Maciej Medyk – COT6930 – Homework 2

Question 1

PageRank score [0.50pt].

Page rank works by creating a random walk that is counting the number and quality of links to determine importance of the website.

Rooted PageRank [0.50pt].

It's a page rank that is countering problems in standard page rank by periodically teleporting to another node and in this way avoiding carefully crafted loops.

Network community [0.50pt].

It's a set of nodes between which the interactions are frequent.

Clique [0.50pt].

Is a subset of the network in which all nodes are more closely and intensely tided to one another and then they are to other members of the network.

k-Clique [0.50pt].

Is a maximal subgraph in which the largest geodesic distance between any nodes.

Low-rank approximation [0.50pt].

It's a method that analyzes the estimation of proximity and closeness of different users which in turn is helpful in link prediction, dimension reduction, compression.

Question 2

Power Iteration PageRank scores for each website [1.00pt].

	A	B	C	D	E	F	G
A	0	1	1	1	1	1	1
B	1	0	0	1	1	0	0
C	1	0	0	0	0	1	1
D	1	1	0	0	0	0	0
E	1	1	0	0	0	0	0
F	1	0	1	0	0	0	0
G	1	0	1	0	0	0	0

	A	B	C	D	E	F	G
A	0	0.333	0.333	0.500	0.500	0.500	0.500
B	0.167	0	0	0.500	0.500	0	0
C	0.167	0	0	0	0	0.500	0.500
D	0.167	0.333	0	0	0	0	0
E	0.167	0.333	0	0	0	0	0
F	0.167	0	0.333	0	0	0	0
G	0.167	0	0.333	0	0	0	0

X_0	X_1	X_2	X_3	X_4	X_5	X_6	X_7
0.1500	0.4000	0.2667	0.3444	0.2963	0.3272	0.3070	0.3203
0.1500	0.1750	0.1417	0.1694	0.1491	0.1633	0.1536	0.1601
0.1500	0.1750	0.1417	0.1694	0.1491	0.1633	0.1536	0.1601
0.1500	0.0750	0.1250	0.0917	0.1139	0.0991	0.1090	0.1024
0.1500	0.0750	0.1250	0.0917	0.1139	0.0991	0.1090	0.1024
0.1500	0.0750	0.1250	0.0917	0.1139	0.0991	0.1090	0.1024
0.1500	0.0750	0.1250	0.0917	0.1139	0.0991	0.1090	0.1024

Eigenvector based approach to calculate PageRank scores for each web page [1.00pt]

Input matrix:

```
0.000 0.330 0.333 0.500 0.500 0.500 0.500
0.167 0.000 0.000 0.500 0.500 0.000 0.000
0.167 0.000 0.000 0.000 0.000 0.500 0.500
0.167 0.333 0.000 0.000 0.000 0.000 0.000
0.167 0.333 0.000 0.000 0.000 0.000 0.000
0.167 0.000 0.333 0.000 0.000 0.000 0.000
0.167 0.000 0.333 0.000 0.000 0.000 0.000
```

Eigenvalues Eigenvectors:

Eigenvalues:

```
( 1.000, 0.000i)
(-0.333, 0.000i)
(-0.667, 0.000i)
(-0.577, 0.000i)
( 0.577, 0.000i)
( 0.000, 0.000i)
( 0.000, 0.000i)
```

Eigenvectors:

```
(-0.717, 0.000i) (-0.816, 0.000i) (-0.634, 0.000i) ( 0.000, 0.000i) ( 0.000, 0.000i) ( 0.000, 0.000i) ( 0.000, 0.000i)
(-0.359, 0.000i) ( 0.409, 0.000i) (-0.315, 0.000i) ( 0.544, 0.000i) ( 0.549, 0.000i) ( 0.000, 0.000i) ( 0.000, 0.000i)
(-0.359, 0.000i) ( 0.409, 0.000i) (-0.315, 0.000i) (-0.551, 0.000i) (-0.547, 0.000i) ( 0.000, 0.000i) ( 0.000, 0.000i)
(-0.239, 0.000i) ( 0.000, 0.000i) ( 0.316, 0.000i) (-0.314, 0.000i) ( 0.317, 0.000i) (-0.707, 0.000i) ( 0.000, 0.000i)
(-0.239, 0.000i) ( 0.000, 0.000i) ( 0.316, 0.000i) (-0.314, 0.000i) ( 0.317, 0.000i) ( 0.707, 0.000i) ( 0.000, 0.000i)
(-0.239, 0.000i) ( 0.000, 0.000i) ( 0.316, 0.000i) ( 0.318, 0.000i) (-0.316, 0.000i) ( 0.000, 0.000i) (-0.707, 0.000i)
(-0.239, 0.000i) ( 0.000, 0.000i) ( 0.316, 0.000i) ( 0.318, 0.000i) (-0.316, 0.000i) ( 0.000, 0.000i) ( 0.707, 0.000i)
```

Question 3

Rooted PageRank to calculate similarity between each pair of nodes. Each time, the random walker has a probability 1 - a where (a = 0.2) to return back to an original node [1.00pt]

Identity Matrix

	A	B	C	D	E	F	G
A	1	0	0	0	0	0	0
B	0	1	0	0	0	0	0
C	0	0	1	0	0	0	0
D	0	0	0	1	0	0	0
E	0	0	0	0	1	0	0
F	0	0	0	0	0	1	0
G	0	0	0	0	0	0	1

1 – a

L
0.8
0.8
0.8
0.8
0.8
0.8
0.8

Degree Matrix * A

	A	B	C	D	E	F	G
A	0	0.167	0.167	0.167	0.167	0.167	0.167
B	0.333	0	0	0.333	0.333	0	0
C	0.333	0	0	0	0	0.333	0.333
D	0.500	0.500	0	0	0	0	0
E	0.500	0.500	0	0	0	0	0
F	0.500	0	0.500	0	0	0	0
G	0.500	0	0.500	0	0	0	0

=

Identity Matrix – (a * Degree Matrix * A)

	A	B	C	D	E	F	G
A	1.000	-0.033	-0.033	-0.033	-0.033	-0.033	-0.033
B	-0.067	1.000	0.000	-0.067	-0.067	0.000	0.000
C	-0.067	0.000	1.000	0.000	0.000	-0.067	-0.067
D	-0.100	-0.100	0.000	1.000	0.000	0.000	0.000
E	-0.100	-0.100	0.000	0.000	1.000	0.000	0.000
F	-0.100	0.000	-0.100	0.000	0.000	1.000	0.000
G	-0.100	0.000	-0.100	0.000	0.000	0.000	1.000

Inverse (Identity Matrix – (a * Degree Matrix * A))

	A	B	C	D	E	F	G
A	1.020	0.041	0.041	0.036	0.036	0.036	0.036
B	0.083	1.017	0.003	0.071	0.071	0.003	0.003
C	0.083	0.003	1.017	0.003	0.003	0.071	0.071
D	0.110	0.106	0.004	1.011	0.011	0.004	0.004
E	0.110	0.106	0.004	0.011	1.011	0.004	0.004
F	0.110	0.004	0.106	0.004	0.004	1.011	0.011
G	0.110	0.004	0.106	0.004	0.004	0.011	1.011

(1 – a) * Inverse (Identity Matrix – (a * Degree Matrix)

	A	B	C	D	E	F	G
A	0.816	0.033	0.033	0.029	0.029	0.029	0.029
B	0.066	0.814	0.002	0.057	0.057	0.002	0.002
C	0.066	0.002	0.814	0.002	0.002	0.057	0.057
D	0.088	0.085	0.003	0.809	0.009	0.003	0.003
E	0.088	0.085	0.003	0.009	0.809	0.003	0.003
F	0.088	0.003	0.085	0.003	0.003	0.809	0.009
G	0.088	0.003	0.085	0.003	0.003	0.009	0.809

Question 4

Jacarrd's Coefficient [0.25pt]

	1	2	3	4	5	6	7	8
1	0	1	1	1	1	0	0	0
6	0	0	1	1	0	0	1	0

Jacarrd's Coefficient Score (1 , 6) = 2 / 5 = 0.400

	1	2	3	4	5	6	7	8
1	0	1	1	1	1	0	0	0
7	0	0	0	1	1	1	0	1

Jacarrd's Coefficient Score (1 , 7) = 2 / 6 = 0.333

Adamic/Adar [0.25pt]

	1	2	3	4	5	6	7	8
1	0	1	1	1	1	0	0	0
6	0	0	1	1	0	0	1	0
3	1	1	0	1	1	1	0	0
4	1	1	1	0	0	1	1	0

Adamic Adar Score (1 , 6) = (1 / log(5)) + (1 / log(5)) = 1.431 + 1.431 = 2.862

	1	2	3	4	5	6	7	8
1	0	1	1	1	1	0	0	0
7	0	0	0	1	1	1	0	1
4	1	1	1	0	0	1	1	0
5	1	0	1	0	0	0	1	0

Adamic Adar Score (1 , 7) = (1 / log(5)) + (1 / log(3)) = 1.431 + 2.096 = 3.527

Preferential attachment (0.25pt)

	1	2	3	4	5	6	7	8
1	0	1	1	1	1	0	0	0
6	0	0	1	1	0	0	1	0

Preferential Attachment Score (1 , 6) = 4 * 3 = 12

	1	2	3	4	5	6	7	8
1	0	1	1	1	1	0	0	0
7	0	0	0	1	1	1	0	1

Preferential Attachment Score (1 , 7) = 4 * 4 = 16

Katz (with b=0.05) (0.25pt)

S-E	L	Count	B^L	(B^L)*Count
1-6	2	2	0.0025000	0.0050000
1-6	3	7	0.0001250	0.0008750
1-6	4	9	0.0000063	0.0000563
1-6	5	19	0.0000003	0.0000059
			Katz	0.00593719

Katz Score (1 , 6) = 0.050000 + 0.0008750 + 0.0000563 + 0.0000059 = 0.00593719

S-E	Len	Count	B^L	(B^L)*Count
1-7	2	2	0.0025000	0.0050000
1-7	3	5	0.0001250	0.0006250
1-7	4	11	0.0000063	0.0000688
1-7	5	16	0.0000003	0.0000050
			Katz	0.00569875

Katz Score (1 , 7) = 0.005000 + 0.006250 + 0.0000688 + 0.0000050 = 0.00569875

SimRank score with C=1 [0.50pt]

Node 1 = { 2 , 3 , 4 , 5 }
Node 6 = { 3 , 4 , 7 }
SimRank Score (1 , 6) = 2 / (4 * 3) = 2 / 12 = 0.1667

Node 1 = { 2 , 3 , 4 , 5 }
Node 7 = { 4 , 5 , 6 , 8 }
SimRank Score (1 , 7) = 2 / (4 * 4) = 2 / 16 = 0.1250

Question 5

Complete set of communities by using 3-cliquie [0.25pt]

{ 5 , 6 , 7 , 8 , 9 , 10 , 11 , 13 , 14 , 15 , 16 , 17 } = 12
{ 7 , 8 , 9 , 10 , 11 , 13 , 14 , 15 , 16 , 17 ,18 } = 11
{ 1 , 2 , 3 , 4 , 5 , 6 , 7 , 8 } = 8

Complete set of communities by 3-club [0.25pt]

{ 4 , 5 , 6 , 7 , 8 , 9 , 10 , 11 , 12 } = 9
{ 7 , 9 , 13 , 14 , 15 , 16 , 17 , 18 } = 8
{ 1 , 2 , 3 , 4 , 5 , 6 , 7 , 8 } = 8

Complete set of communities by 3-core [0.25pt]

{ 1 , 2 , 3 , 4 , 5 , 6 , 7 , 8 , 9 , 10 , 11 , 12 , 13 , 14 , 15 , 16 , 17 , 18 }

Geodesic distance between each pair of nodes, and use Multidimensional Scaling (MDS) to convert the network into a two dimensional space [1.25pt]

Identity matrix – (e * e^T)

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1	0.944	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056
2	-0.056	0.944	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056
3	-0.056	-0.056	0.944	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056
4	-0.056	-0.056	-0.056	0.944	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056
5	-0.056	-0.056	-0.056	-0.056	0.944	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056
6	-0.056	-0.056	-0.056	-0.056	-0.056	0.944	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056
7	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	0.944	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056
8	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	0.944	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056
9	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	0.944	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056
10	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	0.944	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056
11	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	0.944	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056
12	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	0.944	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056
13	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	0.944	-0.056	-0.056	-0.056	-0.056	-0.056
14	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	0.944	-0.056	-0.056	-0.056	-0.056
15	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	0.944	-0.056	-0.056	-0.056
16	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	0.944	-0.056	-0.056
17	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	0.944	-0.056
18	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	-0.056	0.944

Geodesic Distance Matrix Squared

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1	0	1	1	1	4	4	9	9	16	16	16	16	16	25	25	25	25	36
2	1	0	1	1	4	4	9	9	16	16	16	16	16	25	25	25	25	36
3	1	1	0	1	1	4	4	9	16	16	16	16	9	16	16	16	16	25
4	1	1	1	0	1	1	4	4	9	9	9	9	9	16	16	16	16	25
5	4	4	1	1	0	1	1	4	9	9	9	9	4	9	9	9	9	16
6	4	4	4	1	1	0	1	1	4	4	4	4	4	9	9	9	9	16
7	9	9	4	4	1	1	0	4	9	9	9	9	1	4	4	4	4	9
8	9	9	9	4	4	1	4	0	1	1	1	1	9	4	9	4	9	9
9	16	16	16	9	9	4	9	1	0	1	1	4	4	1	4	1	4	4
10	16	16	16	9	9	4	9	1	1	0	1	1	9	4	9	4	9	9
11	16	16	16	9	9	4	9	1	1	1	0	1	9	4	9	4	9	9
12	16	16	16	9	9	4	9	1	4	1	1	0	16	9	16	9	16	16
13	16	16	9	9	4	4	1	9	4	9	9	16	0	1	1	1	1	4
14	25	25	16	16	9	9	4	4	1	4	4	9	1	0	1	4	4	1
15	25	25	16	16	9	9	4	9	4	9	9	16	1	1	0	1	4	4
16	25	25	16	16	9	9	4	4	1	4	4	9	1	4	1	0	1	1
17	25	25	16	16	9	9	4	9	4	9	9	16	1	4	4	1	0	1
18	36	36	25	25	16	16	9	9	4	9	9	16	4	1	4	1	1	0

Input matrix:

```
  9.515  9.015  7.293  6.293  3.710  3.154  0.821  0.654 -2.401 -1.762 -1.762 -0.623 -2.123 -5.985 -5.290 -6.068 -5.290 -9.151
  9.015  9.515  7.293  6.293  3.710  3.154  0.821  0.654 -2.401 -1.762 -1.762 -0.623 -2.123 -5.985 -5.290 -6.068 -5.290 -9.151
  7.293  7.293  6.071  4.571  3.488  1.432  1.599 -1.068 -4.123 -3.485 -3.485 -2.346 -0.346 -3.207 -2.512 -3.290 -2.512 -5.373
  6.293  6.293  4.571  4.071  2.488  1.932  0.599  0.432 -1.623 -0.985 -0.985  0.154 -1.346 -4.207 -3.512 -4.290 -3.512 -6.373
  3.710  3.710  3.488  2.488  1.904  0.849  1.015 -0.651 -2.707 -2.068 -2.068 -0.929  0.071 -1.790 -1.096 -1.873 -1.096 -2.957
  3.154  3.154  1.432  1.932  0.849  0.793  0.460  0.293 -0.762 -0.123 -0.123  1.015 -0.485 -2.346 -1.651 -2.429 -1.651 -3.512
  0.821  0.821  1.599  0.599  1.015  0.460  1.127 -1.040 -3.096 -2.457 -2.457 -1.318  1.182  0.321  1.015  0.238  1.015  0.154
  0.654  0.654 -1.068  0.432 -0.651  0.293 -1.040  0.793  0.738  1.377  1.377  2.515 -2.985  0.154 -1.651  0.071 -1.651 -0.012
 -2.401 -2.401 -4.123 -1.623 -2.707 -0.762 -3.096  0.738  1.682  1.821  1.821  1.460 -0.040  2.099  1.293  2.015  1.293  2.932
 -1.762 -1.762 -3.485 -0.985 -2.068 -0.123 -2.457  1.377  1.821  2.960  2.460  3.599 -1.901  1.238 -0.568  1.154 -0.568  1.071
 -1.762 -1.762 -3.485 -0.985 -2.068 -0.123 -2.457  1.377  1.821  2.460  2.960  3.599 -1.901  1.238 -0.568  1.154 -0.568  1.071
 -0.623 -0.623 -2.346  0.154 -0.929  1.015 -1.318  2.515  1.460  3.599  3.599  5.238 -4.262 -0.123 -2.929 -0.207 -2.929 -1.290
 -2.123 -2.123 -0.346 -1.346  0.071 -0.485  1.182 -2.985 -0.040 -1.901 -1.901 -4.262  2.238  2.377  3.071  2.293  3.071  3.210
 -5.985 -5.985 -3.207 -4.207 -1.790 -2.346  0.321  0.154  2.099  1.238  1.238 -0.123  2.377  3.515  3.710  1.432  2.210  5.349
 -5.290 -5.290 -2.512 -3.512 -1.096 -1.651  1.015 -1.651  1.293 -0.568 -0.568 -2.929  3.071  3.710  4.904  3.627  2.904  4.543
 -6.068 -6.068 -3.290 -4.290 -1.873 -2.429  0.238  0.071  2.015  1.154  1.154 -0.207  2.293  1.432  3.627  3.349  3.627  5.265
 -5.290 -5.290 -2.512 -3.512 -1.096 -1.651  1.015 -1.651  1.293 -0.568 -0.568 -2.929  3.071  2.210  2.904  3.627  4.904  6.043
 -9.151 -9.151 -5.373 -6.373 -2.957 -3.512  0.154 -0.012  2.932  1.071  1.071 -1.290  3.210  5.349  4.543  5.265  6.043  8.182
```

Eigenvalues Eigenvectors:

Eigenvalues:

```
(54.072, 0.000i)
(20.380, 0.000i)
(-5.054, 0.000i)
(-3.050, 0.000i)
( 3.463, 0.000i)
( 3.010, 0.000i)
( 1.890, 0.000i)
(-1.512, 0.000i)
( 1.318, 0.000i)
(-1.167, 0.000i)
(-0.563, 0.000i)
(-0.391, 0.000i)
(-0.156, 0.000i)
( 0.000, 0.000i)
( 0.033, 0.000i)
( 0.448, 0.000i)
( 0.500, 0.000i)
( 0.500, 0.000i)
```

Identity matrix – (e * e^T) * Geodesic Distance Matrix Squared

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1	9.515	9.015	7.293	6.293	3.710	3.154	0.821	0.654	-2.401	-1.762	-1.762	-0.623	-2.123	-5.985	-5.290	-6.068	-5.290	-9.151
2	9.015	9.515	7.293	6.293	3.710	3.154	0.821	0.654	-2.401	-1.762	-1.762	-0.623	-2.123	-5.985	-5.290	-6.068	-5.290	-9.151
3	7.293	7.293	6.071	4.571	3.488	1.432	1.599	-1.068	-4.123	-3.485	-3.485	-2.346	-0.346	-3.207	-2.512	-3.290	-2.512	-5.373
4	6.293	6.293	4.571	4.071	2.488	1.932	0.599	0.432	-1.623	-0.985	-0.985	0.154	-1.346	-4.207	-3.512	-4.290	-3.512	-6.373
5	3.710	3.710	3.488	2.488	1.904	0.849	1.015	-0.651	-2.707	-2.068	-2.068	-0.929	0.071	-1.790	-1.096	-1.873	-1.096	-2.957
6	3.154	3.154	1.432	1.932	0.849	0.793	0.460	0.293	-0.762	-0.123	-0.123	1.015	-0.485	-2.346	-1.651	-2.429	-1.651	-3.512
7	0.821	0.821	1.599	0.599	1.015	0.460	1.127	-1.040	-3.096	-2.457	-2.457	-1.318	1.182	0.321	1.015	0.238	1.015	0.154
8	0.654	0.654	-1.068	0.432	-0.651	0.293	-1.040	0.793	0.738	1.377	1.377	2.515	-2.985	0.154	-1.651	0.071	-1.651	-0.012
9	-2.401	-2.401	-4.123	-1.623	-2.707	-0.762	-3.096	0.738	1.682	1.821	1.821	1.460	-0.040	2.099	1.293	2.015	1.293	2.932
10	-1.762	-1.762	-3.485	-0.985	-2.068	-0.123	-2.457	1.377	1.821	2.960	2.460	3.599	-1.901	1.238	-0.568	1.154	-0.568	1.071
11	-1.762	-1.762	-3.485	-0.985	-2.068	-0.123	-2.457	1.377	1.821	2.460	2.960	3.599	-1.901	1.238	-0.568	1.154	-0.568	1.071
12	-0.623	-0.623	-2.346	0.154	-0.929	1.015	-1.318	2.515	1.460	3.599	3.599	5.238	-4.262	-0.123	-2.929	-0.207	-2.929	-1.290
13	-2.123	-2.123	-0.346	-1.346	0.071	-0.485	1.182	-2.985	-0.040	-1.901	-1.901	-4.262	2.238	2.377	3.071	2.293	3.071	3.210
14	-5.985	-5.985	-3.207	-4.207	-1.790	-2.346	0.321	0.154	2.099	1.238	1.238	-0.123	2.377	3.515	3.710	1.432	2.210	5.349
15	-5.290	-5.290	-2.512	-3.512	-1.096	-1.651	1.015	-1.651	1.293	-0.568	-0.568	-2.929	3.071	3.710	4.904	3.627	2.904	4.543
16	-6.068	-6.068	-3.290	-4.290	-1.873	-2.429	0.238	0.071	2.015	1.154	1.154	-0.207	2.293	1.432	3.627	3.349	3.627	5.265
17	-5.290	-5.290	-2.512	-3.512	-1.096	-1.651	1.015	-1.651	1.293	-0.568	-0.568	-2.929	3.071	2.210	2.904	3.627	4.904	6.043
18	-9.151	-9.151	-5.373	-6.373	-2.957	-3.512	0.154	-0.012	2.932	1.071	1.071	-1.290	3.210	5.349	4.543	5.265	6.043	8.182

Eigenvalues Eigenvectors:

Eigenvalues:

```
(54.072, 0.000i)
(20.380, 0.000i)
(-5.054, 0.000i)
(-3.050, 0.000i)
( 3.463, 0.000i)
( 3.010, 0.000i)
( 1.890, 0.000i)
(-1.512, 0.000i)
( 1.318, 0.000i)
(-1.167, 0.000i)
(-0.563, 0.000i)
(-0.391, 0.000i)
(-0.156, 0.000i)
( 0.000, 0.000i)
( 0.033, 0.000i)
( 0.448, 0.000i)
( 0.500, 0.000i)
( 0.500, 0.000i)
```

Eigenvectors:

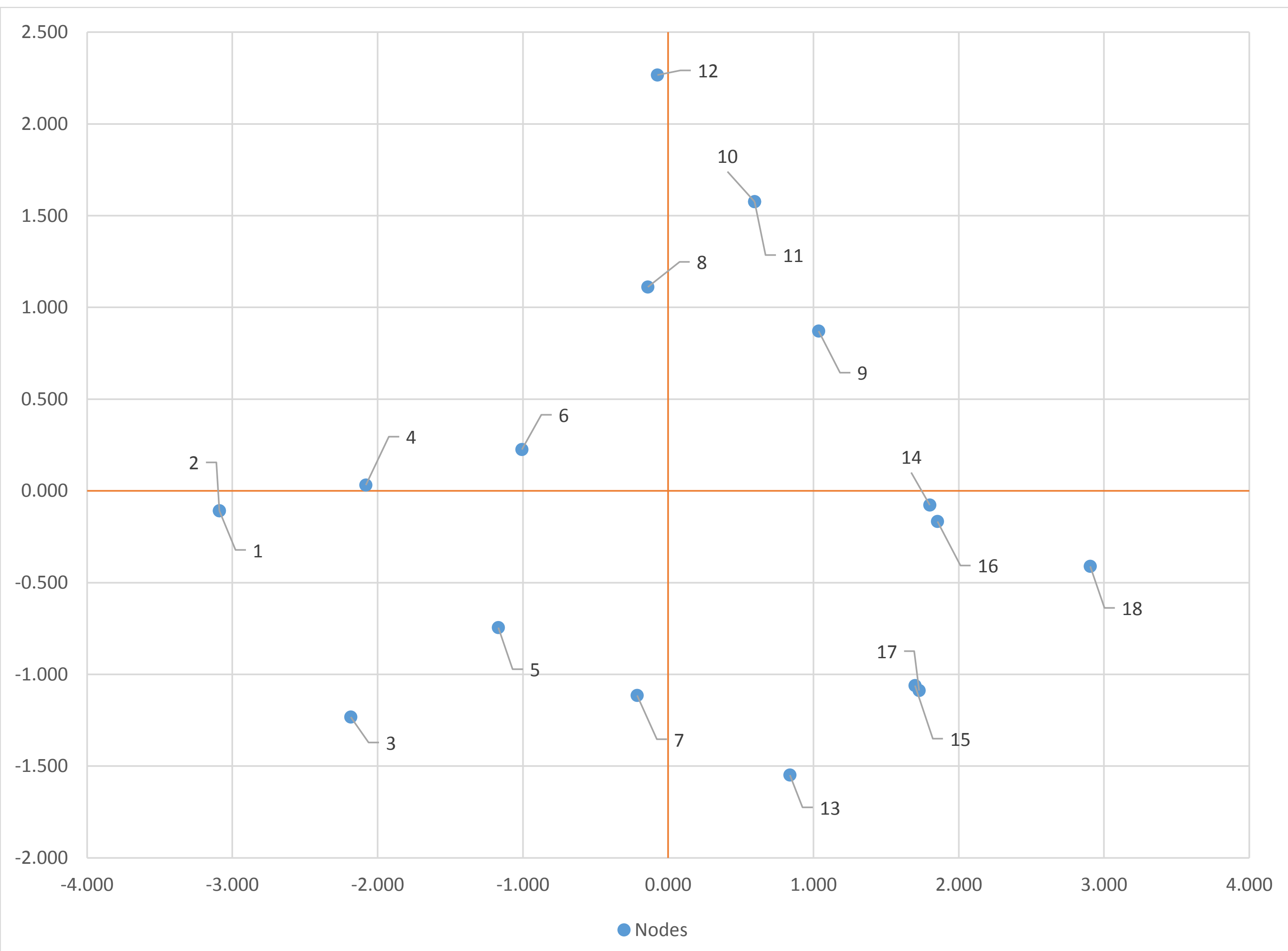
```
(-0.420, 0.000i) (-0.024, 0.000i) ( 0.243, 0.000i) (-0.113, 0.000i) (-0.195, 0.000i) (-0.002, 0.000i) ( 0.084, 0.000i) (-0.159, 0.000i)
(-0.420, 0.000i) (-0.024, 0.000i) ( 0.243, 0.000i) (-0.113, 0.000i) (-0.195, 0.000i) (-0.002, 0.000i) ( 0.084, 0.000i) (-0.159, 0.000i)
(-0.297, 0.000i) (-0.273, 0.000i) (-0.255, 0.000i) ( 0.079, 0.000i) ( 0.155, 0.000i) ( 0.024, 0.000i) ( 0.222, 0.000i) ( 0.070, 0.000i)
(-0.283, 0.000i) ( 0.007, 0.000i) ( 0.199, 0.000i) ( 0.064, 0.000i) (-0.100, 0.000i) (-0.002, 0.000i) (-0.062, 0.000i) ( 0.008, 0.000i)
(-0.159, 0.000i) (-0.165, 0.000i) (-0.130, 0.000i) ( 0.186, 0.000i) ( 0.229, 0.000i) ( 0.027, 0.000i) (-0.010, 0.000i) (-0.004, 0.000i)
(-0.137, 0.000i) ( 0.050, 0.000i) ( 0.142, 0.000i) ( 0.265, 0.000i) (-0.032, 0.000i) (-0.009, 0.000i) (-0.286, 0.000i) ( 0.146, 0.000i)
(-0.029, 0.000i) (-0.247, 0.000i) (-0.344, 0.000i) ( 0.294, 0.000i) ( 0.477, 0.000i) ( 0.052, 0.000i) (-0.124, 0.000i) (-0.156, 0.000i)
(-0.019, 0.000i) ( 0.246, 0.000i) (-0.229, 0.000i) (-0.288, 0.000i) ( 0.121, 0.000i) ( 0.072, 0.000i) ( 0.262, 0.000i) ( 0.447, 0.000i)
( 0.141, 0.000i) ( 0.193, 0.000i) (-0.353, 0.000i) ( 0.361, 0.000i) (-0.527, 0.000i) (-0.074, 0.000i) ( 0.088, 0.000i) (-0.061, 0.000i)
( 0.081, 0.000i) ( 0.349, 0.000i) (-0.169, 0.000i) ( 0.063, 0.000i) (-0.098, 0.000i) (-0.027, 0.000i) (-0.059, 0.000i) (-0.066, 0.000i)
( 0.081, 0.000i) ( 0.349, 0.000i) (-0.169, 0.000i) ( 0.063, 0.000i) (-0.098, 0.000i) (-0.027, 0.000i) (-0.059, 0.000i) (-0.066, 0.000i)
(-0.010, 0.000i) ( 0.502, 0.000i) ( 0.074, 0.000i) (-0.379, 0.000i) ( 0.435, 0.000i) ( 0.064, 0.000i) (-0.147, 0.000i) (-0.154, 0.000i)
( 0.114, 0.000i) (-0.343, 0.000i) (-0.264, 0.000i) (-0.463, 0.000i) (-0.226, 0.000i) (-0.116, 0.000i) (-0.075, 0.000i) ( 0.159, 0.000i)
( 0.245, 0.000i) (-0.017, 0.000i) ( 0.378, 0.000i) ( 0.181, 0.000i) ( 0.172, 0.000i) (-0.515, 0.000i) ( 0.447, 0.000i) ( 0.352, 0.000i)
( 0.231, 0.000i) (-0.234, 0.000i) ( 0.024, 0.000i) (-0.251, 0.000i) (-0.030, 0.000i) (-0.517, 0.000i) (-0.426, 0.000i) (-0.218, 0.000i)
( 0.252, 0.000i) (-0.037, 0.000i) ( 0.364, 0.000i) ( 0.241, 0.000i) ( 0.014, 0.000i) ( 0.301, 0.000i) (-0.413, 0.000i) ( 0.271, 0.000i)
( 0.235, 0.000i) (-0.241, 0.000i) ( 0.066, 0.000i) (-0.200, 0.000i) (-0.154, 0.000i) ( 0.549, 0.000i) ( 0.063, 0.000i) ( 0.188, 0.000i)
( 0.395, 0.000i) (-0.091, 0.000i) ( 0.181, 0.000i) ( 0.010, 0.000i) ( 0.051, 0.000i) ( 0.203, 0.000i) ( 0.411, 0.000i) (-0.598, 0.000i)
```

Calculation of S1 and S2

	1	2
1	-0.420	-0.024
2	-0.420	-0.024
3	-0.297	-0.273
4	-0.283	0.007
5	-0.159	-0.165
6	-0.137	0.050
7	-0.029	-0.247
8	-0.019	0.246
9	0.141	0.193
10	0.081	0.349
11	0.081	0.349
12	-0.010	0.502
13	0.114	-0.343
14	0.245	-0.017
15	0.231	-0.235
16	0.252	-0.037
17	0.235	-0.241
18	0.395	-0.091

	1	2
1	7.353	0
2	0	4.514

	S1	S2
1	-3.088	-0.108
2	-3.088	-0.108
3	-2.184	-1.232
4	-2.081	0.032
5	-1.169	-0.745
6	-1.007	0.226
7	-0.213	-1.115
8	-0.140	1.111
9	1.037	0.871
10	0.596	1.576
11	0.596	1.576
12	-0.074	2.266
13	0.838	-1.548
14	1.802	-0.077
15	1.699	-1.061
16	1.853	-0.167
17	1.728	-1.088
18	2.905	-0.411



Implement a k-means clustering algorithm (selecting k=2 and using node 18 and node 1 as the initial centers), and report the community structures after 10 iterations [2.00pt]

					Iteration 01				Iteration 02			
	s1	s2			c1	c2			c1	c2		
1	-3.088	-0.108	-3.088	-0.108	0.000	6.001	-1.449	0.036	1.645	4.539	-1.449	0.036
2	-3.088	-0.108		1	0.000	6.001	1	9	1.645	4.539	1	9
3	-2.184	-1.232			1.442	5.155	1		1.466	3.826	1	
4	-2.081	0.032			1.017	5.006	1		0.632	3.532	1	
5	-1.169	-0.745			2.022	4.088	1		0.830	2.714	1	
6	-1.007	0.226			2.108	3.964	1		0.481	2.471	1	
7	-0.213	-1.115			3.046	3.196	1		1.689	1.982	1	
8	-0.140	1.111			3.190	3.404	1		1.694	1.961	1	
9	1.037	0.871			4.240	2.266	2		2.623	0.997	2	
10	0.596	1.576			4.051	3.046	2		2.560	1.825	2	
11	0.596	1.576			4.051	3.046	2		2.560	1.825	2	
12	-0.074	2.266			3.837	4.005	1		2.620	2.761	1	
13	0.838	-1.548			4.182	2.359	2		2.782	1.631	2	
14	1.802	-0.077			4.890	1.152	2		3.253	0.354	2	
15	1.699	-1.061			4.881	1.370	2		3.334	1.054	2	
16	1.853	-0.167			4.941	1.080	2		3.309	0.423	2	
17	1.728	-1.088		1	4.915	1.358	2	9	3.370	1.087	2	9
18	2.905	-0.411	2.905	-0.411	6.001	0.000	1.450	-0.037	4.377	1.502	1.450	-0.037

	Iteration 03				Iteration 04				Iteration 05			
	c1	c2			c1	c2			c1	c2		
1	1.645	4.539	-1.449	-0.036	1.645	4.539	-1.449	-0.036	1.645	4.539	-1.449	-0.036
2	1.645	4.539	1	9	1.645	4.539	1	9	1.645	4.539	1	9
3	1.466	3.826	1		1.466	3.826	1		1.466	3.826	1	
4	0.632	3.532	1		0.632	3.532	1		0.632	3.532	1	
5	0.830	2.714	1		0.830	2.714	1		0.830	2.714	1	
6	0.481	2.471	1		0.481	2.471	1		0.481	2.471	1	
7	1.689	1.982	1		1.689	1.982	1		1.689	1.982	1	
8	1.694	1.961	1		1.694	1.961	1		1.694	1.961	1	
9	2.623	0.997	2		2.623	0.997	2		2.623	0.997	2	
10	2.560	1.825	2		2.560	1.825	2		2.560	1.825	2	
11	2.560	1.825	2		2.560	1.825	2		2.560	1.825	2	
12	2.620	2.761	1		2.620	2.761	1		2.620	2.761	1	
13	2.782	1.631	2		2.782	1.631	2		2.782	1.631	2	
14	3.253	0.354	2		3.253	0.354	2		3.253	0.354	2	
15	3.334	1.054	2		3.334	1.054	2		3.334	1.054	2	
16	3.309	0.423	2		3.309	0.423	2		3.309	0.423	2	
17	3.370	1.087	2	9	3.370	1.087	2	9	3.370	1.087	2	9
18	4.377	1.502	1.450	0.037	4.377	1.502	1.450	0.037	4.377	1.502	1.450	0.037

Iteration 06

	c1	c2		
1	1.645	4.539	-1.449	-0.036
2	1.645	4.539	1	9
3	1.466	3.826	1	
4	0.632	3.532	1	
5	0.830	2.714	1	
6	0.481	2.471	1	
7	1.689	1.982	1	
8	1.694	1.961	1	
9	2.623	0.997	2	
10	2.560	1.825	2	
11	2.560	1.825	2	
12	2.620	2.761	1	
13	2.782	1.631	2	
14	3.253	0.354	2	
15	3.334	1.054	2	
16	3.309	0.423	2	
17	3.370	1.087	2	9
18	4.377	1.502	1.450	0.037

Iteration 07

	c1	c2		
1	1.645	4.539	-1.449	-0.036
2	1.645	4.539	1	9
3	1.466	3.826	1	
4	0.632	3.532	1	
5	0.830	2.714	1	
6	0.481	2.471	1	
7	1.689	1.982	1	
8	1.694	1.961	1	
9	2.623	0.997	2	
10	2.560	1.825	2	
11	2.560	1.825	2	
12	2.620	2.761	1	
13	2.782	1.631	2	
14	3.253	0.354	2	
15	3.334	1.054	2	
16	3.309	0.423	2	
17	3.370	1.087	2	9
18	4.377	1.502	1.450	0.037

Iteration 08

	c1	c2		
1	1.645	4.539	-1.449	-0.036
2	1.645	4.539	1	9
3	1.466	3.826	1	
4	0.632	3.532	1	
5	0.830	2.714	1	
6	0.481	2.471	1	
7	1.689	1.982	1	
8	1.694	1.961	1	
9	2.623	0.997	2	
10	2.560	1.825	2	
11	2.560	1.825	2	
12	2.620	2.761	1	
13	2.782	1.631	2	
14	3.253	0.354	2	
15	3.334	1.054	2	
16	3.309	0.423	2	
17	3.370	1.087	2	9
18	4.377	1.502	1.450	0.037

Iteration 09

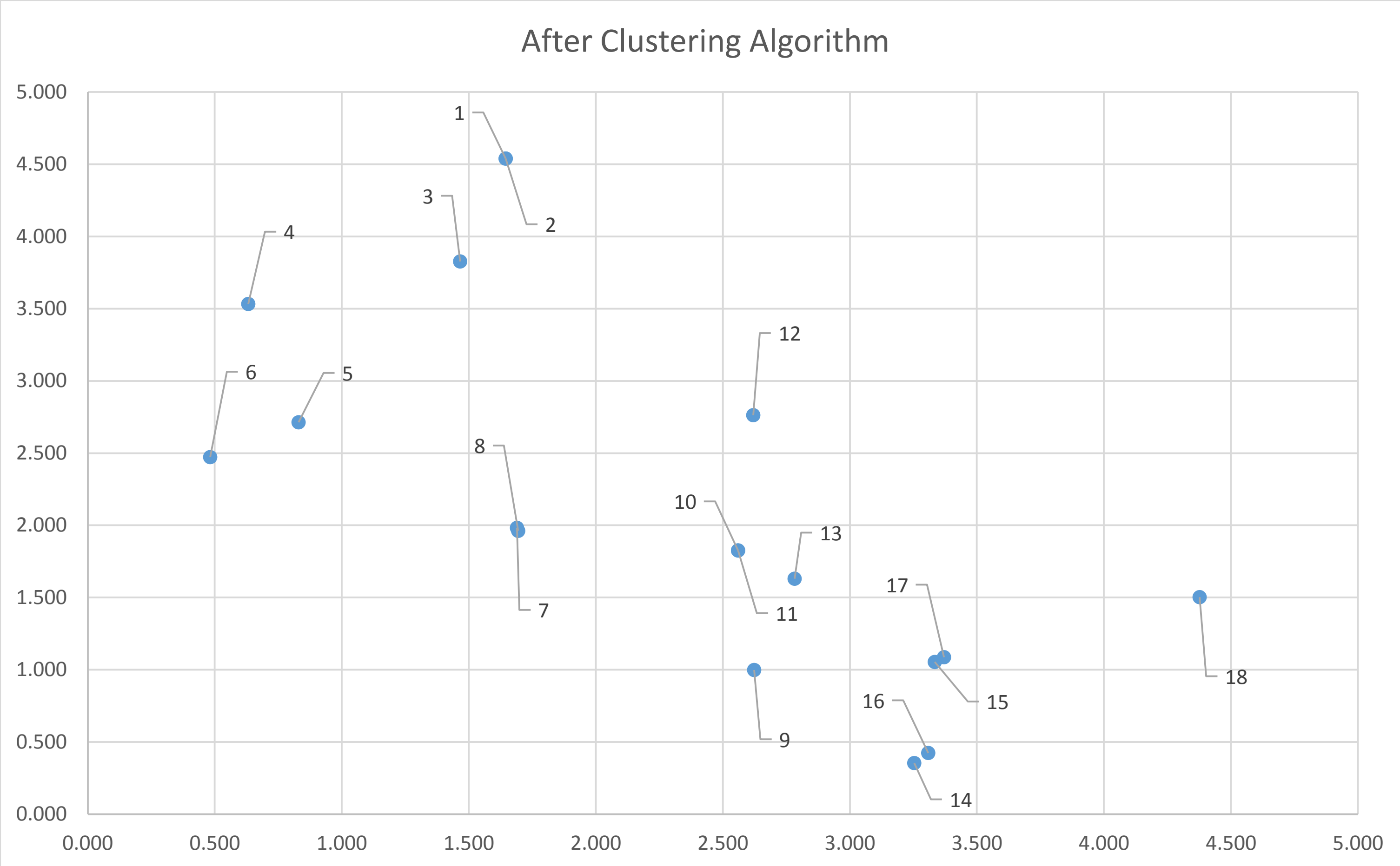
	c1	c2		
1	1.645	4.539	-1.449	-0.036
2	1.645	4.539	1	9
3	1.466	3.826	1	
4	0.632	3.532	1	
5	0.830	2.714	1	
6	0.481	2.471	1	
7	1.689	1.982	1	
8	1.694	1.961	1	
9	2.623	0.997	2	
10	2.560	1.825	2	
11	2.560	1.825	2	
12	2.620	2.761	1	
13	2.782	1.631	2	
14	3.253	0.354	2	
15	3.334	1.054	2	
16	3.309	0.423	2	
17	3.370	1.087	2	9
18	4.377	1.502	1.450	0.037

Iteration 10

	c1	c2		
1	1.645	4.539	-1.449	-0.036
2	1.645	4.539	1	9
3	1.466	3.826	1	
4	0.632	3.532	1	
5	0.830	2.714	1	
6	0.481	2.471	1	
7	1.689	1.982	1	
8	1.694	1.961	1	
9	2.623	0.997	2	
10	2.560	1.825	2	
11	2.560	1.825	2	
12	2.620	2.761	1	
13	2.782	1.631	2	
14	3.253	0.354	2	
15	3.334	1.054	2	
16	3.309	0.423	2	
17	3.370	1.087	2	9
18	4.377	1.502	1.450	0.037

C1 – (- 1.449 , - 0.036)

C2 – (1.450 , 0.037)



After Clustering Algorithm
5.000
4.500
4.000
3.500
3.000
2.500
2.000
1.500
1.000
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0.000
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