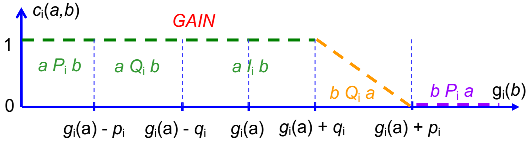

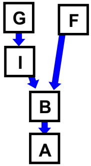

Multiple criteria ranking with the ELECTRE III / IV method

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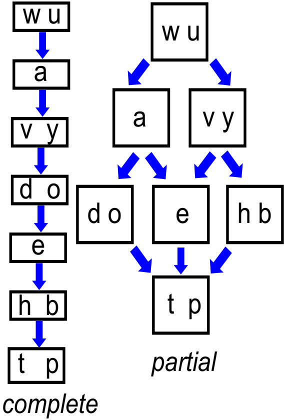
Multiple Criteria Ranking

How to order alternatives from the best to the worst?

ranking



- **Ranking** consists in imposing a preference relation on the set of alternatives
- The desired type of order in the set of alternatives can be **complete** or **partial**
- Using incomparability relation allows deriving partial order of alternatives
- **Partial** order has the advantage of highlighting alternatives that are considerably dissimilar
- The **cardinal** ranking is based on scores
- The **ordinal** ranking is based on binary relations (only the order/ranks of alternatives is meaningful)
- **Choice** refers to the selection of the subset of the best alternative(s)

Our Illustrative Study

INPUT DATA

- finite set of alternatives $A=\{a, b, c, \ldots, m\}$
- consistent family of criteria $G=\{g_1, g_2, \ldots, g_n\}$

AIM

Build an additional electric plant in Europe

Alt.	g_1	...	g_n
a	$g_1(a)$		$g_n(a)$
b	$g_1(b)$		$g_n(b)$
c	$g_1(c)$		$g_n(c)$
...	...		...
m	g_1(m)		$g_n(m)$

Alt.	$g_1 \uparrow$	$g_2 \uparrow$	$g_3 \downarrow$
ITA	90	4	600
BEL	58	0	200
GER	66	7	400
AUT	74	8	800
FRA	98	6	800

ALTERNATIVES AND CRITERIA

- Five possible locations (countries) evaluated in terms of 3 criteria
- g_1 (gain) – Power (in Megawatt)
- g_2 (gain) – Safety level (0-10 scale)
- g_3 (cost) – Construction cost (in million USD)

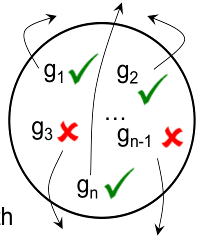
For illustrative purposes, we simplify the problem by taking into account only three criteria. Other relevant criteria include: manpower for running the plant, annual maintenance cost, or ecology (number of villages to evacuate).

- The alternatives are compared by means of **outranking relation *S***
- Outranking relation *S* groups three basic preference relations of indifference *I*, weak preference *Q*, and strict preference *P*:

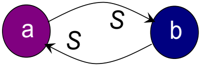
$$S = \{I, Q, P\}$$

- *aSb* means "*a is at least as good as b*"
- *S* is established by verifying the reasons for and against its truth
- *S* is reflexive and non-transitive
- *S* can be **crisp** (binary; 0 (false) or 1 (true)) or **valued** (fuzzy; on a scale from 0 to 1)
- Focus on valued outranking relation, but for simplicity, we start with the crisp one

reasons for S

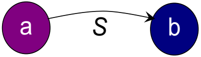


reasons against *S*



a* is indifferent with *b

a I b iff aSb ∧ *bSa*



a* is preferred to *b

a > b iff aSb ∧ *not*(*bSa*)

a* is incomparable with *b

a ? b iff not(*aSb*) ∧ *not*(*bSa*)

ELECTRE III – Simplified Principle

Assume a **crisp relation *S***

S	ITA	BEL	GER	AUT	FRA
ITA	1	0	1	0	0
BEL	0	1	0	0	1
GER	1	1	1	0	0
AUT	0	0	0	1	1
FRA	1	0	1	0	1

Construct **two complete preorders** (descending and ascending) using the distillation procedure

- In the **descending distillation**, one orders the alternatives from the best to the worst
- In the **ascending distillation**, one orders the alternatives from the worst to the best

$s(a) = |b \in A \setminus \{a\} : aSb|$ = the number of alternatives outranked by a

$w(a) = |b \in A \setminus \{a\} : bSa|$ = the number of alternatives that outrank a

quality $q(a)$ = strength $s(a)$ – weakness $w(a)$

S	*s*	*w*	*q*
ITA	1	2	-1
BEL	1	1	0
GER	2	2	0
AUT	1	0	1
FRA	2	2	0

- Once some alternative is added to the preorder, it is eliminated from further consideration, and the same procedure is applied to the remaining alternatives
- In the case of a tie, the internal distillation is performed using the same procedure though limited only to a subset of tied alternatives (with the same quality)

Descending Distillation for Crisp Outranking

Compute the quality of each alternative: ***q*(*a*) = *s*(*a*) – *w*(*a*) = |*b*∈A\{*a*} : *a*S*b*| - |*b*∈A\{*a*} : *b*S*a*|**

- If the alternative with **the greatest quality** is unique, add it to the **currently lowest possible position** in the descending preorder and eliminate from further consideration
- In the case of a tie, try to break it by running the **internal distillation** (embed until it is possible)

Continue until all alternatives are added to the **descending order** (from the best to the worst)

S	**I**	**B**	**G**	**A**	**F**	*s*	*w*	*q*
I	1	0	1	0	0	1	2	-1
B	0	1	0	0	1	1	1	0
G	1	1	1	0	0	2	2	0
A	0	0	0	1	1	1	0	1
F	1	0	1	0	1	2	2	0

A

Add **A** to the lowest position and eliminate

S	**I**	**B**	**G**	**F**	*s*	*w*	*q*
I	1	0	1	0	1	2	-1
B	0	1	0	1	1	1	0
G	1	1	1	0	2	2	0
F	1	0	1	1	2	1	1

A → **F**

Add **F** to the lowest position and eliminate

S	**I**	**B**	**G**	*s*	*w*	*q*
I	1	0	1	1	1	0
B	0	1	0	0	1	-1
G	1	1	1	2	1	1

A → **F** → **G**

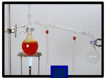
Add **G** to the lowest position and eliminate

S	**I**	**B**	*s*	*w*	*q*
I	1	0	0	0	0
B	0	1	0	0	0

It is impossible to break the tie

A → **F** → **G** → **I, B**

Descending preorder



Add **I** and **B** to the lowest position and eliminate

Ascending Distillation for Crisp Outranking

Compute the quality of each alternative: $\boldsymbol{q(a)} = \boldsymbol{s(a)} - \boldsymbol{w(a)} = |b \in A \setminus \{a\} : aSb| - |b \in A \setminus \{a\} : bSa|$

- If the alternative with **the least quality** is unique, add it to the **currently highest possible position** in the ascending preorder and eliminate from further consideration
- In the case of a tie, try to break it by running the **internal distillation** (embed until it is possible)

Continue until all alternatives are added to the **ascending order** (from the worst to the best)

S	I	B	G	A	F	*s*	*w*	*q*
I	1	0	1	0	0	1	2	-1
B	0	1	0	0	1	1	1	0
G	1	1	1	0	0	2	2	0
A	0	0	0	1	1	1	0	1
F	1	0	1	0	1	2	2	0

Add **I** to the highest position and eliminate

I

S	B	G	A	F	*s*	*w*	*q*
B	1	0	0	1	1	1	0
G	1	1	0	0	1	1	0
A	0	0	1	1	1	0	1
F	0	1	0	1	1	2	-1

F
I

Add **F** to the highest position and eliminate

S	B	G	A	*s*	*w*	*q*
B	1	0	0	0	1	-1
G	1	1	0	1	0	1
A	0	0	1	0	0	0

B
F
I

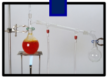
Add **B** to the highest position and eliminate

S	G	A	*s*	*w*	*q*
G	1	0	0	0	0
A	0	1	0	0	0

It is impossible to break the tie

G, A
B
F
I

Ascending preorder



Add **G** and **A** to the highest position and eliminate

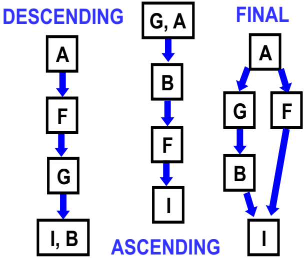
ELECTRE III – Final Results

The **final partial preorder** of alternatives produced by the ELECTRE III method is obtained as the “intersection” of two complete – descending and ascending – preorders:

- *a* is *preferred* to *b* (*aPb*) if *a* is not worse than *b* in both distillations and better in at least one dist.
- *a* and *b* are *indifferent* (*aIb*), if *a* and *b* are indifferent in both distillations
- *a* and *b* are *incomparable* (aRb) if *a* is better than *b* in one distillation and worse in the other one

Rank in the final preorder is determined by the length of the longest path from the alternative to some top-ranked alternative

The **median preorder** is determined by the ranks with ties broken by the difference between ranks in the descending and ascending preorders



RANKS

1. A
2. G, F
3. B
4. I

MEDIAN

1. A
2. G
3. F
4. B
5. I

G and **F** have the same rank, but **G** is better than **F** in the ascending order by two positions, and **F** is better than **G** in the descending order by only one position

- **EL**imination **E**t **C**hoix **T**raduisant la **Re**alite
  (*Elimination and Choice Expressing the Reality*)
- **Choice**: **ELECTRE I**, Iv, Is, ELECTRE^{GKMS}
- **Ranking**: **ELECTRE** II, **III**, and **IV**
- **Sorting**: ELECTRE TRI-B, TRI-C, TRI-rC, TRI-nC, TRI-nB
- **Our focus today**

Bernard Roy

When to use ELECTRE methods?

- Applicable with at least 3 and **preferably less than a dozen or so criteria**
- Handling qualitative performance scale for ordinal criteria
- Dealing with **heterogenous** scales without the need of recoding
- **Not allowing for compensation** between criteria
- Accounting for **imperfect knowledge** and arbitrariness when building criteria
- Implementing intuitive **analogy to voting** procedures
- Ability to represent weak preference (on a per-criterion level)
  and **incomparability** (on a comprehensive level)

Construction of an outranking relation for all pairs of alternatives via **concordance** and **discordance** tests

Directly provided input parameters

compare pairwise on each criterion

Marginal concordance indices c_i

aggregate results from all criteria

Comprehensive concordance indices C

Marginal discordance indices D_i

Credibilities of outranking relation (valued relation) σ

- **Concordance test**: is the support given to *aSb* strong?
- **Discordance test**: is there any strong opposition to *aSb*?

σ	a	b	c
a	1	0.5	0.7
b	0.8	1	0.3
c	0.2	1	1

Exploitation of an outranking relation with **distillation procedures**

Final ranking, ranks, and median order

Indifference and Preference Thresholds

- To take into account the imperfect character of performances, ELECTRE methods make use of **discrimination** (indifference and preference) **thresholds**
  - This leads to a **pseudo-criterion** model on each criterion
- **Indifference threshold q_i** is the maximal difference in performances on g_i, by which two alternatives are judged indifferent
- **Preference threshold p_i** is the minimal difference in performances on g_i, which justifies a strict preference of one alternative over another on g_i
- The performance difference between q_i and p_i can be interpreted as a hesitation (weak preference) between opting for a strict preference or an indifference between the two alternatives
- In ELECTRE III, the thresholds are defined via affine functions: $q_i(a) = \alpha_i^q \times g_i(a) + \beta_i^q$ and $p_i(a) = \alpha_i^p \times g_i(a) + \beta_i^p$, where α_i, $\beta_i \geq 0$ (when $\alpha_i = 0$, the thresholds are constant)

Notation: q_i and p_i

Constraint: $q_i(a) \leq p_i(a)$

Threshold	g_1	g_2	g_3
indifference q_i	4	1	100
preference p_i	12	2	200

Alt.	*I*	*B*	*G*	*A*	*F*
$g_1 \uparrow$	90	58	66	74	98

A ($g_1(A) = 74$) is **indifferent to alternatives *a* with** $g_1(a)$ between 70 and 78, **weakly preferred to *G*** ($g_1(G) = 66$), and **strictly preferred to *B*** ($g_1(B) = 58$)

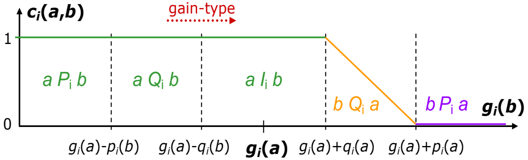
Marginal Concordance for Gain-type Criterion (1)

- Let us start with verifying for each criterion $\boldsymbol{g_i}$ a degree to which it supports the hypothesis that ***a*** outranks ***b***; thus, we consider pair (*a*,*b*)
  - The inverse pair (*b*,*a*) can be considered analogously
- The answer is quantified with **a marginal concordance index $\boldsymbol{c_i(a,b)}$**

y-axis: value of $\boldsymbol{c_i(a,b)}$ ranging from 0 to 1

- $\boldsymbol{P_i}$ – strong preference on g_i
- $\boldsymbol{Q_i}$ – weak preference on g_i
- $\boldsymbol{I_i}$ – indifference on g_i

aim: for different $g_i(b)$ and fixed $\boldsymbol{g_i(a)}$, return $\boldsymbol{c_i(a,b)}$



x-axis: performance of alternative ***a*** unchanged as well as decreased or increased by discrimination thresholds (the thresholds are computed for the worse alternative – *b* to the left from $g_i(a)$ and *a* to the right)

FOR GAIN–TYPE CRITERION:

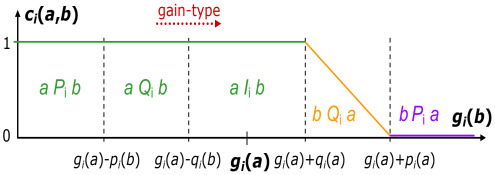
$$c_i(a,b) = \begin{cases} 1, & \text{if } g_i(a) - g_i(b) \geq -q_i(a) \\ 0, & \text{if } g_i(a) - g_i(b) < -p_i(a) \\ \dfrac{p_i(a) - (g_i(b) - g_i(a))}{p_i(a) - q_i(a)}, & \text{otherwise} \end{cases}$$

a is preferred to b or a is indifferent with b (it can be worse, but only by at most $q_i(a)$)

b is strongly preferred to a, being better by more than $p_i(a)$

b is weakly preferred to a, being better by more than $q_i(a)$, but not more than $p_i(a)$

Marginal Concordance for Gain-type Criterion (2)



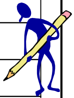
FOR *a*: 54, 62, **66**, 70, 78

FOR *b*: 58, 66, 74, 90 … 98

MARGINAL CONCORDANCE INDEX FOR GAIN-TYPE CRITERION:

$$c_i(a,b) = \begin{cases} 1, & \text{if } g_i(a) - g_i(b) \geq -q_i(a) \\ 0, & \text{if } g_i(a) - g_i(b) < -p_i(a) \\ \dfrac{p_i(a) - (g_i(b) - g_i(a))}{p_i(a) - q_i(a)}, & \text{otherwise} \end{cases}$$

G	**66**	q_1	4	p_1	12
Alt.	$g_1\uparrow$	$c_1(G,b)$			
I	90	0			
B	58	1			
G	66	1			
A	74	0.5			
F	98	0			



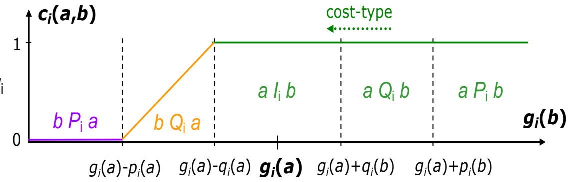
- Let us compare alternative ***a*** with alternatives ***b*** on criterion g_1: $c_1(a,b)$
  - Discrimination thresholds: $q_i = 4$ and $p_i = 12$
- Results:
  - G ($g_1(G) = 66$) is better than B ($g_1(B) = 58$), so $c_1(G,B) = 1$
  - G ($g_1(G) = 66$) is the same as G ($g_1(G) = 64$), so $c_1(G,G) = 1$
  - G ($g_1(G) = 66$) is weakly worse than A ($g_1(A) = 74$), so $c_1(G,A) = 0.5$
  - G ($g_1(G) = 66$) is strictly worse than I ($g_1(I) = 90$), so $c_1(G,I) = 0$
  - G ($g_1(G) = 66$) is strictly worse than F ($g_1(F) = 98$), so $c_1(G,F) = 0$

Marginal Concordance for Cost-type Criterion

- **For cost-type criteria, we need to reason analogously**
  - Bear in mind *lesser performances are more preferred*
- For example, $c_i(a,b) = 1$ (i.e., g_i fully supports aSb), if a has lesser, the same, or negligibly greater performance than b on g_i

y-axis: value of $\boldsymbol{c_i(a,b)}$ ranging from 0 to 1

- $\boldsymbol{P_i}$ – strong preference on g_i
- $\boldsymbol{Q_i}$ – weak preference on g_i
- $\boldsymbol{I_i}$ – indifference on g_i



aim: for different $g_i(b)$ and fixed $\boldsymbol{g_i(a)}$, return $\boldsymbol{c_i(a,b)}$

x-axis: performance of alternative ***a*** unchanged as well as decreased or increased by discrimination thresholds

FOR COST–TYPE CRITERION:

$$c_i(a,b) = \begin{cases} 1, & \text{if } g_i(a) - g_i(b) \leq q_i(a) \\ 0, & \text{if } g_i(a) - g_i(b) > p_i(a) \\ \dfrac{p_i(a) - (g_i(a) - g_i(b))}{p_i(a) - q_i(a)}, & \text{otherwise} \end{cases}$$

a is preferred to b or a is indifferent with b (it can be worse, but only by at most $q_i(a)$)

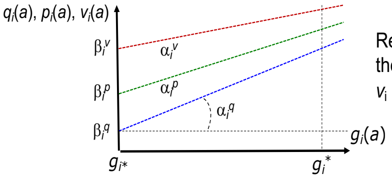
b is strongly preferred to a, being better by more than $p_i(a)$

b is weakly preferred to a, being better by more than $q_i(a)$, but not more than $p_i(a)$

Veto Threshold

- **Veto threshold v_i** is the minimal, absolutely critical difference in performances on g_i, which has an impact on a comprehensive comparison of a pair of objects, irrespective of the remaining criteria
- **If *a* is worse than *b* on g_i by at least veto threshold v_i, then g_i strongly disagrees against a*Sb*, and *a* cannot outrank *b***
  - Veto threshold is treated as an **inter-criteria parameter** (it is defined on a particular criterion, but its impact is more comprehensive and affects an entire comparison)
- Veto threshold is **usually used on the most important criteria** (when not specified, then $v_i^h = \infty$)
- In ELECTRE III, if *a* is worse than *b* on g_i by more than preference threshold p_i and less than veto threshold v_i, then g_i **weakly disagrees** with a*Sb*, decreasing support to the assertion a*Sb*
- In ELECTRE III, the veto thresholds are defined via affine functions: $q_i(a) = \alpha_i^v \times g_i(a) + \beta_i^v$

Threshold	g_1	g_2	g_3
indifference q_i	4	1	100
preference p_i	12	2	200
veto v_i	28	8	600



Relations between the thresholds: $v_i > p_i \geq q_i$

Alt.	*I*	*B*	*G*	*A*	*F*
$g_1 \uparrow$	90	58	66	74	98

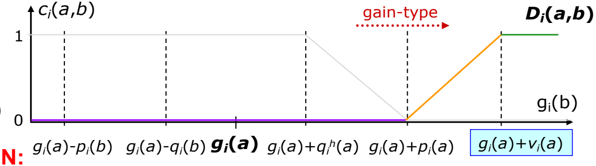
for *G* ($g_1(G) = 66$), g_1 **is strongly discordant with outranking *GSb* for b=*F*** **($g_1(F) = 98$), weakly discordant for *I* ($g_1(I) = 90$),** and **not discordant for *A* ($g_1(A) = 74$)**

Marginal Discordance for Gain-type Criterion (1)

- Let us start with verifying for each criterion $\boldsymbol{g_i}$ a degree to which it disagrees with the hypothesis that alternative ***a*** outranks altern. ***b***; thus, we consider pair (*a*,*b*)
  - The inverse pair (*b*,*a*) can be considered analogously
- The answer is quantified with **a marginal discordance index $\boldsymbol{D_i(a,b)}$**
- Unlike in ELECTRE I, a marginal discordance index is fuzzy (between 0 and 1)

y-axis: value of $\boldsymbol{D_i(a,b)}$ ranging from 0 to 1

aim: for different $g_i(b)$ and fixed $\boldsymbol{g_i(a)}$, return $\boldsymbol{D_i(a,b)}$



FOR GAIN–TYPE CRITERION:

$$
D_i(a,b) = \begin{cases} 1, & \text{if } g_i(a) - g_i(b) \le -v_i(a) \\ 0, & \text{if } g_i(a) - g_i(b) \ge -p_i(a) \\ \dfrac{(g_i(b) - g_i(a)) - p_i(a)}{v_i(a) - p_i(a)}, & \text{otherwise} \end{cases}
$$

a is worse than b by at least $v_i(a)$

a is at least as good as b or not worse than b by more than $p_i(a)$

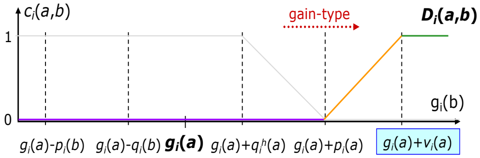
a is worse than b by more than $p_i(a)$, but less than $v_i(a)$

- If $\boldsymbol{c_i(a,b) > 0}$, then $\boldsymbol{D_i(a,b) = 0}$ and if $\boldsymbol{D_i(a,b) > 0}$, then $\boldsymbol{c_i(a,b) = 0}$
  - A criterion cannot send contradicting signals
- Only one point for which $\boldsymbol{c_i(a,b) = 0}$ and $\boldsymbol{D_i(a,b) = 0}$ (*a* is worse than *b* by $p_i(a)$)

Marginal Discordance for Gain-type Criterion (2)

MARGINAL DISCORDANCE INDEX FOR GAIN-TYPE CRITERION:

$$D_i(a,b) = \begin{cases} 1, & \text{if } g_i(a) - g_i(b) \leq -v_i(a) \\ 0, & \text{if } g_i(a) - g_i(b) \geq -p_i(a) \\ \dfrac{(g_i(b) - g_i(a)) - p_i(a)}{v_i(a) - p_i(a)}, & \text{otherwise} \end{cases}$$



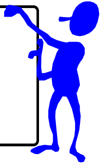
FOR *a*:	54	62	**66**	70	78	94		
FOR *b*:	58		66		74		90	98

G	66	p_1	12	v_1	28
Alt.	g_1 ↑	$D_1(G,b)$			
I	90	3/4			
B	58	0			
G	66	0			
A	74	0			
F	98	1			

- Let us compare alternative ***a*** with alternatives ***b*** on criterion g_1: $D_1(a,b)$
  - Thresholds: $q_i = 4$, $p_i = 12$ and $v_i = 28$
- Results:
  - G ($g_1(G) = 66$) is critically worse than F ($g_1(F) = 98$), so $D_1(G,F) = 1$
  - G ($g_1(G) = 66$) is weakly crit. worse than I ($g_1(I) = 90$), so $D_1(G,I) = 3/4$
  - G ($g_1(G) = 66$) is not critically worse than A ($g_1(A) = 74$), so $D_1(G,A) = 0$
  - G ($g_1(G) = 66$) is not critically worse than G ($g_1(G) = 66$), so $D_1(G,G) = 0$
  - G ($g_1(G) = 66$) is not critically worse than B ($g_1(B) = 58$), so $D_1(G,B) = 0$

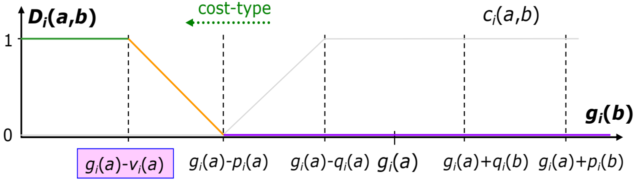
Marginal Discordance for Cost-type Criterion

- For cost-type criteria, we need to reason analogously
  - Bear in mind lesser performances are more preferred
- For example, $D_i(a,b) = 1$ (i.e., g_i fully disagrees with aSb), if a has performance greater than b by at least $v_i(a)$



y-axis: value of $\boldsymbol{D_i(a,b)}$ ranging from 0 to 1

aim: for different $g_i(b)$ and fixed $\boldsymbol{g_i(a)}$, return $\boldsymbol{D_i(a,b)}$



x-axis: performance of alternative ***a*** unchanged as well as decreased or increased by discrimination thresholds

FOR COST–TYPE CRITERION:

$$D_i(a,b) = \begin{cases} 1, & \text{if } g_i(a) - g_i(b) \geq v_i(a) \\ 0, & \text{if } g_i(a) - g_i(b) \leq p_i(a) \\ \dfrac{v_i(a) - (g_i(a) - g_i(b))}{v_i(a) - p_i(a)}, & \text{otherwise} \end{cases}$$

a is worse than b by at least $v_i(a)$

a is at least as good as b or worse than b by at most $p_i(a)$

a is worse than b by more than $p_i(a)$, but less than $v_i(a)$

Marginal Concordances - Need for Aggregation

$c_1(a,b)$	I	B	G	A	F
I	1	1	1	1	0.50
B	0	1	0.50	0	0
G	0	1	1	0.50	0
A	0	1	1	1	0
F	1	1	1	1	1

$c_2(a,b)$	I	B	G	A	F
I	1	1	0	0	0
B	0	1	0	0	0
G	1	1	1	1	1
A	1	1	1	1	1
F	1	1	1	0	1

$c_3(a,b)$	I	B	G	A	F
I	1	0	0	1	1
B	1	1	1	1	1
G	1	0	1	1	1
A	0	0	0	1	1
F	0	0	0	1	1

In view of the marginal concordance indices for all criteria, what is the **comprehensive support** given to hypothesis aSb?

- G seems to be strong compared to A
- I seems to be weak compared to G
- for (B,G) or (I,F) the results are ambiguous

We need some interpretable numbers that would aggregate the answers from the individual criteria

Weights

- **The greater w_i, the more important criterion g_i**
- **Voting power of criterion g_i** in enforcing a supported decision (when the criterion contributes to the majority which is in favor of an outranking)
  - Weights are intra-criteria parameters
- A single weight cannot be interpreted alone; it has a meaning only in the context of all other weights being specified
- Weights do not depend on the ranges nor on the encoding of scales
- In a standard setting, precise weight values need to be provided directly by the DM

Weight	g_1	g_2	g_3
w_i	3	3	4

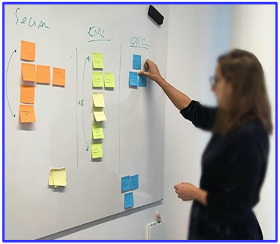
- g_3 is the most important criterion, whereas g_1 and g_2 are less important
- g_3 is 4/3 times more important than g_1 and g_2
- The weights do not need to sum up to 1 (they are normalized later in any case)

The SRF Procedure (1)

- In practice, ELECTRE III is often coupled with the **SRF (Simos-Roy-Figueira) procedure** for determining the criteria weights (also called the method of cards)
- The DM is asked to **rank the elementary criteria** with respect to their relative importance from the least important (a group with raw rank L_1) to the most important (a group with raw rank L_v)
  - Each criterion is assigned to a group with raw rank L_s, $s = 1,\ldots,v$
- To increase the difference of importance between criteria in the subsequent groups L_s and L_{s+1}, the DM can insert some blank cards between them
  - e_s is the number of **blank cards** between L_s and L_{s+1}
- The DM is asked to provide a **ratio *Z*** between the importance of criteria in groups L_v and L_1

	ratio Z	ratio between weights of the most and the least important criteria
L_v	g_4	
	2 white cards	e_{v-1}
L_{v-1}	g_2, g_3, g_7	
…	…	
L_3	g_5	
	0 white cards	e_2
L_2	g_6, g_8	
	3 white cards	e_1
L_1	g_1	

METHOD OF CARDS



The SRF Procedure (2)

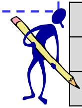
- Compute rank $r(t)$ of each group L_t: $r(t) = t + (\sum_{s=1,\ldots,t-1} e_s)$
  - Sum up the raw rank of group L_t and the number of blank cards below group L_t
- Compute non-normalized weight of elementary criterion in group L_t:

$$w_{Gt}' = 1 + (Z - 1) \cdot \frac{r(t) - 1}{r(v) - 1}$$

  - Relate the rank of group L_t to the rank of the best group L_v and multiply by $(Z - 1)$
- Compute normalized weight by dividing w_t' by the sum of non-normalized weights for all criteria:

$$w_{Gt} = w_{Gt}' / \sum_{i=1,\ldots,m} w_i'$$

raw rank	***ratio Z = 8***	***blank cards***	***rank r(t)***	***Non-normalized weight w_t'***
L_4 $(t = 4)$	g_1		$r(4) = 4 + (3 + 0 + 1) = 8$	$w_{G4}' = 1 + (8 - 1)\cdot(8 - 1)/(8 - 1) = 8$
	1 *white card*	e_3=1		
L_3 $(t = 3)$	g_2		$r(3) = 3 + (3 + 0) = 6$	$w_{G3}' = 1 + (8 - 1)\cdot(6 - 1)/(8 - 1) = 6$
		e_2=0		
L_2 $(t = 2)$	g_3		$r(2) = 2 + (3) = 5$	$w_{G2}' = 1 + (8 - 1)\cdot(5 - 1)/(8 - 1) = 5$
	3 *white cards*	e_1=3		
L_1 $(t = 1)$	g_4		$r(1) = 1$	$w_{G1}' = 1 + (8 - 1)\cdot(1 - 1)/(8 - 1) = 1$



Weight	w_1	w_2	w_3	w_4	Sum
Non-norm.	8	6	5	1	20
Normalized	8/20 = 0.4	6/20 = 0.3	5/20 = 0.25	1/20 = 0.05	1

COMPREHENSIVE CONCORDANCE INDEX

$$C(a,b) = \frac{\sum_{i=1,\ldots,n} w_i \cdot c_i(a,b)}{\sum_{i=1,\ldots,n} w_i}$$

- Criteria have different impacts on the value of ***C*(*a*,*b*)**, dependent on their weights (voting powers)
- ***C*(*a*,*b*)** quantifies the strength of the coalition of criteria supporting ***aSb***
- ***C*(*a*,*b*) is in the range [0,1]**
  1 = all criteria strongly support *aSb*
  0 = none criterion supports *aSb* weakly or strongly

Three example pairs

	g_1	g_2	g_3
$c_j(I,G)$	1	0	0
$c_j(I,F)$	0.5	0	1
$c_j(G,A)$	0.5	1	1
w_i	3	3	4

$$C(I,G) = \frac{3\cdot1 + 3\cdot0 + 4\cdot0}{3+3+4} = 0.3$$

$$C(I,F) = \frac{3\cdot0.5 + 3\cdot0 + 4\cdot1}{3+3+4} = 0.55$$

$$C(G,A) = \frac{3\cdot0.5 + 3\cdot1 + 4\cdot1}{3+3+4} = 0.85$$

Concordance matrix for the study:

- the results are not univocal for all pairs
- the support given to the outranking differs from one pair to another

C(*a*,*b*)	I	B	G	A	F
I	1	0.6	0.3	0.7	0.55
B	0.4	1	0.55	0.4	0.4
G	0.7	0.6	1	0.85	0.7
A	0.3	0.6	0.6	1	0.7
F	0.6	0.6	0.6	0.7	1

Marginal Discordances - Need for Aggregation

- **Discordance** verifies if among criteria discordant with the outranking hypothesis, there is strong **opposition** against *aSb*
- In ELECTRE I, the discordance at the per-criterion and comprehensive levels was binary
- In ELECTRE III, the marginal discordance indices are fuzzy, taking values between 0 and 1
  - $D_i(a,b) = 1$ means that the opposition to *aSb* is the strongest possible
  - $D_i(a,b) = 0$ means that there is no opposition to *aSb*
  - $D_i(a,b) \in (0,1)$ means that the opposition to *aSb* exists, but is not extremely strong
- The results of the concordance and discordance tests are aggregated into a single measure, called outranking credibility

MARGINAL DISCORDANCE INDICES

$D_1(a,b)$	I	B	G	A	F
I	0	0	0	0	0
B	1	0	0	1/4	1
G	3/4	0	0	0	1
A	1/4	0	0	0	3/4
F	0	0	0	0	0

$D_2(a,b)$	I	B	G	A	F
I	0	0	1/6	1/3	0
B	1/3	0	5/6	1	2/3
G	0	0	0	0	0
A	0	0	0	0	0
F	0	0	0	0	0

$D_3(a,b)$	I	B	G	A	F
I	0	1/3	0	0	0
B	0	0	0	0	0
G	0	0	0	0	0
A	0	1	1/2	0	0
F	0	1	1/2	0	0

Outranking Credibility

- **Outranking credibility** $\boldsymbol{\sigma}$ aggregates the comprehensive concordance and marginal discordances:

$$\sigma(a,b) = C(a,b) \prod_{j \in F} \frac{1 - D_j(a,b)}{1 - C(a,b)} \qquad \text{where } F = \{\, j = 1,\ldots,n : D_j(a,b) > C(a,b) \,\}$$

  - *Starting point*: results of the concordance test $\boldsymbol{C(a,b)}$
  - For each criterion for which the marginal discordance is sufficiently great ($D_j(a,b) > C(a,b)$), multiply by the module which is then lower than 1, hence decreasing the outranking credibility
- If there is no discordance on any criterion ($D_j(a,b) = 0$) or it is not sufficiently great, then $\boldsymbol{\sigma(a,b) = C(a,b)}$
- If there is at least one criterion with $\boldsymbol{D_j(a,b) = 1}$, then $\boldsymbol{\sigma(a,b) = 0}$ (multiplying by module = 0)
- If the marginal discordance is sufficiently strong, but not equal to 1, then we multiply by the respective modules for each criterion for which the **condition** is satisfied (then, $\boldsymbol{\sigma(a,b) < C(a,b)}$)

Pair	C	D_1	D_2	D_3	$\sigma(a,b)$
(I,B)	0.6	0	0	1/3	0.6
(B,G)	0.55	0	5/6	0	0.2
(A,F)	0.7	3/4	0	0	0.58
(B,F)	0.4	1	2/3	0	0

$\sigma(I,B) = 0.60$

$$\sigma(B,G) = 0.55 \cdot \frac{1 - 5/6}{1 - 0.55} = 0.2$$

$$\sigma(B,F) = 0.40 \cdot \frac{1 - 1}{1 - 0.40} \cdot \frac{1 - 2/3}{1 - 0.40} = 0$$

σ	I	B	G	A	F
I	1	0.6	0.3	0.7	0.55
B	0	1	0.2	0	0
G	0.58	0.6	1	0.85	0
A	0.3	0	0.6	1	0.58
F	0.6	0	0.6	0.7	1

Distillation Procedure – General Idea

1. Compute **the lower threshold for the credibilities** under interest (based on the upper threshold equal to the maximal observed credibility in the analyzed set of alternatives).
2. If the upper threshold is zero, it is **impossible to distinguish** between alternatives. Then, add all alternatives to the constructed preorder and STOP.
3. Construct an **outranking relation under interest**: save only credibilities exceeding the lower thresh. and being significantly greater than the credibilities for inverse pairs.
4. Compute the **strength, weakness, and quality** of each alternative based on the constructed outranking relation (how many are outranked – how many outrank).
5. If the alternative with the greatest/least qualification is **not unique**, try breaking the tie. Perform the **distillation limited to the tied set** (assume the lower threshold becomes the upper threshold to loosen the requirements on the credibility value).
6. **Add the best/worst alternative(s)** to the preorder and eliminate them from the set.
7. If all alternatives are added to the constructed preorder, STOP. Otherwise, continue.

CREDIBILITY THRESHOLDS

NO ARGUMENTS TO DISTINGUISH

CREDIBILITIES UNDER INTEREST = OUTRANK. REL.

QUALITIES

BREAK TIES

THE BEST / / THE WORST

RANKING CONSTRUCTION

Descending Distillation

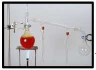
1. Set $k := 0$.
2. Compute the **upper credibility threshold**: $\lambda_k = Max_{a,b \in A, a \neq b}\{\sigma(a,b)\}$.
3. Compute the lower credibility threshold: $\lambda_{k+1} = Max_{a,b \in A: \sigma(a,b) < \lambda_k - s(\lambda_k)}\{\sigma(a,b), 0\}$, where $s(\lambda_k) = \alpha \cdot \lambda_k + \beta$ (by default, $\alpha = -0.15$ and $\alpha = 0.3$).
4. If $\lambda_k = 0$, then place set A **at the bottom of the descend. preorder $\bar{P}$** and **STOP**. Otherwise, $k := k + 1$.
5. Save only relations $aS^{\lambda_k}b$ that satisfy the following conditions:

$$\sigma(a,b) > \lambda_k \text{ and } \sigma(a,b) > \sigma(b,a) + s[\sigma(a,b)].$$

6. Compute the **strength, weakness, and quality**:

$$q_A^{\lambda_k}(a) = s_A^{\lambda_k}(a) - w_A^{\lambda_k}(a) = |b \in A: aS^{\lambda_k}b| - |b \in A: bS^{\lambda_k}a|.$$

7. Identify a subset of alternatives with **maximal quality**:



$$\bar{D}_1^k = \left\{a \in A: q_A^{\lambda_k}(a) = Max_{x \in A}\left\{q_A^{\lambda_k}(x)\right\}\right\}.$$

8. If $|\bar{D}_1^k| = 1$, set $\bar{D}_F^k = \bar{D}_1^k$. Otherwise, call an **internal distillation** procedure resulting in $\bar{D}_F^k \subseteq \bar{D}_1^k$. Go to **step 3**, performing sub-iteration *h* for iteration *k* (start with *h*=0, assume the lower thr. become the upper thr.; change *h*, not *k*).
9. Place set $\bar{D}_F^k$ **at the (current) bottom of the descending preorder $\bar{P}$**. Eliminate $\bar{D}_F^k$ from further consideration: $A := A \setminus \bar{D}_F^k$.
10. If $A = \emptyset$, then **STOP**. Otherwise, go to **step 2**.

σ	I	B	G	A	F
I	1	0.6	0.3	0.7	0.55
B	0	1	0.2	0	0
G	0.58	0.6	1	0.85	0
A	0.3	0	0.6	1	0.58
F	0.6	0	0.6	0.7	1

$\lambda_0 = Max\{\sigma(a,b)\} = 0.85$
$s(\lambda_0) = -0.15 \cdot 0.85 + 0.3 = 0.1725$
$\lambda_1 = Max\{\sigma(a,b)\} < 0.6775$
$\lambda_1 = 0.6$

S^λ	I	B	G	A	F	s	w	q
I				0.7		1	0	1
B						0	0	0
G				0.85		1	0	1
A						0	2	-2
F				~~0.7~~		0	0	0

$\lambda_1^0 = 0.6$ (iter k=1; sub-iter h=0)
$s(\lambda_1^0) = -0.15 \cdot 0.6 + 0.3 = 0.21$
$\lambda_1^1 = Max\{\sigma(a,b)\} < 0.39$
$\lambda_1^1 = 0.3$

σ	I	G
I	1	0.3
G	0.58	1

S^λ	I	G	s	w	q
I			0	1	-1
G	0.58		1	0	1

Descending Distillation – Example

1st iteration

σ	I	B	G	A	F	s	w	q
I	1	0.6	0.3	0.7	0.55	1	0	1
B	0	1	0.2	0	0	0	0	0
G	0.58	0.6	1	0.85	0	1	0	1
A	0.3	0	0.6	1	0.58	0	2	-2
F	0.6	0	0.6	0.7	1	0	0	0

G

Add **G** to the lowest position and eliminate

After the internal distillation (**G** and **I**), **G** is the best

2nd iteration

σ	I	B	A	F
I	1	0.6	0.7	0.55
B	0	1	0	0
A	0.3	0	1	0.58
F	0.6	0	0.7	1

S^{λ}	I	B	A	F	s	w	q
I		0.6	0.7	~~0.55~~	2	0	2
B					0	1	-1
A				~~0.58~~	0	1	-1
F	~~0.6~~		~~0.7~~		0	0	0

G → **I**

Add **I** to the lowest position and eliminate

$\lambda_1 = 0.7$
$s(\lambda_1) = -0.15 \cdot 0.7 + 0.3 = 0.195$
$\lambda_2 = Max\{\sigma(a,b)\} < 0.505$
$\lambda_2 = 0.3$

σ(I,B) = 0.6 >> σ(B,I) = 0.0
σ(I,A) = 0.7 >> σ(A,I) = 0.3
not(σ(F,I) = 0.6 >> σ(I,F) = 0.55)
not(σ(F,A) = 0.7 >> σ(A,F) = 0.58)

Save σ greater than 0.3 and significantly greater than for the inverse pair

3rd iteration

σ	B	A	F
B	1	0	0
A	0	1	0.58
F	0	0.7	1

$\lambda_2 = 0.7$
$s(\lambda_2) = -0.15 \cdot 0.7 + 0.3 = 0.195$
$\lambda_3 = Max\{\sigma(a,b)\} < 0.505$
$\lambda_3 = 0$

Save σ greater than 0 and significantly greater than for the inverse pair

S^{λ}	B	A	F	s	w	q
B				0	0	0
A			~~0.58~~	0	0	0
F		~~0.7~~		0	0	0

Tie between **B**, **A**, and **F**. Call internal distillation.

3rd iteration (internal distillation)

$\lambda_2^0 = 0$ It is impossible to break the tie

σ	B	A	F
B	1	0	0
A	0	1	0.58
F	0	0.7	1

Descending preorder

G → **I** → **B, A, F**

Add **B**, **A**, and **F** to the lowest position and eliminate

Ascending Distillation

1. Set $k := 0$.
2. Compute the **upper credibility threshold**: $\lambda_k = Max_{a,b\in A, a\neq b}\{\sigma(a,b)\}$.
3. Compute the lower credibility threshold: $\lambda_{k+1} = Max_{a,b\in A:\sigma(a,b)<\lambda_k - s(\lambda_k)}\{\sigma(a,b),0\}$, where $\boldsymbol{s(\lambda_k) = \alpha\cdot\lambda_k + \beta}$ (by default, $\alpha = -0.15$ and $\alpha = 0.3$).
4. If $\lambda_k = 0$, then place set A **at the top of the ascending preorder $\overline{P}$** and **STOP**. Otherwise, $k := k+1$.
5. Save only relations $\boldsymbol{aS^{\lambda_k}b}$ that satisfy the following conditions:

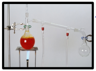
$$\sigma(a,b) > \lambda_k \text{ and } \sigma(a,b) > \sigma(b,a) + s[\sigma(a,b)].$$

6. Compute the **strength, weakness, and quality**:

$$q_A^{\lambda_k}(a) = s_A^{\lambda_k}(a) - w_A^{\lambda_k}(a) = |b\in A: aS^{\lambda_k}b| - |b\in A: bS^{\lambda_k}a|.$$

7. Identify a subset of alternatives with **minimal quality**:

$$\overline{D}_1^k = \left\{a\in A: q^{\lambda_k}(a) = Max_{x\in A}\left\{q_A^{\lambda_k}(x)\right\}\right\}.$$



8. If $|\overline{D}_1^k| = 1$, set $\overline{D}_F^k = \overline{D}_1^k$. Otherwise, call an **internal distillation** procedure resulting in $\overline{D}_F^k \subseteq \overline{D}_1^k$. Go to **step 3**, performing sub-iteration *h* for iteration *k* (start with *h*=0, assume the lower thr. become the upper thr.; change *h*, not *k*).
9. Place set $\overline{D}_F^k$ **at the (current) top of the ascending preorder $\underline{P}$**. Eliminate $\overline{D}_F^k$ from further consideration: $A := A\backslash\overline{D}_F^k$.
10. If $A = \emptyset$, then **STOP**. Otherwise, go to **step 2**.

σ	I	B	G	A	F
I	1	0.6	0.3	0.7	0.55
B	0	1	0.2	0	0
G	0.58	0.6	1	0.85	0
A	0.3	0	0.6	1	0.58
F	0.6	0	0.6	0.7	1

$\lambda_0 = Max\{\sigma(a,b)\} = 0.85$

$s(\lambda_0) = -0.15\cdot 0.85 + 0.3 = 0.1725$

$\lambda_1 = Max\{\sigma(a,b)\} < 0.6775$

$\lambda_1 = 0.6$

The first step in the descending and ascending distillations is the same

S^λ	I	B	G	A	F	s	w	q
I				0.7		1	0	1
B						0	0	0
G				0.85		1	0	1
A						0	2	-2
F				~~0.7~~		0	0	0

Ascending Distillation – Example

1st iteration

σ	I	B	G	A	F	s	w	q
I	1	0.6	0.3	0.7	0.55	1	0	1
B	0	1	0.2	0	0	0	0	0
G	0.58	0.6	1	0.85	0	1	0	1
A	0.3	0	0.6	1	0.58	0	2	-2
F	0.6	0	0.6	0.7	1	0	0	0

Add **A** to the highest position and eliminate

A

After the distillation, **A** is the worst

2nd iteration

σ	I	B	G	F
I	1	0.6	0.3	0.55
B	0	1	0.2	0
G	0.58	0.6	1	0
F	0.6	0	0.6	1

S^λ	I	B	G	F	s	w	q
I		0.6		~~0.55~~	1	1	0
B					0	2	-2
G	0.58	0.6			2	1	1
F	~~0.5~~		0.6		1	0	1

B → A

Add **B** to the highest position and eliminate

$\lambda_1 = 0.6$
$s(\lambda_1) = -0.15 \cdot 0.6 + 0.3 = 0.21$
$\lambda_2 = Max\{\sigma(a,b)\} < 0.39$
$\lambda_2 = 0.3$

For example:

σ(I,B) = 0.6 >> σ(B,I) = 0.0
σ(G,I) = 0.58 >> σ(I,G) = 0.3
not(σ(F,I) = 0.6 >> σ(I,F) = 0.55)

Save σ greater than 0.3 and significantly greater than for the inverse pair

3rd iteration

σ	I	G	F
I	1	0.3	0.55
G	0.58	1	0
F	0.6	0.6	1

$\lambda_2 = 0.6$
$s(\lambda_2) = -0.15 \cdot 0.6 + 0.3 = 0.21$
$\lambda_3 = Max\{\sigma(a,b)\} < 0.39$
$\lambda_3 = 0.3$

Save σ greater than 0.3 and significantly greater than for the inverse pair

S^λ	I	G	F	s	w	q
I			~~0.55~~	0	1	-1
G	0.58			1	1	0
F	~~0.6~~	0.6		1	0	1

I → B → A

Add **I** to the highest position and eliminate

4th iteration

σ	G	F
G	1	0
F	0.6	1

S^λ	G	F	s	w	q
G			0	1	-1
F	0.6		1	0	1

G → I → B → A

$\lambda_3 = 0.6$
$s(\lambda_3) = -0.15 \cdot 0.6 + 0.3 = 0.21$
$\lambda_4 = Max\{\sigma(a,b)\} < 0.39$
$\lambda_4 = 0.0$

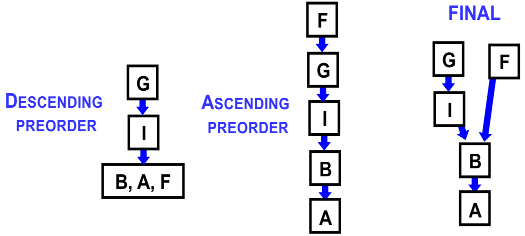
Add **G** to the highest position and eliminate

5th iteration

F → G → I → B → A

Ascending preorder

ELECTRE III – Final Results



RANKS	MEDIAN
1. G, F	1. G
2. I	2. F
	3. I
3. B	4. B
4. A	5. A

Final preorder: the intersection of the descending and ascending preorders:

- G*P*I because G is preferred to I in both preorders
- F*P*B because F is preferred to B in the descending preorder and they are indifferent in the ascending preorder
- I*R*F because I is preferred to F in the descending preorder and F is preferred to I in the ascending preorder

Rank = the length of the longest path from the alternative to some top-ranked alternative increased by one

- For G and F, the rank is one; for I, the rank is two (path to G); for B, the rank is three (path to G (the longest))

Median preorder is determined by the ranks with ties broken by the difference between ranks in the descending and ascending preorders

- G is better than F in the descend. preorder by 2 positions, and worse in the ascending preorder by 1 position

ELECTRE III – Summary

Preference information (i=1,…,n)

ELECTRE III

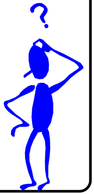
- **Intracriteria**:
  - indifference $q_i(a)$ and preference $p_i(a)$ thresholds (constant or defined with affine functions)
- **Intercriteria**:
  - importance coefficients (weights) of criteria w_i (can be inferred, e.g., with SRF)
  - veto thresholds $v_i(a)$ (constant or defined with affine functions)
  - $0 \leq q_i(a) \leq p_i(a) < v_i(a)$

Preference model:

- valued outranking relation $\boldsymbol{\sigma}$ is constructed for each pair (a,b) for $a,b \in A$ via concordance and discordance tests of ELECTRE III
- Can be generalized to crisp (binary) relation; requires the cutting level (credibility threshold)

Recommendation:

- incomplete, ordinal ranking (final preorder) based on the preorders derived with the descending and ascending distillations
- complete, ordinal rankings (descending and ascending preorders, ranks, median preorder)



ELECTRE IV

ELECTRE IV is a variant of ELECTRE III, where the use of weights w_i is replaced by the definition of **5 embedded outranking relations**

- It obtains outranking credibility matrix $\sigma(\cdot,\cdot)$ for all pairs of alternatives by associating arbitrary credibility level with each embedded outranking relation
- Matrix $\sigma(\cdot,\cdot)$ is exploited by (descending and ascending) distillations
- **Assumptions**:
  - No criterion is more important than all remaining criteria
  - No criterion is negligible compared to any other criteria
- **Notation**:
  - $n_p(a,b)$ – the number of criteria on which a is strictly preferred to b
  - $n_q(a,b)$ – the number of criteria on which a is weakly preferred to b
  - $n_i(a,b)$ – the number of criteria on which a is indifferent to b, while having a better performance than b
  - $n_o(a,b)=n_o(b,a)$ – the number of criteria on which a is indifferent to b, with the same performance of a and b
- $\forall a,b \in A$: $n=n_p(a,b)+n_q(a,b)+n_i(a,b)+n_o(b,a)+n_i(b,a)+n_q(b,a)+n_p(b,a)$

ELECTRE IV – Embedded Outranking Relations

Verify the truth of relations starting from the most demanding (from S_q to S_v)

- Quasi-dominance S_q (if $a\ S_q\ b$, then $\sigma(a,b)=1$):

  $a\ S_q\ b \Leftrightarrow [n_p(b,a)+n_q(b,a)=0]\ \&\ [n_i(b,a)<n_p(a,b)+n_q(a,b)+n_i(a,b)]$
- Canonical dominance S_c (if $a\ S_c\ b$, then $\sigma(a,b)=0.8$):

  $a\ S_c\ b \Leftrightarrow [n_p(b,a)=0]\ \&\ [n_q(b,a) \le n_p(a,b)]\ \&\ [n_q(b,a)+n_i(b,a)<n_p(a,b)+n_q(a,b)+n_i(a,b)]$
- Pseudo-dominance S_p (if $a\ S_p\ b$, then $\sigma(a,b)=0.6$):

  $a\ S_p\ b \Leftrightarrow [n_p(b,a)=0]\ \&\ [n_q(b,a) \le n_p(a,b)+n_q(a,b)]$
- Sub-dominance S_s (if $a\ S_s\ b$, then $\sigma(a,b)=0.4$): $a\ S_s\ b \Leftrightarrow [n_p(b,a)=0]$
- Veto-dominance S_v (if $a\ S_v\ b$, then $\sigma(a,b)=0.2$):

  $a\ S_v\ b \Leftrightarrow [n_p(b,a) \le 1]\ \&\ [n_p(a,b) \ge n/2]\ \&\ [g_i(b)-g_i(a) \le v_i(a),\ i=1,\dots,n]$

Embedded relations: $S_q \subseteq S_c \subseteq S_p \subseteq S_s \subseteq S_v$; if no relation is true, then $\sigma(a,b)=0$

	$g_1\uparrow$	$g_2\uparrow$	$g_3\downarrow$
I	90	4	600
B	58	0	200
G	66	7	400
A	74	8	800
F	98	6	800
q_i	4	1	100
p_i	12	2	200
v_i	28	8	600

Pair	$n_p(I,B)$	$n_q(I,B)$	$n_i(I,B)$	$n_o(I,B)$	$n_i(B,I)$	$n_q(B,I)$	$n_p(B,I)$	(I,B)	$\sigma(I,B)$	(B,I)	$\sigma(B,I)$
(I,B)	2						1	S_v	0.2	-	0.0
Data	g_1, g_2						g_3				

Pair	$n_p(G,A)$	$n_q(G,A)$	$n_i(G,A)$	$n_o(G,A)$	$n_i(A,G)$	$n_q(A,G)$	$n_p(A,G)$	(G,A)	$\sigma(G,A)$	(A,G)	$\sigma(A,G)$
(G,A)	1				1	1		S_p	0.6	-	0.0
Data	g_3				g_2	g_1					

ELECTRE – Summary

- **Research** on ELECTRE methods is still **evolving rapidly**, even though the family of approaches dates back to the 60s of the 20^{th} century
- ELECTRE methods have a long history of successful **real-world applications** with an impact on the life of populations

K. Govindan, M. Jepsen, ELECTRE: A comprehensive literature review on methodologies and applications, *European Journal of Operational Research*, 250(1):1-29, 2016

- **Major application fields**: agriculture and forest management, energy, environment and water management, finance, medicine, military, and transportation
- When applying ELECTRE methods, analysts should pay attention to the characteristics of the context and also to the (theoretical) weaknesses of these methods
  - *All MCDA methods have theoretical limitations.*
- *Limitations of ELECTRE*: no scoring and intransitivities of preferences (the latter only if we impose a priori that preferences should be transitive)

Example Applications of ELECTRE III / IV

- **Ranking of suburban line extension projects** on the Paris metro system (Roy and Hugonnard,1982)
- **Ranking Moroccan villages** as part of the rural electrification program (El Mazouri et al., 2018)
- **Choosing a solid waste management system** in Finland (Hokkanen and Salminen, 1998)
- **Location of a depolluting** (municipal waste incineration) **plant** in Switzerland (Bollinger et al. 1997)
- **Rank websites of tourist destinations** in Catalonia (Del-Vasto Terrientes, 2015)
- **Fan zone localization** during UEFA EURO 2012 in Poznan, Poland (Zmuda-Trzebiatowski et al., 2012)