set of decision rules. Use it to classify non-reference objects O1-O4 using a standard classification algorithm.

Classification - Standard Algorithm



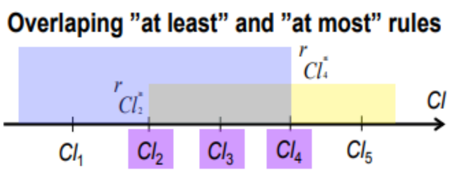
The lowest class of upward union of recommendations

St.	Set of activated decision rules	Recommendation
S9	R1: ⪰ good, R2: ⪰ medium	good



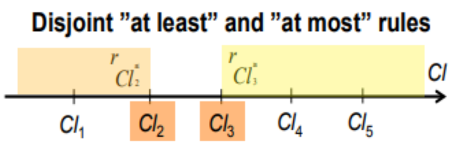
The highest class of downward union of recommendations

St.	Set of activated decision rules	Recommendation
S10	R3: ⪯ medium, R4: ⪯ bad	bad



The range between the lowest class of upward union of recommendations and the highest class of downward union of recommendations

St.	Set of activated decision rules	Recommendation
S11	R2: ⪰ medium, R3: ⪯ medium	[medium, medium]



The range between the lowest class of upward union of recommendations and the highest class of downward union of recommendations

St.	Set of activated decision rules	Recommendation
S12	R1: ⪰ good, R2: ⪰ medium, R3: ⪯ medium	[medium, good]

Rules	Objects	Activated rules (decision parts)	Recommended class(es)
if A ≥ 3 and C ≥ 2 then ≥ C_5	O1: A=2, B=2, C=2	if A ≥ 2 and C ≥ 2 then ≥ C_3 if B ≤ 2 and C ≤ 2 then ≤ C_4	[C3,C4]
if A ≥ 2 and C ≥ 2 then ≥ C_3	O2: A=2, B=1, C=1	if B ≤ 1 then ≤ C_1 if B ≤ 2 and C ≤ 2 then ≤ C_4	C1
if B ≤ 2 and C ≤ 2 then ≤ C_4	O3: A=2, B=3, C=3	if A ≥ 2 and C ≥ 2 then ≥ C_3	C3
if A ≤ 1 and C ≤ 1 then ≤ C_2	O4: A=3, B=2, C=2	if A ≥ 3 and C ≥ 2 then ≥ C_5 if A ≥ 2 and C ≥ 2 then ≥ C_3 if B ≤ 2 and C ≤ 2 then ≤ C_4	[C5,C4]
if B ≤ 1 then ≤ C_1			

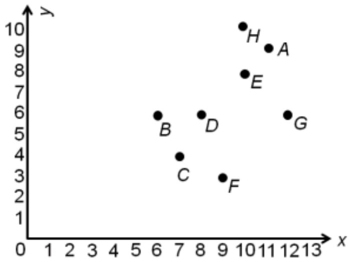
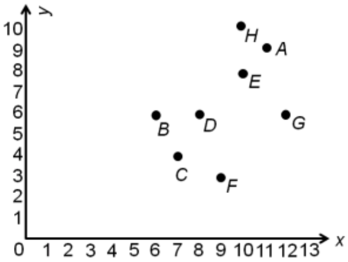
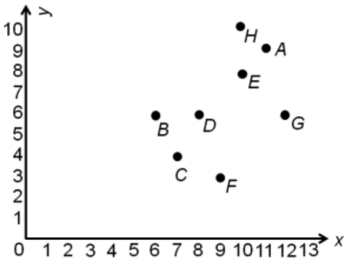
Wykład 9 – DATA ENVELOPMENT ANALYSIS

I. Indicate the truth (T) or falsity (F) for the below statements.

a) For all problems, there can be only one efficient unit in the CCR model	F
b) The CCR model admits convex combinations of existing units	T
c) The BCC model assumes constant returns to scale	F
d) The efficiency score of one denotes an efficient unit	T
e) The input- and output-oriented efficiencies in the CCR model are the same	F
f) All units efficient in the BCC model are also efficient in the CCR model	F

II. For the problem involving eight Decision Making Units (see the table given below), draw an efficient frontier while assuming that:

DMU	A	B	C	D	E	F	G	H
x	11	6	7	8	10	9	12	10
y	9	6	4	6	8	3	6	10

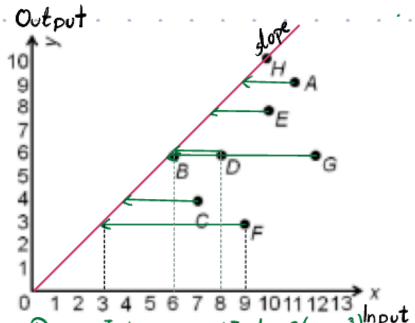




- x is an input and y is an output for the CCR or BCC model; for units D and F compute the efficiency in the input- and output-oriented CCR and BCC model;

DMU	A	B	C	D	E	F	G	H
x	11	6	7	8	10	9	12	10
y	9	6	4	6	8	3	6	10
Ratio	0,81	1	0,57	0,75	0,80	0,33	0,50	1

Input-oriented

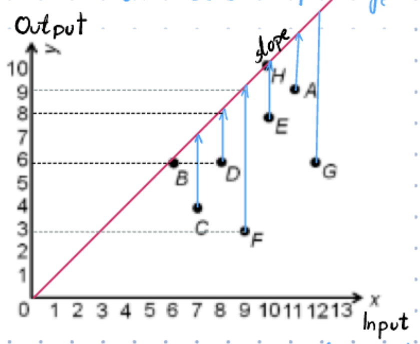
"o ile można zmn. nakład?"



D musiałaby zmn. nakład o 2 (pomn. $\cdot \frac{3}{4}$)
F -ll- o 6 (pomn. $\cdot \frac{1}{3}$)

Output-oriented

"o ile można zw. produkcję?"

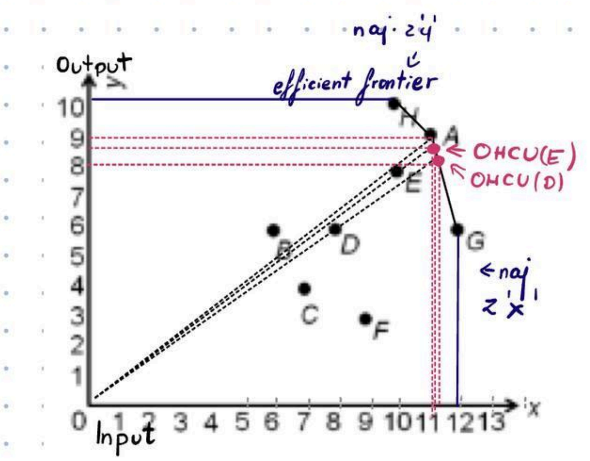


D musiałoby zw. prod. o 2 (pomn. $\cdot \frac{4}{3}$)
F -ll- o 6 (pomn. $\cdot \frac{9}{3} = 3$)

Scenario: 1 input, 1 output CCR

Scenario: 1 input, 1 output BCC

- x and y are outputs, and input is the same for all units; show, graphically, how to compute the efficiency for units A, D, and E;
  Scenario: two outputs, one input



$$\text{eff } A = 1$$

$$y = ax + b$$

$$a = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6-9}{12-11} = -3$$

$$9 = -3 \cdot 11 + b \Rightarrow b = 42$$

$$y = -3x + 42 \Rightarrow 3x - y - 42 = 0$$

$$d = \frac{|ax_0 + by_0 + c|}{\sqrt{a^2 + b^2}}$$

$$OHCU = \frac{|-3 \cdot 0 + (-1)0 - 42|}{\sqrt{9+1}} = \frac{42}{\sqrt{10}}$$

$$\text{eff } D = \frac{OD}{OHCU(D)} = \cancel{10}^{5} \cdot \frac{\sqrt{10}}{\cancel{42}_{21}} \approx 0{,}75$$

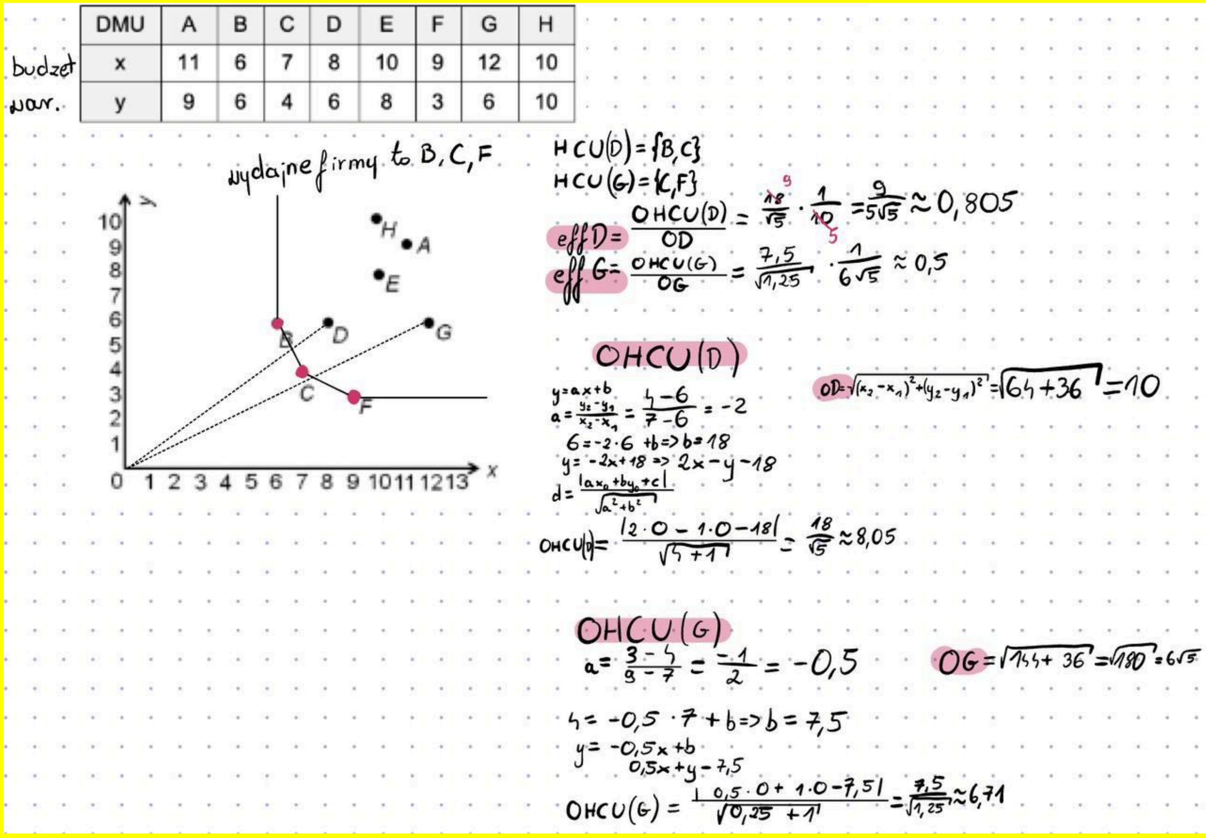
$$OD = \sqrt{64+36} = 10$$

$$\text{eff } E = \frac{OE}{OHCU(E)} = \cancel{2}\sqrt{41} \cdot \frac{\sqrt{10}}{\cancel{42}_{21}} \approx 0{,}96$$

$$OE = \sqrt{100+64} = 2\sqrt{41}$$

- x and y are inputs, and output is the same for all units for the CCR model; show, graphically, how to compute the efficiency for units B, D, and G.

Scenario: 2 inputs, one output



III. Formulate the problem that verifies the efficiency of DMU I according to the input-oriented CCR model. Consider both perspectives: efficiency- and combination-oriented.

DMU	Input 1	Input 2	Output 1	Output 2	Output 3
I	5	14	**9**	**4**	**16**
II	8	15	5	7	10
III	7	12	4	9	13

max 9μ1 + 4μ2 + 16μ3

$s.t.\ 5 * v_1 + 14 * v_2 = 1$

9μ1 + 4μ2 + 16μ3 ≤ 5v1 + 14v2

5μ1 + 7μ2 + 10μ3 ≤ 8v1 + 15v2

$4\mu_1 + 9\mu_2 + 13\mu_3 \leq 7v_1 + 12v_2$

…

μ1, μ2, μ3, v1, v2 ≥ 0

min θ_1

5λ1 + 8λ2 + 7λ3 ≤ $5 \cdot \theta_1$

14λ1 + 15λ2 + 12λ3 ≤ $14\ \theta_1$

9λ1 + 5λ2 + 4λ3 ≥ 9

4λ1 + 7λ2 + 9λ3 ≥ 4

16λ1 + 10λ2 + 13λ3 ≥ 16

λ1, λ2, λ3 ≥ 0

The modifications needed in the BCC model:

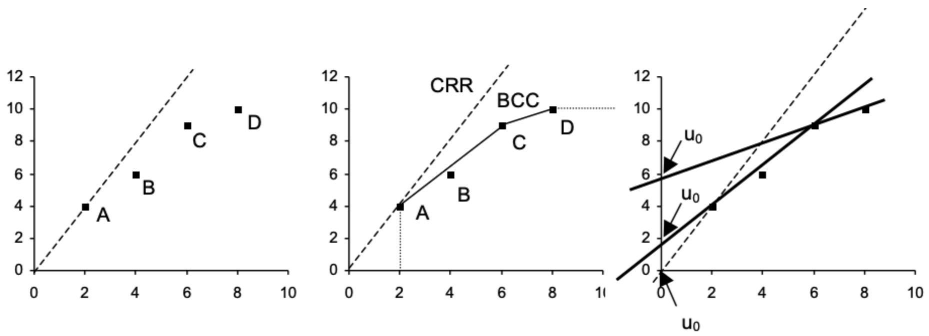
In the space of efficiencies, objective function:

- max 9μ1 + 4μ2 + 16μ3 + u0, and then, in the constraints, e.g.: 5μ1 + 7μ2 + 10μ3 + u0 ≤ 8v1 + 15v2 (u0 free)
- In the space of combinations, we only add: λ1 + λ2 + λ3 = 1, which enforces the convex combinations. Could you interpret the solutions to the above-formulated problems? That is, indicate the efficiency status (efficient or inefficient), the required changes, and for the combination-oriented perspective, point out the reference set.

  Jeśli θ=1i wszystkie zmienne są równe zero w rozwiązaniu optymalnym, wówczas DMU I jest uważane za efektywne.
  Jeśli θ<1, DMU I jest nieefektywne i może potencjalnie zmniejszyć nakłady proporcjonalnie o θ.
  Wartość λ wskazują zbiór referencyjny - inne DMU, do których porównywana jest DMU I.

IV. Compare the results of the CCR (CRS - Constant Returns to Scale) and BCC (VRS – Variable Returns to Scale) models.

DMU	Input	Output
A	2	4
B	4	6
C	6	9
D	8	10



Which units are efficient in the BCC model? Are they all efficient in the CCR model?
Wszystkie jednostki efektywne w modelu CCR są też efektywne w modelu BCC, ale odwrotne zdanie nie jest już prawdziwe.
Answer: BCC - {A,C,D} CCR - {A}
For example, B is inefficient according to BCC, because a convex combination 50%-50% of units A and C consumes 0.5 * 2 + 0.5 * 6 = 4 units of the input and produces 0.5 * 4 + 0.5 * 9 = 6.5 units of the output, which is more than 6 units that are produced by B

- BBC output efficiency for unit B is therefore 1/(6.5/6) = 0.923

Wykład 10 – RANKING METHODS IN DEA (PART I)

I. Indicate the truth (T) or falsity (F) for the below statements.

a) For an efficient unit, super-efficiency is always greater or equal to its efficiency	T
b) For a given unit, cross-efficiency can be greater than its efficiency	F

c) When including additional weight constraints, efficiencies for all units always become lesser	F
d) To estimate the stochastic acceptability indices with 0.01 accuracy and 95% confidence, one needs to consider 1,000 samples	F

II. Write the linear programming model for computing the super-efficiency of A, B, C, or D, while assuming:

- a) input- or output-oriented improvements
- b) CCR (CRS) or BCC (VRS) (i.e., constant or variable returns to scale)

DMU	A	B	C	D
$input_1$	5	8	7	6
$input_2$	14	15	10	12
$output_1$	9	5	3	6
$output_2$	4	8	7	9

Wyznaczanie dla:
-A
-Output oriented
- CCR

max 9 * u_1 + 4 * u_2 #maksymalizacja outputu
s.t. 5 * v_1 + 14 * v_2 = 1 #ograniczenia na wejście

5 * u_1 + 8 * u_2 <= 8 * v_1 + 15 * v_2 #B
3 * u_1 + 7 * u_2 <= 7 * v_1 + 10 * v_2 #C
6 * u_1 + 9 * u_2 <= 6 * v_1 + 12 * v_2 #D

v_1, v_2, u_1, u_2 >= 0

III. Given the matrix of efficiencies attained by different DMUs for the weights vectors being most favorable to other units:

- a) What is the cross-efficiency of unit A? 0.8
- b) What is the super-efficiency of unit B? 0.7
- c) What can we say about the super-efficiencies of units A and C? Są równe co najmniej 1.

DMU	E_{kA}	E_{kB}	E_{kC}
E_{Ak}	1.0	0.2	0.9
E_{Bk}	0.6	0.7	0.8
E_{Ck}	0.8	0.6	1.0

IV. Using the Monte Carlo simulation, three weight vectors (w1,2,3) have been sampled. They implied the efficiency scores given in the below table. Show the respective matrices of efficiency rank acceptability indices (ERAIs) and pairwise efficiency outranking indices (PEOIs) (for w1: A is the best, C – 2nd, B is the worst)

	A	B	C
w_1	1.0	0.2	0.9
w_2	0.6	0.7	0.8
w_3	0.8	0.6	1.0

Dla Erai należy podliczyć które miejsca w wektorach zajmują jednostki - np. jednostka C zajęła pierwsze miejsce w 2 wektorach, oraz drugie miejsce w jednym wektorze przy 3 dostępnych wektorach więc dla kolumny C1 otrzymujemy ⅔, dla C2 ⅓, dla C3 otrzymujemy 0. Expected rank (ER_k) jest wyznaczane jako suma miejsc i ich prawdopodobieństw.

ERAI:

DMU	1	2	3	ER_k
A	1/3	1/3	1/3	2
B	0	1/3	2/3	8/3 = 2,66
C	2/3	1/3	0	4/3 = 1,33

Wyznaczany jest stosunek wag niegorszych od porównywanej jednostki

PEOI (pairwise relations)

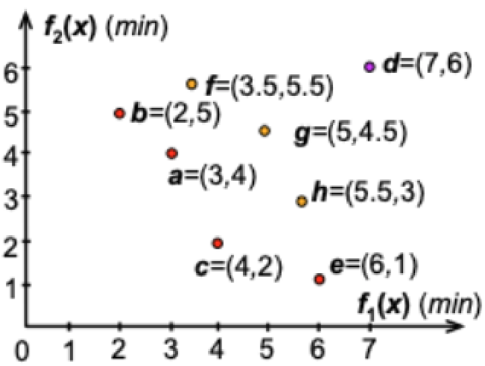
	A	B	C
A	1	2/3	1/3
B	1/3	1	0
C	2/3	1	1

Wykład 10 – CLASSICAL MOO METHODS (PART II)

I. Indicate the truth (T) or falsity (F) for the below statements.

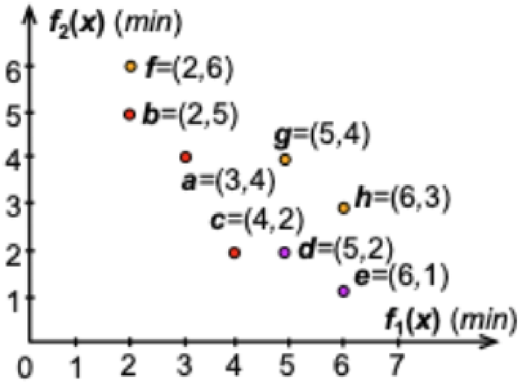
a) Various solutions in the decision space are always translated to different points in the objective space	F
b) Weakly Pareto optimal solutions are always Pareto optimal	F
c) The number of solutions contained in the Pareto frontier may be finite	T
d) The max point attains not better values than the nadir point on all objectives	T
e) Classical optimization methods require multiple runs with different parameter values to approximate the Pareto frontier	T
f) The weighted sum method (WSM) parameterized with positive weights for all objectives identifies the Pareto optimal solution	F/T
g) The epsilon constrain method (ECM) can find non-supported efficient solutions	T

II. Consider a set of solutions a-h in the objective space with two minimized objectives (see figure below).



- a) Compute the ideal point z^{ideal}. **(2,1)**
- b) Compute a utopian point z^{utop} for ε=0.1. **? Nie ma bo nic nie łapie - strzelam**
- c) Compute the max point z^{max}. **(7,6)**
- d) Compute the nadir point z^{nadir}. **(6,5)**

III. Consider a set of solutions a-h in the objective space with two minimized objectives (see figure below).



- a) Identify Pareto optimal and weakly Pareto optimal solutions. **PO: b,a,c,e WPO: f,d**
- b) What would be the solution returned by **WSM** with the following objective function: **Minimize** $0.5 \cdot f_1(x) + 0.5 \cdot f_2(x)$? **c: (4+2)/2=3**
- c) What about **WSM** with: **Minimize** $2/3 \cdot f_1(x) + 1/3 \cdot f_2(x)$? **b: ⅔*2+⅓*5=3**
- d) Solution **a** is Pareto optimal. Can it be discovered by **WSM**? NIE
- e) What would be the solution returned by **ECM** with the following objective function and constraint: **Minimize** $f_1(x)$, $s.t.\ f_2(x) \leq 4.5$? **a**
- What about **ECM** with: **Minimize** $f_2(x)$, $s.t.\ f_1(x) \leq 5.5$?
- How to reformulate the objective function using the augmentation factor to be sure that **ECM** always returns a Pareto optimal rather than a weakly Pareto optimal solution? **c, dodać do f.celu augmentation term: ro*SUM(f_i(x))**
- g) What would be the solution(s) returned by the **ASF** method with the following objective function: **Minimize** $max\{0.5 \cdot f_1(x),\ 0.5 \cdot f_2(x)\}$? **dla a i c = 2**
- h) What about **ASF** with: **Minimize** $max\{2/3 \cdot f_1(x),\ 1/3 \cdot f_2(x)\}$? **dla b = 1.66**
- i) Which solution would be selected for the following order of lexicographic optimization $(f_1(x),\ f_2(x))$? **e (b)**

Wykład 11 – EVOLUTIONARY MULTIPLE OBJECTIVE OPTIMIZATION

I. Indicate the truth (T) or falsity (F) for the below statements.

a) Recombination can introduce new information to the optimization	F
b) The impact of the mutation on the evolutionary search is exploitative rather than explorative	T
c) Evolutionary optimization methods require multiple runs with different parameter values to approximate the Pareto frontier	F

d) Tournament selection belongs to the class of ordinal selection methods	T
e) The max point attains not better values than the nadir point on all objectives	T
f) VEGA applies a generation model of managing the population	T
g) The crowding distance for the non-dominated solutions in NSGA-II is equal to infinity	F
h) SPEA2 includes the archive members in the selection process	T
i) SMS-EMOA is an indicator based evolutionary algorithm for multiple objective optimization	T

II. Given the following chromosome in the binary encoding [1 0 1 1 0 0], representing an example solution for the knapsack problem, present a chromosome obtained after a flip bit mutation:

0	1	0	0	1	1

III. Given the two below presented chromosomes in the binary encoding:

1	0	1	1	0	0

0	1	1	0	0	1

present a pair of chromosomes obtained after applying 2-point crossover with crossover points after the second and fifth genes:

1	0	1	0	0	0

0	1	1	1	0	1

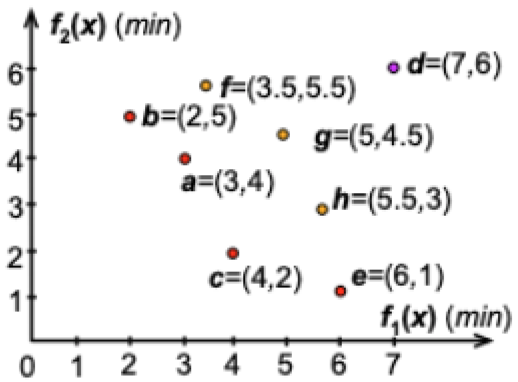
IV. Given the following table of fitness values for seven solutions a - g (fitness Fi to be maximized):

sol	*a*	*b*	*c*	*d*	*e*	*f*	*g*
F_i	3	1	3	2	0.5	1.5	1

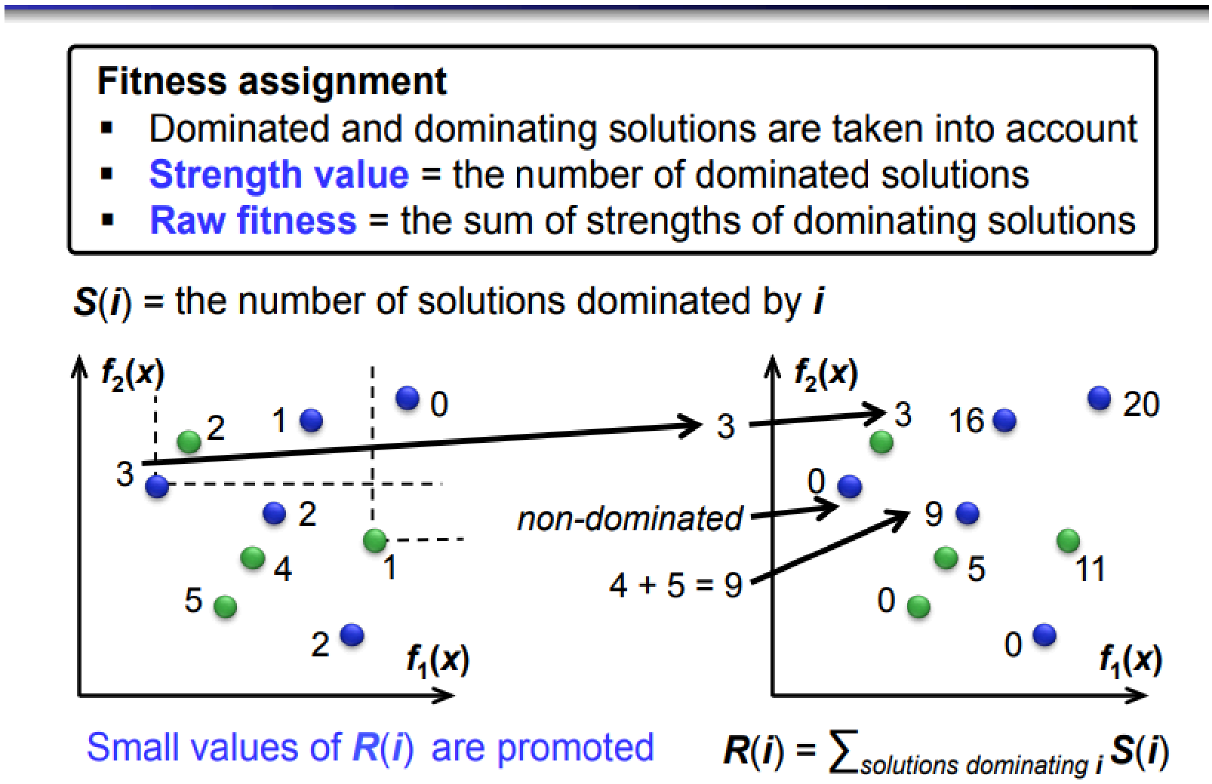
Indicate the parents selected for the recombination operator with the tournament selection of size 4, when the following subsets of solutions participate in each tournament:

- i) {a, d, f, g} - Winner: **a**
- ii) {b, c, e, f} - Winner: **c**

V. Consider a set of solutions a-h in the objective space with two minimized objectives (see figure below).



- a) Use the Kung's method to identify the first non-dominated front. a,b,c,e
- b) Show the Pareto fronts used by NSGA-II as the primary sorting criterion.
  first: a,b,c,e second: f,g,h third: d
- c) For all solutions, compute their raw fitness values (sum of strengths of dominating solutions) according to the rules of SPEA2.



Najpierw szukamy siłę tych rozwiązań, to liczba rozwiązań zdominowanych:

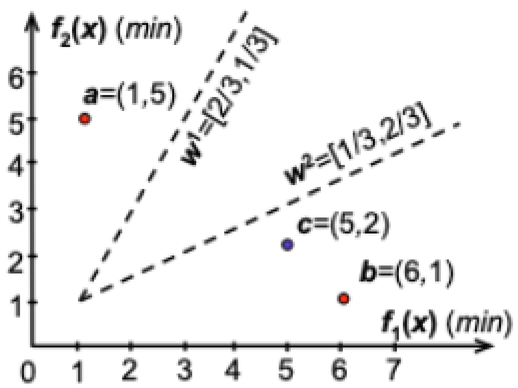
b = 2
a = 3 f = 1
c = 3 g = 1 d = 0
e = 1 h = 1

Teraz możemy obliczyć raw fitness, to liczba dominujących rozwiązań a,b,c,e = 0, bo nikt nie przewyższa

f = a + b = 5
g = a + c = 6
h = c = 3
d = a + b + c + f + g + h + e = 3   + 2 + 3 + 1 + 1 + 1 + 1 = 12

- d) Which solution: a or c would be found more favorable by:
  NSGA-II (draw cuboids related to the respective crowding distances),
  SPEA2 (draw distances to the k=1 nearest neighbor), or
  SMS EMOA (draw individual contributions to hypervolume; assume $d = z^{ref}$).
  wszędzie c

VI. Consider a population composed of just two solutions a and b, which is evolved by MOEA/D with the two uniformly distributed weight vectors provided in the figure. The latter ones are used as the parameters in the weighted Chebyshev distance from the reference point. Solutions a and b are the only ones contained in the current external archive.



- a) Compute the current reference point according to MOEA/D. **(1,1)**
- b) Associate solutions **a** and **b** with the targets (which solution is the best for which target?). **po prostu a: f1, b: f2 ?**
- c) Assume that by recombining **a** and **b** (the neighborhood's size T=2) and further mutating the newly obtained solution, we obtain solution c. Will it become the new best solution for some target(s)? Show the new archive.
  c stanie się najlepszym rozwiązaniem dla w2
  archiwum = [a,b,c]