Interview Questions and Solutions

Mackenzie Qu

September 6, 2024

1 Questions

1.1 Ants and Rope

500 ants are randomly put on a 1 metre string. Each ant randomly moves toward on end of the string with equal probability to the left or the right at constant speed of 1 m/min until it falls off one end of the string. Also assume that the size of the ant is infinitely small. When two ants collide head-on, they both immediately change directions and keep on moving at 1 m/min. What is the expected time for all ants to fall off the string?

1.2 Noodle Bowl

There are 50 noodles in a bowl. You can tie two ends of either one noodle or two different noodles, forming a nod. After 50 nods are tied. What is the expected value of number of circles in the bowl?

1.3

There are two dice. The game is as following: Two dice will be thrown at once. Adding the number together you will get a number ranging from 2 to 12. The probability of getting 12 is 40%, and 6% for getting 2,3,4,5,...11 Person A and B will need to guess what the number would be. The one who has a closer guess will win. A will guess first and then B guess, after they've guessed, two dice will be thrown at once, what number should A guess and what number should be B guess?

2 Solutions

2.1 Ants and Rope

Consider the case where there is only 1 ant on the string. The ant will have a position of d_l, denoting the distance to the left and d_r denoting the distance to the right, such that $d_l + d_r = 1$.
Let t_1 be the time for the ant to fall off the rope. To find the expected time that the aunt will take to fall off, we are looking for $E(t_1)$, where

$$E(t_1) = E(\frac{\text{distance the ant will travel to fall off}}{\text{the volocity the ant is travelling at}}) = \frac{E(\text{distance the ant will travel to fall off})}{1}$$

as the the volocity of ant is constant.

$$
\begin{aligned}
E(\text{distance the ant will travel to fall off}) =& P(\text{travelling to the left direction}) \times \text{distance to the left} + \\
& P(\text{travelling to the right direction}) \times \text{distance to the right} \\
=& \frac{1}{2} \times d_l + \frac{1}{2} \times d_r \\
=& \frac{1}{2} \times (d_l + d_r) = \frac{1}{2}
\end{aligned}
$$

Now consider the case where there are two ants. When two ants collide into each other, the momentum of the two ants carry over - we can imagine it as the two ants have swapped positions and continued going in the same direction.
In this case, we are looking for the expected value of the maximum amout of time each ant spends on the rope. ie. $E(max\{t_1, t_2\})$, which is equivalent to finding the maximum distance an ant is from the edge of the rope.
Since each ant is randomly dropped onto the rope, the distance D_i of each ant toward the direction it is moving is independent and identically distributed on $Uniform(0, 1)$. Let $Y = max\{D_1, D_2\}$, the cumulative distribution function of Y follows:

$$
\begin{aligned}
F_Y(y) &= P(Y < y) \\
&= P(max\{D_1, D_2\} < y) \\
&= P(D_1 < y)P(D_2 < y) \ldots \text{because of independence} \\
&= F_{D_1}(y) * F_{D_2}(y) \\
&= y^2
\end{aligned}
$$

and the probability density function follows:

$$f_Y(y) = \frac{d}{dy} F_Y(y) = 2y$$

$$E(Y) = \int_0^1 y * 2y \ dy = \int_0^1 2y^2 \ dy = \frac{2}{3}$$

To find the answer for 500 ants, let $Y = max\{D_1, ..., D_{500}\}$. It follows:

$$
\begin{aligned}
F_Y(y) &= y^{500}, \\
f_y(y) &= 500 * y^{499}, \\
E(Y) &= \frac{500}{501}
\end{aligned}
$$