

Pseudoentropy

PhD Dissertation Talk

University of Warsaw

May 23, 2023

This talk

- ✓ Overviews the goals, resources, and deliverables of my PhD project.
- ✓ Demonstrates/sketches interesting techniques used in the dissertation.
- ✓ Defends my position on the dissertation form, in view of reviews received 🛡️.
- ✗ Avoids complex definitions and proofs for brevity's sake (see the papers) ⌚.
- ✗ Does not assess my own academic KPIs (see the documentation) 😬.

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Credits

I am particularly grateful:

- ❤️ for love, to my wife Aneta
- 💰 for funding and know-how, to my advisor Stefan Dziembowski
- 💡 for merit support, to my co-advisor Krzysztof Pietrzak
- 👏 for motivation and recognition, to dozens of people with whom I shared ideas: research collaborators, reviewers, audience 😊

Funding

My PhD research received support from numerous funding sources:



Ideas for Poland



WELCOME



TOCNeT



PRELUDIUM



+ several travel grants from various research institutions

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About Pseudoentropy



Introduced in [ILL89, HILL99] as a **computational variant of information-theoretic entropy**.



Recognized as a **useful tool and convenient language** in research around cryptography, computational complexity and information theory. Examples:



Pseudorandom generators from one-way functions [HILL99]






Computational Dense Model Theorem [RTTV08, Zha11], improving upon the result of Green-Tao-Ziegler



Promising but messy: suffers from **contextual definitions** and **insufficiently developed foundations**.

Goals

My PhD project set these goals:






-  **improve understanding of foundational properties** of pseudoentropy notions
-  **demonstrate further technical applications**
-  optionally, identify **new inspirational application areas**

Contribution

Works presented under the scope of this PhD project:

- ✓ **obtained characterizations and manipulation rules** for pseudoentropy notions, using **convex analysis as a toolbox**
- ✓ **simplified some of existing technical proofs**, for instance of Dense Model Theorem and of Computational Simulators
- ✓ **developed machine-learning inspired framework** for proving computational indistinguishability

My self-assesment:

-  these works contributed to the goals  ,  and  respectively.
-  goals were set broadly, leaving still room for improvement

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Background

-  Pseudoentropy at least k when the distribution behaves *nearly as well* as with information-theoretic (min)entropy k in *cryptographic games*.
-  Program-input games used in definitions
 - (a) Distinguish: discriminate between two distributions based on a sample.
 - (b) Predict: guess a sampled outcome
 - (c) Compress: successfully decode after decoding

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Outline

-  Indistinguishability quantifies how close are two distributions under a given class of computationally bounded tests.
-  What is the geometrical meaning of indistinguishability?
-  Computational indistinguishability can be **characterized by inseparability by a class of feasible hyperplanes**. The margin of separation can be analytically characterized too!

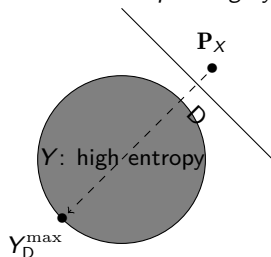
Contribution

The characterizations [Sko15a] has found the following applications:

-  Unifying unpredictability-based and indistinguishability-based pseudoentropy notions [SGP15]
-  Short proof of the Dense Model Theorem [Sko15c]
-  Further applications to key derivation [Sko17b]
-  Simplifies other technical arguments [VZ12]

Technique (Sketch)

- In program-input indistinguishability games, it makes sense to **characterize the optimal input player Y** against a given program player D .
- View D as a *separating hyperplane*, maximize margin with high-entropy Y .



Symbol/Operator	Crypto	Geometry
X	candidate distribution	
$\text{ED}(Y)$	expectation	$D \cdot P_Y$ (dot-product)
D	distinguisher/program player	separating hyperplane
Y	input player	feasible point
$\epsilon = \text{ED}(Y) - \text{ED}(X)$	advantage	separation margin

Figure 1: Geometrical meaning of cryptographic indistinguishability.

- Closed-form solutions found** in interesting cases by **convex optimization**. For pseudoentropy of at least k bits against attackers \mathcal{D} with advantage ϵ :

$$\forall D \in \mathcal{D} : \quad \text{ED}(X) \leq 2^{-k} |D| + \epsilon$$

instead of the standard depth-2 formula $\forall D \exists Y : \mathbf{H}_\infty(Y) \geq k \& \text{ED}(X) \leq \text{ED}(Y) + \epsilon$.



Characterization depend on feasible distinguishers and the baseline entropy.

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Outline

- 📖 Applications of pseudoentropy use different notions, most commonly unpredictability-based and indistinguishability-based.
- ? Are unpredictability and indistinguishability entropies different?
Note: usually, distinguishing is easier than predicting¹
- 👉 Surprisingly, **equivalent in high-entropy regimes!**

¹Think of discriminating between dogs and cats versus predicting the breed. ⏪ ⏩ ⏴ ⏵ ⏶ ⏷ ⏸ ⏹ ⏺ ⏻ ⏼ ⏽ ⏾ ⏿ 🔍 ↺ ↻

Contribution

The following result was obtained [SGP15]:

- 🔑 **equivalence of unpredictability and indistinguishability** pseudoentropy definitions in **high-entropy regimes**, namely $n - O(\log n)$ for n -bit strings,
- 🔑 **geometric characterizations as a workhorse** of the proof.

Technique (Sketch)

The proof strategy is to *constructively convert a distinguisher into a predictor*:

- (a) Indistinguishability fails: $\mathbf{ED}(X) \geq \mathbf{ED}(Y) + \epsilon$ for all Y of min-entropy k .
- (b) $\mathbf{ED}(X) \geq |\mathbf{D}|/2^k + \epsilon$ for boolean \mathbf{D} , by geometrical characterizations (!)
- (c) Sample \mathbf{A} from the image of \mathbf{D} , then $\mathbf{P}\{\mathbf{A} = X\} > 2^{-k} + \frac{\epsilon}{\#\mathbf{D}}$.
- (d) Approximate image sampling by *rejection sampling* ℓ times, then

$$\mathbf{P}\{\mathbf{A} = X\} > \left(2^{-k} + \frac{\epsilon}{\#\mathbf{D}}\right) \cdot \left(1 - \frac{\#\mathbf{D}}{2^n}\right)^\ell.$$

- (e) $\mathbf{P}\{\mathbf{A} = X\} > 2^{-k}$ when $\ell \approx 2^{n-k}/\epsilon$ independently of $\#\mathbf{D}$!

⚠ More sophisticated rejection-sampling handles X with auxiliary input Z .

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Outline

- 📖 Applications of pseudoentropy assume strength parameters that propagate through reduction proofs. Not clear what regimes are non-trivial to start with.
- ? Can we characterize when quality parameters are non-trivial?
- 👉 Yes, by time-advantage tradeoffs - similarly to pseudorandomness!

Contribution

The following result was obtained [Sko17b]:

- 🔑 generic attacks with time t succeed against pseudoentropy amount k with advantage $\epsilon = O\left(\sqrt{t/2^k}\right)$.
- 🔑 generalization of the famous time-advantage tradeoffs against pseudorandomness [DTT10].

Technique (Sketch)

The proof leverages **random walk techniques**, see also [Ber97].

- (a) Let $D : \{0, 1\}^n \rightarrow \{-1, 1\}$ be fully random (Rademacher), then $|\mathbf{ED}(X) - \mathbf{ED}(Y)| \approx \|\mathbf{P}_X - \mathbf{P}_Y\|_2$ w.h.p. (random walks theory [Haa81]).
- (b) By slicing the domain $\{0, 1\}^n$ into t random parts and flipping the signs accordingly, we can have $|\mathbf{ED}'(X) - \mathbf{ED}'(Y)| \approx t \cdot \frac{\|\mathbf{P}_X - \mathbf{P}_Y\|_2}{\sqrt{t}}$ w.h.p., with $O(t)$ extra memory.
- (c) Under mild assumptions on Y (far from having k bits of min-entropy) the attack achieves advantage $\epsilon \approx \sqrt{t/2^k}$. Compare with $\epsilon \approx \sqrt{t/2^n}$ for pseudorandomness!
- (d) "Random" can be weakened to $O(1)$ -**wise independent**, and the construction complexity is indeed $O(t)$, alternatively complexity t yields $\epsilon = O(\sqrt{t/2^k})$!

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Outline


- 📖 Applications of pseudoentropy **heavily rely on manipulation rules**, particularly chain rules and transformations [BSW03, FOR12]. However, their use **weakens security guarantees**, due to tradeoffs in quality parameters **caused by reduction proofs**.
- ? Can we improve known manipulation rules?
- 👉 No, not by black-box reductions!

Contribution

The following results were obtained in [PS16]:

- 🔑 **impossibility of better proofs** by black-box reductions!
- 🔑 the **probabilistic construction of an oracle**, of independent interest, inspired by the earlier work on limitations of dense model theorems [Zha11]
- 🔥 this construction inspired techniques used in research on black-box limitations of auxiliary input simulators [CCL18]

Techniques (Sketch)

- (a) Proofs, in case of indistinguishability-based pseudoentropy, rely on *building distinguishers from distinguishers in other setups*. In particular, we have $D' = \mathbb{I}\{\sum_i w_i D_i > t_0\}$ in the proof of the Dense Model Theorem [Zha11] or transformations [Sko15b].
 - (b) Loosely speaking, black-box reductions *aggregate distinguishers by high-level operations*. To prove the need of many operations (queries) we manipulate distinguishers at low-level by choosing values probabilistically!
 - (c) Examples: $D_i \sim \text{Bern}(1/2 + \epsilon)$ on a small set and $D_i \sim \text{Bern}(1/2)$ elsewhere.
-  High-level aggregations are essential. Without them, Dense Model Theorems fail [IM20].

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- **Simulating Auxiliary Information** 💎

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Outline

- 📖 In security proofs, it helps to model leakages as explicit functions of secrets [JP14].
- ? What leakages can be modelled as functions of secrets?
- 👉 Short leakages can be efficiently simulated!

Contribution

The following important results were obtained [Sko16a]:

- 🔑 Construction of a simulator for m bits of leakage which makes only $2^{O(m)}\epsilon^{-2}$ calls to achieve ϵ -indistinguishability. Significantly improved upon prior works [VZ13, JP14]
- 🏆 The reasoning, inspired by ML techniques, **builds on the gradient descent algorithm** and was recognized with the *best student paper award at TCC'16*.
- 🔥 Inspired follow-up works that solved the simulator problem [CCL18], and generalized the learning framework [Sko16b, Sko17a], and studied quantum pseudoentropy (c.f. Chen's dissertation [Che19]).

Technique (Sketch)

(a) The algorithm below demonstrates the procedure

Algorithm 1: Auxiliary Input Simulator

Data:

- Oracle access to distinguishers/test functions \mathcal{D}

Result: Simulator h of $Z \in \{0, 1\}^m$ given $X \in \{0, 1\}^n$

```

1  $\mathbf{P}\{h(x) = z\} \leftarrow 2^{-m}$  // initialize the solution as uniform
2 while  $\max_{D \in \mathcal{D}} \mathbf{ED}(X, Z) - \mathbf{ED}(X, h(X)) > \epsilon$  // as long as can distinguish...
3 do
4    $\mathbf{P}\{h'(x) = z\} \leftarrow \mathbf{P}\{h(x) = z\} - \gamma D(x, z)$  // improve candidate
5    $\mathbf{P}\{h'(x) = z\} \leftarrow \mathbf{P}\{h'(x) = z\} + \text{Correct}(x, z)$  // guarantee constraints
6    $h \leftarrow h'$ 
7 end
8 return  $h$ 

```

(b) Outputs an efficient simulator p.d.f., with appropriate γ and Correct operation.

(c) Finishes after $2^{O(m)} \epsilon^{-2}$ steps, proved by "energy" arguments.

(d) Resembles boosting: we learn how to (strongly) simulate from (weak) distinguishers

(e) Resembles convex optimization: with D as subgradient, γ as a stepsize, Correct as a projection operation!

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Addressing Reviewers Feedback

R: Editorial changes and reference requests.

M: Addressed, thanks for the feedback!

R: A book-style dissertation would be better than a mixture of conference works.

M: I discussed this form with senior researchers, but found *ineffective*:

- 🥕 Gain citations! 😞 *Time-consuming, better to keep writing papers.*
- 🥕 Get your PhD distinguished. 😞 *Prestigious conferences not enough?*
- 💧 Take your time to present it better! 😞 *Why to work harder? We count conference works when granting junior/senior professorships!*

R: Parts of lengthy works might not have been fully reviewed at conferences.

M: Same can happen for junior professorships, but we had extra reviewers 😊.