COT6405 Analysis of Algorithms
Spring 16

Assignment 1

Posted on Jan 26, due on Feb 9

Maximum total 70 points

10 points

1. Find a Θ notation in terms of *n* for the number of times the statement x = x + 1 is executed in the segment:

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for i = 1 to n
   for j = 1 to i
       for k = 1 to i
           x = x + 1
```

10 points

2. Arrange the following functions in ascending order of growth rate. That is, if function g(n) immediately follows function f(n) in your list, then it should be the case that f(n) = O(g(n)).

$$f_1(n) = \left(\frac{1}{3}\right)^n + (\log_3 n)^{100}$$

$$f_2(n) = n^2 (\log_2 n)^3$$

$$f_3(n) = \sqrt{8n^3}$$

$$f_4(n) = \left(\frac{1}{5}\right)^n + n!$$

$$f_5(n) = 50n - 100 + n^2 \log_2 n$$

$$f_6(n) = 4^{\log_2 n} - 1000$$

20 points

3. Use the formal definitions to show that:
   - $2n^3 - 25n = \Omega(n^2)$. Find some positive values c and n_0.
   - $5n^2 - 8 = o(n^3)$. Find n_0.

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20 points

4. Solve the following recurrences:

    a. $T(n) = 3T(n-1) + 2$ for $n > 1$, $T(1) = 4$.
    Solve using backward substitution.

    b. $T(n) = 2T(\sqrt[3]{n^2}) + (\log_2 n)^3$. Use the change of variable $m = \log_2 n$

10 points

5. There are n bacteria and 1 virus in a Petri dish. Within the first minute, the virus kills one bacterium and produces another copy of itself, and all of the remaining bacteria reproduce, making 2 viruses and $2(n–1)$ bacteria. In the second minute, each of the viruses kills a bacterium and produces a new copy of itself, resulting in 4 viruses and $2(2(n–1) –2) = 4n – 8$ bacteria; again, the remaining bacteria reproduce. This process continues every minute.
    - Derive formulas to compute the number of viruses and the number of bacteria in the k^{th} minute. These formulas are functions of k and n.
    - Will the viruses eventually kill all bacteria? If so, how many steps (minutes) will it take?