# INTELLIGENT DECISION SUPPORT SYSTEMS – EXERCISES II – ROBUST ORDINAL REGRESSION

I. Indicate the truth (T) or falsity (F) for the below statements.

a) $UTA^{GMS}$ uses an outranking-based preference model

b) $UTA^{GMS}$ accepts indirect preference information in the form of pairwise comparisons of reference alternatives

c) The marginal value functions in $UTA^{GMS}$ are general

d) The marginal value functions in $UTA^{GMS}$ need to be strictly monotonic

e) If the set is of value functions compatible with the Decision Maker's comparisons is non-empty, there is always only one compatible value function

f) The truth of the necessary relation for a given pair of alternatives implies the truth of the possible relation

g) The possible preference relation is transitive

h) If alternative a dominates alternative b, then a is necessarily preferred to b

i) When preference information becomes richer, the necessary relation is impoverished

j) GRIP admits comparisons regarding the preference intensity

II. Consider the below-presented performance matrix involving three criteria and six alternatives. Write down the ordinal regression problem considered in the $UTA^{GMS}$ method if the decision maker provided the following two pairwise comparisons: AUT > FRA (AUT is preferred to FRA), SWE > GER (SWE is preferred to GER).

| | ITA | BEL | GER | SWE | AUT | FRA |
|---|---|---|---|---|---|---|
| $g_1 \uparrow$ | 98 | 58 | 66 | 74 | 90 | 82 |
| $g_2 \uparrow$ | 8 | 0 | 5 | 3 | 7 | 10 |
| $g_3 \downarrow$ | 400 | 800 | 1000 | 600 | 200 | 600 |

max **ε** — objective function

[C1] U(AUT) ≥ U(FRA) + **ε**, so $u_1(90) + u_2(7) + u_3(200) \geq u_1(82) + u_2(10) + u_3(600)$ **+ ε** (as we want U(AUT) > U(FRA))

[C2] U(      ) ≥ U(      ) + **ε**, so $u_1(74) + u_2(3) + u_3(600) \geq u_1(66) + u_2(5) + u_3(1000)$ **+ ε** (as we want U(SWE) > U(GER))

[C3] $u_1(58) = 0$ ($g_1$ of gain type), $u_2(0) = 0$ ($g_2$ of gain type), $u_3(1000) = 0$ ($g_3$ of cost type) — normalization

[C4] $u_1(\quad) + u_2(\quad) + u_3(\quad) =$ — normalization

[C5] $u_1(98) \geq u_1(90)$, $u_1(90) \geq u_1(82)$, $u_1(82) \geq u_1(74)$, $u_1(74) \geq u_1(66)$, $u_1(66) \geq u_1(58)$ — monotonicity

[C6] $u_2(\quad) \geq u_2(\quad)$, $u_2(8) \geq u_2(7)$, $u_2(7) \geq u_2(5)$, $u_2(5) \geq u_2(3)$, $u_2(3) \geq u_2(0)$ — monotonicity

[C7] $u_3(\quad) \geq u_3(\quad)$, $u_3(400) \geq u_3(600)$, $u_3(600) \geq u_3(800)$, $u_3(800) \geq u_3(1000)$ — monotonicity

At least one compatible value function exists if the above constraint set is feasible and max **ε** > 0. Otherwise, a set of compatible value functions is empty.

III. Let us denote the constraint set from the previous exercise by $E(A^R)$. Formulate the linear programming models that allow verifying the truth of the necessary relation FRA $\succeq^N$ BEL and the truth of the possible relation ITA $\succeq^P$ AUT.

**Fill in the objective function:**

... d(FRA,BEL) =
s.t. **$E(A^R)$** (ε = arbitrarily small positive value)
*if* min d(FRA,BEL) ≥ 0, *then* FRA $\succeq^N$ BEL

... d(ITA, AUT) =
s.t. **$E(A^R)$** (ε = arbitrarily small positive value)
*if* max d(ITA,AUT) ≥ 0, *then* ITA $\succeq^P$ AUT

**Fill in the missing constraint:**

max ε
p.o. $E(A^R)$

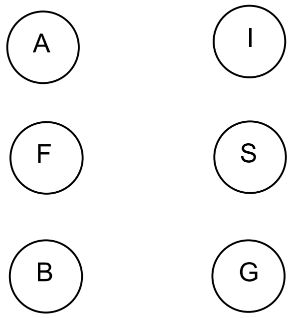
*if* the constraint set is infeasible or max ε ≤ 0,
*then* FRA $\succeq^N$ BEL
*underlying idea*: prove the falsity of the inverse relation

max ε
p.o. $E(A^R)$

*if* the constraint set is feasible and max ε ≥ 0,
*then* ITA $\succeq^P$ AUT
*underlying idea*: prove the feasibility of the preference relation

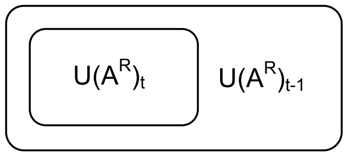
IV. Based on the solutions of a series of optimization problems that minimize the differences between comprehensive values for each pair of alternatives, min d(a,b), draw the Hasse diagram for the necessary relation.

| min d(a,b) | ITA | BEL | GER | SWE | AUT | FRA |
|---|---|---|---|---|---|---|
| ITA | = 0 | < 0 | > 0 | ≥ 0 | < 0 | < 0 |
| BEL | < 0 | = 0 | < 0 | < 0 | < 0 | ≤ 0 |
| GER | < 0 | < 0 | = 0 | < 0 | < 0 | ≤ 0 |
| SWE | ≤ 0 | < 0 | > 0 | = 0 | < 0 | < 0 |
| AUT | < 0 | > 0 | > 0 | < 0 | = 0 | > 0 |
| FRA | < 0 | ≥ 0 | ≥ 0 | < 0 | < 0 | = 0 |




V. Consider the incremental specification of pairwise comparisons. For the preference information provided in iteration t, the set of compatible value functions is denoted by $U(A^R)_t$, and the weak preference relations is denoted by $\succeq_t$ . Fill in the set inclusion relations ($\subseteq$ or $\supseteq$) for the necessary and possible preference relations obtained in the following two iterations. Which relation becomes richer, and which is impoverished?

$\succeq_t^N \quad \succeq_{t-1}^N \qquad\qquad \succeq_t^P \quad \succeq_{t-1}^P$




VI. Formulate the linear constraints that translate the below-provided preference intensity statements, where $\succ^*$ means ”intensity strict preference”, $\sim^*$ denotes ”intensity indifference”, and $\succeq_1^*$ means ”intensity weak preference on criterion $g_1$”.

(AUT,FRA) $\succ^*$ (SWE,GER) U( AUT ) – U( FRA ) …… U( SWE ) – U( GER )

(AUT,ITA) $\sim^*$ (FRA,SWE) U( AUT ) – U( ITA ) …… U( FRA ) – U( SWE )

(AUT,ITA) $\succeq_1^*$ (FRA,SWE) $u_1$ ...

VII. The Decision Maker provided the following four pairwise comparisons: $a_1 > a_4$, $a_5 > a_3$, $a_2 > a_6$, $a_3 > a_1$. Change the formulation of the below mathematical programming model to identify the minimal subset of pairwise comparisons underlying inconsistency of preference information. Select the preference direction (min or max), write down the objective function, change the below conditions by adding appropriate formulations, denote the binary variables (if you use them), do not change *CONSTRAINTS* denoting a constraint set modeling the monotonicity, normalization, and non-negativity constraints.

*min / max*

*s.t.* $U(a_1) > U(a_4)$

$U(a_5) > U(a_3)$

$U(a_2) > U(a_6)$

$U(a_3) > U(a_1)$

*CONSTRAINTS*

Assume that the optimal solution of the problem to the left indicated $a_5 > a_3$ and $a_2 > a_6$ as the minimal subset of pairwise comparisons underlying inconsistency. Which condition must one add in the next iteration to find another (different) minimal subset underlying inconsistency? Refer to the variables you have previously introduced to the left.