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Kalman Filter Summary

For simplicity's sake, I chose to use the model provided in the targeting handout. The following matrices were provided:

$$A_c = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad D_C = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \qquad C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

These matrices are used in the following equations. (Note that since these matrices will not change over time, the k has been dropped from notation involving these matrices)

$$\dot{x}(t) = A_c x(t) + D_c w(t)$$

$$z_k = Cx_k + \theta_k$$

Where w(t) is the driving noise and θ is the sampled additive noise with θ_k = N(0, Θ). Both are considered to be zero mean, white, wide sense stationary process. It can be noted that the common term Bu_k is omitted from the above equations. This is due to the assumption of a constant velocity. Therefore, there is no input into the system, so the term is considered to be 0. z_k is the sampled measurements. It is noted that the first equation is continuous, while the second equation is discrete. To convert the first equation to discrete, the following equations are used

$$x_k = x(kT_s)$$

$$A = e^{A_c T_s} = I + A_c T_s$$

This creates a discrete time model as such

$$x_{k+1} = Ax_k + D\xi_k$$

$$z_k = Cx_k + \theta_k$$

Where ξ_k is a zero mean, wide sense stationary, white, discrete random sequence N(0,W) with:

$$Cov(\xi_i, \xi_j) = W\delta_{ij}$$

Since δ_{ij} is equal to 0 when $i\; != j$ and 1 when $i = j$. This leads to:

$$Cov(\xi_k, \xi_k) = \Xi = WT_s$$

Also defined by the provided model is:

$$Cov(\theta_k, \theta_k) = \Theta$$

Another choice made by the given model is:

$$D = AD_c$$

However, the paper did not give exact values for several important constants needed for the Kalman filter. The values for these constants were chosen based on personal intuition as well as examining the results of the initial filter run. The three tunable variables (W, Θ, $\sum_0$) values and how they change the final results of the filter are discussed below. The initial state vector was chosen by the assignment. Ideally, this vector would be chosen with a starting value close to the actual expected value of the system to improve the tracking of the filter.

$$m_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$x_{0|0} = m_0 + M_0(z_0 - C(m_0))$$

Where M_0 is defined below. The first guess was $\sum_0 = I$ as that was the simplest matrix to test the evolution of $\sum$ with.

Using the above equations and constants, the Kalman filter is as follows, and is implemented in the attached .m file. Since the assignment asked only for the state estimation $\hat{x}_{k|k}$, several of the equations are commented out in the .m file.

$$x_{k+1} = Ax_k + D\xi_k$$

$$z_k = Cx_k + \theta_k$$

$$L_k = A\,C^T(C\,C^T + \Theta\,)^{-1}$$

$$\sum\nolimits_{k+1} = A\left(\sum\nolimits_k - \sum\nolimits_k C^T * \left(C\sum\nolimits_k C^T + \Theta\right)^{-1} C\sum\nolimits_k\right)A^T + D\Xi D^T$$

$$\hat{x}_{k+1} = A\hat{x}_k + L(z_k - C\hat{x}_k)$$

$$\tilde{x}_{k+1} = x_{k+1} - \hat{x}_{k+1}$$

$$\hat{x}_{k+1|k+1} = A\hat{x}_{k|k} + M_{k+1}(z_k - C(A\hat{x}_{k|k}))$$

$$M_k = \sum\nolimits_k C^T\left(C\sum\nolimits_k C^T + \Theta\right)^{-1}$$

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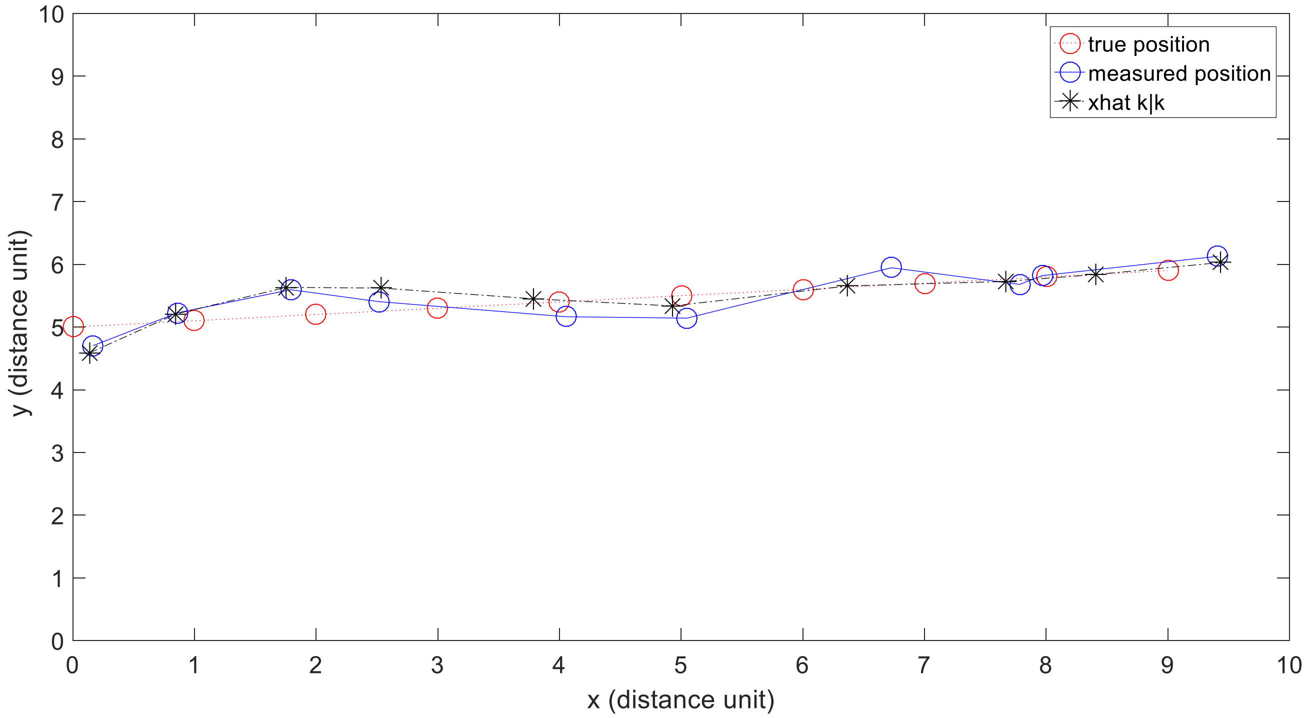
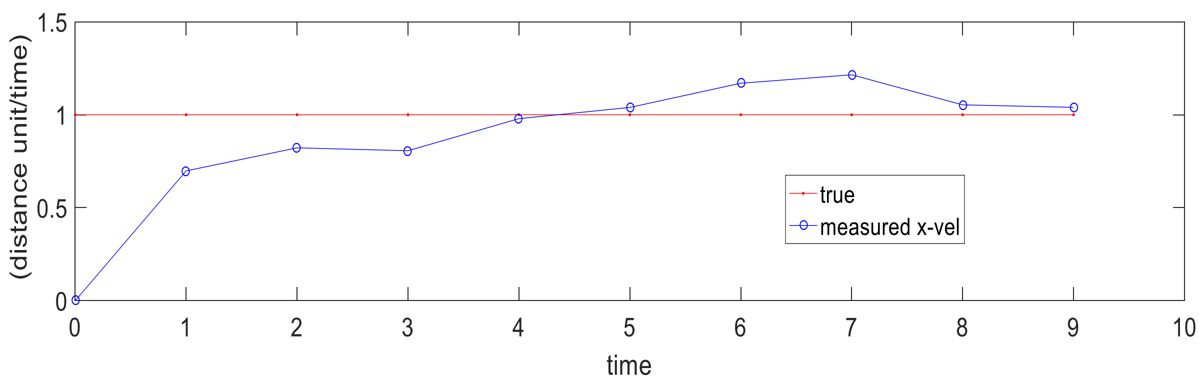
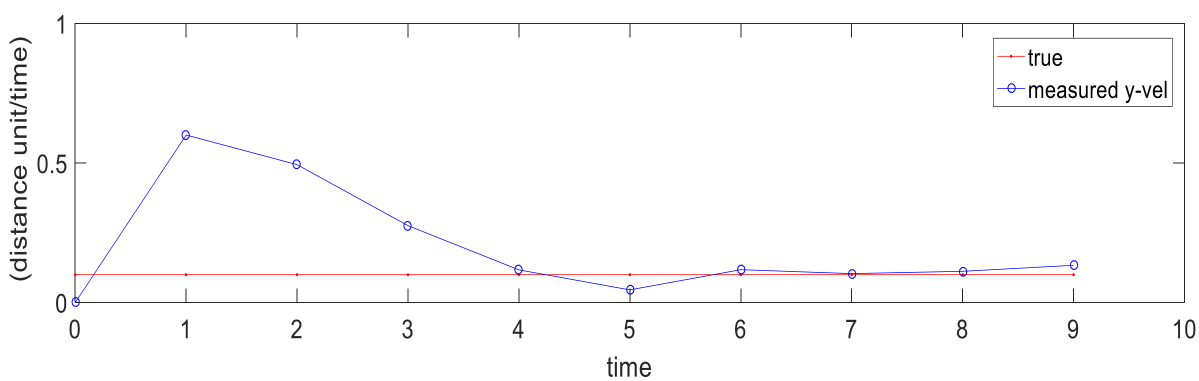
Final Results







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Tuning Parameter Discussion

The final choice for $\sum_0$ was $50I$. This was chosen based off of multiple trials, some of which are shown below. Knowing nothing about the system, I simply guessed I as a starting point, and scaled it until the results were in the acceptable range. Figure 1 shows $1000I$. Figure 2 shows I. Figure 3 shows $.01I$. All have the same and matrices. It can be noted that the higher $\sum_0$ is, the closer the state estimate tends to trend towards the measurement values from the data. As $\sum_0$ grows smaller, the first state estimate starts further and further away from the measurement data. However, it eventually still begins to follow the true value by k = 9.

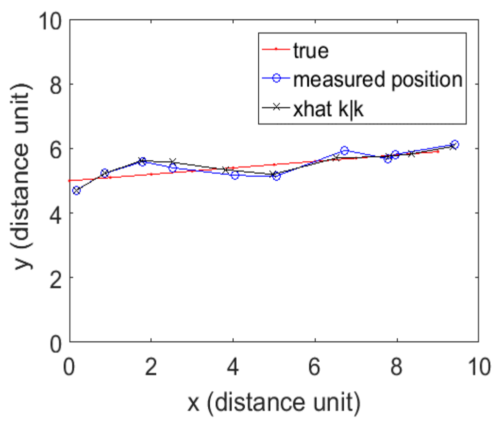

Figure 1 $1000I$

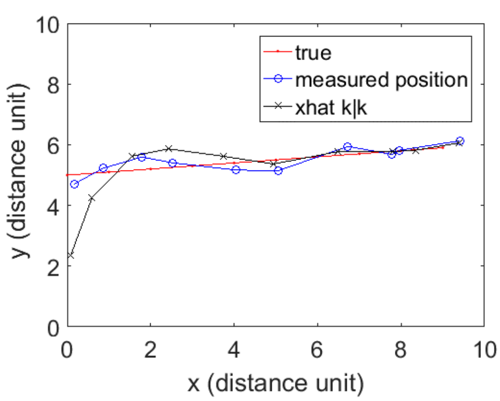

Figure 2 I

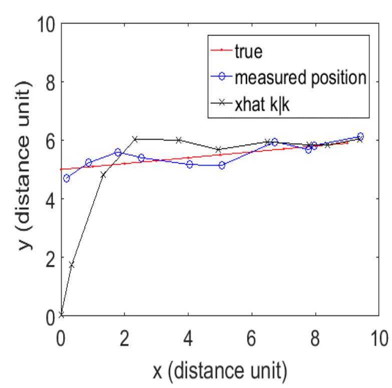

Figure 3 $.01I$

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The final choice for Ξ is $\begin{bmatrix} .0285 & .005 \\ .005 & .046 \end{bmatrix}$. The initial matrix was chosen based off of literature available elsewhere. It is assumed that the model is quite accurate due to its assumed linearity, so therefore Ξ, as it represents the model noise, is quite small. Most of the error present in this model will be from the measurement error. The scalar was chosen based off of testing and comparison with the baseline results. A larger Ξ trends towards a noiser state estimator of the velocity, as shown in Figure 4. A smaller Ξ tends towards a tighter estimation of the velocity, as shown in Figure 5. Figure 4 is done with a value $\Xi = 10 * \begin{bmatrix} .0285 & .005 \\ .005 & .046 \end{bmatrix}$.Figure 5 is done with $\Xi = .001 * \begin{bmatrix} .0285 & .005 \\ .005 & .046 \end{bmatrix}$.

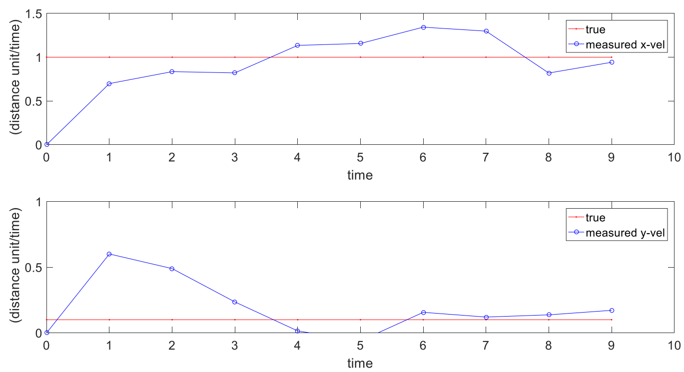

Figure 4 $\Xi = 10 * \begin{bmatrix} .0285 & .005 \\ .005 & .046 \end{bmatrix}$.

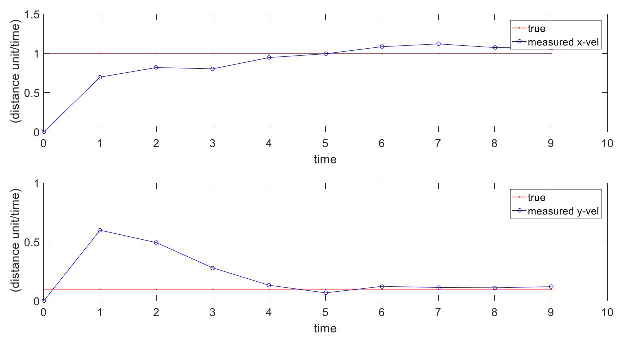

Figure 5 $\Xi = .001 * \begin{bmatrix} .0285 & .005 \\ .005 & .046 \end{bmatrix}$

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The final choice for Θ was determined to be $\begin{bmatrix} 1 & .5 \\ .5 & 2 \end{bmatrix}$. The initial guess for this system was also I, which was then scaled to find an acceptable answer. A larger Θ tends to have the state estimate of the positions to trend towards the measurements instead of the true value. For the state estimates of the velocity, shown below, they tend to spike higher above the true v=lue (especially in the y-pos) before approaching the true value. For smaller Θ values, the ability of the filter to track the state smoothly is greatly reduced as shown below. Figure 6 shows $\Theta = 10 * \begin{bmatrix} 1 & .5 \\ .5 & 2 \end{bmatrix}$. Figure 7 shows $\Theta = .010 * \begin{bmatrix} 1 & .5 \\ .5 & 2 \end{bmatrix}$.

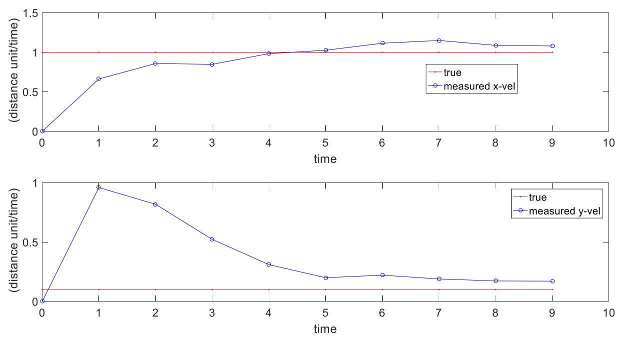

Figure 6 $\Theta = 10 * \begin{bmatrix} 1 & .5 \\ .5 & 2 \end{bmatrix}$

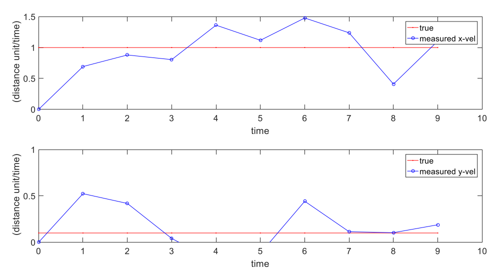

Figure 7 $\Theta = .010 * \begin{bmatrix} 1 & .5 \\ .5 & 2 \end{bmatrix}$