MACIEJ SIKORA 17.03 gr1 (A)

[3] c.d.

$$\det(AB) = \det\left(\begin{bmatrix}1&3\\2&4\end{bmatrix}\begin{bmatrix}1&2\\0&6\end{bmatrix}\right) =$$

$$\begin{array}{cc|cc} & & 1 & 2 \\ & & 0 & 6 \\ \hline 1 & 3 & 25 & 20 \\ 2 & 4 & 34 & 28 \end{array} = \det\left(\begin{bmatrix}25&20\\34&28\end{bmatrix}\right) =$$

$1+24$
$2+18$
$2+32$
$4+24$

$$25\cdot 28 - 20\cdot 34 = 700 - 680 = 20$$

$$\begin{array}{r} 25 \\ \cdot\ 28 \\ \hline 200 \\ +\ 50 \\ \hline 700 \end{array}$$

$$L = P$$

[4] $\det(\mathbb{I} + uv^T) = 1 + u^T v$

$$u = \begin{bmatrix}3\\5\end{bmatrix} \quad v = \begin{bmatrix}3\\4\end{bmatrix}$$

$$\begin{array}{r} 34 \\ \cdot\ 20 \\ \hline 680 \end{array}$$

$$L = \det\left(\begin{bmatrix}1&0\\0&1\end{bmatrix} + \begin{bmatrix}3\\5\end{bmatrix}\begin{bmatrix}3&4\end{bmatrix}\right) = \det\left(\begin{bmatrix}1&0\\0&1\end{bmatrix} + \begin{bmatrix}9&12\\15&20\end{bmatrix}\right) =$$

$$\begin{array}{cc|cc} & & 3 & 4 \\ \hline 3 & 9 & 12 \\ 5 & 15 & 20 \end{array} = \det\left(\begin{bmatrix}10&12\\15&21\end{bmatrix}\right) = 10\cdot 21 - 12\cdot 15 = 30$$

210
$-\ 180$
30

$$P = 1 + \begin{bmatrix}3&5\end{bmatrix}\begin{bmatrix}3\\4\end{bmatrix} = 1 + 9 + 20 = 30$$

$$\begin{array}{r} 12 \\ \cdot\ 15 \\ \hline 60 \\ 12 \\ \hline 180 \end{array}$$

$$L = P$$

1/4

MACIEJ SIKORA 17.07 gr. 1 (A)

1 $\det(cA) = c^n \det(A)$

$\det\begin{bmatrix} 2 & 6 \\ 4 & 2 \end{bmatrix} = \det\left(2\begin{bmatrix} 1 & 3 \\ 2 & 1 \end{bmatrix}\right) = 4 - 24 = -20$

$2^n \cdot \det\begin{bmatrix} 1 & 3 \\ 2 & 1 \end{bmatrix} = 2^n \cdot (1-6) = 2^n \cdot -5 = 4 \cdot (-5) = -20$

$n = 2$

2 $\det(A^T) = \det(A)$

$A = \begin{bmatrix} 2 & 8 \\ 2 & 4 \end{bmatrix}$

$\det(A) = \det\begin{bmatrix} 2 & 8 \\ 2 & 4 \end{bmatrix} = 8 - 16 = -8$

$\det(A^T) = \det\begin{bmatrix} 2 & 2 \\ 8 & 4 \end{bmatrix} = 8 - 16 = -8 \quad L = P$

3 $\det(AB) = \det(A)\det(B)$

$A = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 2 \\ 8 & 6 \end{bmatrix}$

$\det(A)\det(B) = (4-6)(6-16) = (-2)(-10) = 20$

MACIEJ SIKORA 17.07 gr 1 (A)

5 $\det(I + A) = 1 + \det(A) + \mathrm{Tr}(A)$

$A = \begin{bmatrix} 5 & 3 \\ 2 & 4 \end{bmatrix}$

$L = \det\left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 5 & 3 \\ 2 & 4 \end{bmatrix}\right) = \det\begin{bmatrix} 6 & 3 \\ 2 & 5 \end{bmatrix} = 30 - 6 = 24$

$P = 1 + \det\begin{pmatrix} 5 & 3 \\ 2 & 4 \end{pmatrix} + \mathrm{tr}\begin{pmatrix} 5 & 3 \\ 2 & 4 \end{pmatrix} =$

$1 + 20 - 6 + 9 = 24$

$L = P$

6 $\det(I + A) = 1 + \det(A) + \mathrm{Tr}(A) + \frac{1}{2}\mathrm{Tr}(A)^2 - \frac{1}{2}\mathrm{Tr}(A^2)$

$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \end{bmatrix}$

$L = \det I \det\left(\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 2 & 3 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \end{bmatrix}\right) = \det\left(\begin{bmatrix} 2 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 3 & 4 \end{bmatrix}\right) =$

$24 + 3 + 36 - 27 - 4 \cdot 4 - 4 \cdot 3 = 24 - 27 + 18 - 16 = -3 + 2 = -1$

3/4

MACIEJ SIKORA 17.03 gr 1 (A)

6 c.d.

~~$P = 1 + \det(A)$~~

$P = 1 + \det\begin{bmatrix}1&2&3\\2&2&2\\3&3&3\end{bmatrix} + \operatorname{Tr}\begin{pmatrix}1&2&3\\2&2&2\\3&3&3\end{pmatrix} + \frac{1}{2}\operatorname{Tr}\begin{pmatrix}1&2&3\\2&2&2\\3&3&3\end{pmatrix}^2$ ~~Tr~~

$- \frac{1}{2}\operatorname{Tr}\left(\begin{bmatrix}1&2&3\\2&2&2\\3&3&3\end{bmatrix}^2\right) = 1 + 6 + 12 + 18 - 18 - 12 - 6 + 6 +$

$\frac{1}{2}\cdot 6^2 - \frac{1}{2}\cdot 52 = 7 + \frac{36}{2} - 26 = 7 + 18 - 26 = 25 - 26 =$

1

$$\begin{matrix}1&2&3\\2&2&2\\3&3&3\end{matrix}$$

$$A^2 = \begin{array}{c|ccc} 1\,2\,3 & 14 & 15 & 16 \\ \hline 2\,2\,2 & 12 & 14 & 16 \\ \hline 3\,3\,3 & 18 & 21 & 24 \end{array}$$

$\operatorname{Tr}(A^2) = 14 + 14 + 24 = 52$

$L = P$

$1+4+9$

$2+4+9$

$3+4+9$

~~$4+4+4$~~

~~$4+4+6$~~

$2+4+6$

$4\ 4\ 6$

$6\ 4\ 6$

$3\ 6\ 9$

$6\ 6\ 9$

$9\ 6\ 9$

4/4