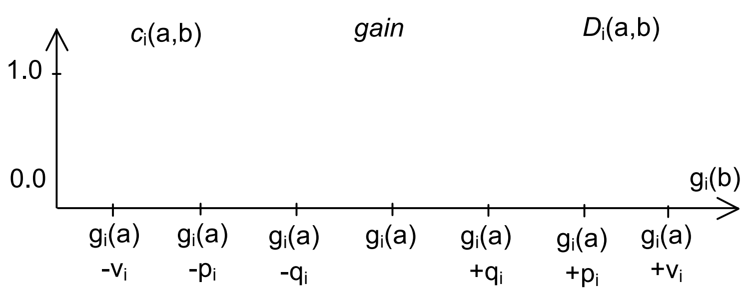
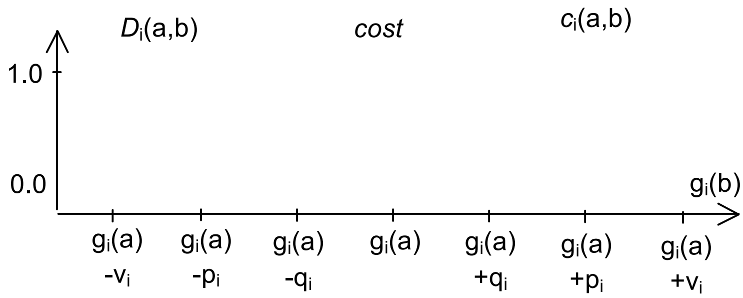
INTELLIGENT DECISION SUPPORT SYSTEMS – EXERCISES V – ELECTRE III AND IV

I. Indicate the truth (T) or falsity (F) for the below statements.

a) ELECTRE III allows dealing with multiple criteria sorting problems	
b) ELECTRE IV requires the Decision Maker to specify the cutting level (credibility threshold)	
c) ELECTRE IV employs a preference model in the form of an additive value function	
d) Weights used in ELECTRE III represent importance coefficients rather than trade-offs between criteria	
e) If alternatives *a* and *b* have the same performances on criterion g_j, then always $c_j(a,b)=1$ and $c_j(b,a)=1$	
f) When marginal concordance $c_j(a,b)$ is greater than zero, then marginal discordance $D_j(a,b)$ is always equal to 0	
g) Outranking credibility $\sigma(a,b)$ cannot be greater than comprehensive concordance $C(a,b)$	
h) Before the distillation ELECTRE III transforms the valued outranking relation into the crisp one	
i) The descending and ascending preorders can be different for the same problem	
j) The final preorder in ELECTRE III admits incomparability	
k) ELECTRE IV does not expect the Decision Maker to specify weights	

II. Draw the plots of marginal concordance $c_i(a,b)$ and discordance $D_i(a,b)$ for criterion g_i that is of either gain (to the left) or cost (to the right) type (in this exercise, the thresholds are assumed constant).





III. Consider three alternatives *a*, *b*, and *e*. They are evaluated on two criteria g_1 and g_2 (the performances are provided in the below table), with the following specification of preference orders as well as intra- and inter-criteria parameters:

- g_1 – gain, weight $w_1=2$, indifference threshold $q_1=10$, preference threshold $p_1=50$, and veto threshold $v_1=100$;
- g_2 – cost, weight $w_2=3$, indifference threshold $q_2=0$, preference threshold $p_2=10$, and veto threshold $v_2=20$.

For pairs (*a*,*e*), (*e*,*a*), (*b*,*e*), (*e*,*b*), compute the marginal concordance c_j and discordance D_j indices, comprehensive concordance indices *C*, and outranking credibilities σ.

	$g_1 \uparrow$	$g_2 \downarrow$
a	145	40
b	240	20
e	150	15

$c_1(a,e)$ =	1	$D_1(a,e)$ =	0	$c_1(e,a)$ =	1	$D_1(e,a)$ =	0
$c_2(a,e)$ =	0	$D_2(a,e)$ =	1	$c_2(e,a)$ =	1	$D_2(e,a)$ =	0
$C(a,e)$ =	$(2\cdot1+3\cdot0)/5 = 0.4$			$C(e,a)$ =	$(2\cdot1+3\cdot1)/5 = 1$		
$\sigma(a,e)$ =	$0.4\cdot(1-1)/(1-0.4) = 0$			$\sigma(e,a)$ =	1		

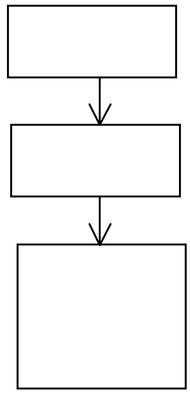
$c_1(b,e)$ =		$D_1(b,e)$ =		$c_1(e,b)$ =		$D_1(e,b)$ =	
$c_2(b,e)$ =		$D_2(b,e)$ =		$c_2(e,b)$ =		$D_2(e,b)$ =	
$C(b,e)$ =				$C(e,b)$ =			
$\sigma(b,e)$ =				$\sigma(e,b)$ =			

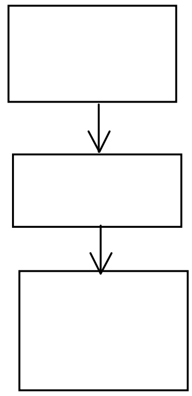
IV. Assume *a* is compared with *b*. The partial results of the concordance and discordance tests are as follows: $C(a,b) = 0.6$, $D_1(a,b) = 0.5$, $D_2(a,b) = 0.0$, $D_3(a,b) = 0.9$. Compute the outranking credibility $\sigma(a,b)$. Recall the meaning of condition $F = \{ j = 1,\ldots,n : D_j(a,b) > C(a,b)\}$ when taking into account the reasons against outranking.

$\sigma(a,b)$ =

V. Conduct the downward and upward distillations for the below credibility matrix.

	P1	P2	P3	P4	P5
P1	1	0	**1**	0.8	**1**
P2	0	1	0	0.9	0.67
P3	0.6	0	1	0.6	0.8
P4	0.25	0.8	0.67	1	0.85
P5	0.67	0	0.8	0.8	1





The first iteration is joint for the downward and upward distillations.

Find the maximal credibility (outside the main diagonal): $\lambda_0 = 1.0$

Compute the lower credibility threshold: $\lambda_1 < 1.0 - (-0.15 \cdot 1.0 + 0.3) = 0.85$, so $\lambda_1 = 0.8$

If $\lambda_0 = 0$, add all (currently considered) alternatives to the ranking. No, so preserve only credibilities greater than λ_1.

	P1	P2	P3	P4	P5	Quality
P1			1		1	**2-0=2**
P2				~~0.9~~		0-0=0
P3						**0-1=-1**
P4					~~0.85~~	0-0= 0
P5						**0-1=-1**

Maintain only these $\sigma(a,b)$, which are significantly greater than $\sigma(b,a)$, i.e., $\sigma(a,b) > \sigma(b,a) + (-0.15 \cdot \sigma(a,b) + 0.3)$.

Cross out the remaining ones

(e.g., $\sigma(P1,P3)$ is OK, but $\sigma(P4,P5)$ needs to be crossed out).

Compute strength: the number of outranked alternatives.

Compute weakness: the number of outranking alternatives.

Compute quality: strength – weakness.

Downward distilllation

Find alternatives with maximal quality: **P1**

It is unique, so **add P1 to the downward preorder (the best position).** Continue without P1.

	P2	P3	P4	P5
P2	1	0	**0.9**	0.67
P3	0	1	0.6	0.8
P4	0.8	0.67	1	0.85
P5	0	0.8	0.8	1

$\lambda_1 = 0.9$ (maximal credibility)

$\mathbf{\lambda_2 = 0.67}$ (max < $0.9 - (-0.15 \cdot 0.9 + 0.3) = 0.735$)

	P2	P3	P4	P5	Qual.
P2			~~0.9~~		0
P3				~~0.8~~	0
P4	~~0.8~~			~~0.85~~	0
P5		~~0.8~~	~~0.8~~		0

The best alternative is not unique, so run **internal distillation**.

$\mathbf{\lambda_1^2 = 0.67}$, $\lambda_2^2 = 0.0$ (max < $0.67 - (-0.15 \cdot 0.67 + 0.3) = 0.47$

	P2	P3	P4	P5	Qual.
P2			~~0.9~~	0.67	1
P3			~~0.6~~	~~0.8~~	0
P4	~~0.8~~	~~0.67~~		~~0.85~~	0
P5		~~0.8~~	~~0.8~~		-1

The best alternative is unique. **Add P2 to the order.**

Continue without P2. See next page.

Upward distilllation

Find alternatives with minimal quality: **P3** and **P5**.

The worst altern. is not unique, so run **internal distillation.**

	P3	P5
P3	1	**0.8**
P5	**0.8**	1

$\lambda_1^1 = 0.8$, $\lambda_2^1 = 0$ (there is nothing less than $0.8 - (-0.15 \cdot 0.8 + 0.3)$)

Maintain only these $\sigma(a,b)$, which are greater than $\lambda_2^1 = 0$

and significantly greater than $\sigma(b,a)$; nothing is left.

	P3	P5	Qual .
P3			0-0=0
P5			0-0=0

$\lambda_2^1 = 0$, so **it is impossible to discriminate.** STOP.

Add P3 and **P5 to the upward preorder (the worst position).** Continue without them.

	P1	P2	P4
P1	1	0	0.8
P2	0	1	**0.9**
P4	0.25	0.8	1

$\lambda_1 = 0.9$ (maximal credibility)

$\lambda_2 = 0.25$ (max < $0.9 - (-0.15 \cdot 0.9 + 0.3)$)

	P1	P2	P4	Qual.
P1				
P2				
P4				

The best alternative is unique. **Add … to the order.**

Continue without P4.

	P3	P4	P5
P3	1	0.6	0.8
P4	0.67	1	**0.85**
P5	0.8	0.8	1

λ_2 = 0.85 (maximal credibility)

λ_3 = 0.67 (max < 0.85 - (-0.15·0.85 + 0.3) = 0.6775

	P3	P4	P5
P3			~~0.8~~
P4			~~0.85~~
P5	~~0.8~~	~~0.8~~	

Qual.
0
0
0

The best altarenative is not unique, so run **internal distillation**.

λ_1^3 = 0.67, λ_2^3 = 0.0 (max < 0.67 - (-0.15·0.67 + 0.3) = 0.47

	P3	P4	P5
P3		~~0.6~~	~~0.8~~
P4	~~0.67~~		~~0.85~~
P5	~~0.8~~	~~0.8~~	

Qual.
0
0
0

The best alternative is not unique, so run **internal distillation**

(second iteration): λ_2^3 = 0.0

It is impossible to discriminate between alternatives.

STOP. Add P3, P4 and **P5 to the order (shared position).**

THE END.

	P1	P2
P1		
P2		

λ_2 = … (maximal credibility)

It is impossible to discriminate between alternatives.

STOP. Add … and **… to the order (shared position).**

THE END.

VI. Using ELECTRE III, we obtained the following downward c > a > b ~ d ~ e and upward a > d > c > b ~ e preorders, where > is the strict preference and ~ is the indifference. Determine the final preorder admitting incomparability, ranks, and complete median preorder.

Final preorder Ranks Median preorder

VII. Identify which outranking relation, if any, holds for pairs of alternatives (a,b) and (b,a) evaluated in terms of four criteria of gain type, according to ELECTRE IV.

	a	b	Parameters:	
				quasi: $a\ S_q\ b \Leftrightarrow [n_p(b,a)+n_q(b,a)=0]\ \&\ [n_i(b,a)<n_p(a,b)+n_q(a,b)+n_i(a,b)]$
$g_1\uparrow$	10	18	g_1 – q=4, p=12, v=20	*canonical:* $a\ S_c\ b \Leftrightarrow [n_p(b,a)=0]\ \&\ [n_q(b,a)\le n_p(a,b)]\ \&\ [n_q(b,a)+n_i(b,a)<n_p(a,b)+n_q(a,b)+n_i(a,b)]$
$g_2\uparrow$	20	10	g_2 – q=2, p=5, v=15	*pseudo:* $a\ S_p\ b \Leftrightarrow [n_p(b,a)=0]\ \&\ [n_q(b,a)\le n_p(a,b)+n_q(a,b)]$
$g_3\uparrow$	10	2	g_3 – q=5, p=10, v=20	*sub:* $a\ S_s\ b \Leftrightarrow [n_p(b,a)=0]$
$g_4\uparrow$	6	8	g_4 – q=3, p=20, v=100	*veto:* $a\ S_v\ b \Leftrightarrow [n_p(b,a)\le 1]\ \&\ [n_p(a,b)\ge n/2]\ \&\ [g_j(b)-g_j(a)\le vj(a),\ j=1,\dots,n]$

$n_p(a,b)$= $n_q(a,b)$= $n_i(a,b)$= $n_o(a,b)$= $n_i(b,a)$= $n_q(b,a)$= $n_p(b,a)$=

Answer: for pair (a,b) ….

for pair (b,a) ….