Pseudoentropy PhD Dissertation Talk

University of Warsaw

May 23, 2023

(University of Warsaw)

This talk

- ✓ Overviews the goals, resources, and deliverables of my PhD project.
- ✓ Demonstrates/sketches interesting techniques used in the dissertation.
- Defends my position on the dissertation form, in view of reviews received .
- X Avoids complex definitions and proofs for brevity's sake (see the papers) \bullet .
- X Does not assess my own academic KPIs (see the documentation) 😳.

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Credits

I am particularly grateful:

- for love, to my wife Aneta
- is for funding and know-how, to my advisor Stefan Dziembowski
- 💡 for merit support, to my co-advisor Krzysztof Pietrzak
- for motivation and recognition, to dozens of people with whom I shared ideas: research collaborators, reviewers, audience of my talks \odot

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Funding

My PhD research received support from numerous funding sources:



Ideas for Poland



WELCOME



TOCNeT



PRELUDIUM



+ several travel grants from various research institutions

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About Pseudoentropy

- Introduced in [ILL89, HILL99] as a computational variant of information-theoretic entropy.
- Recognized as a useful tool and convenient language in research around cryptography, computational complexity and information theory. Examples:
 - Pseudorandom generators from one-way functions [HILL99]
 - Computational Dense Model Theorem [RTTV08, Zha11], improving upon the key ingredient in the results of Green-Tao-Ziegler [GT08, TZ08].
- Promising but messy: suffers from contextual definitions and insufficiently developed foundations.

Goals

My PhD project set these goals:

- ✓ improve understanding of foundational properties of pseudoentropy notions
- demonstrate further technical applications
- optionally, identify new inspirational application areas

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Contribution

Works presented under the scope of this PhD project:

- obtained characterizations and manipulation rules for pseudoentropy notions, using convex analysis as a toolbox, as well as impossibility results
- ✓ simplified some of existing technical proofs, for instance of Dense Model Theorem
 and of Computational Simulators
- developed machine-learning inspired framework for proving computational indistinguishability

My self-assesment:

- these works contributed to the goals \checkmark , and \overrightarrow{v} respectively.
 - 🏃 goals were set broadly, leaving still room for improvement

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- Pseudoentropy at least k when the distribution behaves nearly as well as with information-theoretic (min)entropy k in cryptographic games.
- Program-input games used in definitions
 (a) Distinguish: discriminate between two distributions based on a sample.Pseudoentropy example: for a pseudorandom generator G from d-bit seeds to
 - k-bit outputs, $|\mathbf{ED}(G(U_d)) \mathbf{ED}(U_k)| \le \epsilon$ for small ϵ and comp. bounded D. (b) Predict: guess a sampled outcome. Pseudoentropy example: a one way function f satisfies $\mathbf{P}\{f(\mathsf{D}(f(x))) = f(x)\} \le 2^{-k}$ for computationally bounded D and large k.
 - (c) Compress: decoding / encoding games, see [Yao82, BSW03].
 - (d) ... and even more exotic examples, see [HRVW09].

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- Indistinguishability quantifies how close are two distributions under a given class of computationally bounded tests.
- ? What is the geometrical meaning of indistinguishability?
- Computational indistinguishability can be characterized by inseparability by a class of feasible hyperplanes. The separation margin can be determined analytically too!

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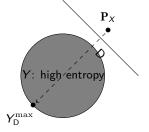
The characterizations [Sko15a] has found the following applications:

- Unifying unpredictability-based and indistinguishability-based pseudoentropy notions [SGP15]
- Short proof of the Dense Model Theorem [Sko15c]
- New proofs of Hardcore Lemmas [Sko15c]
- Further applications to key derivation [Sko17b]
- Simplifies other technical arguments [VZ12]

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Technique (Sketch)

- In program-input indistinguishability games, it makes sense to characterize the optimal input player Y against a given program player D.
- $^{
 m P}$ View D as a separating hyperplane, maximize margin with high-entropy Y.



Symbol/Operator	Crypto	Geometry
X	candidate distribution	
Y	input player	feasible point
D	distinguisher/program player	separating hyperplane
ED(Y)	expectation	$D \cdot P_Y$ (dot-product)
$\epsilon = \mathbf{ED}(Y) - \mathbf{ED}(X)$	advantage	separation margin

Figure 1: Geometrical meaning of cryptographic indistinguishability.

Closed-form solutions found in interesting cases by **convex optimization**. For pseudoentropy of at least k bits against attackers \mathcal{D} with advantage ϵ :

$$\forall D \in \mathcal{D}: \quad \mathbf{E}D(X) \leqslant 2^{-k}|D| + \epsilon$$

instead of the standard depth-2 formula $\forall D\exists Y: \mathbf{H}_{\infty}(Y) \geqslant k \& \mathbf{ED}(X) \leqslant \mathbf{ED}(Y) + \epsilon$. Characterization depend on feasible distinguishers and the baseline entropy.

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- Applications of pseudoentropy use different notions, most commonly unpredictability-based and indistinguishability-based.
- ? Are unpredictability and indistinguishability entropies different? Note: usually, distinguishing is easier than predicting¹.
- Surprisingly, equivalent in high-entropy regimes!

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Contribution

The following result was obtained [SGP15]:

- equivalence of unpredictability and indistinguishability pseudoentropy definitions in **high-entropy regimes**, namely $n - O(\log n)$ for *n*-bit strings,
- geometric characterizations as a workhorse of the proof.

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Technique (Sketch)

The proof strategy is to constructively convert a distinguisher into a predictor.

- (a) Indistinguishability fails: $ED(X) \ge ED(Y) + \epsilon$ for all Y of min-entropy k.
- (b) $ED(X) \ge |D|/2^k + \epsilon$ for boolean D, by geometrical characterizations (!)
- (c) Sample A from the image of D, then $P\{A = X\} > 2^{-k} + \frac{\epsilon}{\#D}$.
- (d) Approximate image sampling by rejection sampling ℓ times, then

$$\mathbf{P}\{A = X\} > \left(2^{-k} + \frac{\epsilon}{\#\mathbf{D}}\right) \cdot \left(1 - \frac{\#\mathbf{D}}{2^n}\right)^{\ell}.$$

- (e) $P\{A = X\} > 2^{-k}$ when $\ell \approx 2^{n-k}/\epsilon$ independently of #D!
- More sophisticated rejection-sampling handles X with auxiliary input Z.

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- Applications of pseudoentropy assume strength parameters that propagate through reduction proofs. Not clear what regimes are non-trivial to start with.
 - ? Can we characterize when quality parameters are non-trivial?
- Yes, by time-advantage tradeoffs similarly to pseudorandomness!

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Contribution

The following result was obtained [Sko17b]:

- P generic attacks with time t succeed against psuedoentropy amount k with advantage $\epsilon = O\left(\sqrt{t/2^k}\right)$.
- generalization of the famous time-advantage tradeoffs against pseudorandomness [DTT10].

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Technique (Sketch)

The proof leverages random walk techniques, see also [Ber97].

- (a) Let D: {0,1}ⁿ → {-1,1} be fully random (Rademacher), then |ED(X) ED(Y)| ≈ ||P_X P_Y||₂ w.h.p. (random walks theory [Haa81]).
 (b) By slicing the domain {0,1}ⁿ into t random parts and flipping the signs accordingly.
- (b) By slicing the domain $\{0,1\}^n$ into t random parts and flipping the signs accordingly, we can have $|\mathbf{ED}'(X) \mathbf{ED}'(Y)| \approx t \cdot \frac{\|\mathbf{P}_X \mathbf{P}_Y\|_2}{\sqrt{t}}$ w.h.p., with O(t) extra memory.
- (c) Under mild assumptions on Y (far from having k bits of min-entropy) the attack achieves advantage $\epsilon \approx \sqrt{t/2^k}$. Compare with $\epsilon \approx \sqrt{t/2^n}$ for pseudorandomness!
- (d) "Random" can be weakened to O(1)-wise independent, and the construction complexity is indeed O(t), alternatively complexity t yields $\epsilon = O(\sqrt{t/2^k})!$

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- Applications of pseudoentropy heavily rely on manipulation rules, particularly chain rules and transformations [BSW03, FOR12]. However, their use weakens security guarantees, due to tradeoffs in quality parameters caused by reduction proofs.
- ? Can we improve known manipulation rules?
- No, not by black-box reductions!

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Contribution

The following results were obtained in [PS16]:

- Impossibility of better proofs by black-box reductions!
- A probabilistic construction of an oracle, of independent interest, inspired by the earlier work on limitations of dense model theorems [Zha11].
- Inspired techniques used in research on black-box limitations of auxiliary input simulators [CCL18].

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Techniques (Sketch)

- (a) Proofs, in case of indistinguishability-based pseudoentropy, rely on building distinguishers from distinguishers in other setups. In particular, we have $D' = \mathbb{I}\{\sum_i w_i D_i > t_0\}$ in the proof of the Dense Model Theorem [Zha11] or transformations [BSW03, Sko15b].
- (b) Loosely speaking, black-box reductions aggregate distinguishers by high-level operations. To prove the need of many operations (queries) we manipulate distinguishers at low-level by choosing values probabilistically and sophistically!
- (c) Examples (from the proof, see also [CCL18]):
 - (1) $D_i \sim \text{Bern}(1/2 + \epsilon)$ on a small set and $D_i \sim \text{Bern}(1/2)$ elsewhere.
 - (2) $D_i \sim \text{Bern}(1/2 + \epsilon)$ and $D_i \sim \text{Bern}(1/2 \epsilon)$ on complementary random subsets.

High-level aggregations are essential. For example, without them Dense Model Theorems can fail [IM20].

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- In security proofs, it helps to model leakages as explicit functions of secrets [JP14].
- ? What leakages can be modelled as functions of secrets?
- Short leakages can be efficiently simulated!

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Contribution

The following important results were obtained [Sko16a]:

- P Construction of a simulator for m bits of leakage which makes only $2^{O(m)}\epsilon^{-2}$ calls to achieve ϵ -indistinguishability. Significantly improved upon prior works [VZ13, JP14]
- The reasoning, inspired by ML techniques, builds on the gradient descent algorithm and was recognized with the best student paper award at TCC'16.
- Inspired follow-up works that solved the simulator problem [CCL18], and generalized the learning framework [Sko16b, Sko17a], and studied quantum pseudoentropy (c.f. Chen's dissertation [Che19]).

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Technique (Sketch)

(a) The algorithm below demonstrates the procedure

```
Algorithm 1: Auxiliary Input Simulator
```

Data:

- Oracle access to distinguishers/test functions ${\cal D}$

```
Result: Simulator h of Z \in \{0,1\}^m given X \in \{0,1\}^n
1 P\{h(x) = z\} \leftarrow 2^{-m}
                                                  // initialize the solution as uniform
2 while \max_{D \in \mathcal{D}} \mathrm{ED}(X,Z) - \mathrm{ED}(X,h(X)) > \epsilon // as long as can distinguish...
3 do
     \mathbf{P}\{h'(x) = z\} \leftarrow \mathbf{P}\{h(x) = z\} - \gamma \mathsf{D}(x, z)
                                                                          // improve candidate
     P\{h'(x) = z\} \leftarrow P\{h'(x) = z\} + Correct(x, z) // guarantee constraints
7 end
8 return h
```

- (b) Outputs an efficient simulator p.d.f., with appropriate γ and Correct operation.
- Finishes after $2^{O(m)} \epsilon^{-2}$ steps, proved by "energy" arguments.
- Resembles boosting: we learn how to (strongly) simulate from (weak) distinguishers
- (e) Resembles convex optimization: with D as subgradient, γ as a step size, Correct as a projection operation!

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Addressing Reviewers Feedback

- R: Editorial changes and reference requests.
- M: Addressed, thanks for the feedback!
- R: A book-style dissertation would be better than a mixture of conference works.
- M: I discussed this form with senior researchers, but found ineffective:
 - 🧪 Gain citations! 🤔 Time-consuming, better to keep writing papers.
 - Get your PhD distinguished. Prestigious conferences not enough?
 - Rake your time to present it better! Why to work harder? We count conference works when granting junior/senior professorships!
- R: Parts of lengthy works might not have been fully reviewed at conferences.
- M: Same can happen for junior professorships, but we had extra reviewers 😌.

