Lab 2: Index of Refraction Customer Data Sheet

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1. Introduction

With the help of the data collection from the technical staff, a Python script was developed to properly analyze this data and extract the index of refraction of the glass, along with its experimental uncertainty. During this process, a one-parameter χ^2 best fit for the index of refraction was calculated, therefore an index of refraction and uncertainty could be extracted. To reaffirm this, the χ^2 was plotted against the range of index of refraction values, so the minimum χ^2 value could be identified, which correlates to the best value for the index of refraction.

2. Physics Background

In order to properly extract the index of refraction of glass from the analyzed refraction data, Snell's Law was used to relate the angle of incidence and angle of refraction, shown below:

$$n_{air} \sin(\alpha) = n_{glass} \sin(\beta) \qquad [1]$$

where α is angle of incidence, while β is angle of refraction. However, by setting $n_{air} = 1$, we can rearrange Snell's law, as follows:

$$\sin(\beta) = \sin(\alpha) / n_{glass} \qquad [2]$$

In order to properly analyze this data, Python was used to find the best fit for the data based on Snell's Law, with corresponding plots to determine a precise value and uncertainty for the index of refraction of the glass.

3. Data Analyzation

Set Up

To begin, in order to properly analyze the dataset, it is best to visualize the relationship between the two measured quantities. The dataset contains repeated angle of refraction measurements (β) at various angles of incidence (α) ranging from 10 to 60 degrees. With this, a histogram was formed for each subset of data relating to each angle of incidence. Figure 1 below displays the results, which was formulated using the NumPy package, and histogram function:

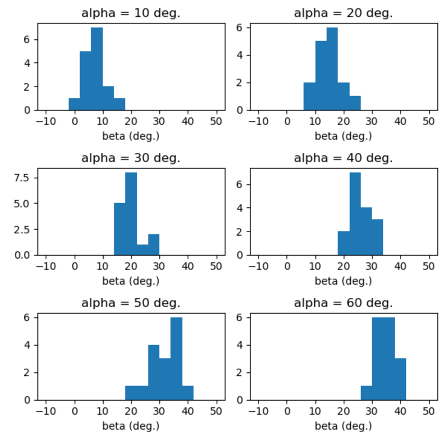

Figure 1: Histogram of refraction data displaying different α to β angle correlations for each angle of incidence.

For the purposes of the data analyzation, both the α and β values were converted to radians. A table of measurements was created to organize the computed values we need as a basis for our fitting process. The table below details each α value, the sine of the mean of β, and the uncertainty of the sine of the mean of β.

α	$sin(\alpha)$	$\bar{\beta}$	$\sin(\bar{\beta})$	$\sigma(\sin(\bar{\beta}))$
0.175	0.174	0.128	0.127	0.019
0.349	0.342	0.258	0.255	0.018
0.524	0.500	0.343	0.337	0.015
0.698	0.643	0.456	0.440	0.017
0.873	0.766	0.540	0.514	0.018
1.047	0.866	0.611	0.573	0.012

Table 1: Computed values used for analyzation

Fitting Process

Snell's Law [equation 2] was used to visualize the relationship between each of the six subsets of data pertaining to each incident angle. Once plotted together, one data point for each angle of incidence, along with their uncertainties, we are able to begin the fitting process. Using Snell's Law as a model function, we fit the data below shown in Figure 2 to extract a proper value for the index of refraction of glass with its uncertainty, using the "curve_fit" function of the SciPy package. With our initial guess for the index of refraction set to one, we used our $sin(\alpha)$ and $\sin(\bar{\beta})$ values as our "x and y" values within our calculations.

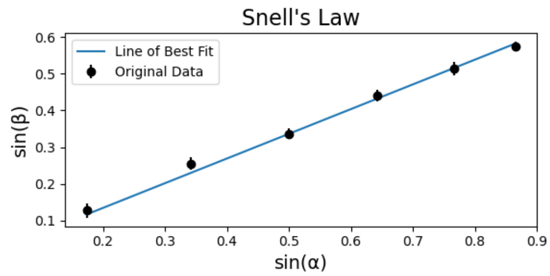

Figure 2: $sin(\beta)$ versus $sin(\alpha)$ with uncertainties provided from Table 1, used to extract the best fit for the index of refraction of glass.

From the one parameter χ^2 fit, the best fit value of the index of refraction is 1.489 ± 0.022.

Chi-Squared Calculations

In order to understand the difference between our data and our Snell's Law model, we conducted a χ^2 test, now using the index of refraction we extracted from the best fit computation inside our model function. By plotting χ^2 as a function of the index of refraction, we are able to visualize where the χ^2 is minimum (the blue curved line) –thus allowing us reaffirm the best fit value we calculated earlier.

In Figure 3, the black vertical line in the middle located at the vertex of the parabola indicate where the minimum in the χ^2 value occurs. The dashed vertical lines on the left and right represent where the (+) and (-) one sigma excursions of the index of refraction (n) occur. Furthermore, the two black horizontal lines also are made to visualize clear excursion boundaries.

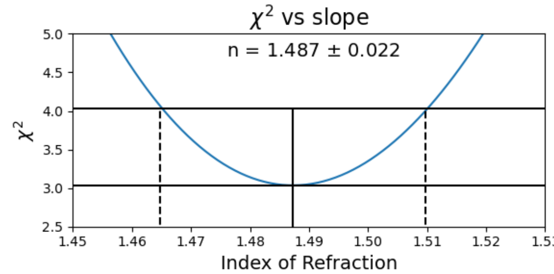

Figure 3: χ^2 as a function of the index of refraction. The blue curve represents the χ^2 value. The black lines indicate maximum and minimum points by mapping out one sigma excursions of each n value.

As indicated by the intersection of the excursion lines to the χ^2 as a function of index of refraction plot, the χ^2 value for the best fit line compared to the data is 3.03. This is visualized where the blue curve hits its lowest point. The p-value for the best fit line compared to the data is 0.69, and the degrees of freedom is 5. The p-value indicates positive agreement between the model and our data, as it is in between $0.05 < p < 0.95$.

4. Summary of Results

In order to understand the dataset at hand, histograms were created to visualize the 96 angle of refraction measurements made. Upon doing so, we were able to compute the values stored in Table 1 to use in the fitting process and χ^2 test to extract the best index of refraction. The best fit value of the index of refraction, the χ^2 value, p-value and degrees of freedom are all underlined throughout the report.