MACIEJ SIKORA Gr (1)

(A)

Zad 1

$$(AB)^T = B^T A^T$$

$$L = \left(\begin{bmatrix} 1 & 3 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} 2 & 6 \\ 2 & 3 \end{bmatrix} \right)^T = \begin{bmatrix} 8 & 15 \\ 8 & 18 \end{bmatrix}^T = \begin{bmatrix} 8 & 8 \\ 15 & 18 \end{bmatrix}$$

$$P = \begin{bmatrix} 1 & 3 \\ 2 & 2 \end{bmatrix}^T \begin{bmatrix} 2 & 6 \\ 2 & 3 \end{bmatrix}^T = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} 2 & 2 \\ 6 & 3 \end{bmatrix} =$$

$$P = \begin{bmatrix} 2 & 6 \\ 2 & 3 \end{bmatrix}^T \begin{bmatrix} 1 & 3 \\ 2 & 2 \end{bmatrix}^T = \begin{bmatrix} 2 & 2 \\ 6 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} 8 & 8 \\ 15 & 18 \end{bmatrix}$$

$L = P$

Zad 2

$$(A+B)^H = A^H + B^H$$

$$L = \left(\begin{bmatrix} 1+3i & 2+6i \\ 2+2i & 2+3i \end{bmatrix} \begin{bmatrix} 11+13i & 12+16i \\ 12+12i & 12+13i \end{bmatrix} \right)^H = \rightarrow$$

$11+13i \quad 12+16i$
$12+12i \quad 12+13i$

$1+3i \quad 2+6i$
$2+2i \quad 2+3i$

$-76+142i \quad -42+150i$

$(1+3i)(11+13i) = 11+13i+33i-39 = -28+46i$

$(2+6i)(12+12i) = 24+24i+72i-72 = -48+96i$

$$(1+3i)(12+16i) + (2+6i)(12+13i) = 12 + 16i + 36i - 48 + 24 + 26i + 72i - 78 = -42 + 150i$$

ZAD 2

$$(A+B)^H = A^H + B^H$$

$$L = \left(\begin{bmatrix} 1+3i & 2+6i \\ 2+2i & 2+3i \end{bmatrix} + \begin{bmatrix} 11+13i & 12+16i \\ 12+12i & 12+13i \end{bmatrix} \right)^H =$$

$$\begin{bmatrix} 12+16i & 14+22i \\ 14+14i & 14+16i \end{bmatrix}^H = \begin{bmatrix} 12-16i & 14-14i \\ 14-22i & 14-16i \end{bmatrix}$$

$$P = \begin{bmatrix} 1+3i & 2+6i \\ 2+2i & 2+3i \end{bmatrix}^H + \begin{bmatrix} 11+13i & 12+16i \\ 12+12i & 12+13i \end{bmatrix}^H =$$

$$\begin{bmatrix} 1-3i & 2-6i \\ 2-6i & 2-3i \end{bmatrix} + \begin{bmatrix} 11-13i & 12-12i \\ 12-16i & 12-13i \end{bmatrix} =$$

$$\begin{bmatrix} 12-16i & 14-14i \\ 14-22i & 14-16i \end{bmatrix} = L$$

(A) ZAD 3 $(AB)^H = B^H A^H$

MACIEJ SIKORA

$$L = \left(\begin{bmatrix} 1+3i & 5+7i \\ 2+4i & 6+8i \end{bmatrix} \begin{bmatrix} 11+13i & 12+14i \\ 15+17i & 16+18i \end{bmatrix} \right)^H =$$

$(1+3i)(11+13i) + (5+7i)(15+17i) =$

$11 + 13i + 33i - 39 + 50 + 25 + 50i + 35i + 70i - 35i - 70 - 49 =$

$-28 + 46i + 75 - 70 - 49 + 155i + 35i =$

$-28 + 46i + 5 - 49 + 190i = -72 + 236i$

$(1+3i)(12+14i) + (5+7i)(16+18i) =$

$12 + 14i + 36i - 42 + 50 + 30 + 50i + 40i + 70i + 42i - 126 =$

$-76 + 252i$

$(2+4i)(11+13i) + (6+8i)(15+17i) =$

$22 + 26i + 44i - 82 + 90 + 60i + 82i + 120i - 80 - 96 =$

$32 - 108 + 172i + 120i = -76 + 292i$

A) ZAD 3 c.d. MACIEJ SIKORA

$(2+4i)(12+14i) + (6+8i)(16+18i) =$

$24 + 28i + 48i - 56 + 96 + 108i + 128i - 144 =$

$-80 + 312i$

$$L = \begin{bmatrix} -72+236i & -76+252i \\ -76+292i & -80+312i \end{bmatrix}^H =$$

$$\begin{bmatrix} -72-236i & -76-292i \\ -76-252i & -80-312i \end{bmatrix}$$

$$P = \begin{bmatrix} 11+13i & 12+14i \\ 15+17i & 16+18i \end{bmatrix}^H \cdot \begin{bmatrix} 1+3i & 5+7i \\ 2+4i & 6+8i \end{bmatrix}^H =$$

$$\begin{bmatrix} 11-13i & 15-17i \\ 12-14i & 16-18i \end{bmatrix} \begin{bmatrix} 1-3i & 2-4i \\ 5-7i & 6-8i \end{bmatrix} =$$

$(11-13i)(1-3i) + (15-17i)(5-7i) = -72 + 236i$

$(11-13i)(2-4i) + (15-17i)(6-8i) = -76 - 292i$

MACIEJ SIKORA

(A)

ZAD 3 $(12-14i)(1-3i) + (16-18i)(5-7i) = -76-252i$

c.d. $(12-14i)(2-4i) + (16-18i)(6-8i) = -80-312i$

$$P = \begin{bmatrix} -72-236i & -76-292i \\ -76-252i & -80-312i \end{bmatrix} = L$$

ZAD 4 $\overline{Tr}(A) = \overline{Tr}(A^T)$

$$L = \overline{Tr}\begin{bmatrix} 5 & 7 \\ 6 & 8 \end{bmatrix} = 5+8 = 13$$

$$P = \overline{Tr}\begin{bmatrix} 5 & 7 \\ 6 & 8 \end{bmatrix}^T = \overline{Tr}\begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix} = 5+8 = 13$$

MACIEJ SIKORA (A)

ZAD 5 $\quad Tr(AB) = Tr(BA)$

$$L = Tr\left(\begin{bmatrix}1&2\\3&4\end{bmatrix}\begin{bmatrix}5&7\\6&8\end{bmatrix}\right) = 5 + 2\cdot 6 + 3\cdot 7 = 5 + 12 + 21 = 38 + 32 = 70$$

$$P = Tr\left(\begin{bmatrix}5&7\\6&8\end{bmatrix}\begin{bmatrix}1&2\\3&4\end{bmatrix}\right) = 5 + 21 + 6\cdot 2 + 8\cdot 4 = 5 + 21 + 12 + 32 = 70$$

$L = P$

ZAD 6

$$Tr(A+B) = Tr(A) + Tr(B)$$

$$L = Tr\left(\begin{bmatrix}1&3\\2&2\end{bmatrix} + \begin{bmatrix}2&6\\2&3\end{bmatrix}\right) = Tr\begin{bmatrix}3&9\\4&5\end{bmatrix} = 8$$

$$P = Tr\begin{bmatrix}1&3\\2&2\end{bmatrix} + Tr\begin{bmatrix}2&6\\2&3\end{bmatrix} = 3 + 5 = 8$$

$L = P$

(A) MACIEJ SIKORA

ZAD 8 $\quad a^T a = \overline{tr}(a a^T)$

$$L = \begin{bmatrix} 1 & 3 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix} = 1 + 9 + 25 = 35$$

$$P = \overline{tr}\left(\begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix} \begin{bmatrix} 1 & 3 & 5 \end{bmatrix} \right) = \overline{tr} \begin{pmatrix} 1 & 3 & 5 \\ 3 & 9 & 15 \\ 5 & 15 & 25 \end{pmatrix} =$$

	1	3	5
1	1	3	5
3	3	9	15
5	5	15	25

$1 + 9 + 25 = 35$

$L = P$