Evolving New State-of-the-Art for Voronoi Diagram and Distance Transform From JFA Using AutoML

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This paper studies a practical usage of machine learning (AutoML) to automate research towards discovering efficient Voronoi Diagram and Distance Transform algorithms. As the baseline we used the Jump Flooding Algorithm (JFA) - by finding new mutations which works best for specific data, and then ensembling them into one, we create new state-of-the-art algorithm in this field named **Vorotron** with best-case $O(1)$ time complexity and $O(N)$ work complexity - in addition, we introduce **JFAStar**, a single variant that in most cases outperforms original JFA with proven worst-case $O(\log^* N)$ time complexity. The algorithm is faster and produces more accurate approximations. It could be extended into 3D space in a slice-by-slice manner. We started from the assumption that JFA has potential for improvement - some benefits can be observed for specific data by adding random noise and adjusting the step size in JFA. This showed us that, AutoML could examine this space, and find the best possible algorithm in each case. In the further part of the work, we discuss the results, compare the variants and ensemble for creating the final algorithm.

CCS Concepts: • **Computing methodologies** → **Computer graphics**; **Parallel computing methodologies**; • **Theory of computation** → *Randomness, geometry and discrete structures*; **Data structures design and analysis**.

Additional Key Words and Phrases: Voronoi Diagram, Distance Transform, Code Generation

1 INTRODUCTION

This paper[1] studies a practical usage of machine learning to automate research towards discovering efficient Distance Transform algorithms (utilizing technique known as AutoML). Thus, by finding mutations which works best for specific data, and then ensembling them into one, we create new state-of-the-art algorithm in this field named Vorotron with best-case $O(1)$ time complexity and $O(N)$ work complexity. In addition, we introduce JFAStar, a single variant that in most cases outperforms original JFA with proven worst-case $O(\log^* N)$ time complexity.[2]

Notable contribution to the quick algorithm that makes Distance Transform (DT) using graphics hardware includes [5] that creates a cone for each input (point/seed) and renders those cones to obtain the Voronoi diagram as the lower envelope of these cones. [4] use planes tangent to a paraboloid and thus avoid the errors caused by the tessellation of the cones. Unfortunately, the drawback of this approach is the significant amount of computation and the implementation complexity.

Jump flooding algorithm (JFA)[3] is an interesting way to utilize the graphics processing unit to efficiently compute Voronoi diagrams and distance transforms [9]. This method is faster and produces more accurate results [10], and furthermore, it could be extended into 3D space in a slice-by-slice manner. This is more effective than the previous research carried out by [12], because the speed of JFA is almost independent to the number of seeds [10].

[1] the original title for this paper was "Lord Vorotron: Finding the Best JFA Variant for the Coming Winter"
[2] source code is available at https://github.com/maciejczyzewski/fast_gpu_voronoi
[3] a novel pattern of communication

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Based on this research and findings, several efficient GPU-based algorithms which are either work optimal or time optimal have been proposed including SKW [11], PBA [2], FastGPU [3], Honda's algorithm [6] and WTO [7]. The main question that needs to be addressed is whether JFA has potential for improvement. We found some benefits for specific data by adding random noise and adjusting the step size in JFA. Therefore, this shows that, AutoML could examine this unknown space, and find the best possible algorithm in each case.

For convenience, this work focus on the Voronoi diagram only - because this problem can be translated to DT [9]. The algorithm would be an approximation of the output, thus we suggest using WTO [7] for exact DT (EDT). The major contributions of this paper are thus:

(1) Presenting new envolved state-of-the-art variants of algorithm for Voronoi Diagram and Distance Transform: **JFAStar** - Multi-domain single variant, replacement of JFA; **Vorotron** - Ensemble of domain-specific variants, production ready; and
(2) Analyzing all possible variants of JFA: comparing error and speedup relative to bruteforce method in different domains.

2 RELATED WORK

Several efficient GPU-based algorithms which are either work optimal or time optimal have been proposed including JFA [9], SKW [11], PBA [2], FastGPU [3], Honda's algorithm [6] and WTO [7].

Reference	Algorithm	Exactness	Time	Work
[3]	FastGPU	Exact	$O(n^3/p)$	-
[2]	PBA	Exact	$O(n)$	$O(mN)$
[6]	based on SKW	Exact	$O(n)$	$O(N)$
[7]	WTO	**Exact**	$\mathbf{O(\log n)}$	$O(N)$
[11]	SKW	Approximate	$O(n)$	$O(N)$
[9]	JFA	Approximate	$O(\log n)$	$O(N \log n)$
In this paper	JFAStar	**Approximate**	$\sim\mathbf{O(\log^* n)}$	$\sim O(N \log^* n)$
	Vorotron	**Approximate**	$\sim\mathbf{O(1)}$	$\sim O(N)$

Table 1. Different GPU algorithms for computing EDT

The original author of JFA[4] defined some variants and modifications [10], in this work we continue on this subject and examine the relationship between them. We chose this algorithm because it is the simplest to implement and it has a wide variety of improvements.

2.1 Jump Flooding

Algorithm for Voronoi Diagram that uses jump flooding as communication pattern propagates the information (2D coordinates and ID) of all sites to all the pixels in the matrix. It is based on the observation that while flooding an area with a seed, each seeded pixel can transmit its information, instead of just the ones on the boundaries of the Voronoi cell.

In each round, the site information stored in each pixel (x, y) is propagated to at most eight other pixels at $(x + i, x + j)$ where $i, j \in \{-k, 0, k\}$, and k is the step length of the current round.

[4]Guodong Rong

In the first round, we use $n/2$ as the initial step length to ensure that each pixel is reached by at least one site. The step length k is halved in each of the following round. All pixels are processed in parallel by the GPU. This ensures an exponential increase in the number of seeded pixels in a grid and thus, can be flooded in $O(\log n)$ rounds, for an $n \times n$ grid [13].

2.2 AutoML

Program synthesis is a class of regression problems where one seeks a solution, in the form of a source-code program, mapping the inputs to their corresponding outputs errorless [8]. If we have a broad range of acceptable solutions, another aspect is how efficient the produced code is.

There are two key ingredients to a synthesis problem: a domain specific language (DSL for short) and a specification. The DSL defines a space of candidate programs which serve as the model class. DSL-based models create different grammar rules for common code statements (e.g., control flow, comments, and brackets). The specification is commonly expressed as a set of input-output examples which the candidate program needs to fit exactly. In our problem, these examples are produced by bruteforce algorithm.

Given the precise and combinatorial nature of synthesis, gradient-descent based approaches perform poorly and an explicit search over the solution space is required [1].

3 PROPOSED METHOD

In this section we outline the general approach that we follow in this work. When testing a JFA on different domains, we found some performance improvements by adding random noise and adjusting the step size between flooding rounds. In order to systematize our research, we adopted the following scheme:

- Developing manually new modifications that can be made to the JFA (such as using a circle instead of interacting with neighbours in a square pattern) and designing them as parameters.
- Heuristic search algorithm[5] - to examine enormous space of possible variants:
  - Implements a new variant from a combination of several modifications (mutation).
  - Tests current variant on various domains and scores the results to update gradient descent.
- Knowing the best variant in a specific domain, we can use it in ensemble (when entering the input, ensemble algorithm will decide which algorithm will solve it).

3.1 Domain Space

The Voronoi Diagram has a stronger relationship with the initial density of input seeds than the matrix shape. Intuitively, if we have a dense matrix, there is no need to deliver information about a particular seed to every pixel of the matrix, only to its surrounding neighbours. Hence, less passes of information can be performed, therefore the algorithm can be terminated earlier.

Density \ **Shape**	Small (32-128×)	Medium (256-448×)	Large (512-1536×)
Low (ρ=0.00005-0.001)	Bruteforce	JFA	JFA
High (ρ=0.01-0.1)	JFA	JFA	JFA

Table 2. State-of-the-art for specific domains (before our work)

[5] Bayesian optimization using Gaussian Processes.

In order to find out which variants are domain-specific, we have to divide the domain into subdomains as in the Table 2. Unfortunately, we can already identify that the jump flooding algorithm would not be efficient in cases with low density of seeds. If there are less than 8 seeds to iterate[6], the bruteforce algorithm is ideal for use.

3.2 Search Space

UWAGA: opisac ze szukamy 4 podprzestrzenie w turach! (moze tez o nowym wzorku ktory szuka tylko prawidlowych

bridge z score function gdziekolwiek to bedzie obliczyc ile jest aktulanie wersji algosow np. czy jest to juz 2do14 jak mamy 3xreal w wielomianie AKTUALNIE JEST okolo 7,200?

jakie modyfikacje, na to osobna sekcja? wiec co tu napisac chyba tylko o zlozonosci problemu i ze kod jest skladany i testowany a niektore wersje sa pomijane zgodnie z dzialaniem gp_minimize (Bayesian optimization using Gaussian Processes).

w naszym wypadku zdefiniowalismy pewien zbior wariantow pewnych czesci algorytmu (Search Space), modul testujacy dana mutacje/wariant - sklada kod kernela a pozniej go weryfikuje na naszej Domain Space.

3.3 Score Function

JEST PROBLEM - zdazaja sie warianty ktore 0zeruja rzadkie i sa bardzo szybkie na duzy matrycach - trzeba do ponownie zbalansowac roznica w pikselach pomiedzy bruteforce a algorytmem - napisac o tym / tez ze to wszystko to ilorazy do bruteforce

dla voronoi-a interesuja nas 2 parametry Error oraz Szybkosc, aby wyniki byly wiarygodne porownujemy je z bruteforcem (a wiec bedzie to iloraz). aby ocenic dana mutacje musimy przypisac jakis Score danej wersji, wiec uzylismy wzor ponizej

$$S(x, y) = max\{0, \sqrt{x} \cdot (100 - y^2)\}, \tag{1}$$

$$0 \leq y \leq 100, 0 < x \tag{2}$$

ktory kaze za zbyt wysokie errory, dajac zerowy wynik - skladnik przy y rosnie szybciej niz x wiec gdy przekroczy 100 da nam ujemny wynik - czyli 0.

3.4 Optimizer

opisac dwie osobne taktyki optymalizacji dla best single vs. ensemble

mozemy napisac ze korzystalismy z forest/gp minimize, ale tez wspomniec ze aby miec najlepszy best single to trzeba bylo optymalizowac rownoczesnie cala przestrzen (od malych do duzych, gestych po rzadkie), a zeby miec najlepszego Vorotrona - czyli ensembla to trzeba bylo dla kazdej domeny z optymalizowac a pozniej jedynie zrobic balancera!!!!!!!!!!!

trzeba to przeszukiwac tak aby nie sfiksowal na zadanych parametrach - bo niechcacy ocenia na poczatku ze szum/dual jest nie fajny i pozniej go juz nie rozwaza

3.5 Ensemble

mozna zapisac tabele dla kazdej domeny (shape) / i zrobic przewidywania parametrow (slownik wariantu) - bo dla malych oplacalne sa Circle6 a dla wiekszych Circle12 i tak dalej - jak to zrobic?

patrzac na rezultaty mozemy znalesc jaki algorytm najlepiej sprawdza sie w zadanej domenie. np. widac ze dla malej ilosci seedow (malo gestych przypadkow, ktore maja mala powierzchnie) oplaca sie uzyc bruteforce. Dla kolejnych wiekszych przypadkow innych wariantow JFA. Jak

[6] each thread in original JFA needs 8 iterations to communicate with neighbours

wybrac algorytm? Kazdy przypadek ma ‘shape‘ oraz ‘num‘ wiec mozna na CPU wysemplowac pare punktow albo odrazu obliczyc gestosc i wybrac odpowiedni algorytm. To takich ensemblacji najlepiej sprawdzi sie drzewo decyzyjne (moze byc boostowane).

4 VARIANTS

ZROBIC LADNE RYSUNECZKI w Google Slides - eksport to pdf!

To compute the Voronoi diagram for a 2D grid of size n×n with a given set of seeds at some grid points, we are interested to propagate the content (in particular, position information) of each seed s to each grid point so that each grid point can decide which seed is its closest one.

Niektore operacje propagacji informacji sa zbedne - tylko w przypadkach rzadkich macierzy potrzeba jest log(n) krokow aby uzyskac prawidlowy wynik. Rozne warianty omawiane w [10] pozwalaja zredukowac blad klasycznego JFA. Nie zostaly jednak omawiane przypadki gdzie poszczegolne modyfikacje sa uzywane z innymi.

Dlatego w tej pracy prezentujemy dodatkowe modyfikacje ktore mozna zastosowac aby stworzyc nowe warianty. Pewne modyfikacje sa oczywiste i wynikaja z alternatywnego podejscia (zamiast anchoru[7] kwadratowego mozna uzyc kola), informacje mozna wstepnie rozpropagowac losowo - w nadzie ze pozwoli nam skonczyc algorytm w mniejszej ilosci krokow.

Aby badania byly bardziej przejrzyste trzymalismy sie pewnej konfencji nazewniczej:

$$[\boldsymbol{anchor\_type}][\boldsymbol{anchor\_num}][\boldsymbol{anchor\_double}] \quad - \quad [\boldsymbol{step\_function}] \quad + \quad [\boldsymbol{noise}]$$

dla przykladu Circle11(1/3)Dual(1/4)-Factor3+Noise ktore mozna przeczytac jako:

$$\begin{aligned}
\text{[anchor_type]} &= Circle,\\
\text{[anchor_num]} &= 11,\\
\text{[anchor_number_ratio]} &= 1/3,\\
\text{[anchor_double]} &= True,\\
\text{[anchor_distance_ratio]} &= 1/4,\\
\text{[step_function]} &= Factor3,\\
\text{[noise]} &= True
\end{aligned}$$

4.1 Noise

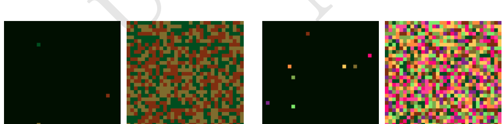

Fig. 1. Noise for 32 × 32

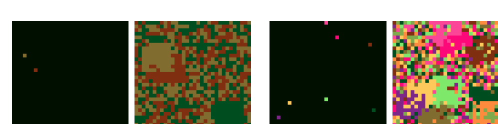

Fig. 2. Local Noise for 32 × 32

FIXME: figure z przykladami szumu (+local) i jak to wyglada i jak wygladalo instancja!

Zamiast zaczynac od pustej macierzy z seed-ami poczatkowymi mozna ja losowa uzupelnic szumem - tworzac przypadkowe short-cuty. Mozna tego dokonac osobnym kernelem ktory zostatnie wywolany przed wykonaniem glownej czesci algorytmu. Interpretacja jest taka ze pewne rejony ktora w JFA sa wypelnione zerami podczas pierwszych iteracji nie podejmuja zadnych decyzji.

[7]anchorem nazywamy metode ktora pobiera sasiadow do przekazania informacji

Uzupelniajac szumem moga one przypadkowo ustawic sie na prawidlowa wartosc i propagowac w kolejnej rundzie najlepsza wartosc w swoim otoczeniu (zgodnie ze stepem).

dowod kamila tutaj????????????????

4.1.1 Local Noise. przesunieta ciezkosci??? czyli srednia z wagami poprawila? **a moze uzaleznisc wage od rozmiaru pierwszego stepu?**

Mozna tez szum uzupelniac nie losowo tylko w otoczeniu. Wiec gdy w punkcie (x, y) wylosujemy losowego seed-a o wartosci (x_{rand}, y_{rand}) to wyliczamy nowa pozycje (x', y') ktora znajduje sie w polowie drogi w nastepujacy sposob: $x' = \frac{x+3x_{rand}}{4}$, analogicznie dla y'. Dodatkowo jesli (x', y') jest pusta to tez uzupelniamy to pole ta informacja. Nie przejmujemy sie wyscigiem w dostepie do danych. Nadpisania beda losowe - a szum tez.

4.2 Anchor Type

losowane punkty na okregu???

Zamiast pobierac informacje od 8 sasiadow o step size from grid points at $(x + i, y + j)$ where $i, j \in \{-\text{step}, 0, \text{step}\}$. Mozna zastosowac okrag - otwiera nam to nowe mozliwosci na swobodna modyfikacje ilosci punktow od ktorych bedziemy pobierac informacje. Naturalnie wydaje sie ze mala ilosc punktow w anchorze spowoduje wzrost bledu, a duza ilosc punktow spowoduje zmalenie bledu.

4.2.1 Anchor Number. Dlatego kolejnym parametrem bedzie mozliwosc kontrolowania ilosci punktow. Niestety nie rozwazalismy wariantu kwadratow o dowolnej ilosci punktow (poniewaz byly by to wielokrotnosci 2x2=4, 3x3=9, 4x4=16, 5x5=25) bo i tak nie dalo by sie wybrac uniformly tej wartosci. Dla okregu punkty sasiada (x_i, y_i) byly liczone nastepujaco:

$$x_i = x + \text{step} \cdot \cos\left(\frac{2\pi}{[\text{anchor\_num}]} \cdot i\right), y_i = y + \text{step} \cdot \sin\left(\frac{2\pi}{[\text{anchor\_num}]} \cdot i\right)$$

4.3 Anchor Double

Oprocz pojedynczego anchora, mozliwe jest uzycie podwojnej warstwy anchorow (czyli np. male kolko i wieksze). Idea za tym stojaca to ze male kolko wewnetrzne jest dokladne (dziala jak w JFA) - a wielkie zewnetrzne jest skautujace lub aby poprawic error wynikajacy np. z mniejszej ilosci anchor_num (w sumie to podobny mechanizm jak w Lookahead - wolny/szybki)

4.3.1 Anchor Distance Ratio. Parametr mowiacy o stosunku dlugosci step size od wewnetrzengo anchora do zewnetrznego.

4.3.2 Anchor Number Ratio. Parametr mowiacy o statusnku ilosci detektorow od wewnetrznego anchora do zewnetrznego.

4.4 Step Function

Gdy nasza informacja propaguje sie szybciej lub jest bardziej zageszczona dlatego sasiedzi szybciej dostaja prawidlowa informacje - to oznacza ze mozna skrocic ilosc round wykonania algorytmu.

Step size jak i ich ilosc mozna okreslic za pomoca 2 podstawowych parametrow: shape and number of points - z ktorych pozniej mozemy okreslic np. srednia gestosc. Zaimplementowalismy 2 warianty ktore sa uzaleznione jedynie od shape: defaultowy z JFA, z JFA o podstawie 3; oraz jeden uzaleniony od shape oraz od num: logstar. Jednak aby wygeneralizowac problem stworzylismy tez mozliwosc wygenerowania dowolnego polynomialu.

4.4.1 Special Polynomial. powinno byc ograniczone do 3 PARAMETROW! wymyslis nowa funkcje problem z Special - on overfituje przyklady zmieniajac 5 miejsce po przecinsku aby 2 zamienialo sie np. w 1

Implementacja nie jest wazna - chodzi o idea zwiazania shape oraz num. Oraz modyfikowanie wartosci, szybkosci spadku, ksztaltu (np. piloksztnego) - jakimis parametrami. Wada tego rozwiazania jest ze trzeba optymalizowac ta funkcje na calej dziedzinie (malej, duzej, gestej, zadkiej) - bo inaczej z overfituje ona ilosc krokow i wiekosc stepu pod rozmiar.

```
def mod_step_function__special(shape, num=None, config=None):
    # [EXAMPLE]
    # Special(1.51/0.92/0.92/1.08/0.42)
    # ------- A -- B -- C -- D -- X ---

    A = config["A"] # <1, 2>
    B = config["B"] # <0, 1>
    C = config["C"] # <0, 1>
    D = config["D"] # <1, 2>
    X = config["X"] # <0.2, 1>

    q = num / (shape[0] * shape[1])
    qm = ((shape[0] + shape[1]) / 2) * q**(1 / 2)
    S = B * qm + (1 - B) * (max(shape) / 2)
    St = math.log2(S)

    steps = []
    for i in range(1, int(X * St * 2), 1):
        f = round(1 / (D**(i**A) + i % max(1, int(C * St))), 4)
        ffm = int(f * S)
        if ffm >= 1:
            steps.append(ffm)
    if len(steps) == 0:
        return [1]

    return steps
```

5 RESULTS

przeniesc legende? JAKO OSOBNY PDF? i podac w tej sekcji - tak sie nie robi ale bylo by ok i czytelnie + wiecej miejsca na wykresy a przypadkow bedzie wiecej czy wykres loss oraz score dla przypadkow powinnien byc nalozony? albo polaczony subfigurem tak aby osie byly sync. i dalo sie porownac

performance plot[8]

UWAGE: usunac z tabelek PODOBNE ALGORYTMY i ich slabsze rezultaty

[8]wykres zostal zrobiony poprzez posortowanie scorow - dzieki temu widac roznice w przyroscie i latwo dostrzec ktory algorytm ma najwyzszy score lub jaka ma chaktersytyke (np. jest bardzo skuteczny dla waskiej grupy przykladow)

Density \ Shape	Small (32-128×)	Medium (256-448×)	Large (512-1536×)
Low (ρ=0.00005-0.001)	Bruteforce	?	?
High (ρ=0.01-0.1)	?	?	?

Table 3. Found in this paper state-of-the-art for specific domains

5.1 Multi-domain Variant (JFAStar)

- **shapes**: {32x32, 64x64, 96x96, 128x128, 256x256, 320x320, 384x384, 448x448, 512x512, 768x768, 1024x1024, 1536x1536}
- **cases**:
  - gen_uniform: seeds=1,
  - gen_uniform: seeds=3,
  - gen_uniform: density=0.0001,
  - gen_uniform: density=0.001,
  - gen_uniform: density=0.01,
  - gen_uniform: density=0.02,
  - gen_uniform: density=0.03,
  - gen_uniform: density=0.04,
  - gen_uniform: density=0.05,
  - gen_uniform: density=0.1,

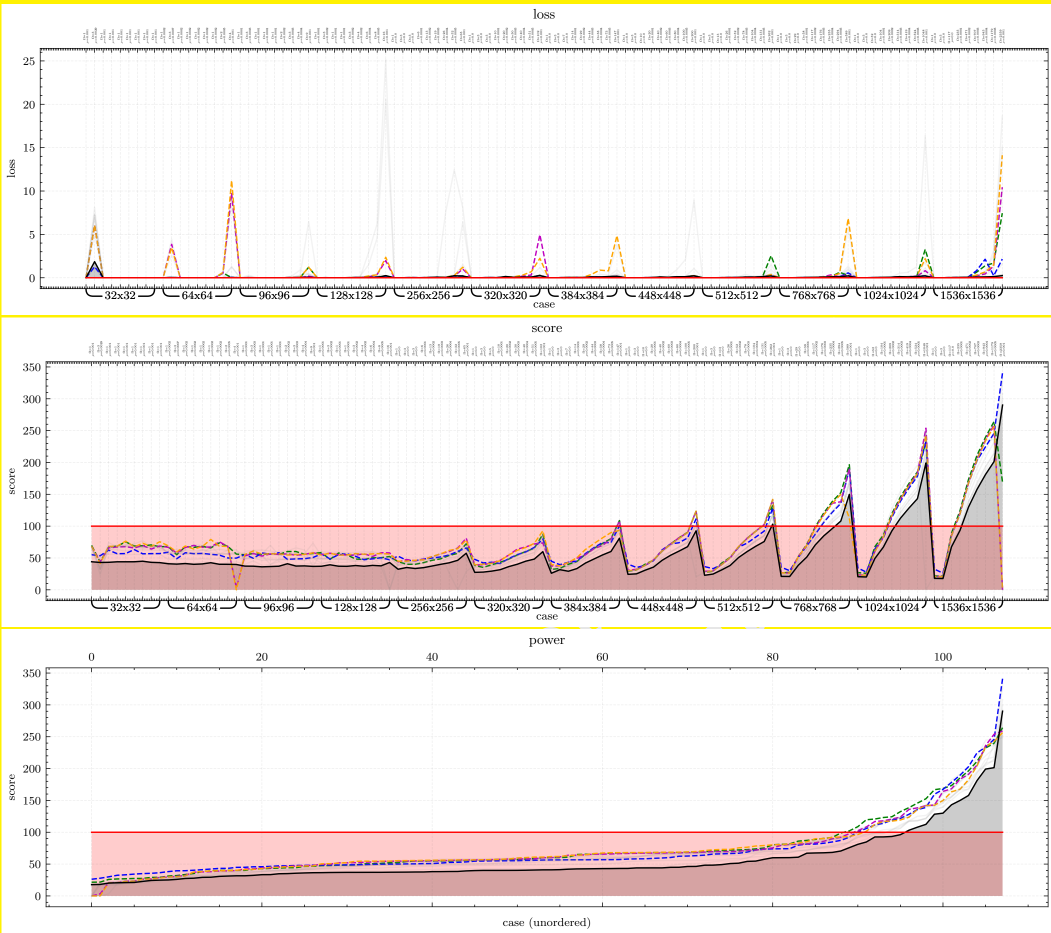

Fig. 3. bla bla bla

Algorithm	ρ=0.0003	ρ=0.0002	ρ=0.0003	ρ=0.0004	ρ=0.0008	Score
Square|Special(1.92/0.9/0.68/1.2/0.67)+Noise	40.4	51.1	74.8	95.1	**113.6**	74
Square|Star+Noise	44.5	50.5	71.2	87.9	**115.8**	73
Square|Special(1.98/0.8/0.95/1.2/0.46)+Noise	40.8	51.5	74.7	95.5	**102.6**	72
Square|Special(1.47/0.68/0.75/1.52/0.41)+Noise	41	51.9	75.5	96.7	98.3	69
Square|Special(1.46/0.81/0.68/1.33/0.5)+Noise	39.6	50.9	73	88.1	**112.1**	69
Square|Special(1.98/0.69/0.74/1.22/0.49)+Noise	37.4	49.5	71.2	91.7	**102.6**	66
Square|Special(1.14/0.8/0.61/1.38/0.62)+Noise	34.6	44.9	64.7	80.4	**108.7**	66
SquareDual(1/3)|Special(1.56/0.88/0.57/1.64/0.39) +Noise	39.1	48.5	67.5	85.7	98.2	65
Square|Special(1.97/0.79/0.61/1.24/0.61)+Noise	37.9	49.5	72.8	93.4	88.2	64
Square|Special(1.92/0.55/0.61/1.1/0.53)+Noise	33.7	41.8	60.9	77.5	**105.4**	63
Square|Special(1.83/0.69/0.81/1.24/0.45)+Noise	38.3	49.5	72.5	92.7	**102.5**	62
Square|Factor3+Noise	31	39.4	58.9	76.2	**105.3**	61
Square|Special(1.84/0.65/0.84/1.26/0.52)+Noise	37.2	48.8	70.3	89.7	**101.9**	60
Square|Special(1.8/0.65/0.58/1.27/0.59)+Noise	36.9	46	67.5	86.8	98.0	59
Square|Special(1.94/0.64/0.8/1.28/0.4)+Noise	40.1	52	77.7	96.3	88.2	58
Square|Special(1.82/0.71/0.73/1.2/0.42)+Noise	40.4	51.5	75.1	94.4	82.5	58
Square|Special(1.33/0.5/0.7/1.26/0.46)+Noise	35	46.1	65.8	80.9	81.3	57
SquareDual(1/3)|Special(1.71/0.06/0.95/1.46/0.34) +Noise	32	38	54.2	68.8	92.2	56
Square|Special(1.93/0.64/0.92/1.21/0.98)+Noise	34.6	43.9	65.6	85.4	96.7	56
JFA (original)	30.2	37.2	54.2	69.2	94.4	56
SquareDual(1/3)|Special(1.51/0.46/0.97/1.88/0.57) +Noise	31.7	39.1	56.7	72.8	83.4	55
SquareDual(3/4)|Star+Noise	34.8	38.6	52.9	65.7	84.8	54
SquareDual(1/3)|Star+Noise	35.4	38	52.5	65.1	83.5	54
SquareDual|Star+Noise	34.6	38	52	65.2	83.3	54
SquareDual(1/3)|Special(1.2/0.9/0.84/1.96/0.44) +Noise	38.6	47.2	65.7	73.5	95.8	53
Square|Default+Noise	26.6	33.6	49.9	64.1	88.7	52
Circle10(1/4)|Special(1.58/0.9/0.64/1.16/0.6)+Noise	29.3	34.7	46.8	57.1	72.9	47
SquareDual(1/3)|Special(1.45/0.85/0.74/1.95/0.34) +LNoise	40.2	44.2	45.2	54.6	56.7	45
Circle11(1/3)|Star+Noise	30	32.5	44.5	51.7	67.4	44
SquareDual|Special(1.48/0.39/0.84/1.34/0.72) +Noise	24	29.9	42.8	54.7	74.6	44
Circle11(3/4)Dual(2/3)|Factor3+Noise	24.7	30	43.1	56.1	69.9	44
Circle9(1/4)|Star+Noise	31.8	35.1	47.6	49.9	61.9	44
Circle9(3/4)|Star+Noise	31.9	34.6	47.1	50.8	61.8	44
Circle12(1/3)|Star+Noise	28.6	32	42.7	51.2	65.5	43
Circle14(1/4)|Special(1.38/0.71/0.93/1.77/0.95) +Noise	24.3	29.4	41.5	51.5	62.2	41
Square|Special(1.56/0.7/0.63/1.29/0.43)+Noise	40.9	51.6	73.7	92.9	73.2	40
SquareDual(3/4)|Default	22.5	27.6	39.1	49.7	66.9	40
SquareDual|Default	22.6	27.3	38.8	49.2	65.7	40

5.2 Domain-specific Variants

5.2.1 Small Shape: 32×32, 64×64, 96×96, 128×128. Square-Special(1.44/0.96/0.17/1.63/0.86)+Noise and Square-Special(1.07/0.24/0.9/1.88/0.64)

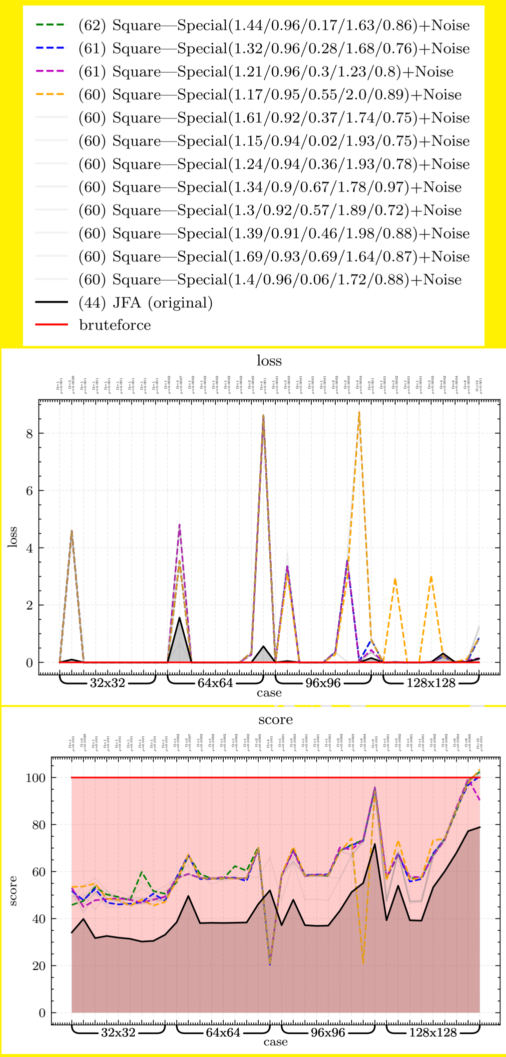

Fig. 4. Low Density

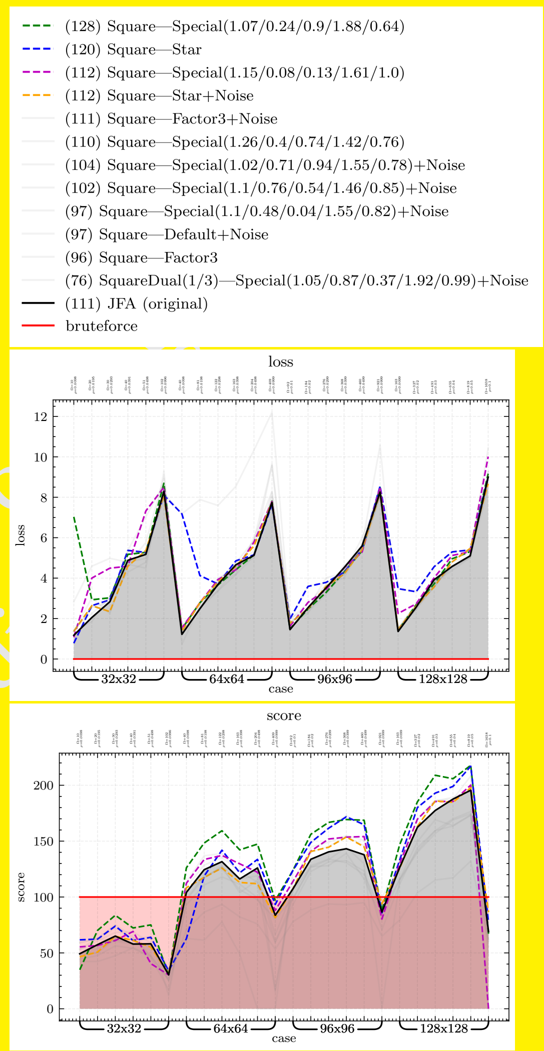

Fig. 5. High Density

5.2.2 Medium Shape: 256 × 256, 320 × 320, 384 × 384, 448 × 448.

5.2.3 Large Shape: 512 × 512, 768 × 768, 1024 × 1024, 1536 × 1536.

5.2.4 Low Density.

5.2.5 High Density.

5.3 Ensemble of Domain-specific (Vorotron)

bla bla

5.4 Objectives

naprawic generowanie tego wykresu napisac co nie moze byc uzyte z czym?

czyli co ma wplyw na co (w sumie to najwazniejsze mialo byc w pracy)

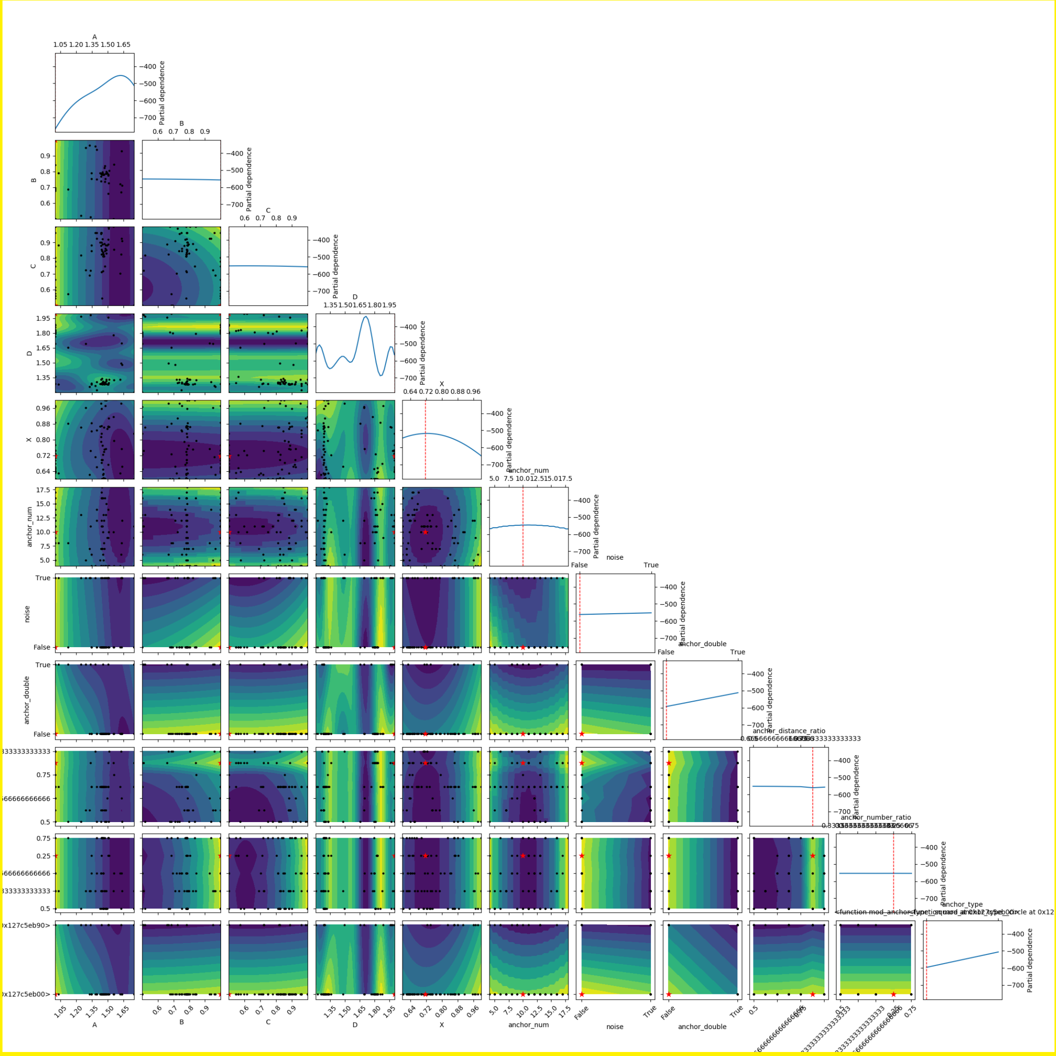

Fig. 6. bla bla bla

6 PRACTICAL USAGE

polaczyc z Conclusions

Jest wiele projektow ktore potrzebuje DT lub voronoi-a. Jedyne dwa praktyczne przyklady z tej pracy to SOTA dla JFA - czyli JFAstar, oraz praktyczny Ensemble (uwzgledniajacy np. bruteforce dla malych instancji).

7 CONCLUSIONS

This paper presents the GPU's effective, almost constant, algorithm for calculating the Euclidean distance transform (DT) approximation for 2D and higher dimensional images. As mentioned in [2], it remains challenging to balance the workload in such an approach. *Vorotron* does not explicitly solve this issue but, by constructing an alternative solution utilizing random shortcuts and parameter estimation, it makes it a reasonable approximation. In practice, such a constant time algorithm is useful in many interactive applications, such as tessellations, rendering, and image processing, involving [9].

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Dziekuje swojemu psu!Dziekuje swojemu psu! Dziekuje swojemu psu!

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