

MACIEJ SIKORA

Gm (1)

(A)

Zad 1

$$(AB)^T = B^T A^T$$

$$L = \begin{pmatrix} \begin{bmatrix} 1 & 3 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} 2 & 6 \\ 2 & 3 \end{bmatrix} \end{pmatrix}^T = \begin{matrix} 2 & 6 \\ 2 & 3 \\ 13 & 8 & 15 \\ 22 & 8 & 78 \end{matrix} \begin{pmatrix} \begin{bmatrix} 8 & 15 \\ 8 & 18 \end{bmatrix} \end{pmatrix}^T = \begin{bmatrix} 8 & 8 \\ 15 & 18 \end{bmatrix}$$

$$12 + 6 = 18$$

$$\begin{matrix} 2 & 2 \\ 6 & 3 \end{matrix}$$

$$\begin{matrix} 12 \\ 36 \end{matrix}$$

$$2 + 12 = 14$$

$$P = \begin{bmatrix} 1 & 3 \\ 2 & 2 \end{bmatrix}^T \begin{bmatrix} 2 & 6 \\ 2 & 3 \end{bmatrix}^T = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} 2 & 2 \\ 6 & 3 \end{bmatrix} =$$

$$P = \begin{bmatrix} 2 & 6 \\ 2 & 3 \end{bmatrix}^T \begin{bmatrix} 1 & 3 \\ 2 & 2 \end{bmatrix}^T = \begin{bmatrix} 2 & 2 \\ 6 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} 8 & 8 \\ 15 & 18 \end{bmatrix}$$

$$\begin{matrix} 1 & 2 \\ 2 & 2 \\ 22 & 8 \\ 63 & 45 & 78 \end{matrix}$$

$$L = P$$

Zad 2

$$(A+B)^H = A^H + B^H$$

$$L = \begin{pmatrix} \begin{bmatrix} 1+3i & 2+6i \\ 2+2i & 2+3i \end{bmatrix} \begin{bmatrix} 11+13i & 12+16i \\ 12+12i & 12+13i \end{bmatrix} \end{pmatrix}^H \Rightarrow$$

$$\begin{matrix} 11+13i & 12+16i \\ 12+12i & 12+13i \end{matrix}$$

$$-76 + 142i$$

$$-42 + 150i$$

$$\begin{matrix} (1+3i)(11+13i) = \\ 11 + 13i + 33i - 39 = \\ -28 + 46i \end{matrix}$$

$$\begin{matrix} 1+3i & 2+6i \\ 2+2i & 2+3i \end{matrix}$$

$$(1+3i)(12+16i) + (2+6i)(12+13i) = 12 + 16i + 36i - 48 + 24 + 72i - 78 = -42 + 150i$$

$$\begin{matrix} (2+6i)(12+12i) = \\ 24 + 24i + 72i - 72 = \\ -48 + 96i \end{matrix}$$

QVE

$$\boxed{ZAD2}$$

$$(A+B)^H = A^H + B^H$$

$$L = \begin{bmatrix} 1+3i & 2+6i \\ 2+2i & 2+3i \end{bmatrix} + \begin{bmatrix} 11+13i & 12+16i \\ 12+12i & 12+13i \end{bmatrix}^H =$$

$$\begin{bmatrix} 12+16i & 14+22i \\ 14+14i & 14+16i \end{bmatrix}^H = \begin{bmatrix} 12-16i & 14-22i \\ 14-14i & 14-16i \end{bmatrix}$$

$$P = \begin{bmatrix} 1+3i & 2+6i \\ 2+2i & 2+3i \end{bmatrix}^H + \begin{bmatrix} 11+13i & 12+16i \\ 12+12i & 12+13i \end{bmatrix}^H =$$

$$\begin{bmatrix} 1-3i & 2-6i \\ 2-2i & 2-3i \end{bmatrix} + \begin{bmatrix} 11-13i & 12-16i \\ 12-12i & 12-13i \end{bmatrix} =$$

$$\begin{bmatrix} 12-16i & 14-14i \\ 14-22i & 14-16i \end{bmatrix} = L$$

A

ZAD 3

$$\boxed{\text{ZAD 3}} \quad (AB)^H = B^H A^H$$

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$$C = \begin{pmatrix} 1+3i & 5+7i \\ 2+4i & 6+8i \end{pmatrix} \begin{pmatrix} 11+13i & 12+14i \\ 15+17i & 16+18i \end{pmatrix} \quad (H)$$

$$\begin{array}{r} 14 \\ 76 \\ \hline 50 \\ 50 \\ 40 \\ \hline 140 \\ 70 \\ \hline 210 \end{array} \quad \begin{array}{l} (1+3i)(11+13i) + (5+7i)(15+17i) = \\ 11+13i+33i+39 + 50+25+50i+35i + \cancel{10i} 35i + \cancel{70i} -49 = \\ -28 + 46i + 75 - 70 - 49 + 155i + 35i = \\ -28 + 46i + 5 - 49 + 190i = \underline{-72 + 236i} \end{array}$$

$$\frac{+71}{72} \quad (1+3i)(12+14i) + (5+7i)(16+18i) =$$

$$\frac{12+14i+36i-42}{72} + \frac{50+30+50i+40i+70i+126}{72} =$$

$$\frac{-76+252i}{72} =$$

$$\begin{array}{r} 76 = \\ \cancel{14} \\ \cancel{24} \\ \cancel{14} \\ \cancel{12} \\ 12 \\ -42 \\ \hline -30 \\ +50 \\ \hline 20 \\ +30 \\ \hline 50 \\ 70 \\ +56 \\ \hline 126 \\ -126 \\ \hline 826 \\ -126 \\ \hline 700 \\ -50 \\ \hline 650 \end{array}$$
$$-76 + 252i$$
$$(2+4i)(11+13i) + (6+8i)(15+17i) =$$
$$\begin{array}{l} 22 + 26i + 44i - 52 \\ + 90 + 60i + 102i + 120i - 80 - 56 = \end{array}$$
$$32 - 108 + 172i + 120i = -76 + 292i$$

① ZAD 3

c.d. MACIEJ SIKORA

36
12
48
108
80

$$(2+4i)(12+14i) + (6+8i)(16+18i) =$$

$$\underline{24} + \underline{28i} + \underline{48i} - \underline{56} + \underline{96} + \underline{108i} + \underline{128i} - \underline{144} =$$

$$\underline{-80 + 312i}$$

24
24
128
80

24
+ 96
120
- 56
64

$$L = \begin{bmatrix} -72 + 236i & -76 + 252i \\ -76 + 292i & -80 + 312i \end{bmatrix}^H =$$

144
- 64
80

$$\begin{bmatrix} -72 - 236i & -76 - 292i \\ -76 - 252i & -80 - 312i \end{bmatrix}$$

76
108
184
128
312

$$P = \begin{bmatrix} 11 + 13i & 12 + 14i \\ 15 + 17i & 16 + 18i \end{bmatrix}^H \begin{bmatrix} 1 + 3i & 5 + 7i \\ 2 + 4i & 6 + 8i \end{bmatrix}^H =$$

$$\begin{bmatrix} 11 - 13i & 15 - 17i \\ 12 - 14i & 16 - 18i \end{bmatrix} \begin{bmatrix} 1 - 3i & 2 - 4i \\ 5 - 7i & 6 - 8i \end{bmatrix} =$$

$$(11 - 13i)(1 - 3i) + (15 - 17i)(5 - 7i) = -72 + 236i$$

$$(11 - 13i)(2 - 4i) + (15 - 17i)(6 - 8i) = -76 - 292i$$

(A)

ZAD 3

$$(12-14i)(1-3i) + (16-18i)(5-7i) = -76-252i$$

$$c.d. (12-14i)(2-4i) + (16-18i)(6-8i) = -80-312i$$

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$$P = \begin{bmatrix} -72-236i & -76-292i \\ -76-252i & -80-312i \end{bmatrix} = L$$

ZAD 4 $\text{Tr}(A) = \text{Tr}(A^T)$

$$L = \text{Tr} \begin{bmatrix} 5 & 7 \\ 6 & 8 \end{bmatrix} = 5+8 = 13$$

$$P = \text{Tr} \begin{bmatrix} 5 & 7 \\ 6 & 8 \end{bmatrix}^T = \text{Tr} \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix} = 5+8 = 13$$

MACIETS

SIKORX

A

$$\boxed{ZAD 5} \quad \text{Tr}(AB) = \text{Tr}(BA)$$

$$L = \text{Tr} \left(\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 5 & 7 \\ 6 & 8 \end{bmatrix} \right) = \begin{matrix} 5 & 7 \\ 6 & 8 \end{matrix} \begin{matrix} 1 & 2 \\ 3 & 4 \end{matrix} = 5 + 2 \cdot 6 + 3 \cdot 7 = 5 + 12 + 21 = 38 + 32 = 70$$

$$P = \text{Tr} \left(\begin{bmatrix} 5 & 7 \\ 6 & 8 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \right) = 5 + 21 + 6 \cdot 2 + 8 \cdot 4 = 5 + 21 + 12 + 32 = 70$$

$$L = P$$

$$\boxed{ZAD 6}$$

$$\text{Tr}(A+B) = \text{Tr}(A) + \text{Tr}(B)$$

$$L = \text{Tr} \left(\begin{bmatrix} 1 & 3 \\ 2 & 2 \end{bmatrix} + \begin{bmatrix} 2 & 6 \\ 2 & 3 \end{bmatrix} \right) = \text{Tr} \begin{bmatrix} 3 & 9 \\ 4 & 5 \end{bmatrix} = 8$$

$$P = \text{Tr} \begin{bmatrix} 1 & 3 \\ 2 & 2 \end{bmatrix} + \text{Tr} \begin{bmatrix} 2 & 6 \\ 2 & 3 \end{bmatrix} = 3 + 5 = 8$$

$$L = P$$

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ZAD 8

$$a^T a = \text{Tr}(a a^T)$$

$$L = [1 \ 3 \ 5] \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix} = 1 + 9 + 25 = 35$$

$$P = \text{Tr} \left(\begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix} [1 \ 3 \ 5] \right) = \text{Tr} \begin{pmatrix} 1 & 3 & 5 \\ 3 & 9 & 15 \\ 5 & 15 & 25 \end{pmatrix} =$$

$$1 + 9 + 25 = 35$$

1	3	5	
1	1	3	5
3	3	9	15
5	5	15	25

$$L = P$$