BMD ENG 301
Quantitative Systems Physiology
(Nervous System)

Lectures 6: Neural Signals in Dendrites

2022_v1

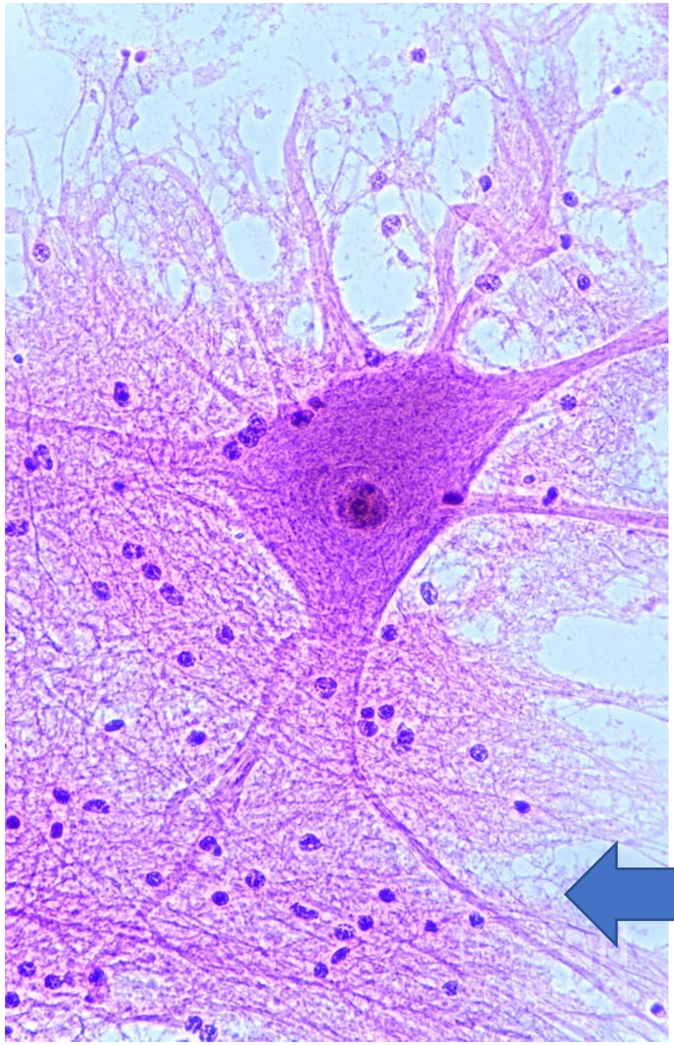
Professor Malcolm A. MacIver

Never Too Late to Up One's Game

Anne Bronte (1820-1849) *The Tenant of Wildfell Hall (1849)*
Charlotte Bronte (1816-1855) *Jane Eyre (1847)*
Emily Bronte (1818-1848) *Wuthering Heights (1847)*

James Bronte Gatenby (1892-1960) – zoologist known for his study of Golgi bodies

Spinal Motor Neuron



How can we model summation of synaptic inputs?

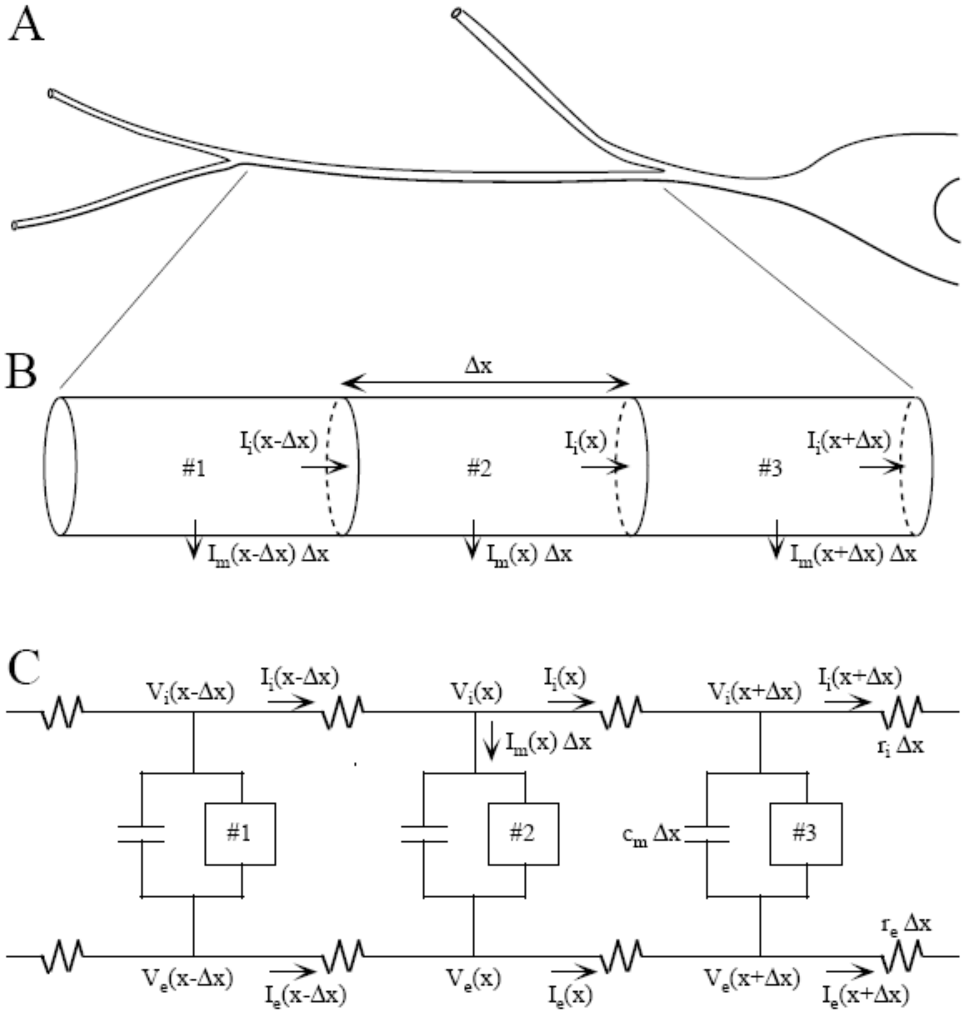
Axon

History of neural cable theory

- Cable theory developed for signal flow along electrical cables in the 19th century
- Original neural application by Hodgkin and Rushton (1946) to the conduction of potential along an axon
- Rall (1959) extended theory and applied it to the dendritic trees of neurons.

Wilfred Rall

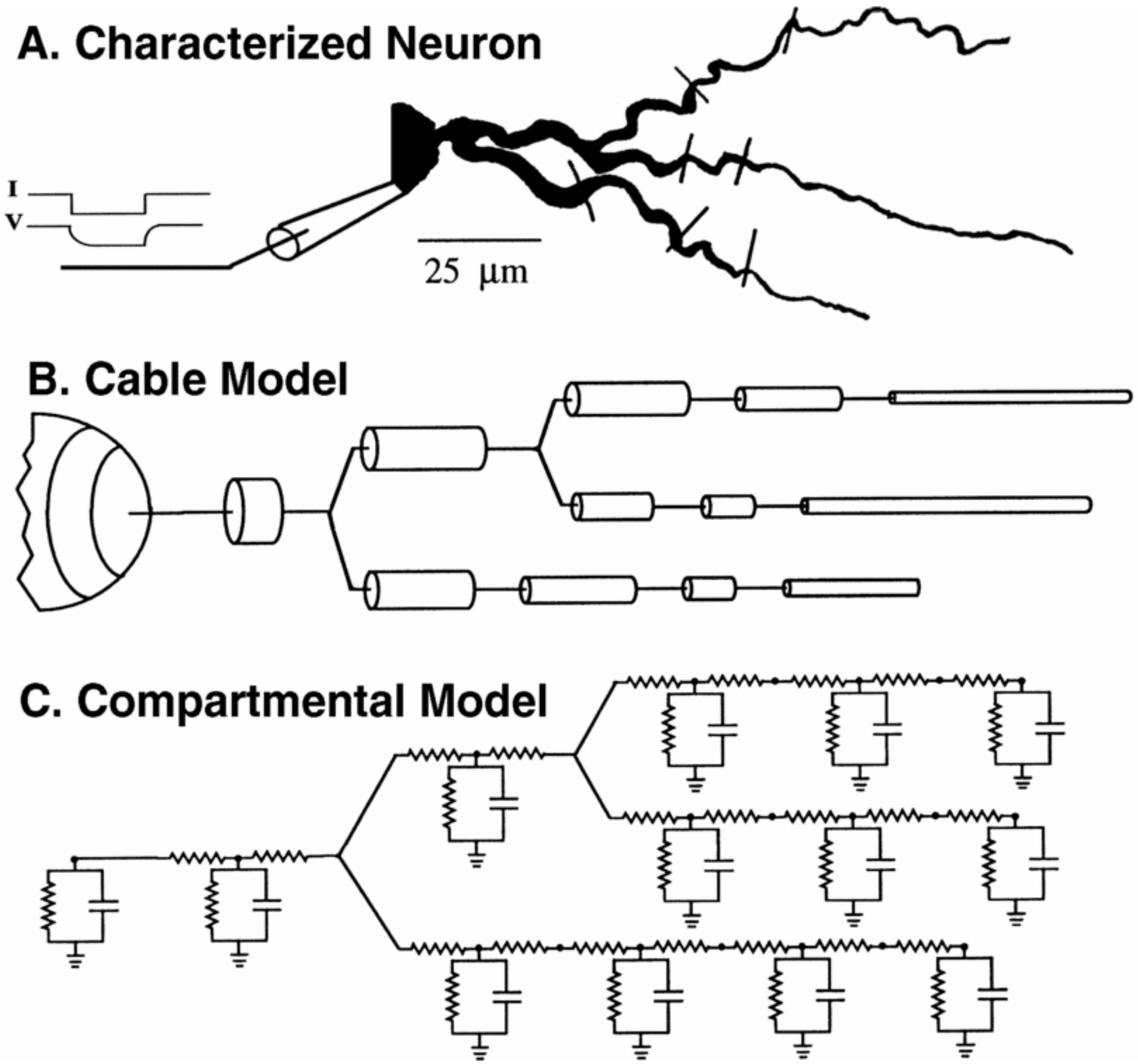
Dendritic Cable



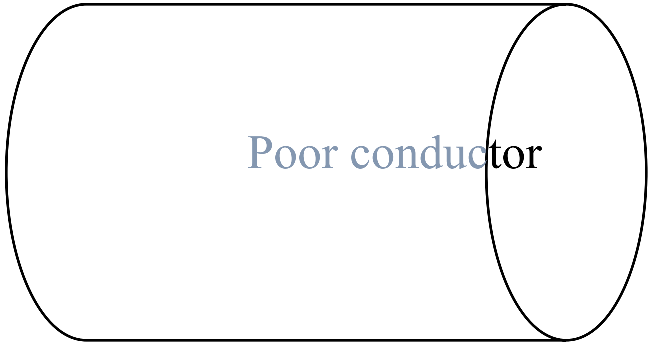
A. One portion of dendrite

B. divided into three sub-cylinders.

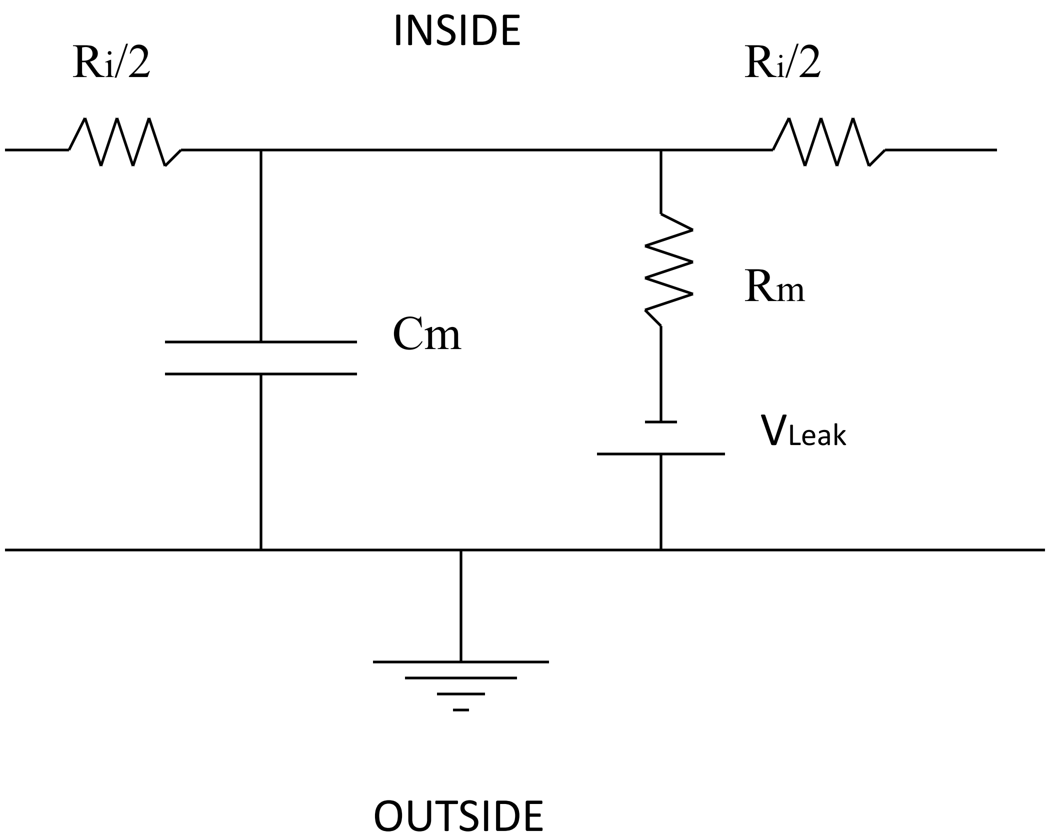
C. Discrete electrical model for the three sub-cylinders.


A. Characterized Neuron
I
V
25 μm
B. Cable Model
C. Compartmental Model

Equivalent electrical circuit



Leaky insulator



Parallel RC Circuit

V_{Leak} is the resting membrane potential

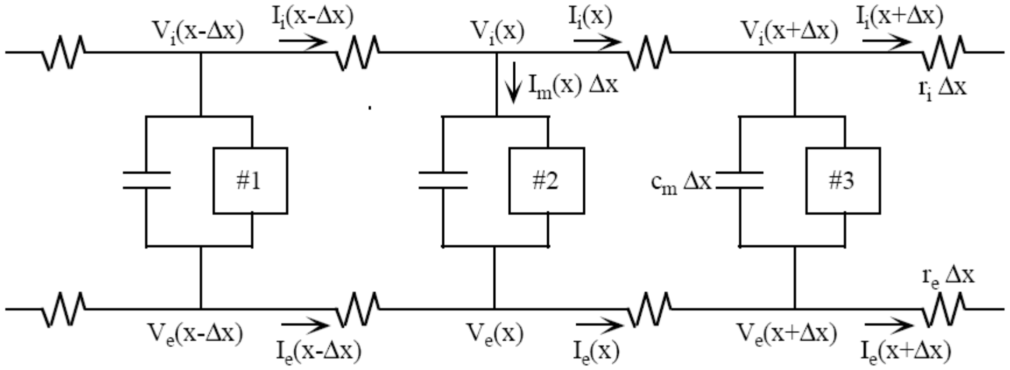
Cable equation

$$\boxed{\frac{\partial^2 V}{\partial x^2}} = R_a \left[\boxed{\frac{(V - E_{leak})}{R_m}} + \boxed{C_m \frac{\partial V}{\partial t}} \right]$$

Internal Resistance

Membrane Resistance

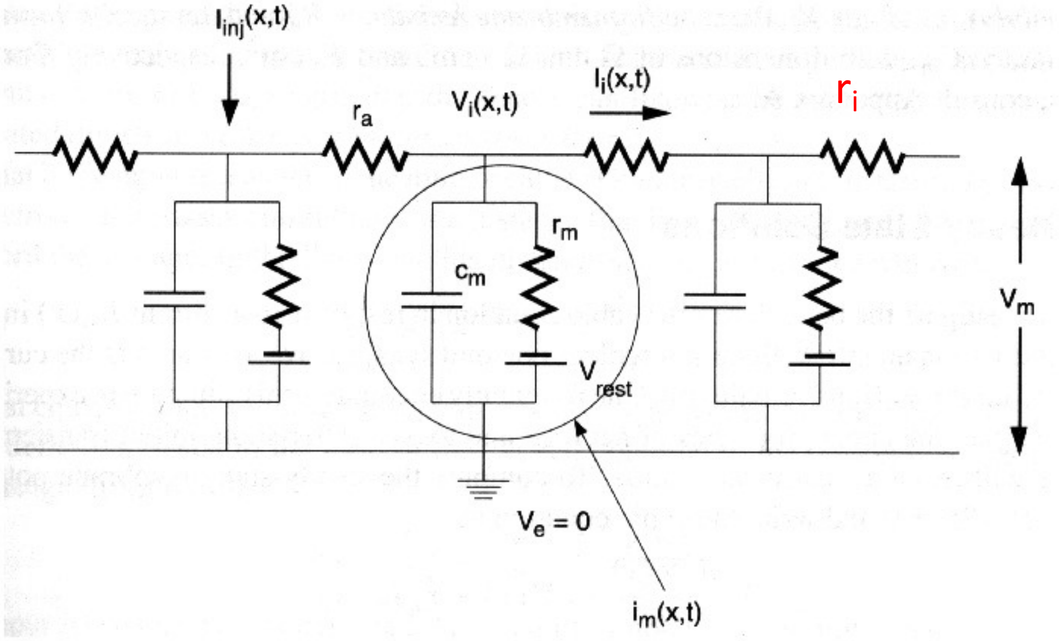
Membrane Capacitance



$$\frac{\partial V_i}{\partial x} = -i_i r_i \Rightarrow \frac{\partial^2 V_i}{\partial x^2} = -r_i \frac{\partial i_i}{\partial x}$$

$$i_m = -\frac{\partial i_i}{\partial x}$$

$$\frac{r_m}{r_i}\frac{\partial^2 V}{\partial x^2} = i_m r_m$$



$$\lambda^2 \frac{\partial^2 V}{\partial x^2} = \tau_m \frac{\partial V}{\partial t} + V$$

$$i_m = c_m \frac{\partial V}{\partial t} + \frac{(V_i - V_e - E_r)}{r_m} \longrightarrow i_m r_m = \tau_m \frac{\partial V}{\partial t} + V \longrightarrow$$

$$\lambda = \sqrt{\frac{r_m}{r_i}} \qquad \tau_m = r_m c_m$$

N.B.: r_a is often used to mean axial (internal) resistance but many use r_i instead

d/dt=0 d/dx=0

Assume conductor is a semi-infinite long cable that extends from x=0 to x=inf

$$\frac{r_m}{r_i}\frac{\partial^2 V}{\partial x^2} = \tau_m \frac{\partial V}{\partial t} + V \xrightarrow{ss} \frac{r_m}{r_i}\frac{\partial^2 V}{\partial x^2} = V$$

$$V = A_1 e^{x/\lambda} + A_2 e^{-x/\lambda}$$

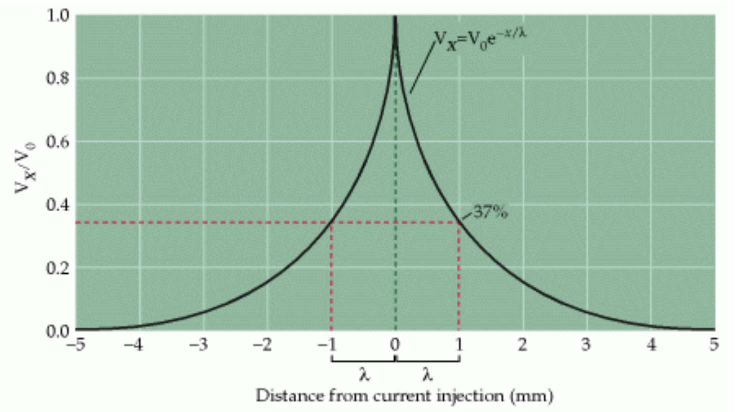
$$V(0) = V_o$$

$$|V(\infty)| < \infty$$

$$A_1 = 0$$

$$A_2 = V_o$$

$$V(x) = V_o e^{-|x|/\lambda}$$

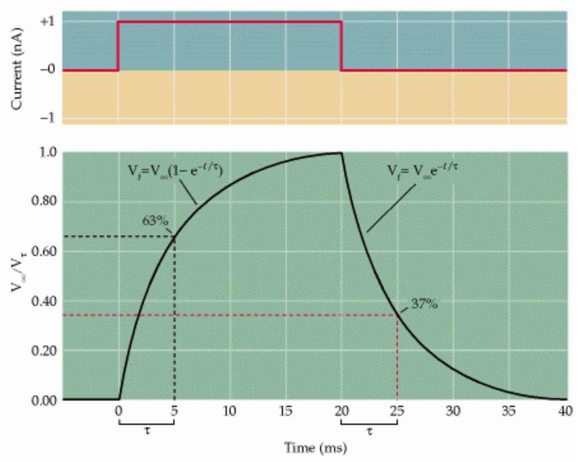


$$\frac{r_m}{r_i}\frac{\partial^2 V}{\partial x^2} = \tau_m \frac{\partial V}{\partial t} + V \longrightarrow 0 = \tau_m \frac{\partial V}{\partial t} + V$$

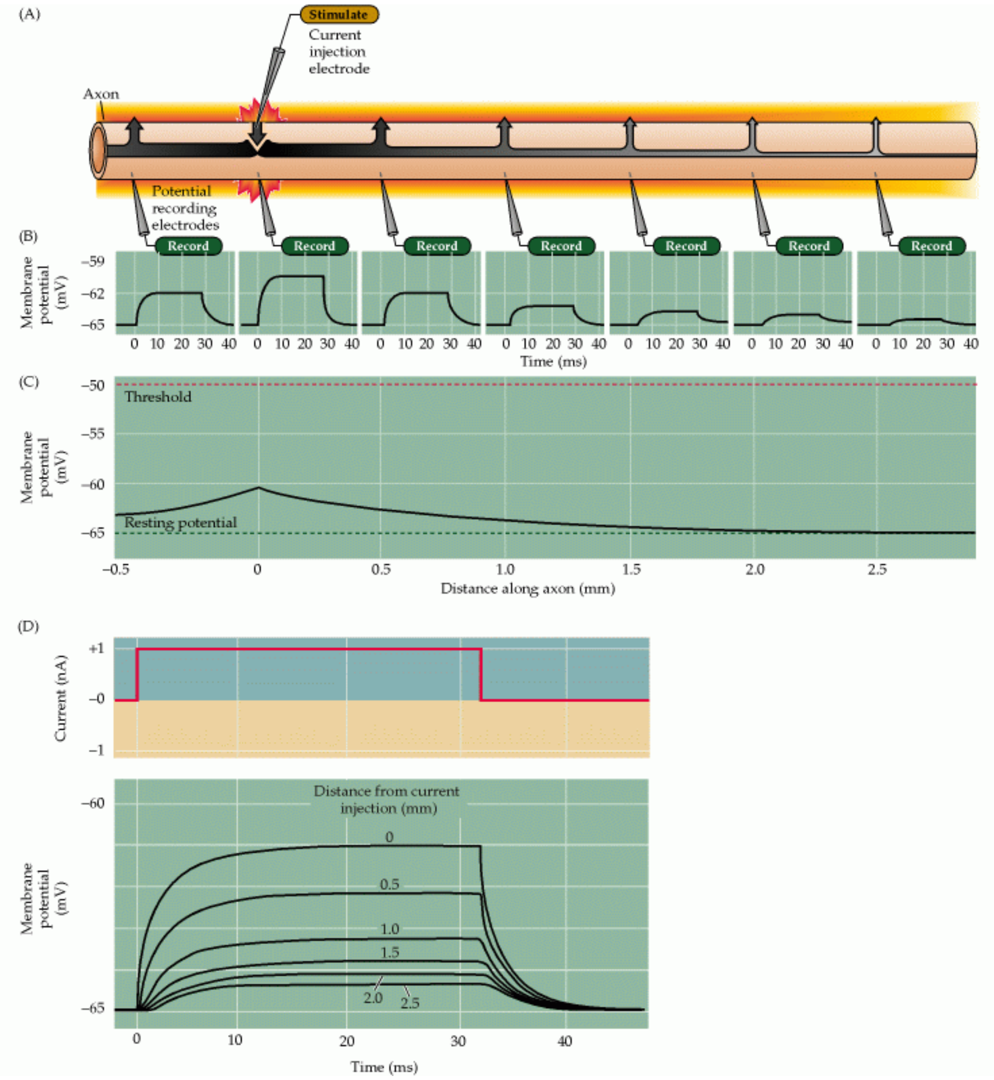
$$\frac{\partial V}{\partial t} = -\frac{V}{\tau_m} \qquad \ln V = -\frac{t}{\tau_m}$$

$$\frac{1}{V}\partial V = -\frac{1}{\tau_m}\partial t$$

$$V(t) = Ae^{-t/\tau_m}$$



Signal propagation



Charge Degradation over Time

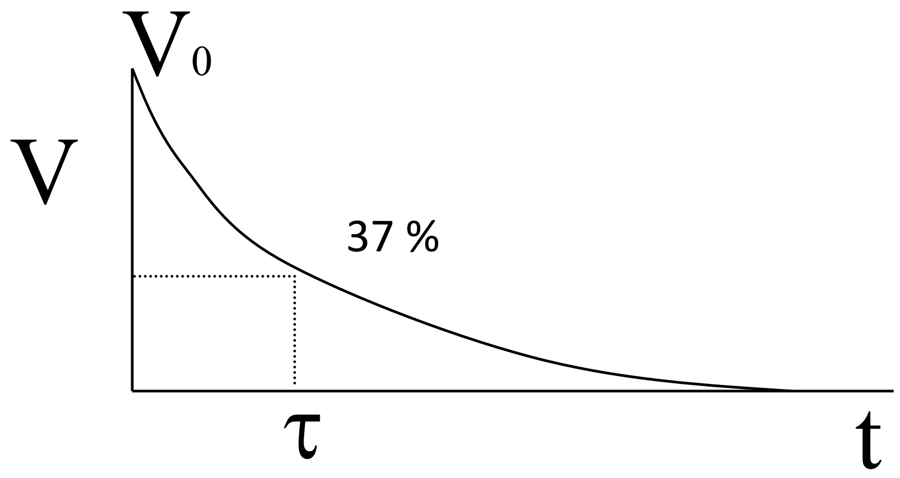
Ignore space: $$\frac{\partial^2 V}{\partial x^2} = 0$$

$$\frac{\partial^2 V}{\partial x^2} = R_i \left[\frac{(V - V_{leak})}{R_m} + C_m \frac{\partial V}{\partial t} \right]$$

So: $$V = V_0 \exp\left(\frac{-t}{R_m . C_m}\right)$$

Where $R_m.C_m = \tau =$ time constant

Representation of time constant



Charge Degradation over Distance

Ignore time: $$\frac{\partial V}{\partial t} = 0$$

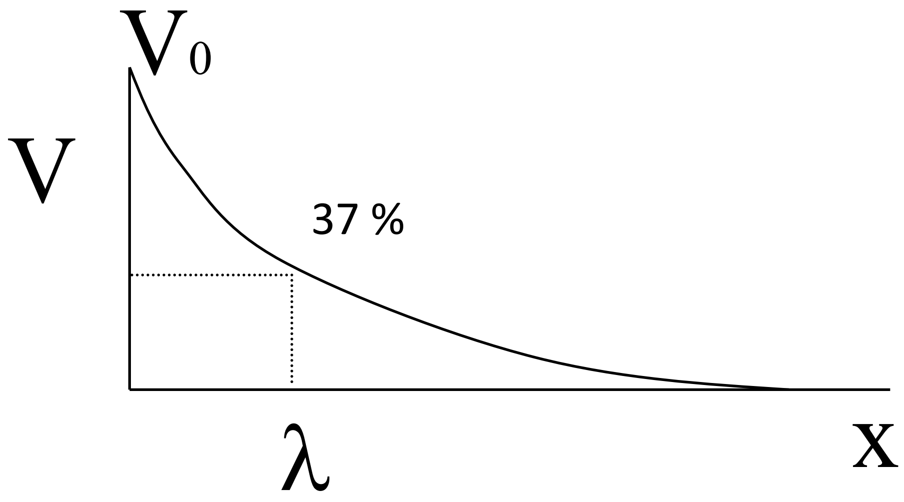
$$\frac{\partial^2 V}{\partial x^2} = R_i \left[\frac{(V - V_{leak})}{R_m} + C_m \frac{\partial V}{\partial t} \right]$$

So: $$V(x) = A_1 e^{(x/\lambda)} + A_2 e^{(-x/\lambda)}$$

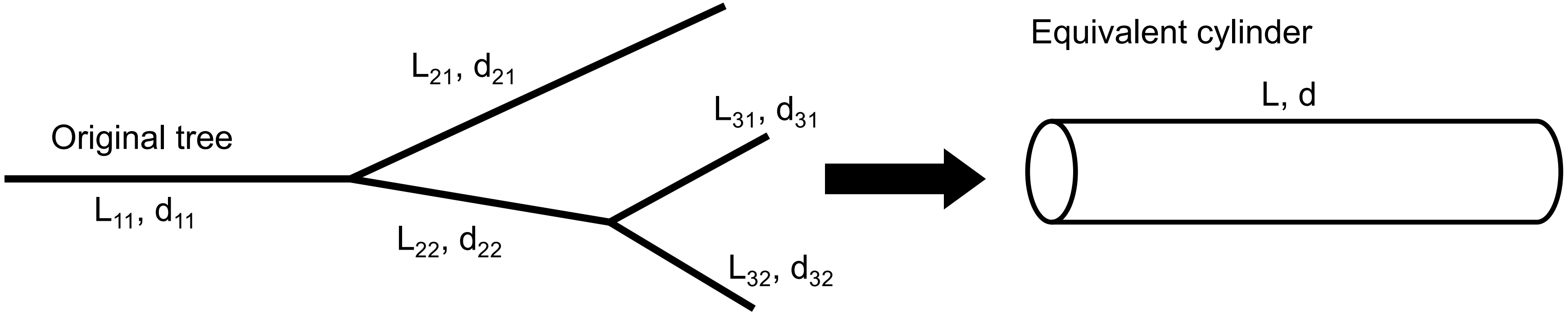
Where λ = length constant

λ = sqroot(R_m/R_i)

Representation of length constant



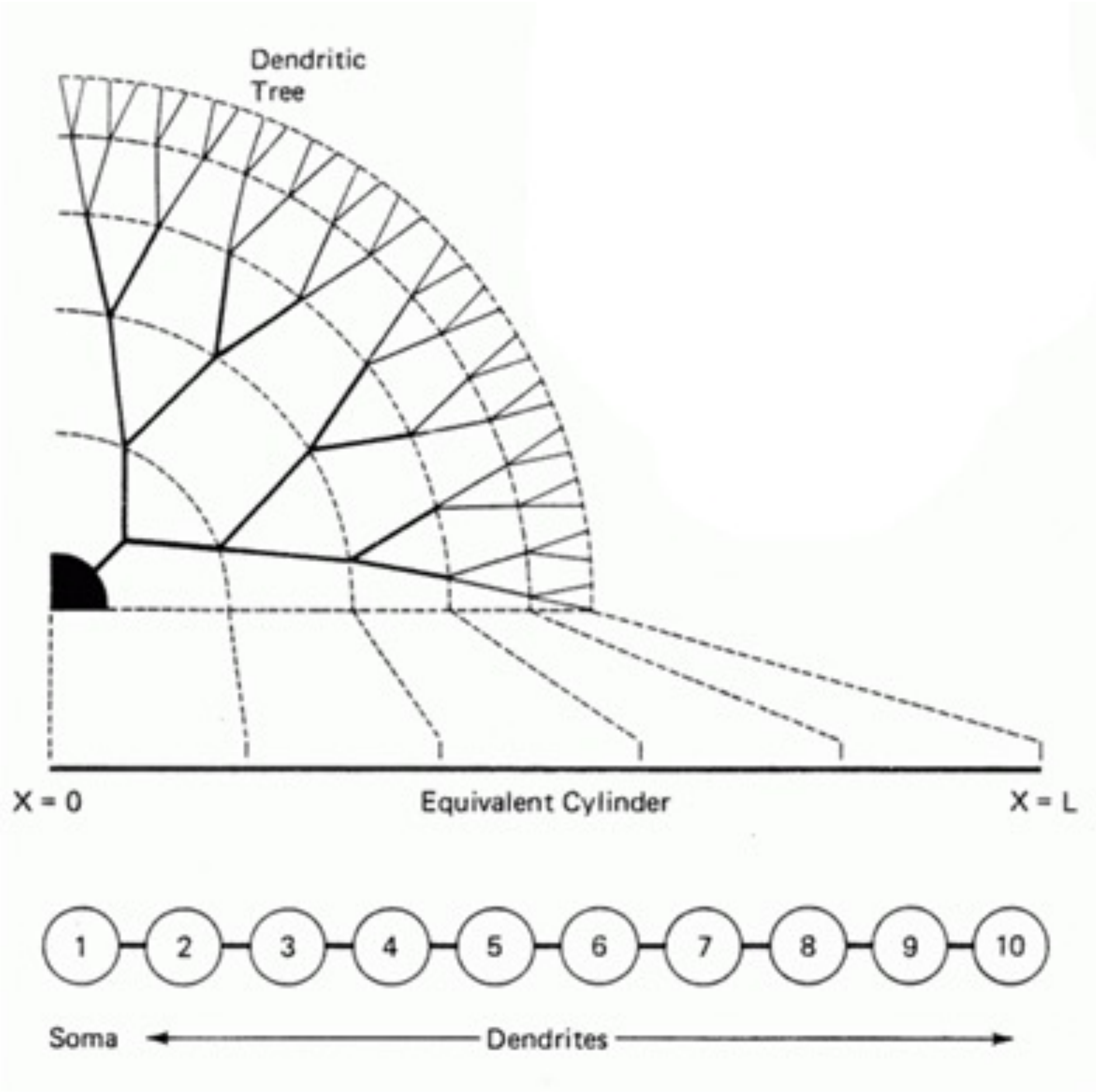
Equivalent Cylinder Theorem



- Equivalent Cylinder Theorem: an entire class of dendritic trees can be reduced into a single equivalent cylinder if the following conditions are met:
  - The values of r_i and r_m are the same in all branches;
  - All terminals end at the same boundary condition;
  - All terminal branches end at the same electrotonic distance from the origin in main branch
  - The 3/2 power law at every branch point

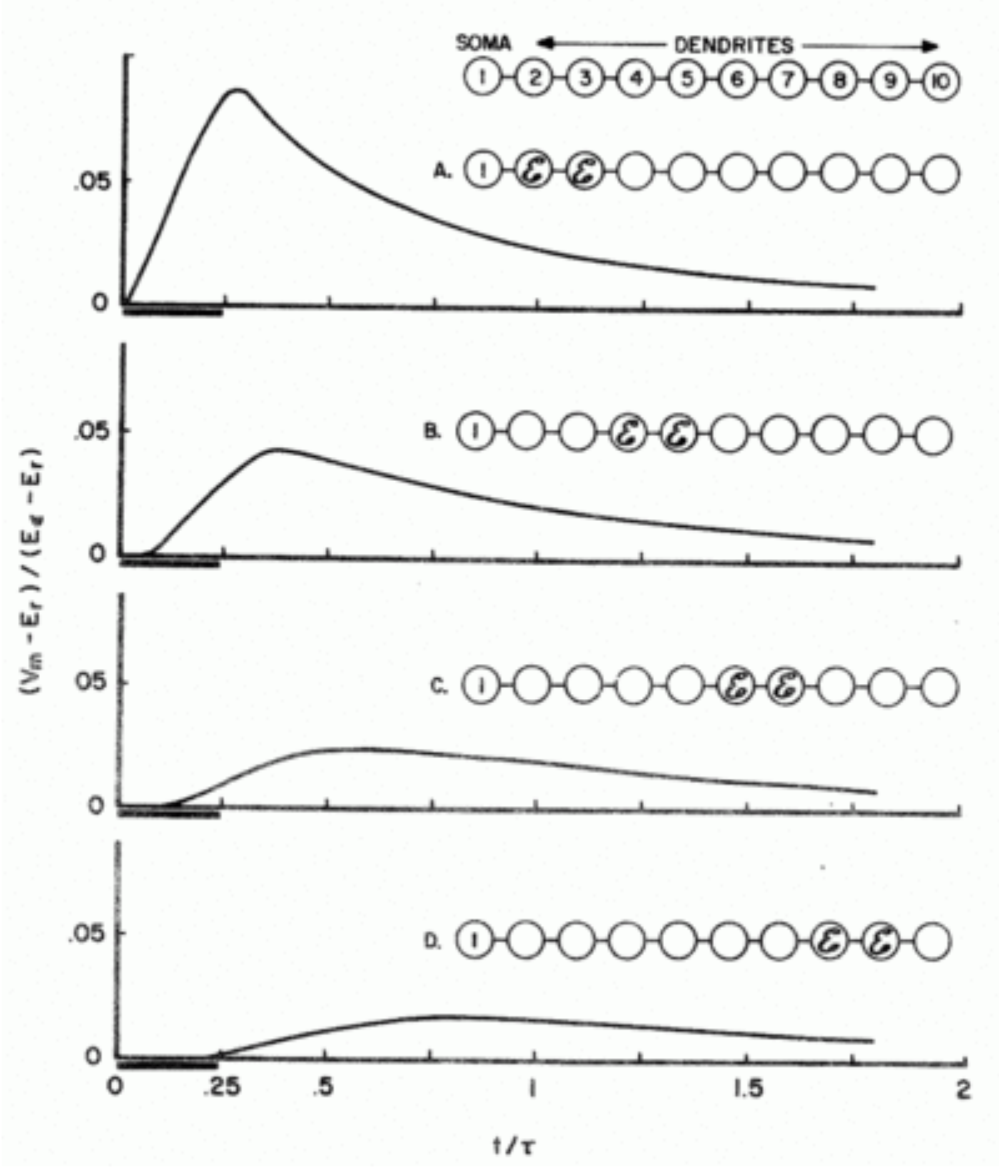
$$d_{parent}^{3/2} = \sum_{\substack{\text{all child} \\ \text{branches } j}} d_j^{3/2}$$

Rall Model

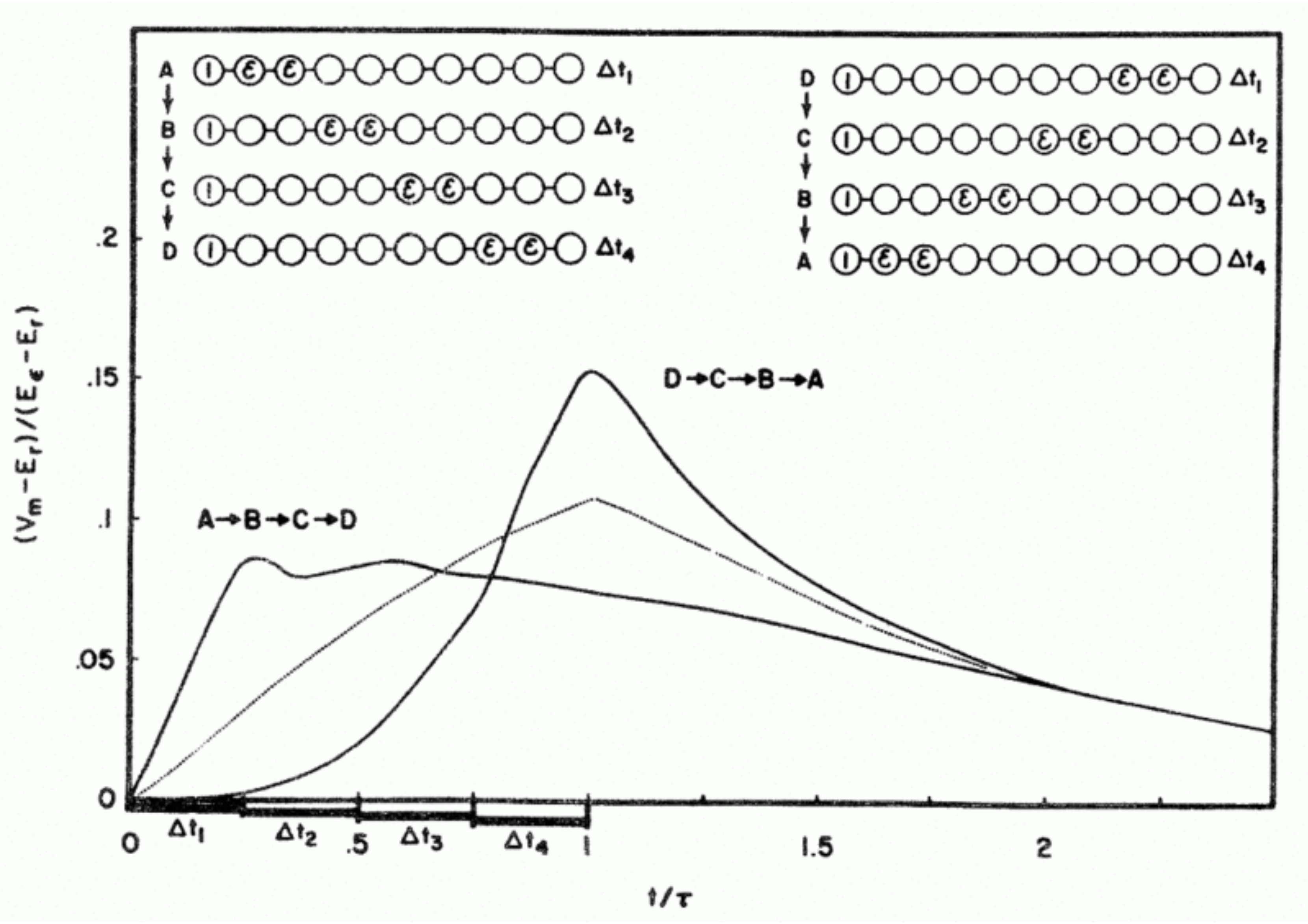

Dendritic
Tree
X = 0
Equivalent Cylinder
X = L
1
2
3
4
5
6
7
8
9
10
Soma
Dendrites

Rall Model

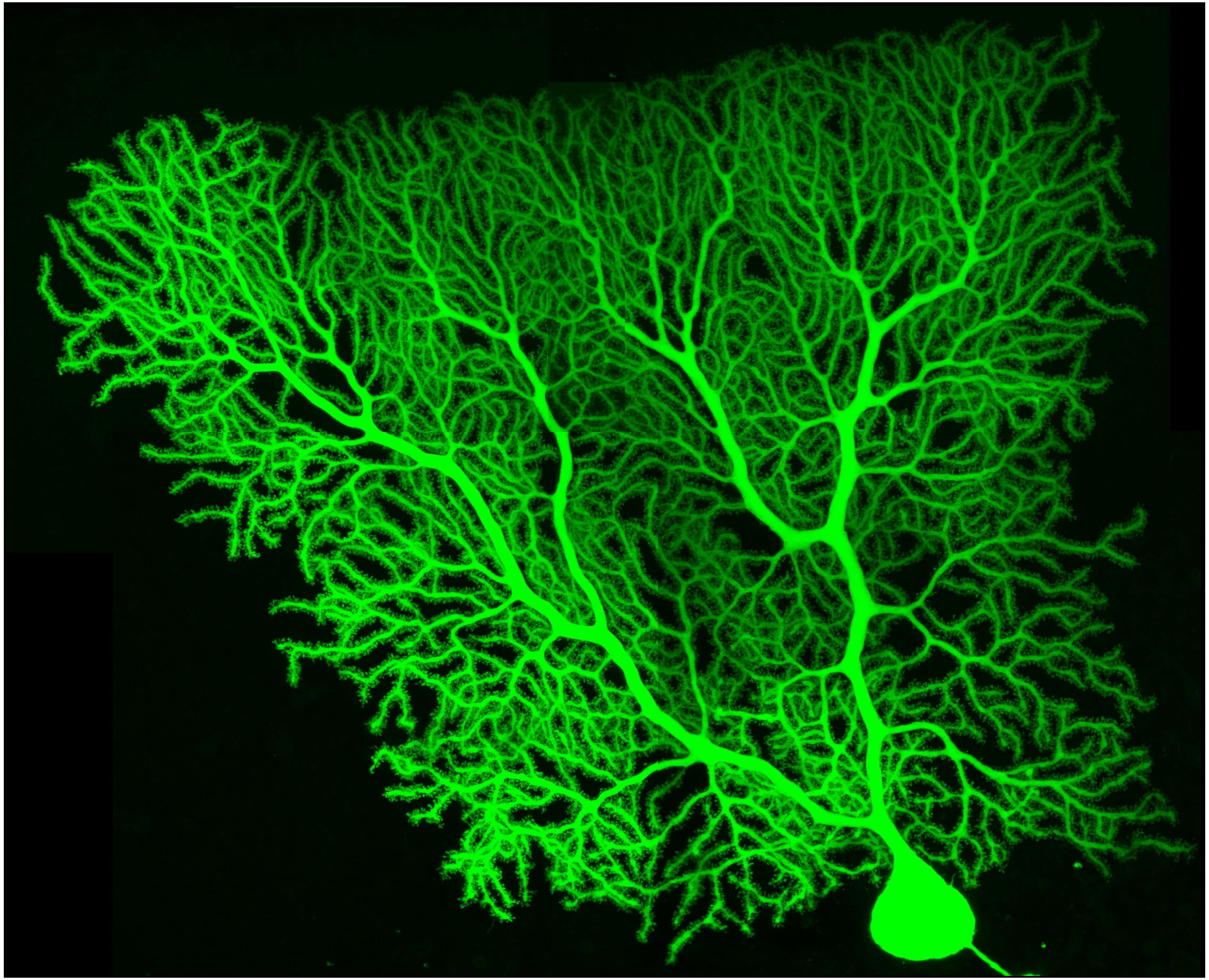
- Allowed modeling of dendritic behavior
  - Confirmed by experimental results
- Stimulation at different distances along arbor
- Modeling of EPSPs



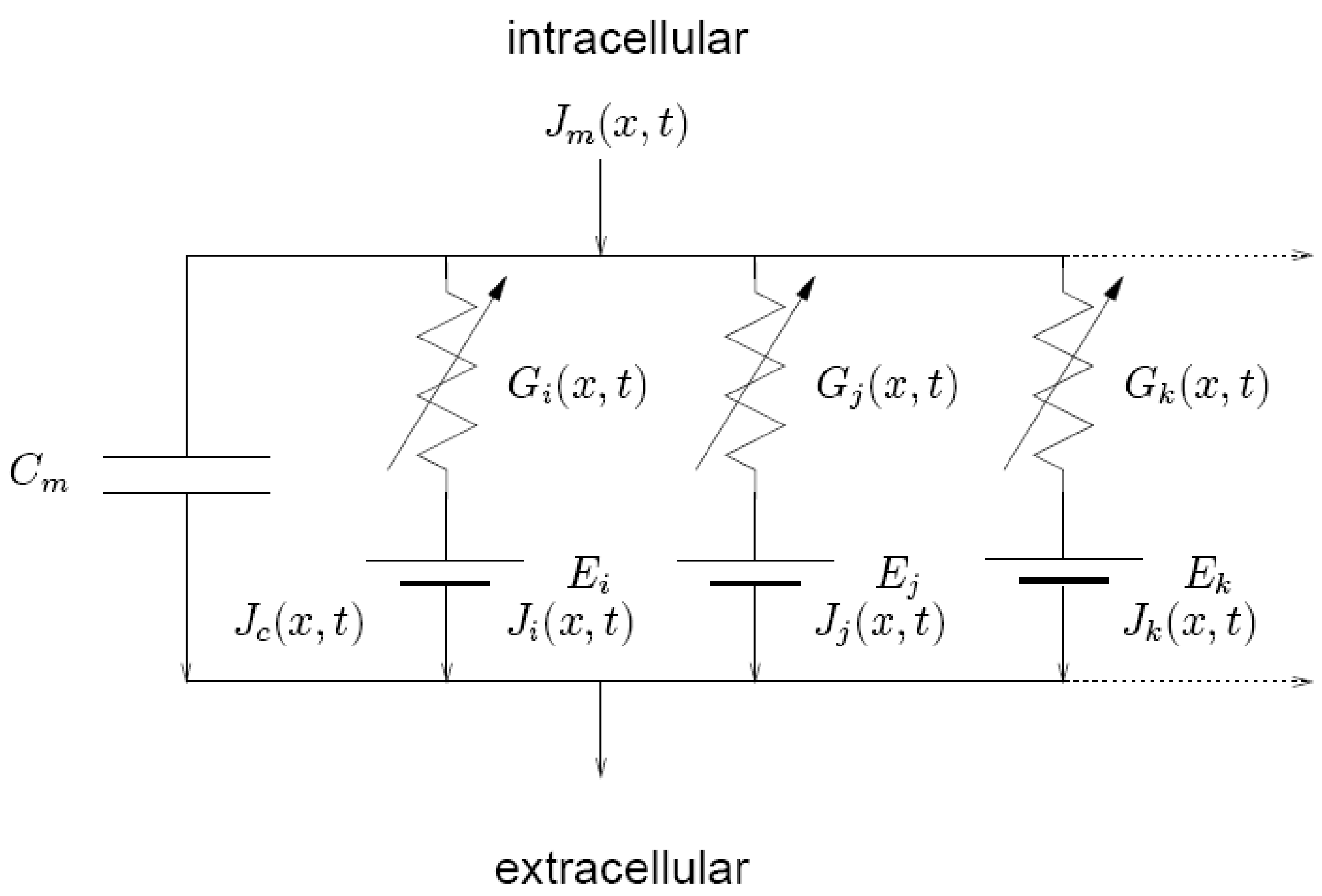
Rall Model



A
B
C
D
Δt1
Δt2
Δt3
Δt4
D
C
B
A
D→C→B→A
A→B→C→D
(Vm − Er)/(Ee − Er)
.2
.15
.1
.05
0
0
.5
1
1.5
2
t/τ



What if R_m is not constant, depending on x, V and t?



$$\lambda^2 \frac{\partial^2 V}{\partial x^2} = \tau_m \frac{\partial V}{\partial t} + V \rightarrow \lambda^2 \frac{\partial^2 V}{\partial x^2} = \tau_m \frac{\partial V}{\partial t} + \sum_k (V - E_k) G(x,t,V)$$