

INTELLIGENT DECISION SUPPORT SYSTEMS – EXERCISES X (PART I) – RANKING METHODS IN DEA

I. Indicate the truth (T) or falsity (F) for the below statements.

- a) For an efficient unit, super-efficiency is always greater or equal to its efficiency
- b) For a given unit, cross-efficiency can be greater than its efficiency
- c) When including additional weight constraints, efficiencies for all units always become lesser
- d) To estimate the stochastic acceptability indices with 0.01 accuracy and 95% confidence, one needs to consider 1,000 samples

II. Write the linear programming model for computing the super-efficiency of A, B, C, or D, while assuming:

- a) input- or output-oriented improvements
- b) CCR (CRS) or BCC (VRS) (i.e., constant or variable returns to scale)

DMU	A	B	C	D
<i>input</i> ₁	5	8	7	6
<i>input</i> ₂	14	15	10	12
<i>output</i> ₁	9	5	3	6
<i>output</i> ₂	4	8	7	9

III. Given the matrix of efficiencies attained by different DMUs for the weights vectors being most favorable to other units:

- a) What is the cross-efficiency of unit A?
- b) What is the super-efficiency of unit B?
- c) What can we say about the super-efficiencies of units A and C?

DMU	E_{kA}	E_{kB}	E_{kC}
E_{Ak}	1.0	0.2	0.9
E_{Bk}	0.6	0.7	0.8
E_{Ck}	0.8	0.6	1.0

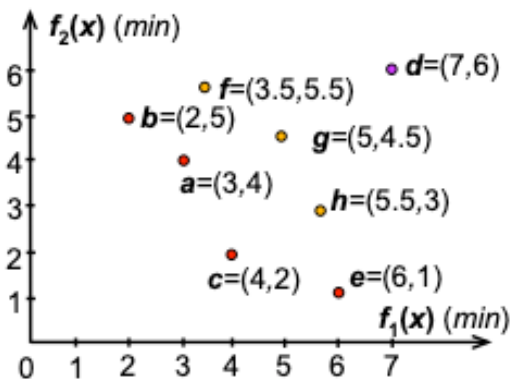
IV. Using the Monte Carlo simulation, three weight vectors ($w_{1,2,3}$) have been sampled. They implied the efficiency scores given in the below table. Show the respective matrices of efficiency rank acceptability indices (ERAI) and pairwise efficiency outranking indices (PEOIs) (for w_1 : A is the best, C – 2nd, B is the worst)

	A	B	C
w_1	1.0	0.2	0.9
w_2	0.6	0.7	0.8
w_3	0.8	0.6	1.0

I. Indicate the truth (T) or falsity (F) for the below statements.

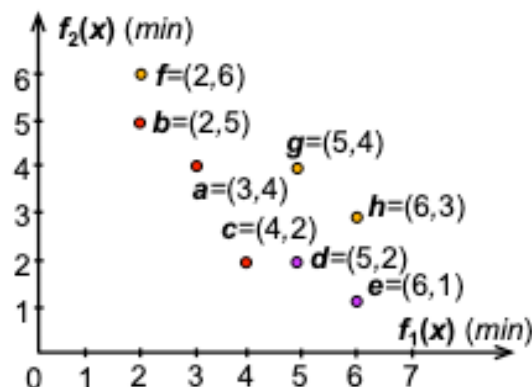
- Various solutions in the decision space are always translated to different points in the objective space
- Weakly Pareto optimal solutions are always Pareto optimal
- The number of solutions contained in the Pareto frontier may be finite
- The max point attains not better values than the nadir point on all objectives
- Classical optimization methods require multiple runs with different parameter values to approximate the Pareto frontier
- The weighted sum method (WSM) parameterized with positive weights for all objectives identifies the Pareto optimal solution
- The epsilon constrain method (ECM) can find non-supported efficient solutions

II. Consider a set of solutions **a-h** in the objective space with two minimized objectives (see figure below).



- Compute the ideal point $\mathbf{z}^{\text{ideal}}$.
- Compute a utopian point \mathbf{z}^{utop} for $\epsilon=0.1$.
- Compute the max point \mathbf{z}^{max} .
- Compute the nadir point $\mathbf{z}^{\text{nadir}}$.

III. Consider a set of solutions **a-h** in the objective space with two minimized objectives (see figure below).



- Identify Pareto optimal and weakly Pareto optimal solutions.
 - What would be the solution returned by **WSM** with the following objective function: **Minimize** $0.5 \cdot f_1(x) + 0.5 \cdot f_2(x)$?
 - What about **WSM** with: **Minimize** $2/3 \cdot f_1(x) + 1/3 \cdot f_2(x)$?
 - Solution **a** is Pareto optimal. Can it be discovered by **WSM**?
 - What would be the solution returned by **ECM** with the following objective function and constraint:
Minimize $f_1(x)$, s.t. $f_2(x) \leq 4.5$?
 - What about **ECM** with: **Minimize** $f_2(x)$, s.t. $f_1(x) \leq 5.5$?
- How to reformulate the objective function using the augmentation factor to be sure that **ECM** always returns a Pareto optimal rather than a weakly Pareto optimal solution?
- What would be the solution(s) returned by the **ASF** method with the following objective function:
Minimize $\max\{0.5 \cdot f_1(x), 0.5 \cdot f_2(x)\}$?
 - What about **ASF** with: **Minimize** $\max\{2/3 \cdot f_1(x), 1/3 \cdot f_2(x)\}$?
 - Which solution would be selected for the following order of lexicographic optimization ($f_1(x)$, $f_2(x)$)?