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Zad 1 →

$$X = \begin{bmatrix} y^2 & 2y^3 \end{bmatrix} \qquad Y = \begin{bmatrix} 5y^2 & 6y \end{bmatrix}$$

Zad 2-5

$$X = \begin{bmatrix} 8y^2 & 4y^2 \\ 6y & 18y \end{bmatrix}$$

Zad 1 $\frac{\partial X \cdot Y}{\partial y} = \frac{\partial X}{\partial y} Y + X \frac{\partial Y}{\partial y}$

$$\frac{\partial \begin{bmatrix} 5y^4 & 12y^4 \end{bmatrix}}{\partial y} = \begin{bmatrix} 20y^3 & 48y^3 \end{bmatrix}$$

$$\begin{bmatrix} 2y & 6y^2 \end{bmatrix}\begin{bmatrix} 5y^3 & 6y \end{bmatrix} + \begin{bmatrix} y^2 & 2y^3 \end{bmatrix}\begin{bmatrix} 10y & 6 \end{bmatrix} =$$

$$\begin{bmatrix} 10y^3 & 36y^3 \end{bmatrix} + \begin{bmatrix} 10y^3 & 12y^3 \end{bmatrix} = \begin{bmatrix} 20y^3 & 48y^3 \end{bmatrix}$$

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ZAD 2

$$X = \begin{bmatrix} 8y^2 & 4y^2 \\ 6y & 18y \end{bmatrix}$$

ZAD 2

$$\frac{\partial X^{-1}}{\partial y} = -X^{-1} \cdot \frac{\partial X}{\partial y} \cdot X^{-1}$$

$$144y^2 - 24y^2 = 120y^3$$

$$X^{-1} = \begin{bmatrix} \frac{18y}{120y^3} & \frac{-4y^2}{120y^3} \\ \frac{-6y}{120y^3} & \frac{8y^2}{120y^3} \end{bmatrix} = \begin{bmatrix} \frac{18}{120y^2} & \frac{-1}{30y} \\ \frac{-1}{20y^2} & \frac{1}{15y} \end{bmatrix}$$

$-\frac{4}{120y}$ $\quad$ $\frac{-6}{120y^2}$ $\quad$ $\frac{8}{120y}$

$$\frac{\partial X^{-1}}{\partial y} = \begin{bmatrix} \frac{18}{120} \cdot -2 \cdot y^{-3} & -\frac{1}{30} \cdot -1 \cdot y^{-2} \\ -\frac{1}{20} \cdot -2 \cdot y^{-3} & \frac{1}{15} \cdot -1 \cdot y^{-2} \end{bmatrix} =$$

$-\frac{36}{120}$ $\quad$ $\frac{4}{120}$

$$\begin{bmatrix} -\frac{18}{60} y^{-3} & \frac{1}{30} \cdot y^{-2} \\ \frac{1}{10} \cdot y^{-3} & -\frac{1}{15} \cdot y^{-2} \end{bmatrix}$$

$\frac{12}{120}$ $\quad$ $\frac{8}{120}$

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Zad 2 c.d.

$$P = \begin{bmatrix} \frac{-18}{120y^2} & \frac{1}{30y} \\ \frac{1}{20y^2} & \frac{-1}{15y} \end{bmatrix} \begin{bmatrix} 16y & 8y \\ 6 & 18 \end{bmatrix} \begin{bmatrix} \frac{18}{120y^2} & \frac{-1}{30y} \\ \frac{-1}{20y^2} & \frac{1}{15y} \end{bmatrix} =$$

$$\begin{bmatrix} \frac{6}{30y} + \frac{-16\cdot 18}{120y^2} & \frac{18}{30y} + \frac{-18\cdot 8}{120y^2} \\ \frac{-6}{15y} + \frac{16}{20y} & \frac{-18}{15y} + \frac{8}{20y} \end{bmatrix} \begin{bmatrix} \frac{18}{120y^2} & \frac{-1}{30y} \\ \frac{-1}{20y^2} & \frac{1}{15y} \end{bmatrix} =$$

$$\begin{bmatrix} \frac{-264}{120y} & \frac{-72}{120y} \\ \frac{48}{120y} & \frac{-96}{120y} \end{bmatrix} \begin{bmatrix} \frac{18}{120y^2} & \frac{-4}{120y} \\ \frac{-8}{120y^2} & \frac{-8}{120y} \end{bmatrix} =$$

$$\begin{bmatrix} \frac{(72\cdot 6)+(-264\cdot 18)}{120^2 y^3} & \frac{(-72)\cdot(-8)+(264\cdot 4)}{120^2 y^2} \\ \frac{(-96\cdot 6)+(48\cdot 18)}{120^2 y^3} & \frac{(-96)\cdot(-8)+(48\cdot 4)}{120^2 y^2} \end{bmatrix} = \begin{bmatrix} \frac{-36}{120y^3} & \frac{-4}{120y^2} \\ \frac{12}{120y^3} & \frac{8}{120y^2} \end{bmatrix} = L$$

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ZAD 3 $\frac{\partial \det X}{\partial y} = \det X \, tr\left(X^{-1} \frac{\partial X}{\partial y}\right)$

$$L = \frac{\partial (120y^3)}{\partial y} = 360y^2$$

$$P = 120y^3 \cdot tr\left(\begin{bmatrix} \frac{18}{120y^2} & \frac{-4}{120y} \\ \frac{-6}{120y^2} & \frac{8}{120y} \end{bmatrix} \cdot \begin{bmatrix} 16y & 8y \\ 6 & 18 \end{bmatrix}\right) =$$

$$120y^3 \cdot tr\left(\begin{bmatrix} \frac{(-6\cdot 4) + 16\cdot 18}{120y} & \frac{(-4\cdot 18) + 18\cdot 8}{120y} \\ \frac{(8\cdot -6) + 16\cdot(-6)}{120y} & \frac{(8\cdot 18) + (8\cdot(-6))}{120y} \end{bmatrix}\right) =$$

$$120y^3 \cdot \left[\frac{-24 + 288 + 144 - 48}{120y}\right] = 360y^2 = L$$

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ZAD 4 $\frac{\partial(\ln(\det X))}{\partial y} = tr\left(X^{-1}\frac{\partial X}{\partial y}\right)$

$$L = \frac{\partial \ln 120y^3}{\partial y} = \frac{1}{120y^3}\cdot\frac{\partial 120y^3}{\partial y} = \frac{360y^2}{120y^3} =$$

$$\frac{3}{y}$$

$$P = tr\left(\begin{bmatrix} \frac{18}{120y^2} & \frac{-4}{120y} \\ \frac{-6}{120y^2} & \frac{8}{120y} \end{bmatrix}\begin{bmatrix} 16y & 8y \\ 6 & 18 \end{bmatrix}\right) =$$

$$\frac{6\cdot 4}{120y} + \frac{18\cdot 16}{120y} + \frac{18\cdot 8}{120y} + \frac{-6\cdot 8}{120y} = \frac{360}{120y} =$$

$$\frac{3}{y} = L$$

C. n. u.

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ZAD 5 $\frac{\partial X^T}{\partial y} = \left(\frac{\partial X}{\partial y}\right)^T$

$$L = \frac{\partial \begin{bmatrix} 8y^2 & 6y \\ 4y^2 & 18y \end{bmatrix}}{\partial y} = \begin{bmatrix} 16y & 6 \\ 8y & 18 \end{bmatrix}$$

$$P = \begin{bmatrix} 16y & 8y \\ 6 & 18 \end{bmatrix}^T = \begin{bmatrix} 16y & 6 \\ 8y & 18 \end{bmatrix} = L$$

C. n. u.