Assignment 1

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1 Fundamental Questions

1.1 1.

1.1.1 (a)

When governments need money, they would issue bonds as a way to "borrow" money with a fixed maturity, thus eliminates the possible inflation caused by printing more money.

1.1.2 (b)

A yield curve might appear to flatten in the long term when, for example, it yields 4% on a 5 year bond, and 4.3% on a 10 year bond.

1.1.3 (c)

Quantitative easing is when a nation's central bank purchases long term securities to inject money back to the economy to lower the interest rates and stabilizes inflation, for example, in response of COVID, the Fed(Federal Reserve) bought $500 billion in Treasury securities and $200 billion in government-guaranteed mortgage-backed securities in order to stimulate the economy.

1.2 2.

The 11 bonds we have chosen are: "CAN 0.5 Feb22," "CAN 0.25 Jul22," "CAN 0.25 Apr23," "CAN 0.25 Jul23," "CAN 0.75 Jan24," "CAN 7.5 Sep24," "CAN 1.25 Feb25," "CAN 0.5 Aug25," "CAN 0.25 Feb26," "CAN 1 Aug26," "CAN 1.25 Feb27." We have selected these 11 bonds to construct a "0-5 year" yield and spot curve as the 11 bonds have maturity date in half year intervals. Moreover, the bonds have similar lower coupons, guaranteeing the consistency when constructing the curves.

Table 1: 11 Selected Bonds

	Name	Coupon	ISIN	Issue.date	Maturity.date
13	CDA 2022	0.50	CA135087G328	10/10/2016	2/28/2022
18	CANADA 20/22	0.25	CA135087L286	5/3/2020	7/31/2022
21	CANADA 21/23	0.25	CA135087L856	2/4/2021	4/30/2023
22	CANADA 21/23	0.25	CA135087M359	5/13/2021	7/31/2023
25	CANADA 21/24	0.75	CA135087M920	10/21/2021	1/31/2024
23	CANADA 21/24	0.75	CA135087M508	7/11/2021	9/30/2024
4	CANADA 19/25	1.25	CA135087K528	10/10/2019	2/28/2025
5	CANADA 20/25	0.50	CA135087K940	4/2/2020	8/31/2025

	Name	Coupon	ISIN	Issue.date	Maturity.date
7	CANADA 20/26	0.25	CA135087L518	10/8/2020	2/28/2026
8	CANADA 21/26	1.00	CA135087L930	4/15/2021	8/31/2026
10	CANADA 21/27	1.25	CA135087M847	10/14/2021	2/28/2027

1.3 3.

If we have several stochastic processes for which each process represents a unique point along a stochastic curve (assume points/processes are evenly distributed along the curve), we can look at the eigenvalues and the eigen vectors of the associated co-variance matrix to find out which linear combination of the stochastic processes gives us the most information. The diagonal matrix of descending eigenvalues projects the linear combination of the variance of the stochastic processes onto higher dimensions, each uncorrelated. The largest eigenvalue denotes the linear combination that explains the highest proportion of the variance, and vise versa. We can then reduce the dimension of the stochastic process with the corresponding eigenvalues that gives us the most information.

2 Empirical Questions

2.1 Assumptions

The bonds data is from Business Insider website(Insider 2022)

It is assumed that time follows actual/365 format. The data is cleaned such that each row contains the one observation of each bond. Table 2 provides a glimpse of our cleaned data.

Table 2: Head of cleaned bond data

Name	Coupon	ISIN	Issue.date	Maturity.date	maturity	fullname	term_to_maturi
CDA 2022	0.5	CA135087G328	10/10/2016	2/28/2022	Feb 2022	CAN 0.5 Feb 2022	
CDA 2022	0.5	CA135087G328	10/10/2016	2/28/2022	Feb 2022	CAN 0.5 Feb 2022	
CDA 2022	0.5	CA135087G328	10/10/2016	2/28/2022	Feb 2022	CAN 0.5 Feb 2022	
CDA 2022	0.5	CA135087G328	10/10/2016	2/28/2022	Feb 2022	CAN 0.5 Feb 2022	
CDA 2022	0.5	CA135087G328	10/10/2016	2/28/2022	Feb 2022	CAN 0.5 Feb 2022	
CDA 2022	0.5	CA135087G328	10/10/2016	2/28/2022	Feb 2022	CAN 0.5 Feb 2022	

The data cleaning process is done with tidyverse(Wickham et al. 2019) package, and the manipulation of time is done with chron(James and Hornik 2020) package.

In the following section, each bond i on the j-th observation date is denoted as $bond_{ij}$

To review the code or replicate this process, view github

2.2 4

2.2.1 (a) Yield Curve

To construct the YTM curve for the 11 bonds, we first need the dirty price of $bond_{ij}$ with the formula:

$$AI_{ij} = \frac{n_j}{365} * Coupon$$

$$DP_{ij} = AI_{ij} + yield_{ij}$$

where n denotes the number of days since the last coupon payment.

The process below follows datacamp tutorial(datacamp, n.d.), using uniroot function to fins YTM.

1. define cash flow vector for each $bond_{ij}$ as $c(DP_{ij}, Coupon_i, \cdots, FV_i)$, where $FV_i = 100 + Coupon_i/2$.
2. create bond valuation function that uses the cash flow vector.
3. use uniroot(R Core Team 2021) function to find YTM, within the interval c(0,1).

Figure 1 shows the 5-year yield curve on each day, using ggplot2 package(Wickham 2016)

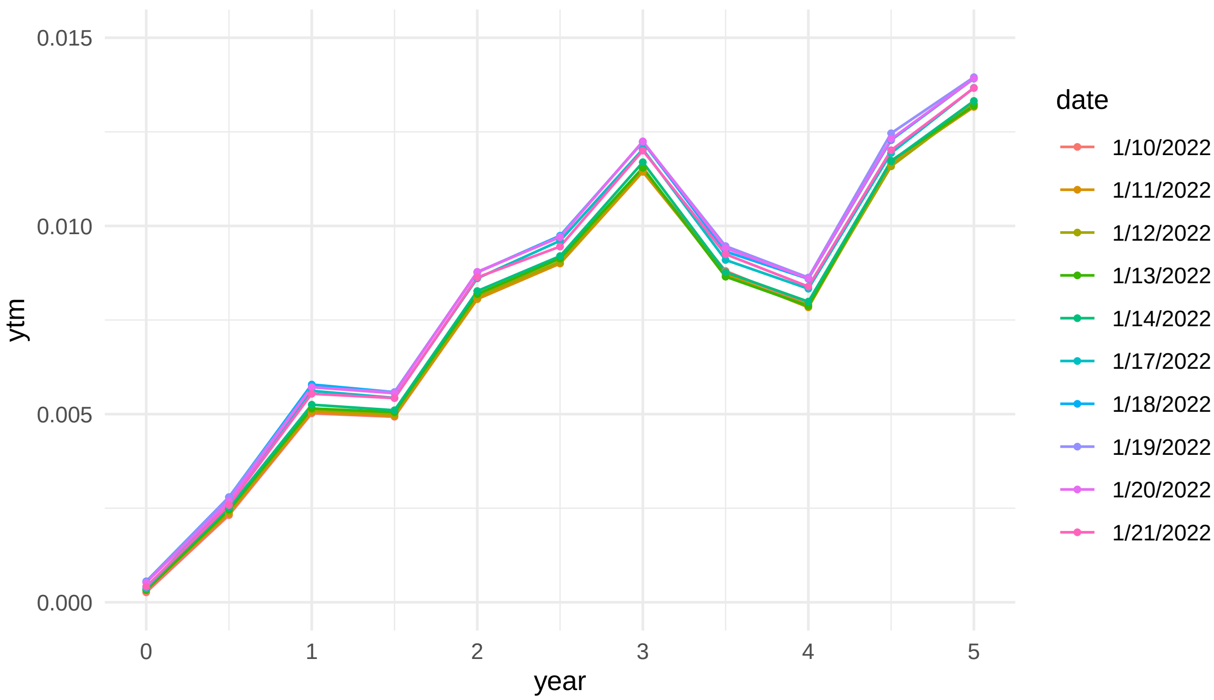

Figure 1: 5-Year Yield Curve

2.2.2 Spot Curve

The method of bootstrapping is used to calculate the short rate of $bond_{ij}$. The general formula to solve for the short rate of bond i on j-th day, r_ij is

$$DP_{ij} = \frac{1}{2}Coupon_i \cdot e^{-r_{1j} \cdot t_{1j}} + \sum_{k=2}^{i-1} \frac{1}{2}Coupon_i \cdot e^{-r_{kj} \cdot t_{kj}} + FV_{ij} \cdot e^{-r_{ij} \cdot t_{ij}}$$

, where t_{ij} denotes the time to maturity in years for $bond_{ij}$.

The process is as follows:

1. Use "CAN 0.5 Feb22," the zero-coupon bond matures in less than half a year to find the yield with

$$r_1(t_j) = -\frac{log(DP_{1j}/FV_1 j)}{t_j}$$

2. Looping through the data for each observation $date_j$ using the previous calculated $r_{(1:i-1)j}$, r_ij is found by solving the formula above.

The 5-year spot curve is plotted in Figure 2.

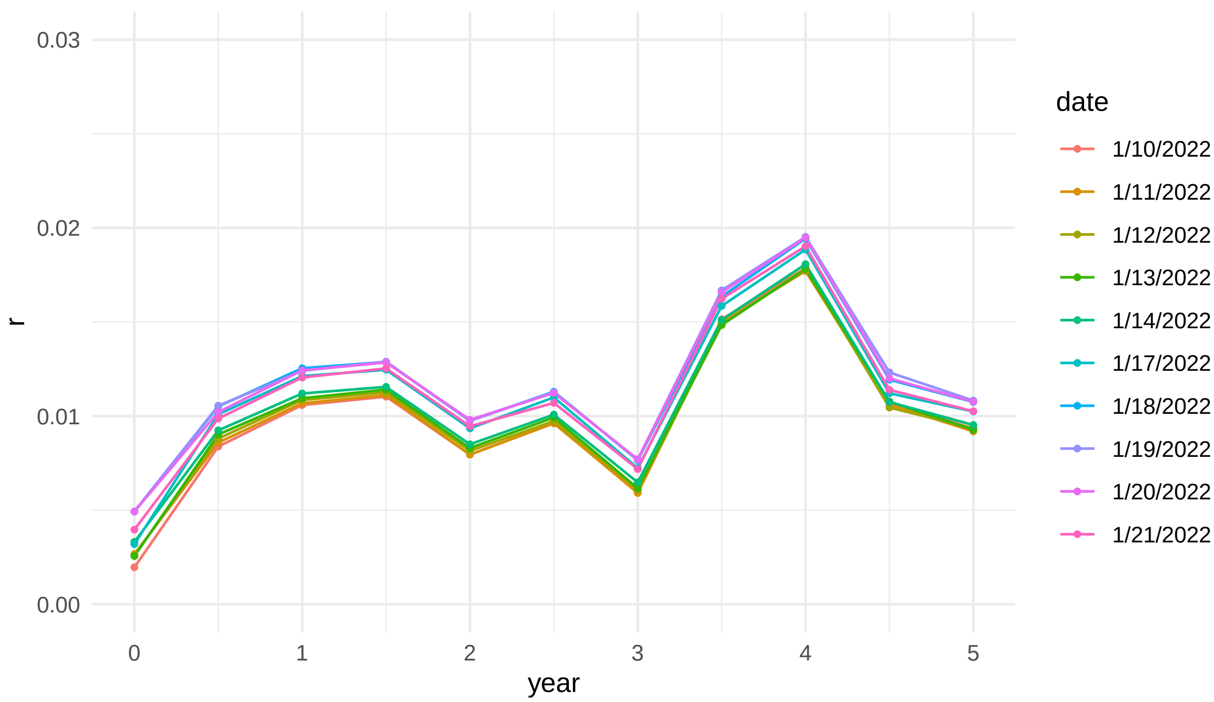

Figure 2: 5-Year Spot Curve

2.2.3 Forward Rate

The 1-year forward rate can be derived from the spot rate, for k=2,...5, with the formula $f_{kj} = \frac{r_{kj}\cdot(T_k-t_j)-r_{1j}\cdot(T_1-t_j)}{T_k-T_1}$, where $T_1 = 1$.

1. The bond data is filtered to contain bonds with only full year interval, ie. k=1,2,3,4,5.
2. Define the 1-yr T_1 and loop through $bond_{kj}$, giving the 1yr-yr to 1-yr-4yr forward rate.

The 1-year forward curve with terms ranging from 2-5 years is plotted in Figure 3.

2.3 5

The daily log-returns of yield's covariance matrix of each full year interval $bond_{(1:5)j}$ is plotted in table 3, and similarly, the forward rates's covariance matrix is shown in table 4.

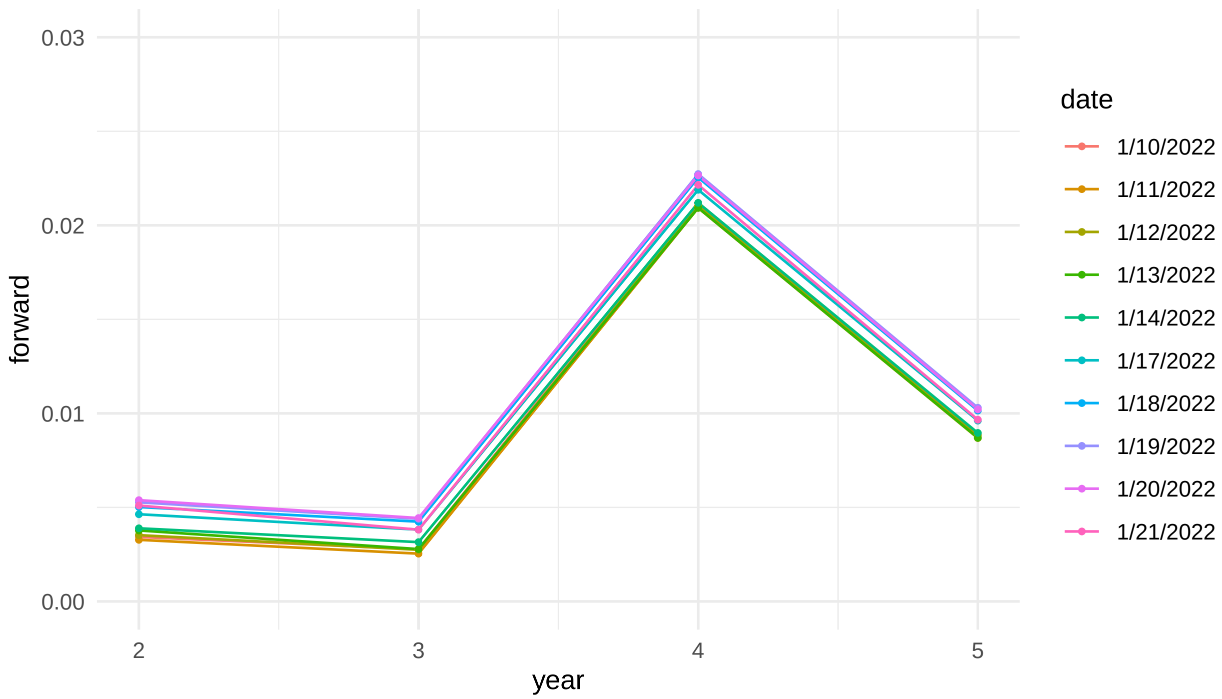

Figure 3: 1-Year Forward Curve

Table 3: Covariance Matrix of daily log return of Yield

V1	V2	V3	V4	V5
0.0007617	0.0004239	0.0003609	0.0005192	0.0003325
0.0004239	0.0002447	0.0002097	0.0003041	0.0001944
0.0003609	0.0002097	0.0001905	0.0002763	0.0001781
0.0005192	0.0003041	0.0002763	0.0004224	0.0002695
0.0003325	0.0001944	0.0001781	0.0002695	0.0001731

Table 4: Covariance Matrix of daily log return of Forward Rate

V1	V2	V3	V4
0.0048138	0.0065649	0.0010696	0.0023561
0.0065649	0.0112643	0.0017927	0.0040207
0.0010696	0.0017927	0.0003287	0.0007140
0.0023561	0.0040207	0.0007140	0.0015752

2.4 6

Table 5 and table 6 provides the eigen vectors and values for the covariance matrix of daily log return of yield and forward rate respectively. The largest eigen value and its corresponding eigen vector in each table provides the direction of the largest variability, it essentially is the first coomponent of PCA.

Table 5: Eigen Values of Covariance Matrix of daily log return of Yield

Eigen Value 1	Eigen Value 2	Eigen Value 3	Eigen Value 4	Eigen Value 5
0.00172376940856266	5.88735055366595e-05	5.76284599409873e-06	3.55389063705611e-06	4.24883765128757e-07
Eigen Vector 1	Eigen Vector 2	Eigen Vector 3	Eigen Vector 4	Eigen Vector 5
0.653449643320847	0.649720629111656	-0.331304047125128	0.201959801188036	-0.0177914252720602
0.373168663735633	0.13412441148582	0.553761729168151	-0.717712403781482	0.144888382613138
0.327383823729149	-0.183500656562476	0.660346128636596	0.590258667846954	-0.273285631579528
0.481263131907755	-0.611852436966624	-0.374704010096858	-0.24234172577413	-0.44146323290903
0.308146623990008	-0.389663766303053	-0.084412504282273	0.192266318274033	0.842090256048735

Table 6: Eigen Values of Covariance Matrix of daily log return of Forward rate

Eigen Value 1	Eigen Value 2	Eigen Value 3	Eigen Value 4
0.0170710516857489	0.000751910560250894	0.000155970934121691	2.94339457630431e-06
Eigen Vector 1	Eigen Vector 2	Eigen Vector 3	Eigen Vector 4
-0.49891974506671	0.866370176390588	0.0150028727131864	0.0160224609177923
-0.805834539142137	-0.456852878894366	-0.374717071085247	-0.0387718868357501
-0.130565985130404	-0.0667759851096038	0.453175390524312	-0.879275586356579
-0.290967879730867	-0.190339518543872	0.808699014387172	0.474462290150258

Reference

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