A , zad 1-4 $a_1 a_2 . b_1 b_2 . c_1 c_2 d_1 d_2$

$$A = \begin{bmatrix} a_1 + a_2 & b_1 + b_2 \\ c_1 + c_2 & d_1 + d_2 \end{bmatrix}$$

05.12.1999 + 3 od wyjściowej

08.12.1999   daty

$$B = \begin{bmatrix} 8 & 3 \\ 10 & 18 \end{bmatrix} , \text{ zad 1}$$

24.03.2022

$$C = \begin{bmatrix} 3 & 2 & 3-2 \\ 7 & 4 & 7-4 \\ 7-3 & 4-2 & \bullet \end{bmatrix} \quad \dots \quad C = \begin{bmatrix} 8 & 3 & 5 \\ 10 & 18 & 8 \\ 2 & 15 & 16 \end{bmatrix}$$

13 + 3

suma różnic

$$A = \begin{bmatrix} 8 & 3 \\ 10 & 18 \end{bmatrix} \quad\quad B = \begin{bmatrix} 6 & 3 \\ 2 & 4 \end{bmatrix}$$

1. $(AB)^{-1} = B^{-1} A^{-1}$

2. $(A^T)^{-1} = (A^{-1})^T$

3. eliminacja Gaussa

4. $\det(A^{-1}) = \dfrac{1}{\det(A)}$

5. $\det(C^{-1}) = \dfrac{1}{\det(C)}$

Zad 1 $(AB)^{-1} = B^{-1} A^{-1}$

$$L = \left(\begin{bmatrix} 8 & 3 \\ 10 & 18 \end{bmatrix} \begin{bmatrix} 6 & 3 \\ 2 & 4 \end{bmatrix} \right)^{-1} = \begin{bmatrix} 54 & 36 \\ 96 & 102 \end{bmatrix}^{-1} =$$

$$\begin{array}{cc} & 6 \quad 3 \\ & 2 \quad 4 \\ 8 \quad 3 & 54 \quad 36 \\ 10 \quad 18 & 96 \quad 102 \end{array} \quad \frac{1}{54 \cdot 102 - 36 \cdot 96} \begin{bmatrix} 102 & -36 \\ -96 & 54 \end{bmatrix} = \begin{bmatrix} \dfrac{102}{-2448} & \dfrac{36}{2448} \\ \dfrac{96}{2448} & \dfrac{54}{-2448} \end{bmatrix}$$

$18 + 6 = 54$
$24 + 12 = 36$
$60 + 36 = 96$
$30 + 72 = 102$

$$P = \begin{bmatrix} 6 & 3 \\ 2 & 4 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 8 & 3 \\ 10 & 18 \end{bmatrix}^{-1} =$$

$$\begin{bmatrix} \dfrac{4}{18} & \dfrac{-3}{18} \\ \dfrac{-2}{18} & \dfrac{6}{18} \end{bmatrix} \cdot \begin{bmatrix} \dfrac{18}{114} & \dfrac{-3}{114} \\ \dfrac{-10}{114} & \dfrac{8}{114} \end{bmatrix} =$$

$$\begin{array}{cc} \dfrac{18}{114} & \dfrac{-3}{114} \\ \dfrac{-10}{114} & \dfrac{8}{114} \end{array}$$

$$\begin{array}{cc} \dfrac{4}{18} & -\dfrac{3}{18} \\ -\dfrac{2}{18} & \dfrac{6}{18} \end{array} \begin{bmatrix} \dfrac{-102}{2448} & \dfrac{36}{2448} \\ \dfrac{96}{2448} & \dfrac{54}{-2448} \end{bmatrix} = L \qquad c.n.u.$$

right column calculations:
$$\begin{array}{r} 1\,3 \\ 114 \cdot \\ \underline{\quad 18} \\ 912 \\ + 114 \\ \hline 2052 \end{array}$$

$$\begin{array}{r} 1 \\ 102 \\ \cdot \quad 54 \\ \hline 408 \\ + 60 \\ \hline 1008 \end{array}$$

$$\begin{array}{r} 5 \\ 3 \\ 36 \\ \cdot 96 \\ \hline 216 \\ + 324 \\ \hline 3456 \end{array}$$

$1008 - 3456 =$

-2448

$\dfrac{4}{114} + \dfrac{30}{114 \cdot 18}$

$\dfrac{1}{24 - 6} = \dfrac{1}{18}$

$80 + 64 = 114$

$\dfrac{1}{8 \cdot 18 - 30} = \dfrac{1}{114}$

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$\boxed{ZAD\ 2}$ $\left(A^T\right)^{-1} = \left(A^{-1}\right)^T$

$$L = \begin{bmatrix} 8 & 10 \\ 3 & 18 \end{bmatrix}^{-1} = \frac{1}{\uparrow} \begin{bmatrix} 18 & -10 \\ -3 & 8 \end{bmatrix} =$$

$18 \cdot 8 - 30$

$\frac{1}{114}$

$$\begin{bmatrix} \dfrac{18}{114} & \dfrac{-10}{114} \\[2mm] \dfrac{-3}{114} & \dfrac{8}{114} \end{bmatrix}$$

$$P = \left(\begin{bmatrix} 8 & 3 \\ 10 & 18 \end{bmatrix}^{-1} \right)^T = \left(\frac{1}{114} \begin{bmatrix} 18 & -3 \\ -10 & 8 \end{bmatrix} \right)^T = \begin{bmatrix} \dfrac{18}{114} & \dfrac{-10}{114} \\[2mm] \dfrac{-3}{114} & \dfrac{8}{114} \end{bmatrix}$$

$L = P$ c. n. u.

Maciej Sikora   24.03   gr 1

$$\boxed{ZAD3} \quad \text{Eliminacja Gaussa}$$

$$A = \begin{bmatrix} 8 & 3 \\ 10 & 18 \end{bmatrix}$$

$$\begin{bmatrix} 8 & 3 \\ 10 & 18 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & \frac{3}{8} \\ 10 & 18 \end{bmatrix}\begin{bmatrix} \frac{1}{8} & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & \frac{3}{8} \\ 0 & \frac{114}{144} \end{bmatrix}\begin{bmatrix} \frac{1}{8} & 0 \\ \frac{-10}{8} & 1 \end{bmatrix} =$$

$$= \begin{bmatrix} 1 & \frac{3}{8} \\ 0 & 1 \end{bmatrix}\begin{bmatrix} \frac{1}{8} & 0 \\ \dfrac{-1440}{\underset{912}{8\cdot 114}} & \dfrac{144}{114} \end{bmatrix} =$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} \frac{1}{8} + \dfrac{1440}{918}\cdot\frac{3}{8} & -\dfrac{\overset{18}{144\cdot 8}}{\underset{38}{114\cdot 8}} \\ -\dfrac{1440}{912} & \dfrac{144}{114} \end{bmatrix} =$$

$$\begin{bmatrix} \dfrac{5232}{7344} & -\dfrac{18}{38} \\[2ex] -\dfrac{1440}{912} & \dfrac{144}{114} \end{bmatrix}$$

ZAD 4 $\det(A^{-1}) = \dfrac{1}{\det(A)}$

$$A = \begin{bmatrix} 8 & 3 \\ 10 & 18 \end{bmatrix} \qquad A^{-1} = \begin{bmatrix} \dfrac{18}{114} & \dfrac{-3}{114} \\ \dfrac{-10}{114} & \dfrac{8}{114} \end{bmatrix}$$

$$L = \det(A^{-1}) = \frac{144}{114^2} - \frac{30}{114^2} = \frac{114}{114^2} = \frac{1}{114}$$

$$\det(A) = 144 - 30 = 114$$

$$P = \frac{1}{\det A} = \frac{1}{114}$$

$$L = P$$

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$\boxed{\text{ZAD 5}}$ $\quad det(C^{-1}) = \dfrac{1}{det(C)}$

$$C = \begin{bmatrix} 8 & 3 & 5 \\ 10 & 18 & 8 \\ 2 & 15 & 16 \end{bmatrix}$$

$det(C) = 8 \cdot 18 \cdot 16 + 3 \cdot 8 \cdot 2 + 10 \cdot 5 \cdot 15$
$\qquad\qquad - 5 \cdot 18 \cdot 2 - 10 \cdot 3 \cdot 16 - 15 \cdot 8^2 =$

$\overset{144+3=147}{}$

$2(8(18 \cdot 8 + 3) + 25 \cdot 15) = 2(8 \cdot 147 + 375) =$

$2 \cdot 1551 = 2102$

$-5(18 \cdot 2 + 8(6 \cdot 2 + 3 \cdot 8)) =$

$-5(36 + 8(12 + 24)) =$

$-5(36 + 8 \cdot 36) =$

$5 \cdot 36 \cdot 9 = 1620$

$2102 - 1620 = 482 = det(C)$

$P = \dfrac{1}{482}$

$\begin{array}{r} 25 \\ \cdot\ 15 \\ \hline 123 \\ 125 \\ \hline 375 \end{array}$

$\begin{array}{r} 147 \\ \cdot\quad 8 \\ \hline 1176 \\ \cdot\ 375 \\ \hline 1551 \\ 2 \\ \hline 2102 \end{array}$

$\begin{array}{r} 360 \\ -\ 36 \\ \hline 324 \\ \cdot\quad 5 \\ \hline 1620 \end{array}$

$\begin{array}{r} 2102 \\ -1620 \\ \hline 482 \end{array}$

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ZAD 5   c. d. 1

$$\begin{bmatrix} 8 & 3 & 5 \\ 10 & 18 & 8 \\ 2 & 15 & 16 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} =$$

$\frac{6 \cdot 4 - 5}{4} = \frac{59}{4}$

$18 - \frac{15}{4}$

$\frac{18 \cdot 4 - 15}{4} =$

$$\begin{bmatrix} 8 & 3 & 5 \\ 0 & 18 - \frac{30}{8} & 8 - \frac{50}{8} \\ 0 & 15 - \frac{3}{4} & 16 - \frac{5}{4} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ -\frac{10}{8} & 1 & 0 \\ -\frac{1}{4} & 0 & 1 \end{bmatrix} =$$

$\frac{72 - 15}{4} =$

$\frac{63}{4}$

$\frac{64 - 50}{8} =$

$$\begin{bmatrix} 8 & 3 & 5 \\ 0 & \frac{63}{4} & \frac{7}{4} \\ 0 & \frac{57}{4} & \frac{59}{4} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ -\frac{10}{8} & 1 & 0 \\ -\frac{1}{4} & 0 & 1 \end{bmatrix}$$

$\frac{14}{8} = \frac{7}{4}$

$\frac{60 - 3}{4} = \frac{57}{4}$

$$\begin{bmatrix} 8 & 3 & 5 \\ 0 & \frac{63}{4} & \frac{7}{4} \\ 0 & 0 & \frac{59}{4} - \frac{7}{4} \cdot \frac{57}{63} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ -\frac{10}{8} & 1 & 0 \\ -\frac{1}{4} + \frac{10}{8}\cdot\frac{57}{63} & -\frac{57}{63} & 1 \end{bmatrix}$$

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$\boxed{\text{ZAD 5}}$ c. d. 2

Left margin calculations:
$$\begin{array}{r} 2 \\ 63 \\ \underline{\cdot 8} \\ 504 \end{array}$$

$$\begin{array}{r} 1 \\ 63 \\ \underline{-59} \\ 567 \\ 315 \\ 3717 \end{array}$$

$$\begin{array}{r} 2 \\ 63 \\ \underline{\cdot 7} \\ 441 \\ +57 \\ \overline{498} \end{array}$$

$$\begin{array}{r} 3717 \\ -498 \\ \overline{3219} \end{array}$$

$$\begin{array}{r} 1 \\ 63 \\ \underline{\cdot 4} \\ 242 \end{array}$$

$$\begin{array}{r} 4 \\ 57 \\ \underline{\cdot 7} \\ 399 \end{array}$$

$$\begin{array}{r} 3717 \\ -400 \\ \underline{1} \\ 3318 \end{array}$$

$$\frac{59 \cdot 63 - ((7 \cdot 63) + 57)}{4 \cdot 63} =$$

$$\frac{3717 - 498}{242} = \frac{3219}{242}$$

$$-\frac{1}{4} + 570$$

$$\frac{3717 - 57 \cdot 7}{242} = \frac{3318}{242}$$

$$-\frac{1}{4} + \frac{570}{504} = \frac{570 - 63}{504} = \frac{507}{504}$$

$$\begin{bmatrix} 8 & 3 & 5 \\ 0 & \frac{63}{4} & \frac{7}{4} \\ 0 & 0 & \frac{3318}{242} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ -\frac{10}{8} & 1 & 0 \\ -\frac{507}{504} & -\frac{57}{63} & 1 \end{bmatrix}$$

$$2052 \;|\; 2$$
$$1026 \;|\; 2$$
$$513 \;|\; 3$$
$$171 \;|\; 3$$
$$57 \;|\; 3$$
$$17$$

$$2448 \;|\; 2$$
$$1224 \;|\; 2$$
$$612 \;|\; 2$$
$$306 \;|\; 2$$
$$153 \;|\; 3$$
$$51 \;|\; 3$$
$$17$$

O.d.

$$17 \cdot 2^2 \cdot 3^3 \qquad\qquad 17 \cdot 2^4 \cdot 3^2$$

To oznacza, że da się przejść z
2052 do 2448 w mianowniku
mnożąc raz $\dfrac{2^2}{3}$