

BMD ENG 301
Quantitative Systems Physiology
(Nervous System)

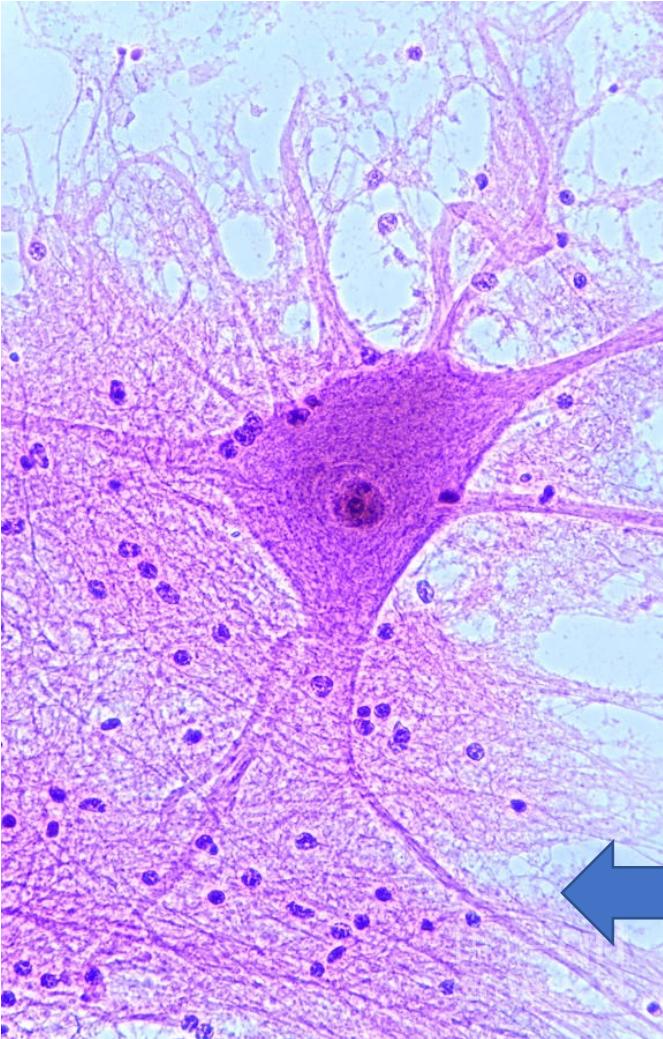
Lecture 6: Neural Signals in Dendrites
2022_v2

Professor Malcolm A. MacIver

Switch to Doc camera

- Notes to be scanned into PDF

Spinal Motor Neuron



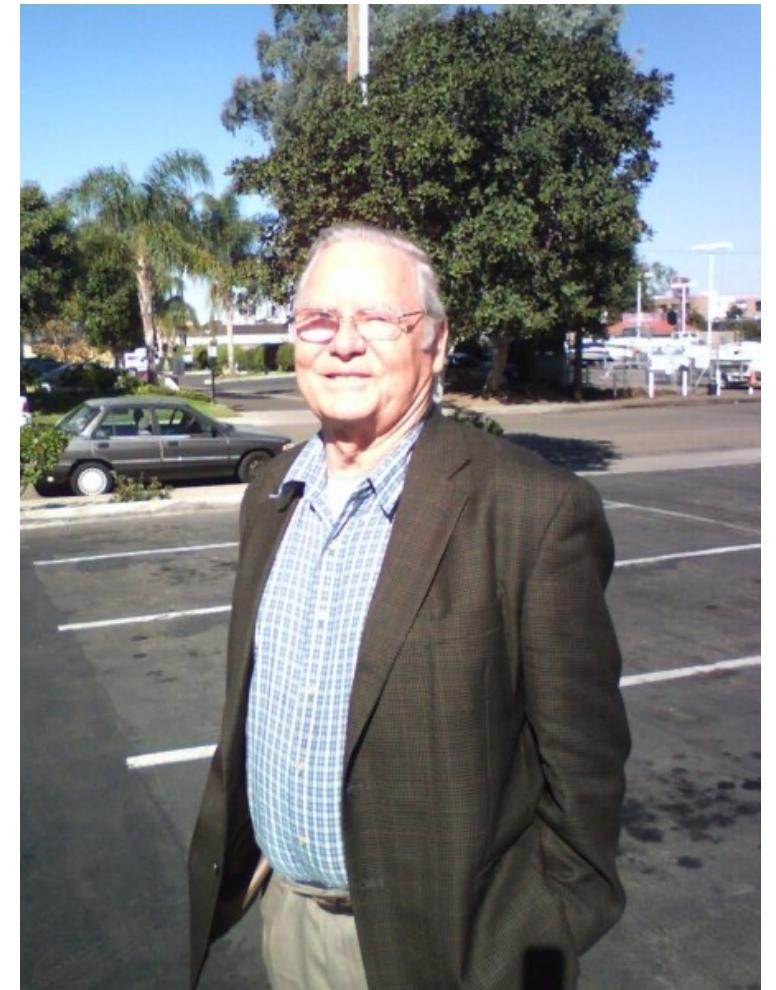
How can we model summation of synaptic inputs?



Axon

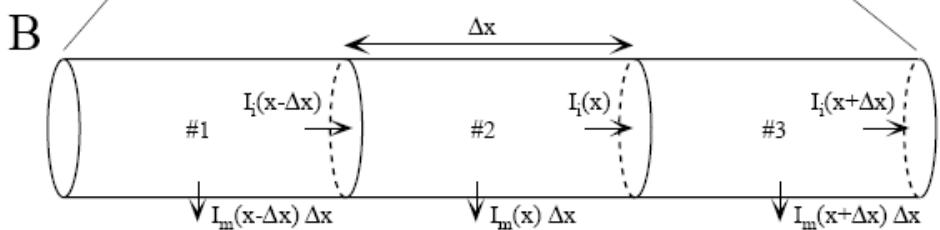
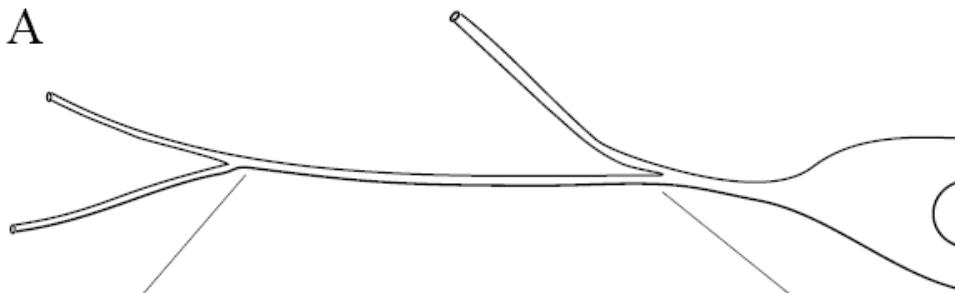
History of neural cable theory

- Cable theory developed for signal flow along electrical cables in the 19th century
- Original neural application by Hodgkin and Rushton (1946) to the conduction of potential along an axon
- Rall (1959) extended theory and applied it to the dendritic trees of neurons.

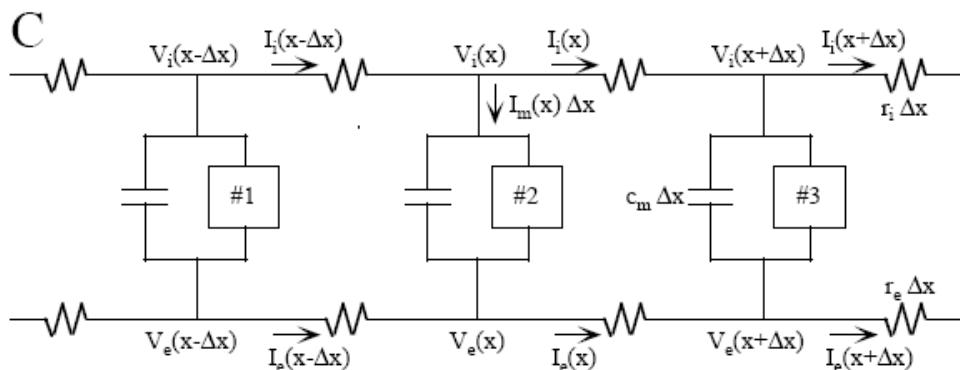


Wilfred Rall

Dendritic Cable



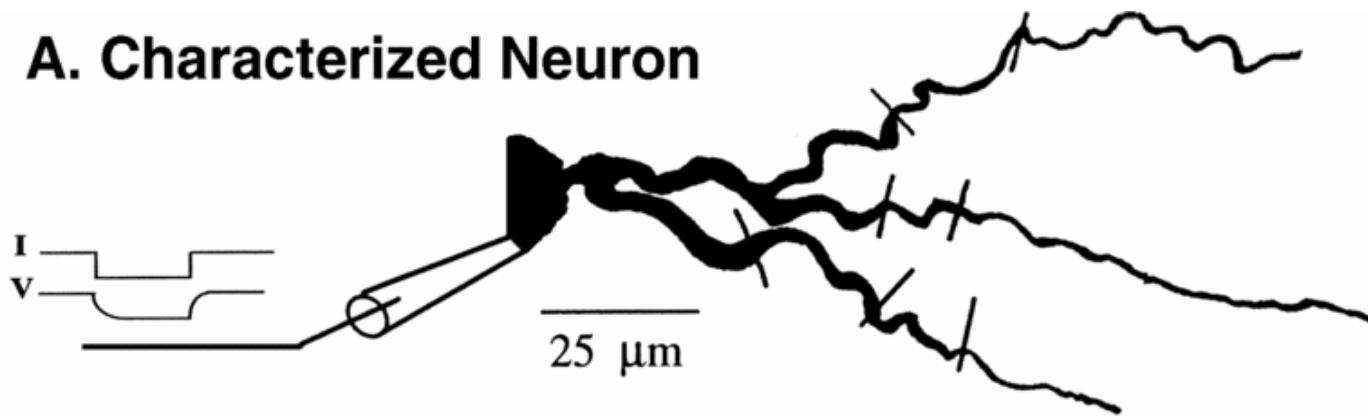
A. One portion of dendrite



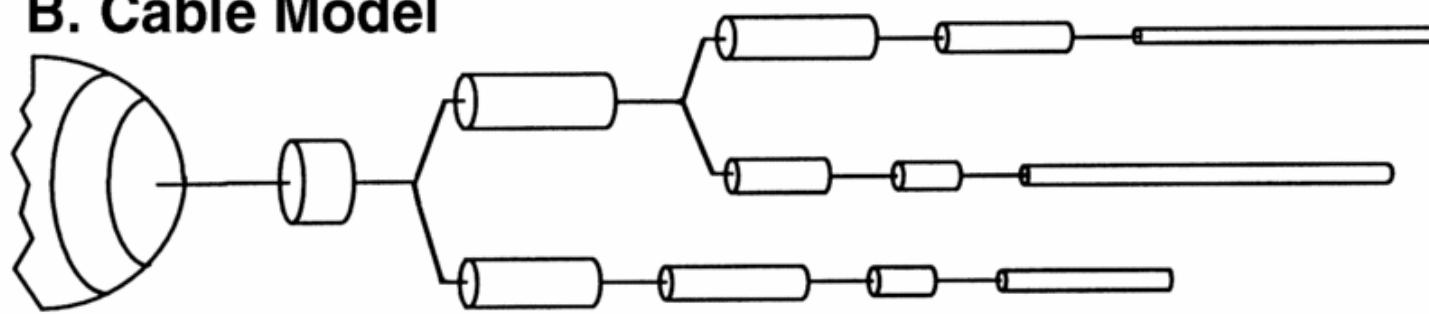
B. divided into three sub-cylinders.

C. Discrete electrical model for the three sub-cylinders.

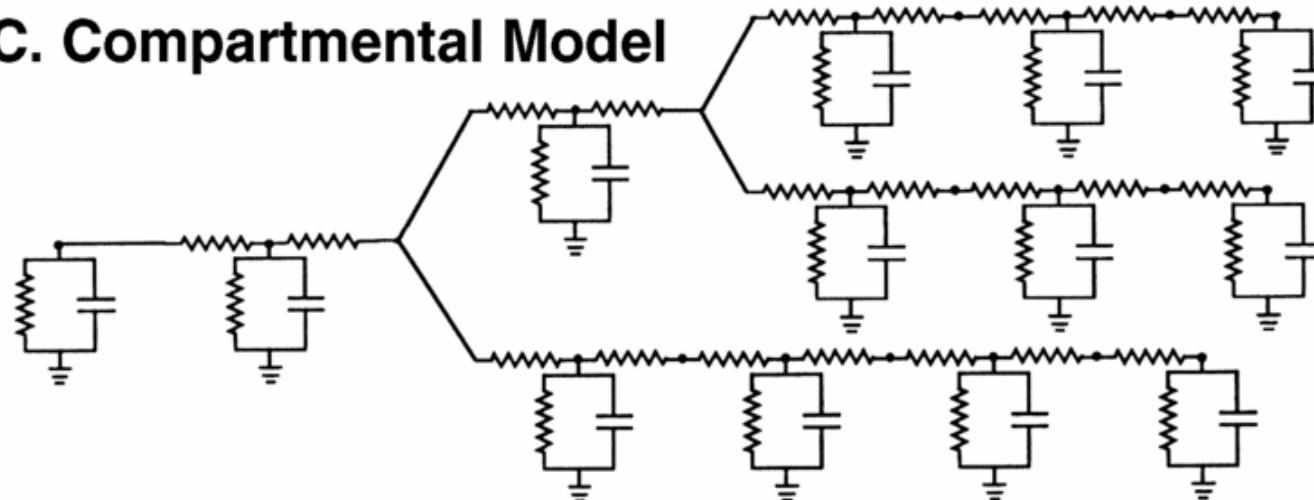
A. Characterized Neuron



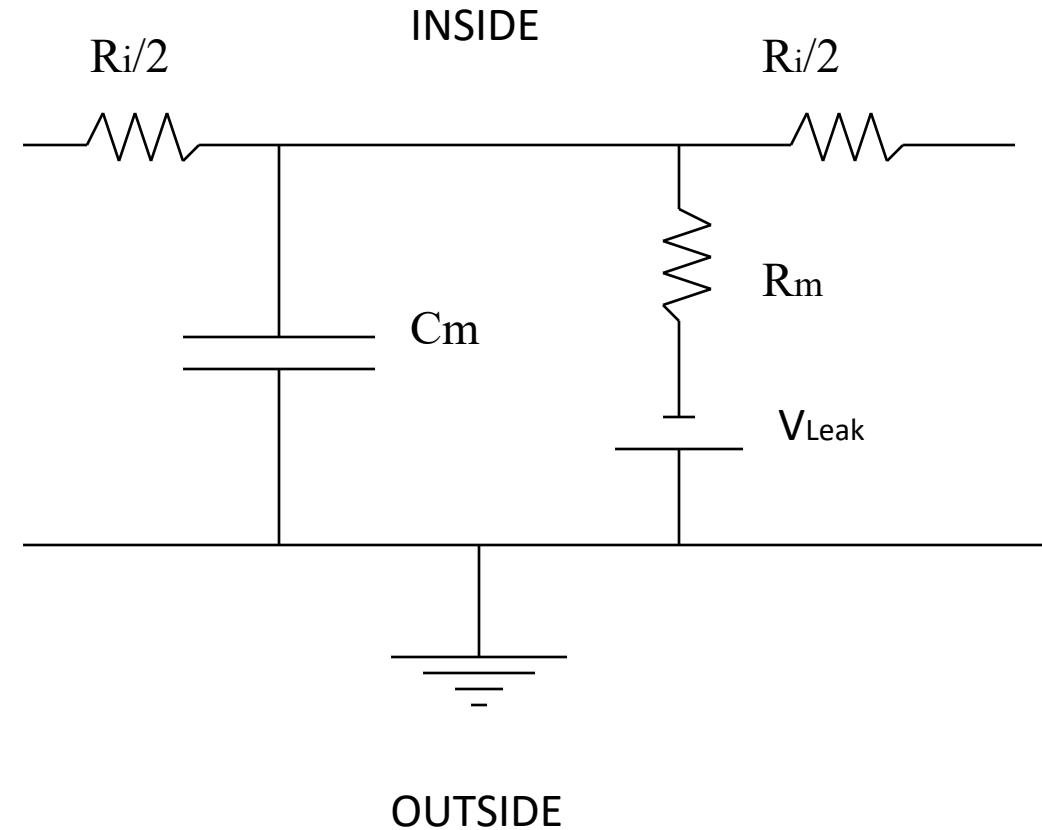
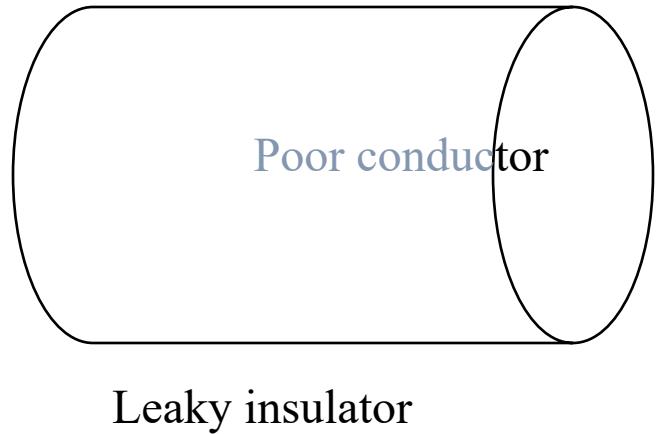
B. Cable Model



C. Compartmental Model



Equivalent electrical circuit



Parallel RC Circuit

V_{Leak} is the resting membrane potential

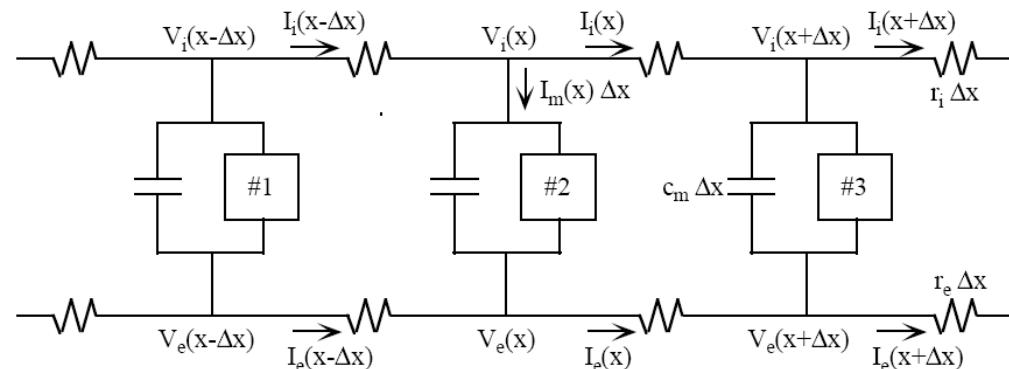
Cable equation

$$\frac{\partial^2 V}{\partial x^2} = R_a \left[\frac{(V - E_{\text{leak}})}{R_m} + C_m \frac{\partial V}{\partial t} \right]$$

Internal
Resistance

Membrane
Resistance

Membrane
Capacitance



$$\frac{\partial V_i}{\partial x} = -i_i r_i \Rightarrow \frac{\partial^2 V_i}{\partial x^2} = -r_i \frac{\partial i_i}{\partial x}$$

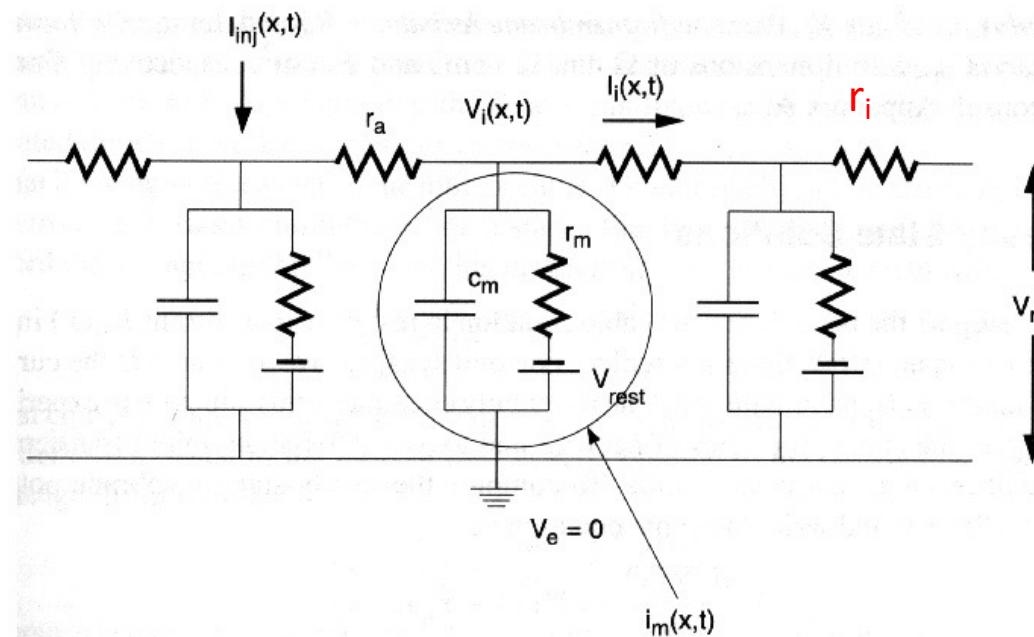
$$i_m = -\frac{\partial i_i}{\partial x}$$

$$\frac{r_m}{r_i} \frac{\partial^2 V}{\partial x^2} = i_m r_m$$



$$\lambda^2 \frac{\partial^2 V}{\partial x^2} = \tau_m \frac{\partial V}{\partial t} + V$$

$$\lambda = \sqrt{\frac{r_m}{r_i}} \quad \tau_m = r_m c_m$$



$$i_m = c_m \frac{\partial V}{\partial t} + \frac{(V_i - V_e - E_r)}{r_m} \longrightarrow i_m r_m = \tau_m \frac{\partial V}{\partial t} + V \longrightarrow$$

N.B.: r_a is often used to mean axial (internal) resistance but many use r_i instead

$$\frac{d}{dt} = 0$$

$$\frac{d}{dx} = 0$$

Assume conductor is a semi-infinite long cable that extends from $x=0$ to $x=\infty$

$$\frac{r_m}{r_i} \frac{\partial^2 V}{\partial x^2} = \tau_m \frac{\partial V}{\partial t} + V \xrightarrow{\text{ss}} \frac{r_m}{r_i} \frac{\partial^2 V}{\partial x^2} = V$$

$$\frac{r_m}{r_i} \frac{\partial^2 V}{\partial x^2} = \tau_m \frac{\partial V}{\partial t} + V \longrightarrow 0 = \tau_m \frac{\partial V}{\partial t} + V$$

$$V = A_1 e^{x/\lambda} + A_2 e^{-x/\lambda}$$

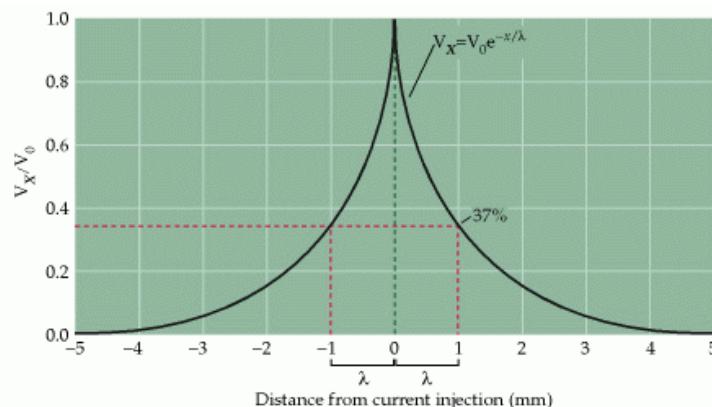
$$V(0) = V_o$$

$$|V(\infty)| < \infty$$

$$A_1 = 0$$

$$A_2 = V_o$$

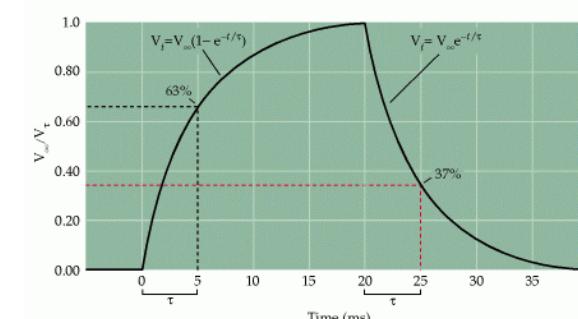
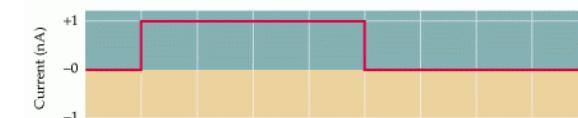
$$V(x) = V_o e^{-|x|/\lambda}$$



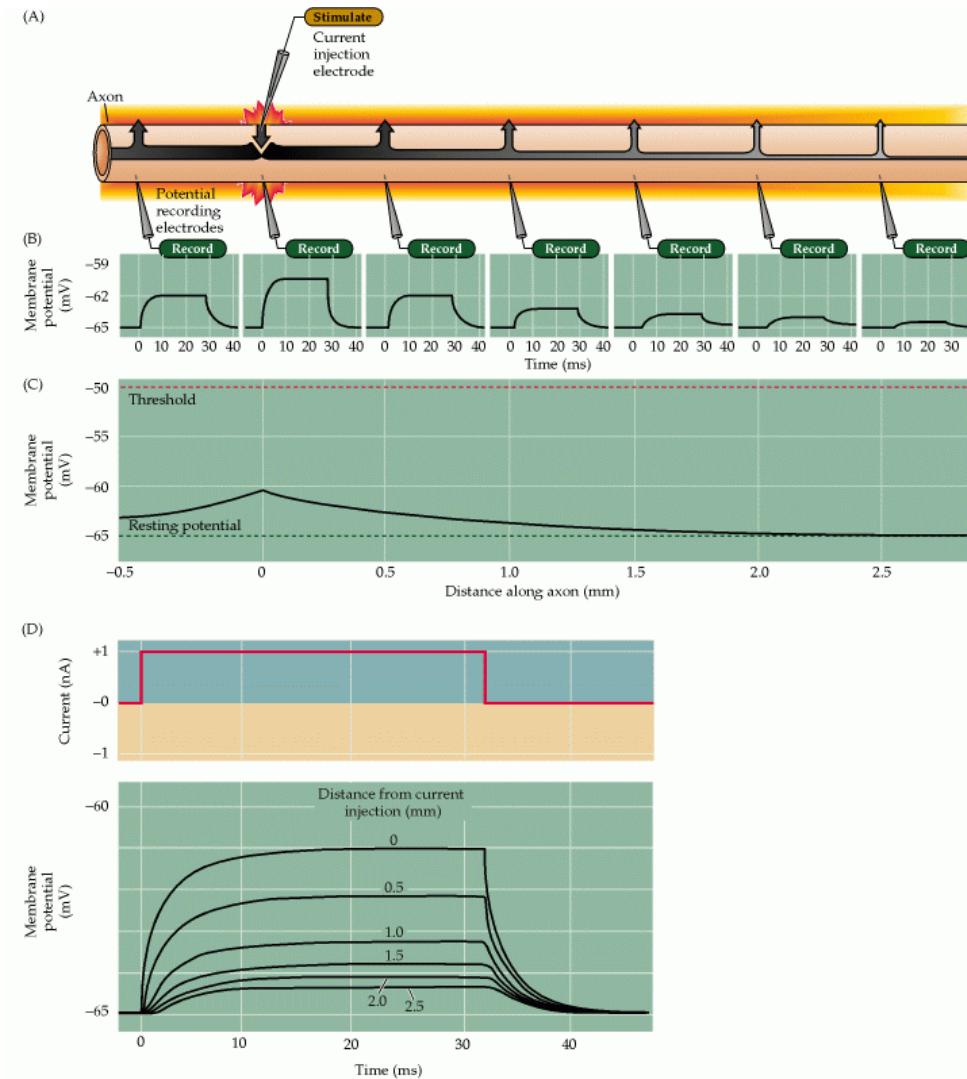
$$\frac{\partial V}{\partial t} = -\frac{V}{\tau_m} \quad \ln V = -\frac{t}{\tau_m}$$

$$\frac{1}{V} \partial V = -\frac{1}{\tau_m} \partial t$$

$$V(t) = A e^{-t/\tau_m}$$



Signal propagation



Charge Degradation over Time

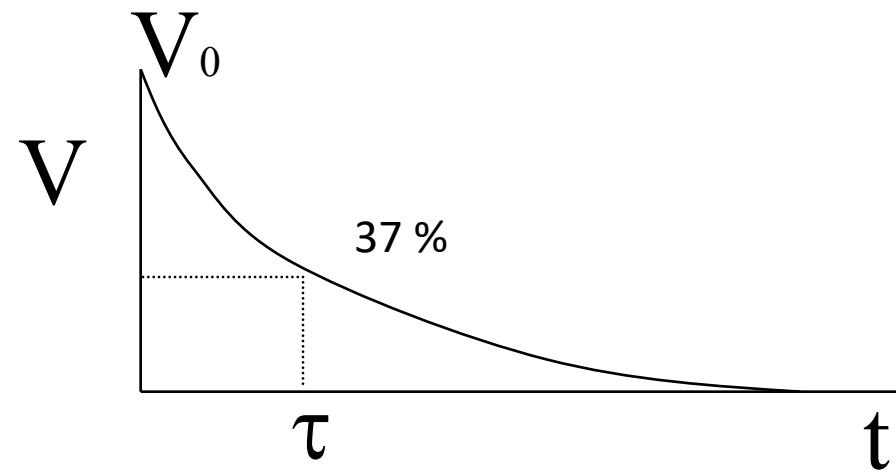
Ignore space: $\frac{\partial^2 V}{\partial x^2} = 0$

$$\frac{\partial^2 V}{\partial x^2} = R_i \left[\frac{(V - V_{\text{leak}})}{R_m} + C_m \frac{\partial V}{\partial t} \right]$$

So: $V = V_0 \exp\left(\frac{-t}{R_m \cdot C_m}\right)$

Where $R_m \cdot C_m = \tau$ = time constant

Representation of time constant



Charge Degradation over Distance

Ignore time: $\frac{\partial V}{\partial t} = 0$

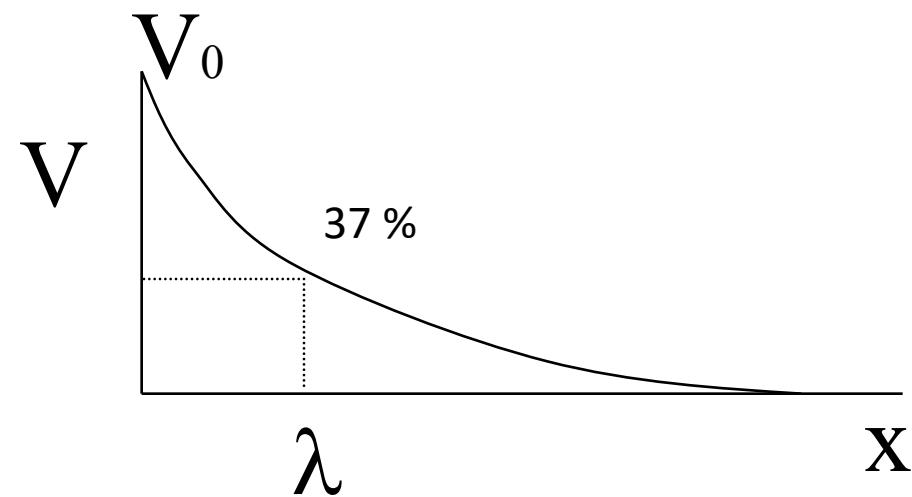
$$\frac{\partial^2 V}{\partial x^2} = R_i \left[\frac{(V - V_{\text{leak}})}{R_m} + C_m \frac{\partial V}{\partial t} \right]$$

So: $V(x) = A_1 e^{(x/\lambda)} + A_2 e^{(-x/\lambda)}$

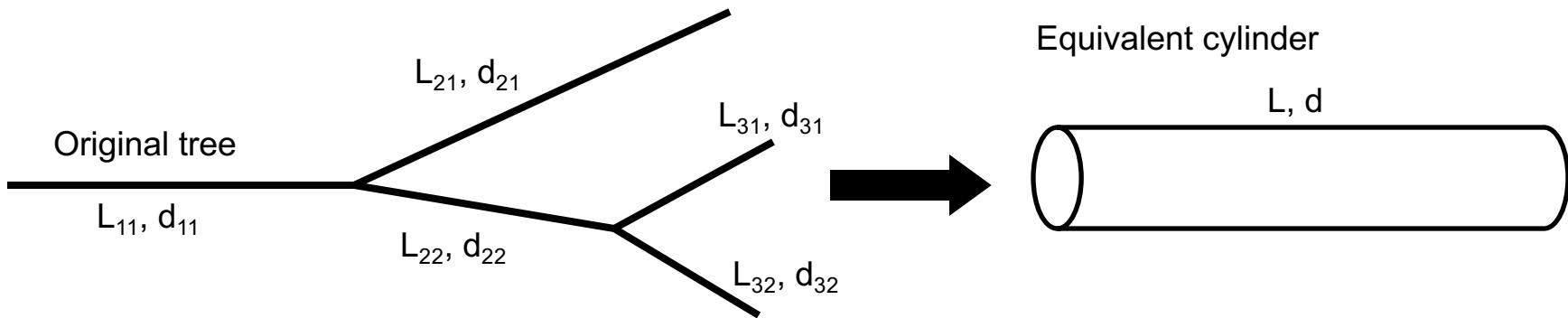
Where λ = length constant

$$\lambda = \text{sqrt}(R_m/R_i)$$

Representation of length constant



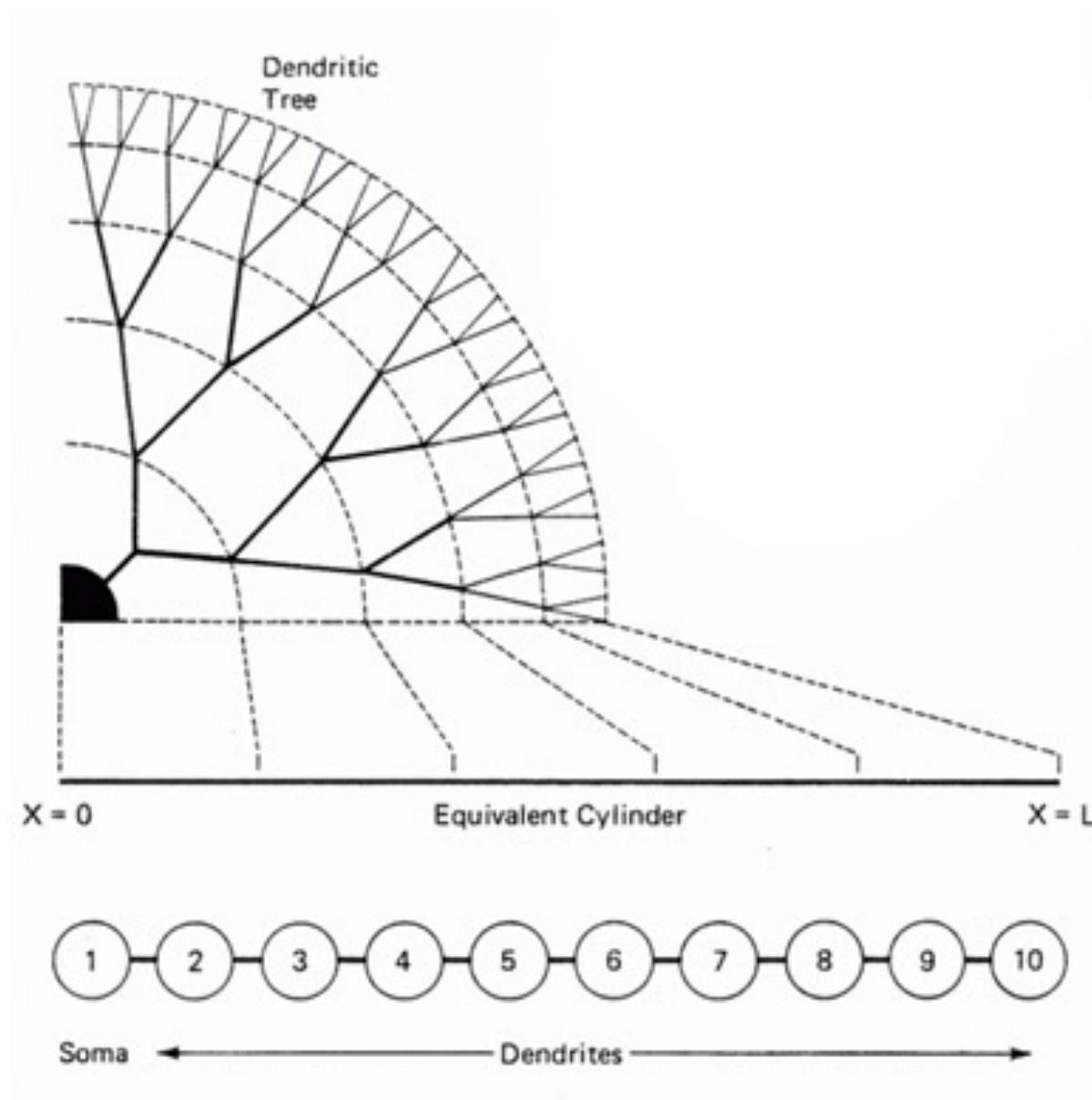
Equivalent Cylinder Theorem



- Equivalent Cylinder Theorem: an entire class of dendritic trees can be reduced into a single equivalent cylinder if the following conditions are met:
 - The values of r_i and r_m are the same in all branches;
 - All terminals end at the same boundary condition;
 - All terminal branches end at the same electrotonic distance from the origin in main branch
 - The 3/2 power law at every branch point

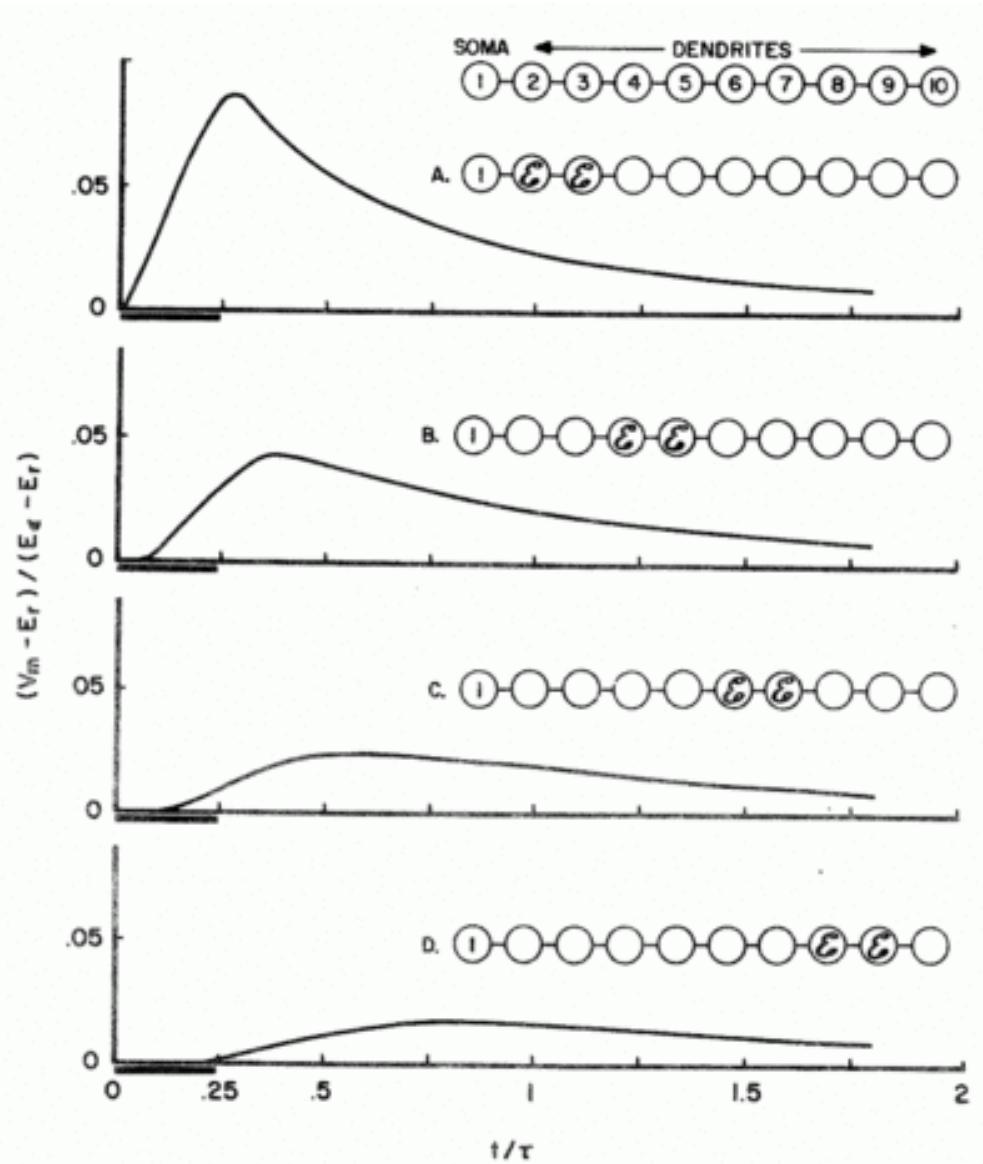
$$d_{parent}^{3/2} = \sum_{\text{all child branches } j} d_j^{3/2}$$

Rall Model

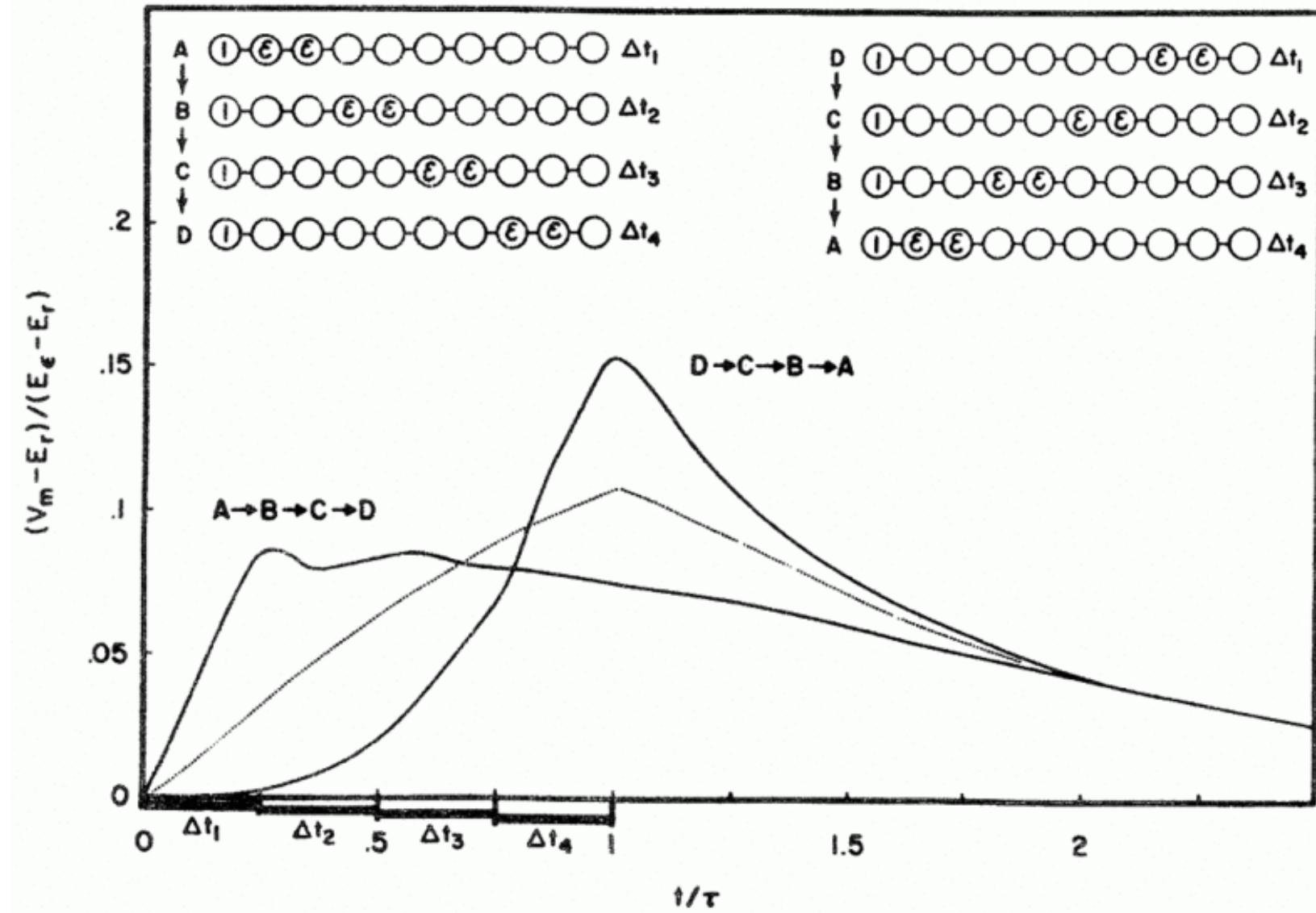


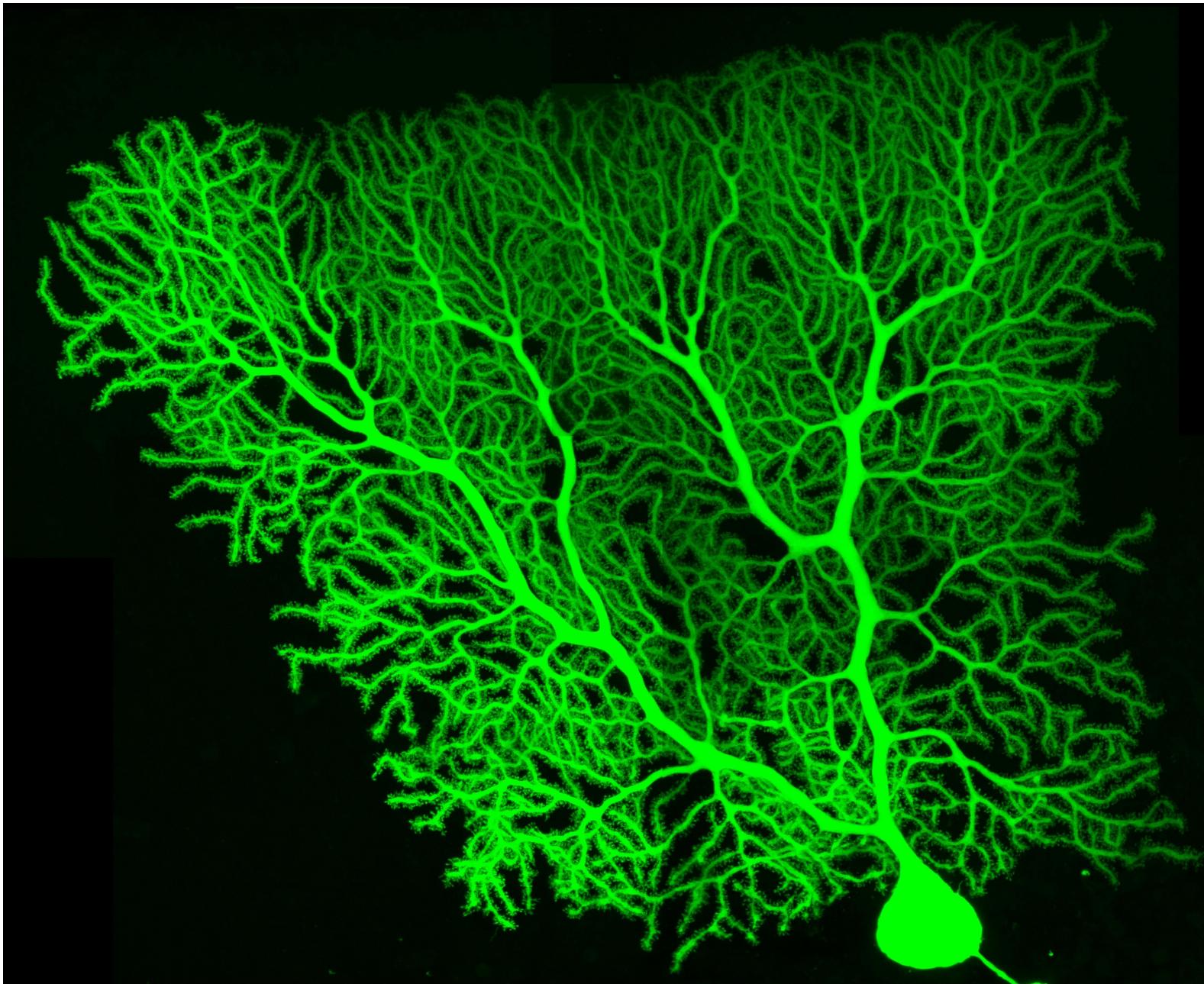
Rall Model

- Allowed modeling of dendritic behavior
 - Confirmed by experimental results
- Stimulation at different distances along arbor
- Modeling of EPSPs

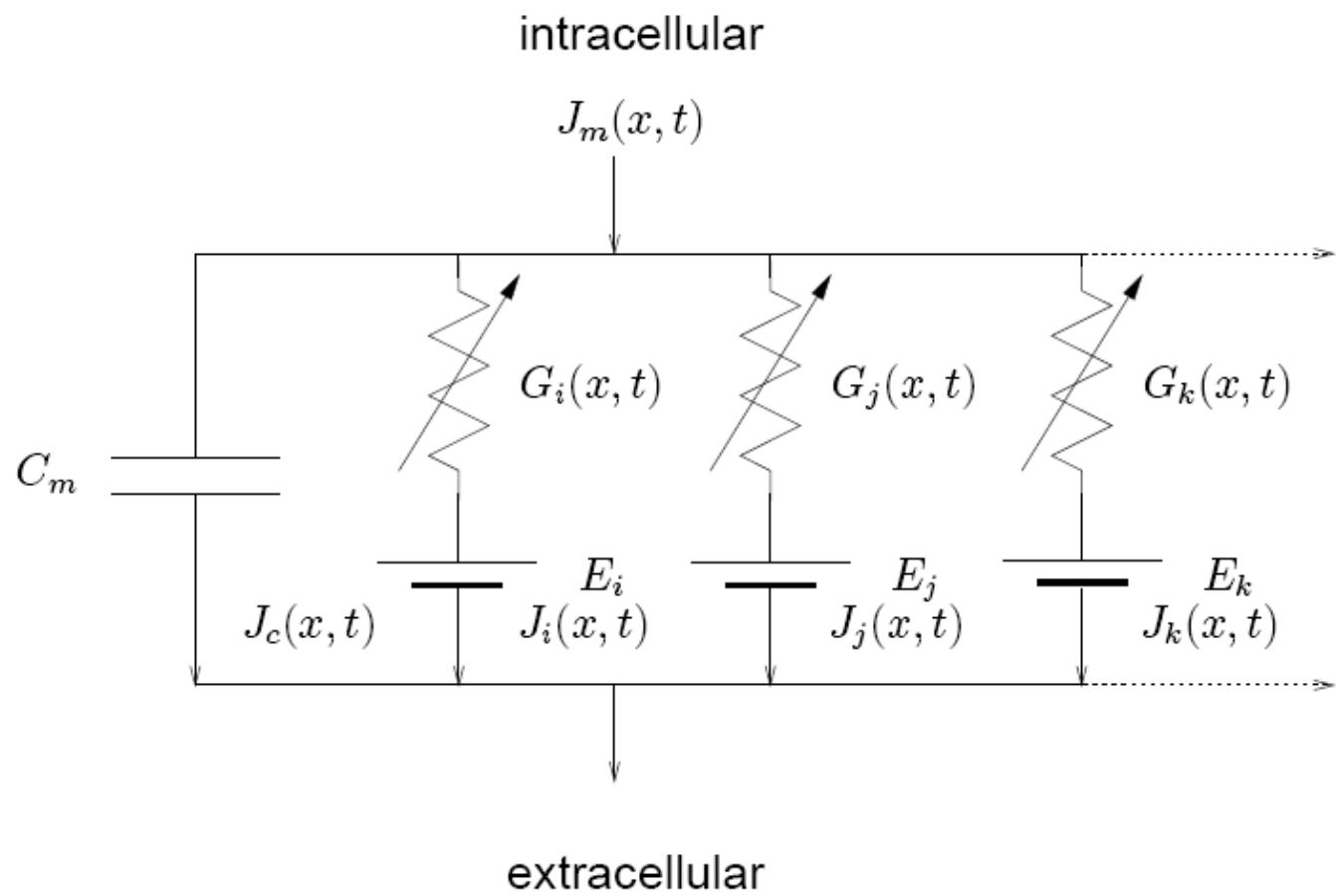


Rall Model





What if R_m is not constant, depending on x , V and t ?



$$\lambda^2 \frac{\partial^2 V}{\partial x^2} = \tau_m \frac{\partial V}{\partial t} + V \rightarrow \lambda^2 \frac{\partial^2 V}{\partial x^2} = \tau_m \frac{\partial V}{\partial t} + \sum_k (V - E_k) G(x, t, V)$$