INTELLIGENT DECISION SUPPORT SYSTEMS – EXERCISES VI – AHP AND CHOQUET INTEGRAL

I. Indicate the truth (T) or falsity (F) for the below statements.

a) AHP uses a rule-based preference model

b) The preference model of AHP is formed by priorities of all elements at all hierarchy levels

c) The minimal number of hierarchy levels in AHP is three

d) AHP enforces cardinal consistency condition

e) The typical Saaty's scale is between 1 and 5

f) AHP requires pairwise comparisons of all pairs of elements with a common predecessor

g) Pairwise comparisons in AHP are based on the nominal scale

h) To rank five criteria with the common predecessor, in AHP, it is required to make 10 pairwise comparisons

i) AHP estimates the priorities by computing the principal eigenvector of a pairwise comparison matrix

j) AHP is vulnerable to the rank reversal phenomenon

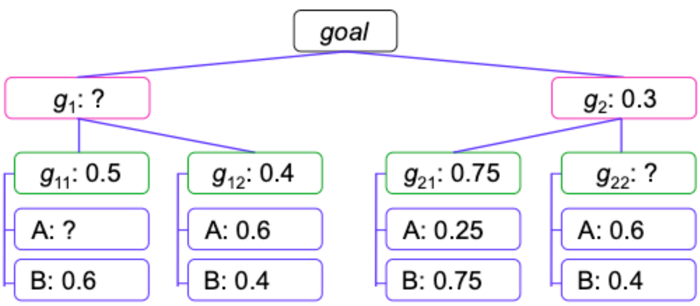
k) AHP maintains the condition of order preservation

l) When consistency ratio CR is greater than 0.1, the consistency is judged satisfactory

II. Given the below incomplete pairwise comparison matrix, make it complete to satisfy the consistency condition of pairwise comparisons (CCPC) and cardinal consistency condition (CCC).

	a_1	a_2	a_3	a_4
a_1	1	3		
a_2			2	
a_3				
a_4	3			

III. Consider the hierarchy consisting of four levels, with two major criteria, each consisting of two sub-criteria, and two alternatives, A and B. Fill in the hierarchy by replacing the question marks (?) so that the hierarchy becomes consistent with the assumptions of AHP. Then, compute the comprehensive scores of A and B. Without computing the exact values, what is the sum of *Sc*(A) and *Sc*(B)?



Sc(A) =

Sc(B) =

IV. Consider the below entirely consistent pairwise comparison matrix. Compute the priorities (w_1 – w_4) corresponding to the compared alternatives.

	a_1	a_2	a_3	a_4
a_1	1	1/2	1	3
a_2	2	1	2	6
a_3	1	1/2	1	3
a_4	1/3	1/6	1/3	1

w_1 =

w_2 =

w_3 =

w_4 =

V. Consider the below inconsistent pairwise comparison matrix. Compute the priorities (w_1 – w_4) corresponding to the compared alternatives by approximating the principal eigenvector using the methods based on the arithmetic mean of the normalized matrix.

	a_1	a_2	a_3	a_4	
a_1	1	1/3	1	3	w_1 =
a_2	3	1	2	5	w_2 =
a_3	1	1/2	1	3	w_3 =
a_4	1/3	1/5	1/3	1	w_4 =

VI. The maximal eigenvalue of the above matrix is 4.034. Compute the consistency index CI and consistency ratio CR. Is the inconsistency level of this matrix acceptable according to a default rule of AHP?

CI = CR =

VII. Consider four alternatives X, Y, W, and Z evaluated in terms of three criteria g_1, g_2, g_3 of gain type. For each statement, indicate its truth (T) or falsity (F) (> denotes a preference relation).

Alternative	g_1	g_2	g_3			
X	8	4	7		Relations X > W and Y > Z can be represented using a weighted sum model	
Y	8	6	5		Relations W > X and Z > Y can be represented using a weighted sum model	
W	3	4	7		Relations X > Y and W > Z can be represented using an additive value function	
Z	3	6	5		Relations X > Y and Z > W can be represented using an additive value function	
					Relations X > Y and Z > W can be represented using the Choquet integral	

VIII. Consider the capacities for all subsets of criteria: u(ø) = 0, u({g_1}) = 0.3, u({g_2}) = 0.4, u({g_3}) = 0.5, u({g_1, g_2}) = 0.8, u({g_1, g_3}) = 0.6, u({g_2, g_3}) = 0.7, u({g_1, g_2, g_3}) = 1. Compute the Choquet integral for alternative A = [3, 6, 5].

Ch(A) =

IX. Consider the capacities for various subsets of criteria: u(ø) = 0, u({g_1}) = 0.3, u({g_2}) = 0.4, u({g_3}) = 0.5, u({g_1, g_2, g_3}) = 1. Provide the example capacities for pairs of criteria so that g_1 and g_2 interact positively, g_1 and g_3 interact negatively, and there is no interaction between g_2 and g_3.

u({g_1, g_2}) = u({g_1, g_3}) = u({g_2, g_3}) =

X. Consider the weights for individual criteria and criteria pairs: m(ø) = 0.3, m({g_1}) = 0.3, m({g_2}) = 0.4, m({g_3}) = 0.5, u({g_1, g_2}) = 0.1, u({g_1, g_3}) = -0.1, u({g_2, g_3}) = -0.2. Compute the Choquet integral for alternative A = [3, 6, 5] using the Möbius representation. Verify if these weights satisfy the normalization and monotonicity constraints.

Ch(A) =