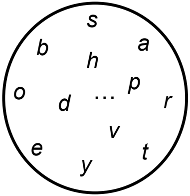

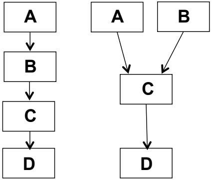

Introduction to Multiple Criteria Decision Analysis and the PROMETHEE methods

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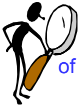
What is Decision Analysis?

- **Operational Research** (OR): “the science of better”; applying advanced analytical methods to help make better decisions
- **Decision Analysis** (DA) includes tools for identifying, representing, and formally assessing important aspects of a decision
  - DA has been historically considered as a *branch of OR*, but its links to Artificial Intelligence become stronger each year
  - DA is inherently *interdisciplinary* (computer science, mathematics, economics, philosophy, cognitive and political sciences)
- Traditionally, DA aims at giving an “objectively” best recommendation (prescribing a course of action maximizing the expected utility)
- More general definition: philosophy, methodology, and professional practice necessary to address important decisions in a formal manner
- There is no agreement for a unique meaning of “better” decision
  - Influenced by the operational approaches and the way of reaching the recommendation

Our course on Decision Analysis - Agenda

- In a decision-making context, we **cannot** scientifically prove that the recommended decision is "the best one"
- The concepts, models, and methods **must not** be considered as a means of discovering pre-existing, universally accepted truth
- **Keys to doors** giving access to elements of **knowledge contributing to the acceptance of a final recommendation**



1st half of semester

I. Multiple Criteria Decision Aiding: approaches for explicitly evaluating decision alternatives in the presence of multiple criteria

II. Game theory: study of strategic interactions between rational agents; the science of logical decision making

DECISION ANALYSIS *our course*

III. Data envelopment analysis: a framework for evaluating efficiency of decision making units

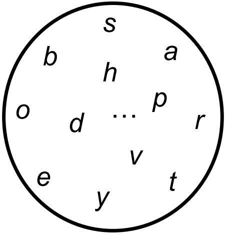
IV. Multiple objective optimization: methods for solving optimization problems involving multiple objectives

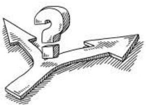
Multiple Criteria Decision Analysis

- Tools that support structuring, analysis, and solving **complex decision problems**, involving **multiple decision alternatives** and **multiple evaluation criteria**
- Models appropriate for **answering questions** asked by stakeholders in a decision making process
- **Decision Maker (DM)** – a stakeholder in whose name or for whom decision analysis is performed
- MCDA involves a **rich interplay** between human judgment, data analysis, and computational processes
- Processing **data and preferences to conclusions** useful in decision making
- It involves the DM in exploring, interpreting, debating, and arguing, to recommend a course of action that is consistent with the DM's objectives and values system

Multiple Criteria Decision Problem

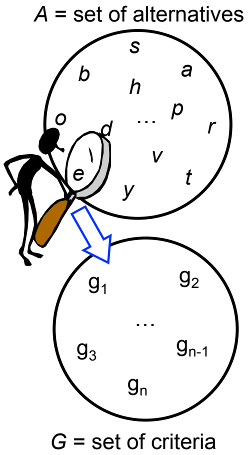
A = set of alternatives





- There is an **objective** or objectives **to be attained**
- The **current state** (of the world or system) is **undesired**, and we are looking for ways to change this situation
- *Intuitive examples:* think of your (future) apartment, or financial portfolio management, or public transportation means, or service industry, or environmental pollution
- There are usually many alternative ways for attaining the goal(s) – they **constitute a set of alternatives *A*** (actions, solutions, variants, acts, …)
- There exist **multiple pertinent factors** that affect a final decision (which can be formalized as evaluation criteria)

Criteria



Criterion is a function g_i defined on A, reflecting a worth/quality of alternatives from a particular point of view, such that in order to compare any two alternatives $a,b \in A$ from this point of view, it is sufficient to compare two values: $g_i(a)$ and $g_i(b)$, called evaluations or performances

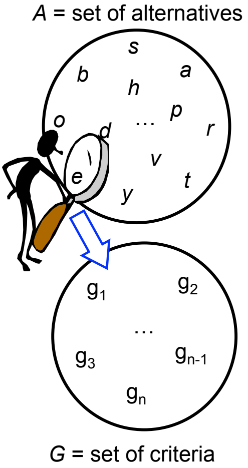
Scales of criteria:

- *Ordinal* – only the order of values matters; a distance in ordinal scale has no meaning of intensity (e.g., school marks)
- *Cardinal* – a distance / difference has a meaning of intensity
  - *Interval scale* – “zero” has no absolute meaning, but one can compare differences (e.g., Celsius scale)
  - *Ratio scale* – “zero” has an absolute meaning, so a ratio of evaluations has a meaning too (e.g., weight)

Criteria need to be monotonic with respect to preference:

- *Gain-type* – greater evaluations / performances are preferred (non-decreasing direction of preference)
- *Cost-type* – lesser evaluations / performances are preferred (non-increasing direction of preference)

Family of Criteria



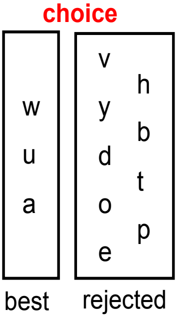
- In many problems, one needs to consider economic, technological, environmental, social, education, … criteria
- It makes no sense, and it is not fair to decide based only on a single evaluation criterion; diversity of relevant viewpoints

Consistent family of criteria $G = \{g_1, g_2, \ldots, g_n\}$:

- *Complete* (*exhaustive*): if two alternatives have the same performances on all criteria, they have to be indifferent
- *Monotonic:* if alternative *a* is preferred to alternative *b*, and there is alternative *c* with at least as good performances as *a* on all criteria, then *c* needs to be preferred to *b*
- *Non-redundant: elimination of any criterion from the family G should violate at least one of the above properties*
- *Conflicting criteria are typical*: cost or price and some measure of quality are easily in conflict
- The diversity and conflicting character of criteria imply that there is usually no single objectively best solution or decision

Types of Decision Problems (1) - Choice

How to choose the best, most preferred (subset of) alternatives?



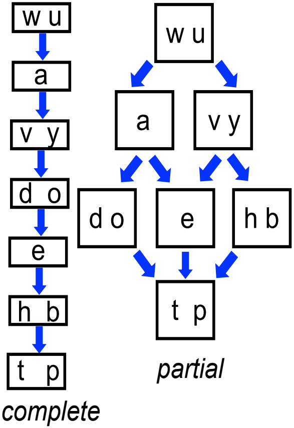
sometimes a single alternative needs to be selected

- Whenever you buy some product, you solve a multiple criteria choice problem (without even realizing it)
- Choice of a **cloud service provider**: reliability, throughput, response time, reputation, price, security
- **Project selection** based on budget, principal investigator evaluation, scientific excellence, justification for forming a new team, and the possibility of realizing the project
- **Supplier selection** based on *economic* (e.g., cost, quality performance, relationship, and communication), *environmental* (e.g., environmental management, pollution control, green product), and *social* (e.g., employment practices, human rights, ethics) *criteria*
- Annual maintenance of **infrastructure assets** or funding new infrastructure investments at the state level

Types of Decision Problems (2) - Ranking

How to order alternatives from the best to the worst?

ranking



- Rankings of **universities** (scientific efficiency, graduates in the job market, scientific potential, internationalization, prestige, learning conditions, and innovation)
- "**Learning to rank**" for information retrieval based on query dependent (e.g., TF-IDF) and independent (e.g., PageRank) features
- Rankings of programming languages, scholars, airports, saving accounts, touristic destinations, credit offers, car insurance quotes, pension funds
- **International rankings** on world competitiveness, ease of doing business (*economy*), climate change performance, sustainability, happy planet index (*environment*), press freedom, corruption perception, level of democracy (*politics*), human development, quality of life (*society*), Webometrics, educational achievements (*education*)

Types of Decision Problems (3) - Sorting

How to assign alternatives to pre-defined and ordered decision classes?

sorting

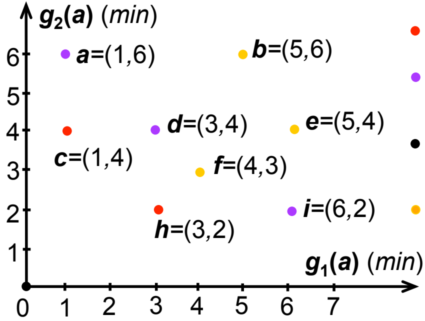
C_3	w	u	a		
C_2	v	y	d	o	e
C_1	h	b	t	p	

C_3=good	C_3=leading
C_2=med.	C_2=average
C_1=bad	C_1=poor

- **Financial management:** credit risk assessment for countries, firms, and consumers; business failure prediction; stock evaluation
- **Marketing:** customer satisfaction measurement
- **Medicine:** diagnosis (e.g., the severity of illness), **Environmental and energy management, ecology**: assessment of the environmental impact of human activity on the ecosystem, efficiency of energy policies, level of pollution
- **Education:** Parametric evaluation of research units: A^+, A, B and C
- Safety classes, technical diagnosis, attractiveness, implementation priorities, exposure level
- *In standard classification, the classes are pre-defined though non-ordered in terms of preference*

Vilfredo Pareto

- Alternative $a \in A$ is **non-dominated** (**Pareto optimal**) iff there is no other alternative $b \in A$ such that:
  b is at least as good as a on all criteria g_i, $i=1,\ldots,n$, *and*
  b is strictly better than a for some criterion, g_i, $i \in \{1,\ldots,n\}$
- Alternative $a \in A$ is **weakly non-dominated** (**weakly Pareto optimal**) iff there is no other alternative $b \in A$ such that
  b is strictly better than a on all criteria g_i, $i=1,\ldots,n$

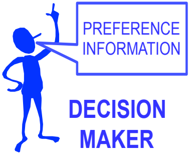


- ***c*** and ***h*** are non-dominated (Pareto optimal)
- ***a, d*** and ***i*** are weakly non-dominated, but they are not Pareto optimal
- non-dominated alternatives are also weakly non-dominated
- ***b, f*** and ***e*** are dominated

Preference Information

Alt.	g_1	.	g_m
a	$g_1(a)$		$g_m(a)$
b	$g_1(b)$		$g_m(b)$
...	...		...
n	g_1(n)		$g_m(n)$

PERFORMANCE TABLE

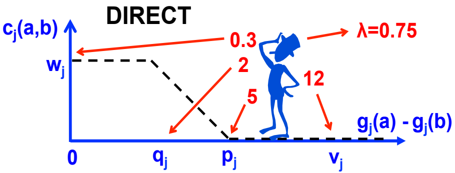


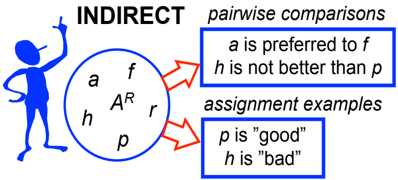
DECISION MAKER

- Decision problems involving multiple criteria (e.g., identifying an alternative optimizing all criteria at the same time) are **ill-posed**
- **Dominance relation is too poor** – it leaves many alternatives incomparable
- One can ”enrich” the dominance relation, using **preference information**
  - Preferences represent the DM’s value system, needs, aspirations, or requirements are needed to provide useful decision aid
- Preference information permits to build a **preference model** that aggregates the evaluations of each alternative
  - Due to the aggregation, the alternatives in A become more comparable
  - A proper exploitation of the preference model leads to a **final recommendation**
- The solution of multiple criteria problems depends on both the basic data included in the performance (evaluation) table and the DM’s preferences

Types of Preference Information

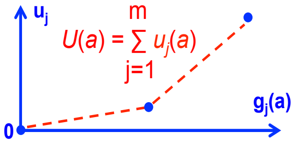
- Elements of the DM's value system or decision policy used to compare alternatives
- **Direct elicitation** of numerical values of model parameters by DMs:
  - Weights giving the relative importance of criteria, substitution rates (trade-offs between criteria), thresholds fixing preference limits or critical differences, …
  - Direct elicitation demands rather high cognitive effort
- **Indirect elicitation** through holistic judgments, i.e., decision examples
  - Pairwise preferences between alternatives, classification examples, …
  - Decision aiding based on decision examples is gaining popularity because …
    - Decision example is a relatively "easy" preference information
    - Decisions can be observed without the active participation of DMs

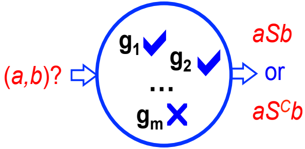




Preference Models

- A mathematical tool for representing the DM's preferences within the method
- Its application on the set of alternatives leads to a recommendation consistent with the DM's viewpoint through aggregating the performances of alternatives
- **Three families of preference modeling (aggregation) methods**:
- **Multiple Attribute Value (Utility) Theory** (MAVT, MAUT) using a value function, assigning a score / utility to each alternative
  - e.g., $U(a) = \Sigma_{j=1,\ldots n}\, w_j \cdot g_j(a)$; $U(a) = \Sigma_{j=1,\ldots n}\, u_j(a)$; Choquet integral
- **Outranking methods** using an outranking relation *S* based on pairwise comparisons
  - *aSb* = "*a* is at least as good as *b*"
- **Decision rule approach** using a set of "*if* … then …" decision rules
  - **Decision rule model is the most general of all three**





$r1 \equiv$ if $g_1(a) \geq 5$ & $g_3(a) \geq 10$, then *a* is at least good

$r2 \equiv$ if $g_1(a) \leq 2$ & $g_2(a) \leq 2$, then *a* is at most bad

Preference Structure

To establish a realistic model of DM's preference, **four basic relations** are sufficient:

- **Indifference** ($\sim$, I) – being "the same" with respect to a decision problem
- **Strict preference** ($>$, P) – being "better" with respect to a decision problem
- **Weak preference** ($\triangleright$, Q) – an intermediate level between indifference and strict preference or being at least as good
- **Incomparability** ($?$, R) – lack of sufficiently strong arguments supporting either side

Axiom of limited comparability

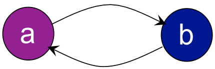
> "Whatever the alternatives considered, the criteria used to compare them, and the information available, one can develop a satisfactory model of DM's preferences by assigning one or a group of two or three of the four basic situations, to any pair of alternatives"

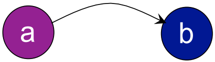
Value function U distinguishes only two possible relations between alternatives

- preference: $a > b$ ($a\ P\ b$) iff $U(a) > U(b)$ and indifference: ($a \sim b$) ($a\ I\ b$) iff $U(a) > U(b)$

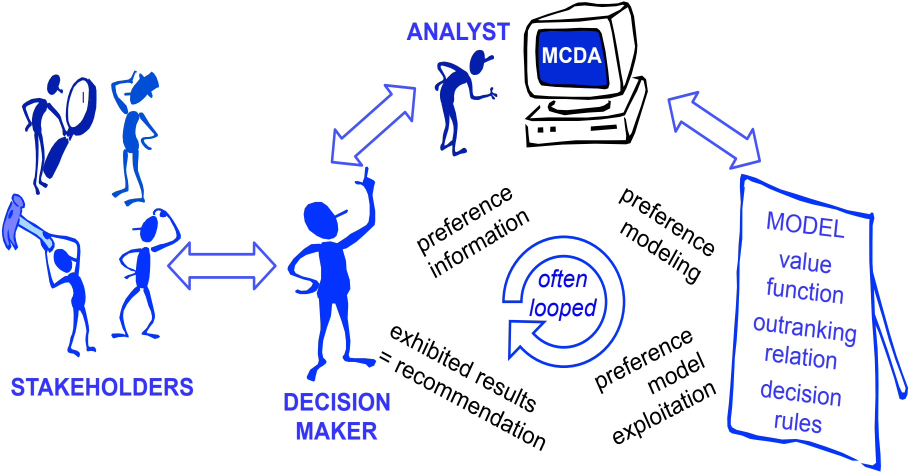
Outranking relation S groups three basic preference relation $S = \{I, P, Q\}$

- It allows to model incomparability in case not(a S b) and not(b S a)





General Scheme of MCDA Methods



The Old Challenge and New Aspirations of MCDA

Aggregation of vector evaluation in line with the DM's preferences, i.e., preference modeling:

- Till early 80's: "**model-centric**": model first, then preference information in terms of model parameters
- Since 80's: more and more "**human-centric**": PC allowed human-computer interaction – "trial-and-error"
- In the 21st century: "**knowledge-driven**": more data about human choice; holistic preference information first, then model building; explanation of past decision, and prediction of future decision

Focus on "fair" aggregation:

- Ensure faithful representation of a value system of the DM
- Act in a constructive and transparent way
- Handle "imperfect" information

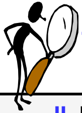
What Can One Reasonably Expect from MCDA?

- **Analyzing the decision making** context by identifying the actors, the various alternatives (possibilities of actions), their consequences, and the stakes
- Taking into account a **broad spectrum of points of view** liable to structure a decision making process for all the relevant actors, while preserving the original concrete meaning of the corresponding evaluation
- Clearing the way for a discussion on the respective **roles that each criterion may play** during the decision aiding process
  - e.g., weight, veto, aspiration level, rejection level
- **Organizing** how the **decision making process** will unfold to increase consistency between the values underlying the objectives and the quality of the final decision
- **Drawing up recommendations** based on results from models and computational procedures designed with respect to some working hypotheses
- Participating in the process to legitimate the final decision

STRUCTURE ⇨ **PROGRESS** ⇨ **RECOMMENDATION**

Our Course on MCDA - Agenda

- Hundreds of MCDA methods have been proposed over the last decades
- They vary according to preference information, preference model, and type of delivered recommendation



TODAY

I. PROMETHEE I, II, V:
ranking and choice, direct preferences, outranking

II. Robust Ordinal Regression
ranking, indirect preferences, value-based

III. ELECTRE III, IV
ranking, direct preferences, outranking

MCDA
our plan

VI. Preference learning
ranking and sorting, indirect preferences, score-based

V. Dominance-based Rough Set Approach
sorting and ranking, indirect, decision rules

IV. Weighted Sum, Choquet Integral, Analytic Hierarchy Process
ranking, direct preferences, score-based

PROMETHEE Methods

- **P**reference **R**anking **O**rganization **METH**od for **E**nrichment
- **Outranking-based methods** (preference model = outranking relation); **though deriving comprehensive scores in the end**
- **Ranking**: **PROMETHEE I, II**, III, PROMETHEE^{GKS}
- **Choice**: **PROMETHEE V**
- **Sorting**: FlowSort, PromSort, SORT-PROMETHEE
  - ***Our focus today***

Jean-Pierre Brans

Bertrand Mareschal

first presentation in 1982

Basic prerequisites

- The method should be understandable for DMs to **avoid the "black box" effect**
- Technical parameters of the method should have practical (economic) significance
- The **differences between** the **evaluations** of alternatives should be taken into account
- The **scales** in which the evaluations are expressed cannot influence conclusions
- Preference, indifference, and **incomparability** are admissible (the incomparabilities may be reduced, but not when it is unrealistic)

Our Running Example

INPUT DATA

- finite set of actions $A=\{a, b, c, \ldots, h\}$
- consistent family of criteria $G=\{g_1, g_2, \ldots, g_m\}$

AIM

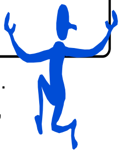
Build an additional electric plant in Europe

Alt.	g_1	…	g_m
a	$g_1(a)$		$g_m(a)$
b	$g_1(b)$		$g_m(b)$
c	$g_1(c)$		$g_m(c)$
d	$g_1(d)$		$g_m(d)$
…	…		…
n	$g_1(n)$		$g_m(n)$

Alt.	$g_1 \uparrow$	$g_2 \uparrow$	$g_3 \downarrow$
ITA	98	8	400
BEL	58	0	800
GER	66	5	1000
SWE	74	3	600
AUT	80	7	200
FRA	82	10	600

ALTERNATIVES AND CRITERIA

- Six possible locations (countries) evaluated in terms of 3 criteria
- g_1 (gain) – Power (in Megawatt)
- g_2 (gain) – Safety level (0-10 scale)
- g_3 (cost) – Construction cost (in million USD)



For illustrative purposes, we simplify the problem by taking into account only three criteria. Other relevant criteria include: manpower for running the plant, annual maintenance cost, or ecology (number of villages to evacuate).

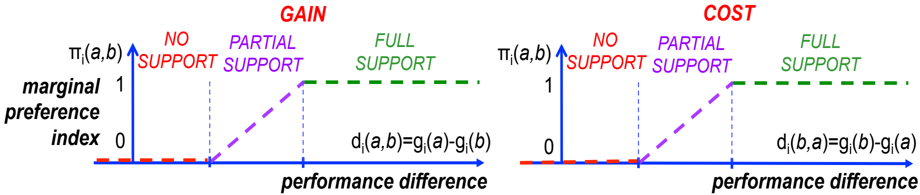
Main principle: pairwise comparisons

- PROMETHEE is not allocating a utility to each alternative, neither globally nor on each criterion
- **The preference structure of PROMETHEE is based on pairwise comparisons**
- Usually, for small differences, the DM will allocate a small preference to a better alternative (possible even no preference)
- The more significant the difference, the larger the preference

Alt.	$g_1 \uparrow$	$g_2 \uparrow$	$g_3 \downarrow$
BEL	58	0	800
GER	66	5	1000

Concerning g_1 – Power, **GER** is better than **BEL**
How can we quantify this advantage?
What does it mean?

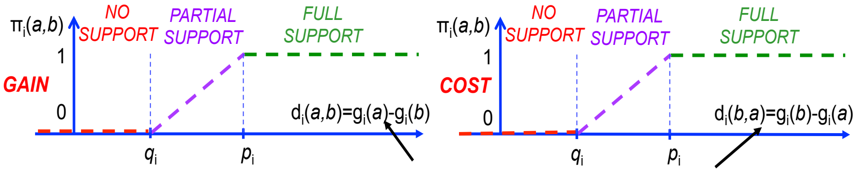
Preference Function



- The preferences are considered to be real numbers varying between 0 and 1
- This means that for each criterion of gain type the DM has in mind a **preference function**:

  $\boldsymbol{\pi_i(a,b) = F_i[d_i(a,b)]}$ in the range [0,1], where $d_i(a,b) = g_i(a) - g_i(b)$
- $\pi_i(a,b)$ gives the **preference of *a* over *b*** for observed performance difference on criterion g_i
- $\pi_i(a,b)$ is called a **marginal preference index** (marginal preference degree)
- For criterion of cost type: $\pi_i(a,b) = F_i[d_i(b,a)]$
- The pair $\{g_i, \pi_i(a,b)\}$ is called a **generalized criterion** associated with g_i
- In order to facilitate the identification of a **preference function**, six types have been proposed

Indifference and Preference Thresholds



the difference in favor of a (first alternative in the pair (*a*,*b*)*) when compared to b* (second alternative)

- *Always*: when one alternative is not better than another, the preference degree is equal to 0
- In some cases, the difference in evaluations on g_i in favor of one alternative exists, but it is so small that in the eyes of DM, it does not justify any preference
- **Indifference threshold q_i** is the maximal difference in performances on g_i which is negligible w.r.t. preference
- An immediate transition from no preference to strict (full) preference on g_i is unrealistic
- **Preference threshold p_i** $\geq q_i$ is the minimal difference in performances on g_i, which is considered as sufficient to generate a full preference of one alternative over another on g_i

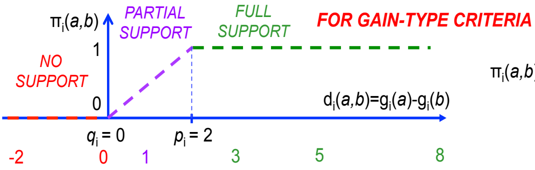
example indifference and preference thresholds

	q_i	p_i
g_2	0	2
g_3	100	300

	ITA	BEL	GER	SWE	AUT	FRA
$g_2 \uparrow$	8	0	5	3	7	10
$g_3 \downarrow$	400	800	1000	600	200	600

- a is better than b by more than preference threshold p_i, implying a strict preference: $\pi_i(a,b) = 1$
- a is not better than a or it is better by not more than indif. threshold q_i, *implying no preference:* $\pi_i(a,b) = 0$
- a is better than a by more than indifference threshold q_i but not more than preference threshold p_i, implying weak preference: $\pi_i(a,b)$ between 0 and 1

	$g_2 \uparrow$
q_2	0
p_2	2



$$\pi_i(a,b) = \begin{cases} 1, & \text{if } d_i(a,b) > p_i \\ 0, & \text{if } d_i(a,b) \le q_i \\ \dfrac{d_i(a,b) - q_i}{p_i - q_i} & \text{if } q_i < d_i(a,b) \le p_i \end{cases}$$

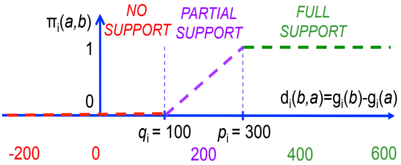
Let us compare ITA with all the remaining alternatives on g_2 given the marginal preference function

Alt. (b)	$g_2 \uparrow$	d_2(ITA,b)	π_2(ITA,b)
ITA	8	0	0
BEL	0	8	1
GER	5	3	1
SWE	3	5	1
AUT	7	1	0.5
FRA	10	-2	0

- π_2(ITA,BEL) = 1, because g_2(ITA) – g_2(BEL) = 8 – 0 = 8 > 2
  ITA is better than BEL by 8, which is greater than $p_2 = 2$
- π_2(ITA,FRA) = 0, because g_2(ITA) – g_2(AUT) = 8 – 10 = -2 ≤ 0
  ITA is worse than FRA by 2, i.e., it is not better by at least $q_2 = 0$
- π_2(ITA,AUT) = 0.5, because 0 < g_2(ITA) – g_2(AUT) = 8 – 7 = 1 ≤ 2
  ITA is better than AUT by 1, which is greater than $q_2 = 0$
  and less than $p_2 = 2$ (between the two, exactly in half)

Preference Function for Cost-type Criterion

	$g_3 \downarrow$
q_3	100
p_3	300



FOR COST-TYPE CRITERIA

$$\pi_i(a,b) = \begin{cases} 1, & \text{if } d_i(b,a) > p_i \\ 0, & \text{if } d_i(b,a) \le q_i \\ \dfrac{d_i(b,a) - q_i}{p_i - q_i} & \text{if } q_i < d_i(b,a) \le p_i \end{cases}$$

Let us compare ITA with all the remaining alternatives on g_3 given the marginal preference function

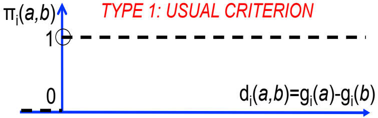
Alt. (b)	$g_3 \downarrow$	$d_3(b,\text{ITA})$	$\pi_3(\text{ITA},b)$
ITA	400	0	0
BEL	800	400	1
GER	1000	600	1
SWE	600	200	0.5
AUT	200	-200	0
FRA	600	200	0.5

- $\pi_3(\text{ITA,BEL}) = 1$, because $g_3(\text{BEL}) - g_3(\text{ITA}) = 800 - 400 = 400 > 300$
  ITA is better than BEL by 400, which is greater than $p_3 = 300$
- $\pi_3(\text{ITA,AUT}) = 0$, because $g_3(\text{AUT}) - g_3(\text{ITA}) = 200 - 400 = -200 \le 300$
  ITA is worse than AUT by 200, i.e., it is not better by at least $q_3 = 100$
- $\pi_3(\text{ITA,SWE}) = 0.5$, bec. $100 < g_3(\text{SWE}) - g_3(\text{ITA}) = 600 - 400 = 200 \le 300$
  ITA is better than SWE by 200, which is greater than $q_3 = 100$
  and less than $p_3 = 300$ (between the two, exactly in half)

So far, we have used just the most standard marginal preference function. Where are the 6 types?

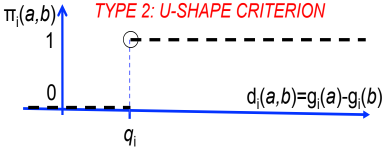
Six Types of Preference Functions (1)

The intensity of preference of *a* over *b* on criterion g_i is a function $\pi_i(a,b)$ of the difference (amplitude) $g_i(a) - g_i(b)$



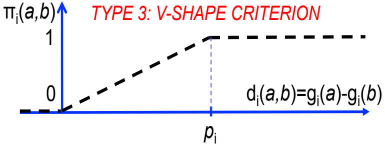
$$\pi_i(a,b) = \begin{cases} 1, \text{ if } d_i(a,b) > 0 \\ 0, \text{ if } d_i(a,b) \le 0 \end{cases}$$

No parameters to be determined
Immediate strict preference
Useful for ordinal performance scales



$$\pi_i(a,b) = \begin{cases} 1, \text{ if } d_i(a,b) > q_i \\ 0, \text{ if } d_i(a,b) \le q_i \end{cases}$$

There exists an indifference q_i threshold which must be fixed



$$\pi_i(a,b) = \begin{cases} 1, & \text{if } d_i(a,b) > p_i \\ 0, & \text{if } d_i(a,b) \le 0 \\ \dfrac{d_i(a,b)}{p_i} & \text{if } 0 \le d_i(a,b) \le p_i \end{cases}$$

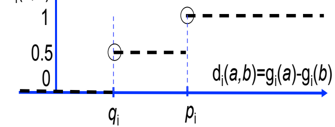
Preference increases up to a preference threshold p_i to be determined

Always, i.e., for all types of preference functions, when *a* is not better than *b*, then $\pi_i(a,b) = 0$

Six Types of Preference Functions (2)

The intensity of preference of *a* over *b* on criterion g_i is a function $\pi_i(a,b)$ of the difference (amplitude) $g_i(a)–g_i(b)$

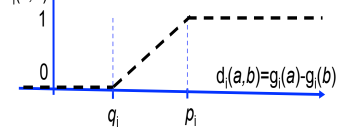
TYPE 4: LEVEL CRITERION



$$\pi_i(a,b) = \begin{cases} 1, & \text{if } d_i(a,b) > p_i \\ 0, & \text{if } d_i(a,b) \le q_i \\ 0.5, & \text{if } q_i < d_i(a,b) \le p_i \end{cases}$$

There exists an indifferent and a preference threshold which must fixed; between the two preference is "average"

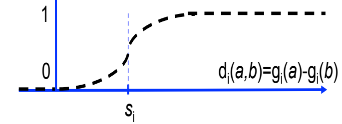
TYPE 5: V-SHAPE WITH INDIF. CRITERION



$$\pi_i(a,b) = \begin{cases} 1, & \text{if } d_i(a,b) > p_i \\ 0, & \text{if } d_i(a,b) \le q_i \\ \dfrac{d_i(a,b) - q_i}{p_i - q_i} & \text{if } q_i < d_i(a,b) \le p_i \end{cases}$$

There exists an indifference and a preference threshold which must be fixed; between the two, preference increases gradually

TYPE 6: GAUSSIAN CRITERION

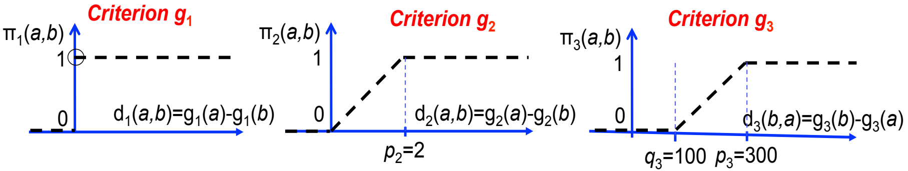


$$\pi_i(a,b) = \begin{cases} 0, & \text{if } d_i(a,b) \le 0 \\ 1 - e^{-\frac{d_i(a,b)}{2s_j^2}} & \text{if } d_i(a,b) > 0 \end{cases}$$

Preference increases following a normal distribution, the standard deviation of which must be fixed

Marginal Preference Indices - Example

Concerning g_1 – Power, **GER** is better than **BEL**. How can we quantify this advantage? Well …



Let us compare BEL and GER all criteria given the marginal preference functions

Alt.	$g_1\uparrow$	$g_2\uparrow$	$g_3\downarrow$
BEL	58	0	800
GER	66	5	1000

Prefer. index	g_1	g_2	g_3
π_i(**BEL**,**GER**)	0	0	0.5
π_i(**GER**,**BEL**)	1	1	0

- g_1: **BEL is not better than GER** → π_1(**BEL**,**GER**)=0
  GER is better than BEL by 8 → π_1(**GER**,**BEL**)=1
- g_2: **BEL is not better than GER** → π_2(**BEL**,**GER**)=0
  GER is better than BEL by 5 → π_2(**GER**,**BEL**)=1
- g_3: **BEL is better than GER by 200** → π_3(**BEL**,**GER**)=0.5
  GER is not better than BEL → π_3(**GER**,**BEL**)=0

Properties: $\pi_i(a,b)$ is expressing with which degree a is preferred to b on criterion g_i

For all (a,b) and for all i: $\pi_i(a,a)=0$; $0 \leq \pi_i(a,b)$, $\pi_i(b,a) \leq 1$ and $0 \leq \pi_i(a,b) + \pi_i(b,a) \leq 1$

Marginal Preference Indices - Need for Aggregation

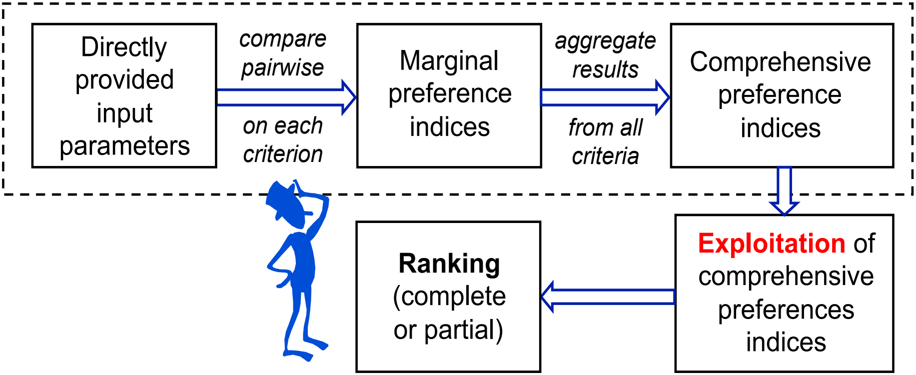
$\pi_1(.,.)$	ITA	BEL	GER	SWE	AUT	FRA
ITA	0	1	1	1	1	1
BEL	0	0	0	0	0	0
GER	0	1	0	0	0	0
SWE	0	1	1	0	0	0
AUT	0	1	1	1	0	1
FRA	0	1	1	1	0	0

$\pi_2(.,.)$	ITA	BEL	GER	SWE	AUT	FRA
ITA	0	1	1	1	0.5	0
BEL	0	0	0	0	0	0
GER	0	1	0	1	0	0
SWE	0	1	0	0	0	0
AUT	0	1	1	1	0	0
FRA	1	1	1	1	1	0

$\pi_3(.,.)$	ITA	BEL	GER	SWE	AUT	FRA
ITA	0	1	1	0.5	0	0.5
BEL	0	0	0.5	0	0	0
GER	0	0	0	0	0	0
SWE	0	0.5	1	0	0	0
AUT	0.5	1	1	1	0	1
FRA	0	0.5	1	0	0	0

- In view of the marginal preference indices, what is the **comprehensive support** given to the hypothesis that *a is preferred to b*?
- AUT seems to be very strong compared to BEL
- SWE seems to be very weak compared to FRA
- For ITA and AUT the results are not univocal
- In the real world, in most cases there are criteria on which *a is better than b*, and criteria on which *b is better than a*

Construction of an outranking relation



Weights

- In many methods, aggregation of results from various criteria requires incorporation of weights
- In PROMETHEE, weights represent the **relative importance** of different criteria independent from their measurement units
- **The greater w_i, the more important criterion g_i**
- Weights are **inter-criteria parameters** (information between criteria)
- A single weight cannot be interpreted alone; it has a meaning only in the context of all other weights being specified

Weight	g_1	g_2	g_3
w_i	3	2	5

- g_3 is the most important criterion, whereas the least important one is g_2
- g_1 is one and a half times as important as g_2
- Users are allowed to introduce arbitrary (non-normalized) numbers for the weights
- The weights do not need to sum up to 1 (they are normalized later in any case)

Comprehensive Preference Index

COMPREHENSIVE PREFERENCE INDEX

$$\pi(a,b) = \frac{\sum_{i=1,\ldots,m} w_i \cdot \pi_i(a,b)}{\sum_{i=1,\ldots,m} w_i}$$

- Criteria have different impacts on the value of $\pi(a,b)$, dependent on their weights
- **$\pi(a,b)$ is in the range [0,1]**
- 1 = all criteria strongly support that *a is preferred to b*
- 0 = none criterion supports that *a is preferred to b* weakly or strongly
- $\pi(a,b)$ quantifies the strength of the coalition of criteria supporting that ***a is preferred to b***
- For all pairs of alternatives (a,b):
  $\pi(a,a)=0$; $0 \le \pi(a,b), \pi(b,a) \le 1$
  $0 \le \pi(a,b) + \pi(b,a) \le 1$

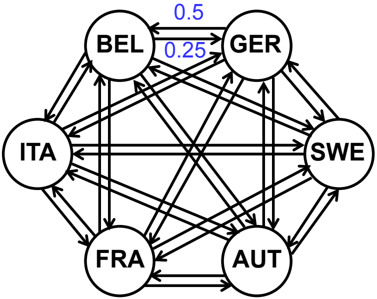
$\pi(.,.)$	ITA	BEL	GER	SWE	AUT	FRA
ITA	0	1	1	0.75	0.4	0.55
BEL	0	0	0.25	0	0	0
GER	0	0.5	0	0.2	0	0
SWE	0	0.75	0.8	0	0	0
AUT	0.25	1	1	1	0	0.8
FRA	0.2	0.75	1	0.5	0.2	0

	g_1	g_2	g_3
weight w_i	3	2	5
π_i(BEL,GER)	0	0	0.5
π_i(GER,BEL)	1	1	0

$$\pi(\text{BEL,GER}) = \frac{3\cdot 0 + 2\cdot 0 + 5\cdot 0.5}{3 + 2 + 5} = \mathbf{0.25}$$

$$\pi(\text{GER,BEL}) = \frac{3\cdot 1 + 2\cdot 1 + 5\cdot 0}{3 + 2 + 5} = \mathbf{0.50}$$

- $\pi(a,b)$, $\pi(b,a)$ are real numbers (without units) completely **independent of the scales of criteria**
- $\pi(a,b)$ can be interpreted as a **valued outranking relation**
- It can be represented as a graph with an arc flow equal to $\pi(a,b)$ from node a to node b (i.e., two arcs between each pair of nodes)



How can we exploit this graph
(matrix of preference degrees)
in order to obtain a ranking
(complete or partial)**?**

π(.,.)	ITA	BEL	GER	SWE	AUT	FRA
ITA	0	1	1	0.75	0.4	0.55
BEL	0	0	0.25	0	0	0
GER	0	0.5	0	0.2	0	0
SWE	0	0.75	0.8	0	0	0
AUT	0.25	1	1	1	0	0.8
FRA	0.2	0.75	1	0.5	0.2	0

Positive and Negative Flows

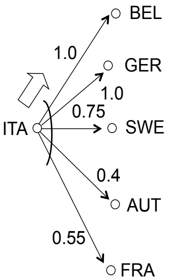
- Each alternative is facing (n-1) other alternatives (note that $\pi(a,a)$ is always 0)
- **The positive (outgoing) outranking flow** expresses how an alternative is outranking all the others:

$$\Phi^+(a) = \Sigma_{j=1,\ldots n}\ \pi(a,b)$$

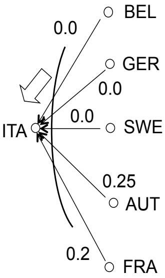
- It is its power (strength); its outranking character; the higher $\Phi^+(a)$, the better alternative a
- **The negative (ingoing) outranking flow** expresses how an alternative is outranked by all the others:

$$\Phi^-(a) = \Sigma_{j=1,\ldots n}\ \pi(b,a)$$

- It is its weakness; its outranked character; the lower $\Phi^-(a)$, the better alternative a
- Sometimes, the flows are defined as averages and not sums (then, they need to be divided by (n-1))



$\pi(.,.)$	ITA	BEL	GER	SWE	AUT	FRA	$\Phi^+(.)$
ITA	0	1	1	0.75	0.4	0.55	**3.7**
BEL	0	0	0.25	0	0	0	**0.25**
GER	0	0.5	0	0.2	0	0	**0.7**
SWE	0	0.75	0.8	0	0	0	**1.55**
AUT	0.25	1	1	1	0	0.8	**4.05**
FRA	0.2	0.75	1	0.5	0.2	0	**2.65**
$\Phi^-(.)$	**0.45**	**4.0**	**4.05**	**2.45**	**0.6**	**1.35**	



Φ^+(**ITA**) = 1 + 1 + 0.75 + 0.4 + 0.55 = 3.7

Φ^-(**ITA**) = 0 + 0 + 0 + 0.25 + 0.2 = 0.45

Two complete rankings of the alternatives from *A* are built:

- Ranking following the decreasing order of the positive flows $\Phi^+(a)$
- Ranking following the increasing order of negative flows $\Phi^-(a)$

The PROMETHEE I **partial ranking** (P^I, I^I, R^I) is obtained from the positive and negative flows:

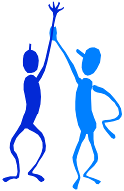
- P^I = preference; I^I = indifference; R^I = incomparability
- Both flows usually do not induce the same rankings
- **The ranking derived by PROMETHEE I is their intersection**

The greater $\Phi^+(a)$, the better
The lesser $\Phi^-(a)$, the better

Alt.	$\Phi^+(.)$	$\Phi^-(.)$
ITA	3.70 (2)	0.45 (1)
BEL	0.25 (6)	4.00 (5)
GER	0.70 (5)	4.05 (6)
SWE	1.55 (4)	2.45 (4)
AUT	4.05 (1)	0.60 (2)
FRA	2.65 (3)	1.35 (3)

Ranking based on $\Phi^+(.)$

AUT → ITA → FRA → SWE → GER → BEL



Ranking based on $\Phi^-(.)$

ITA → AUT → FRA → SWE → BEL → GER

Ranking PROMETHEE I

AUT, ITA → FRA → SWE → GER, BEL

Ranking of PROMETHEE I (2)

- **Preference P^I**: better strength and weakness, or the same strength and better weakness, or the same weakness and better strength (overall, at least as good strength and better weakness or at least as good weakness and better strength)
- The information of both outranking flows is consistent and may therefore be considered as sure
  $\boldsymbol{aP^Ib}$ *iff* $[\Phi^+(a) > \Phi^+(b)$ and $\Phi^-(a) < \Phi^-(b)]$ or $[\Phi^+(a) = \Phi^+(b)$ and $\Phi^-(a) < \Phi^-(b)]$
  or $[\Phi^+(a) > \Phi^+(b)$ and $\Phi^-(a) = \Phi^-(b)]$
- **Indifference I^I**: both strengths and weaknesses are the same
  $\boldsymbol{aI^Ib}$ *iff* $[\Phi^+(a) = \Phi^+(b)$ and $\Phi^-(a) = \Phi^-(b)]$
- **Incomparability R^I**: better strength and worse weakness, or better weakness and worse strength (the information provided by both flows is inconsistent; the PROMETHEE I is prudent)
  $\boldsymbol{aR^Ib}$ *iff* $[\Phi^+(a) > \Phi^+(b)$ and $\Phi^-(a) > \Phi^-(b)]$ or $[\Phi^+(a) < \Phi^+(b)$ and $\Phi^-(a) < \Phi^-(b)]$

The greater $\Phi^+(a)$, the better
The lesser $\Phi^-(a)$, the better

Alt.	$\Phi^+(.)$	$\Phi^-(.)$
ITA	3.70 (2)	0.45 (1)
BEL	0.25 (6)	4.00 (5)
GER	0.70 (5)	4.05 (6)
SWE	1.55 (4)	2.45 (4)
AUT	4.05 (1)	0.60 (2)
FRA	2.65 (3)	1.35 (3)

Ranking based on $\Phi^+(.)$

AUT → **ITA** → **FRA** → **SWE** → **GER** → **BEL**

Ranking based on $\Phi^-(.)$

ITA → **AUT** → **FRA** → **SWE** → **BEL** → **GER**

Ranking PROMETHEE I

$$\begin{tikzcd}
\textbf{AUT} \arrow[dr] & & \textbf{ITA} \arrow[dl] \\
& \textbf{FRA} \arrow[d] & \\
& \textbf{SWE} \arrow[dl] \arrow[dr] & \\
\textbf{GER} & & \textbf{BEL}
\end{tikzcd}$$

Ranking of PROMETHEE II

$\pi(.,.)$	$\Phi^{+}(.)$	$\Phi^{-}(.)$	$\Phi^{-}(.)$
ITA	3.7	0.45	3.25 (2)
BEL	0.25	4.0	-3.75 (6)
GER	0.7	4.05	-3.35 (5)
SWE	1.55	2.45	-0.90 (4)
AUT	4.05	0.6	3.45 (1)
FRA	2.65	1.35	1.30 (3)

Φ^{+}(**ITA**) = 3.7 – 0.45 = 3.25

- The net outranking flow is the **balance** between the positive and negative flows:
  $$\Phi(a) = \Phi^{+}(a) - \Phi^{-}(a)$$
- For each alternative a:
  $$-(n-1) \leq \Phi(a) \leq n-1$$
- The higher the net flow, the better the alternative
- The sum of all net flows is zero:
  $$\Sigma_{j=1,\ldots n}\ \Phi(a) = 0$$

$$\textbf{AUT} \rightarrow \textbf{ITA} \rightarrow \textbf{FRA} \rightarrow \textbf{SWE} \rightarrow \textbf{GER} \rightarrow \textbf{BEL}$$

The ranking of PROMETHEE II consists of (P^{II}, I^{II})

- **Preference P^{II}**: better new flow implies a strict preference - $aP^{II}b$ *iff* $\Phi(a) > \Phi(b)$
- **Indifference I^{II}**: the same flows imply indifference - $aI^{II}b$ *iff* $\Phi(a) = \Phi(b)$

When PROMETHEE II is considered, all the alternatives are comparable

- No incomparabilities, but the resulting information can be more disputable
- When $\Phi(a) > 0$, a is more preferred over the remaining alternatives on all criteria
- When $\Phi(a) < 0$, a is more outranked by the remaining alternatives on all criteria

PROMETHEE I and II - Summary

PROMETHEE

Preference information (i=1,…,n)

- **Intracriteria**:
  - 6 shapes of the intensity of preference on each criterion
    indifference thresholds q_i, preference thresholds p_i, standard deviation on Gaussian shape s_i
- **Intercriteria**:
  - importance coefficients (weights) of criteria w_i

Preference model: an outranking relation in set *A* characterized by the comprehensive preference degrees $\pi(a,b)$

Recommendation: complete or partial ranking based on the Net Flow Score procedures

- In real-world applications, it is recommended to consider both PROMETHEE I and PROMETHEE II
- The complete ranking is easy to use, but the analysis of incomparabilities often helps to finalize a proper decision
- *Samuel Karlin*: “The purpose of models is not to fit the data, but to sharpen the questions”

PROMETHEE V for Choice Problems

- PROMETHEE I and II are appropriate to select one alternative
- In some applications, **a subset of alternatives must be identified**
- Let $\{a_i, i=1,\ldots,n\}$ be the set of alternatives
- Associate a Boolean variable x_i to each alternative
  - $x_i = 1$, if a_i is selected
  - $x_i = 0$, if a_i is not selected

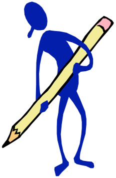
- Use PROMETHEE II to obtain net flows $\Phi(a_i)$ (**neglect constraints**)
- Solve the following **binary linear programming model** accounting for additional constraints

$$\max \ \sum_{i=1,\ldots,n} x_i \cdot \Phi(a_i)$$

subject to:

$$\sum_{i=1,\ldots,n} \lambda_{p,i} \cdot x_i \ \{\geq, =, \leq\} \ \beta_p, \quad p = 1,\ldots,P$$

$$x_i \in \{0,1\}, \quad i = 1,\ldots,n$$



where $\Phi(a_i)$ are the coefficients of the objective function are net flows and $\lambda_{p,l}$ are the coefficients of the P linear constraints

- ***Aim***: collect as much net flow as possible, while taking into account the constraints

- The constraints may concern cardinality, budget, return, investment, marketing, …
- They can be related to all alternatives, but possibly also only to some subsets (clusters)

Example:

- Consider the results of the previously conducted analysis with PROMETHEE II
- Select a subset of 2 alternatives with comprehensive maintenance cost not exceeding 40 million E.
- Having solved the binary linear program, a subset of alternatives a_i associated with $x_i = 1$ is selected

Alternative	$\Phi(.)$	Maintenance cost
ITA (a_1)	3.25	22
BEL (a_2)	-3.75	17
GER (a_3)	-3.35	25
SWE (a_4)	-0.90	28
AUT (a_5)	3.45	20
FRA (a_6)	1.30	18

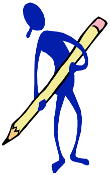
max $3.25 \cdot x_1 + (-3.75) \cdot x_2 + (-3.35) \cdot x_3 + (-0.90) \cdot x_4 + 3.45 \cdot x_5 + 1.30 \cdot x_6$

subject to:

$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 \leq 2$

$22 \cdot x_1 + 17 \cdot x_2 + 25 \cdot x_3 + 28 \cdot x_4 + 20 \cdot x_5 + 18 \cdot x_6 \leq 40$

$x_1, x_2, x_3, x_4, x_5, x_6 \in \{0,1\}$



Optimal solution: select **AUT** and **FRA**

$x_1 = 0,\ x_2 = 0,\ x_3 = 0,\ x_4 = 0,\ x_5 = 1,\ x_6 = 1$

objective function = 3.45 + 1.30 = 4.75

cardinality = 2 ($x_5 = 1$, $x_6 = 1$)

comprehensive maintenance cost = 20 + 18 = 38

PROMETHEE - Summary

Review: Behzadian, M., Kazemzadeh, R.B., Albadvi, A., Aghdasi, M., PROMETHEE: A comprehensive literature review on methodologies and applications, *European Journal of Operational Research,* 200(1),198-215, 2010

- More than 200 papers published in more than 100 journals

The PROMETHEE bibliographic database: far over 2000 references

- `http://en.promethee-gaia.net/bibliographical-database.html`
- The success of the methodology is mainly due to its mathematical properties and particular friendliness of use
- **Major areas of application**: environmental management, hydrology and water management, business and financial management, chemistry, logistics and transport, energy management, health care, manufacturing and assembly, sports, …

PROMETHEE - Example Applications

- Ranking the sites for radioactive waste disposal facilities in Croatia (Petras, 1997)
- Assessing Vietnamese rice quality according to its properties (Kokot and Phuong, 1999)
- Ranking forestry strategies on state-owned lands in Finland (Kangas et al., 2001)
- Ranking the various renewable energy technologies for the development of a wind park in Greece (Polatidis and Haralambopoulos, 2007)
- Selecting the best equipment milling machines to be purchased in an international company in Turkey (Dagdeviren, 2008)
- Prioritization of green suppliers in food supply chain in India (Govindan et al., 2017)