Physics
*ANYTHING NEW WILL GET A DEDICATED PAGE - THIS IS A HOME PAGE FOR THE CLASS: PHYS 106 = A GLORIFIED TAG*

ADD Euler Method

Lecture 1 = rough notes
Lectures 2 - 8 = screenshots of material (essentially review of high school / IB physics)

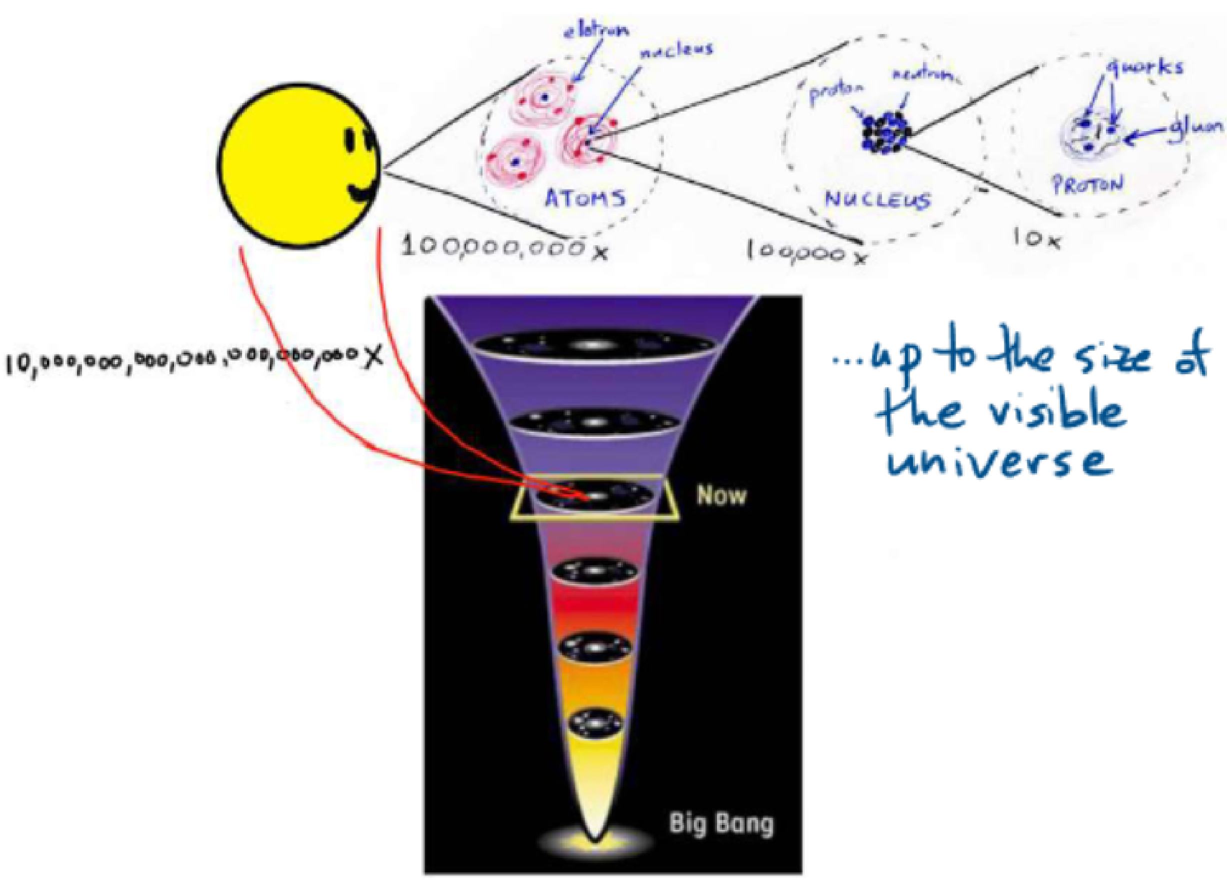
# Introduction (Lecture 1)

*What is physics*
The goal of physics is to observe the universe and to understand its rules.
*"Workings of everything in the universe are governed by a few simple basic principles = elegant mathematical rules"* Occam's Razor Empiricism
We have managed to understand these rules (mostly) + how to manipulate these rules to our advantage.
*Predict the future (outcomes and experiments)*
Deduce the past
* Design technology

# Describing a Universe

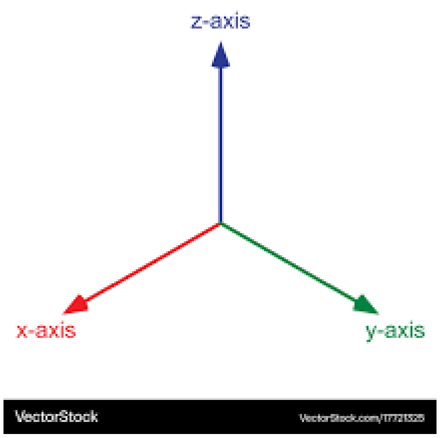
To determine the rules of the universe we must:

1. Have a mathematical description of possible configurations of the system.
   - E.g. In the happy face example: a 2D, discrete time intervals, periodic (torus) is the environment/ and observation.
2. Understand the rules for evolution/propagation: Can predict all future configurations given
   - In the happy face example: Initial velocity $(x_0, y_0)$, the initial velocity, clockwise by $90^\circ$ (on a gray tile), next position = current position + velocity, next velocity (if white = previous velocity, if gray current velocity rotated clockwise).



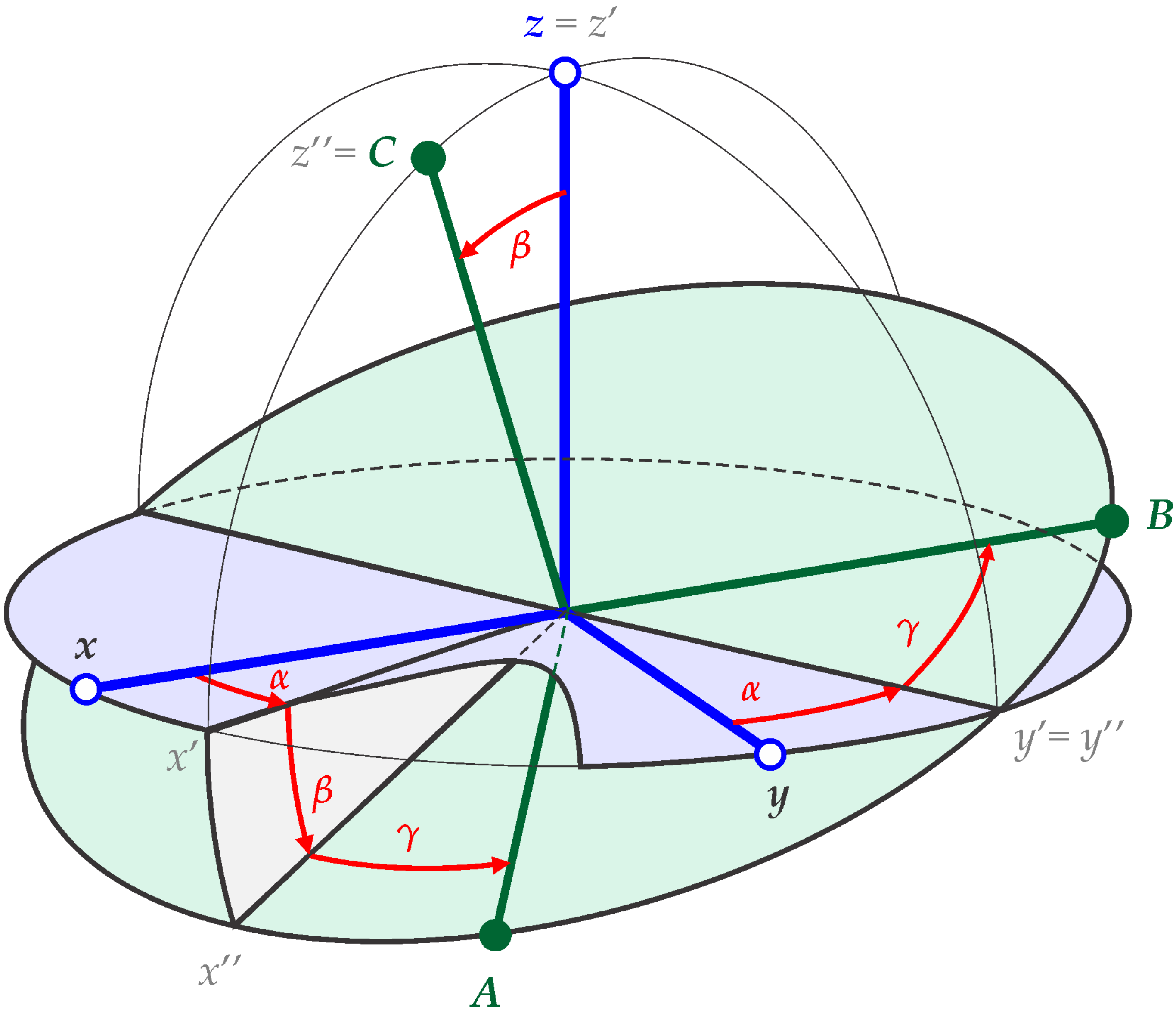
# Coordinates of an Object

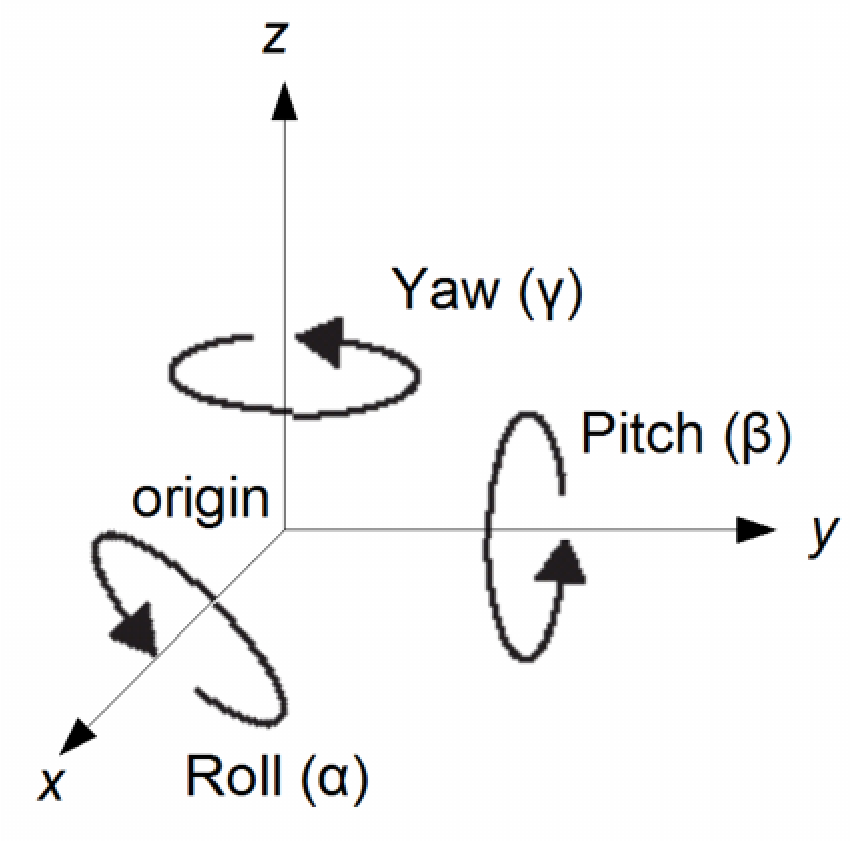
This is similar to the universe we live in where: the previous state of an object is contingent on the environment, previous states etc. in the field of Classical mechanics. This is true for Euclidean geometry, but on larger scales, with larger masses, this falls apart according to Relativistic mechanics (differential geometry to assign coordinates a non-Euclidean plane). = 3 dimensions to describe the central point of an object: x, y, and z are all mutually perpendicular to each other.




# Orientation of an Object

There are 3 angles needed to describe orientation:


z = z′
z″= C
β
B
x
α
γ
α
x′
β
γ
y′= y″
y
x″
A

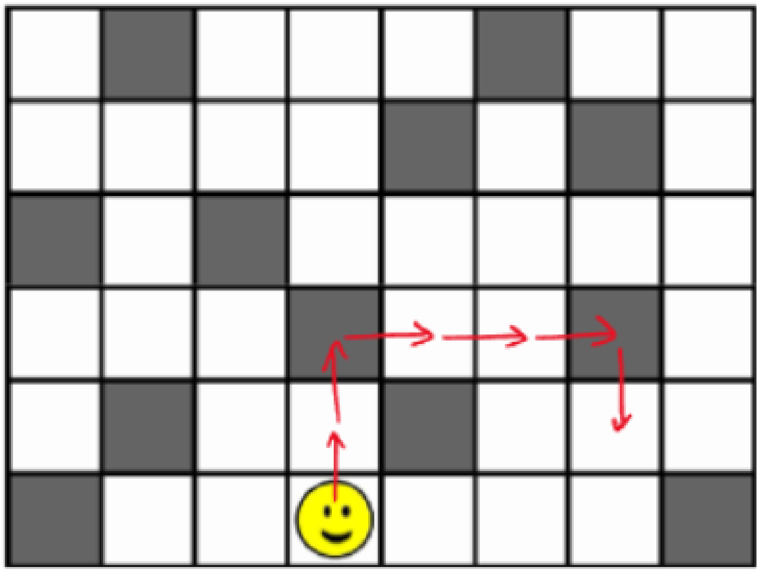



What is mass?

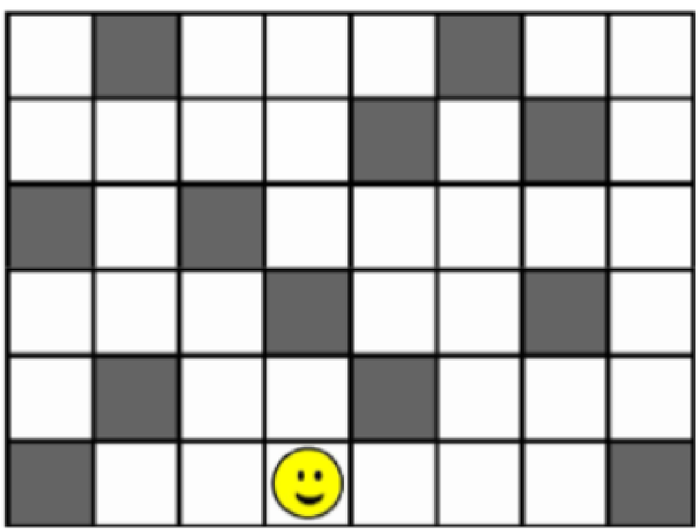
> In deep space where we can ignore gravity, is it possible to tell how massive things are? If so, how can you tell?

# Representing Motion (Lecture 2) [ss]

## Constructing a Universe from Scratch

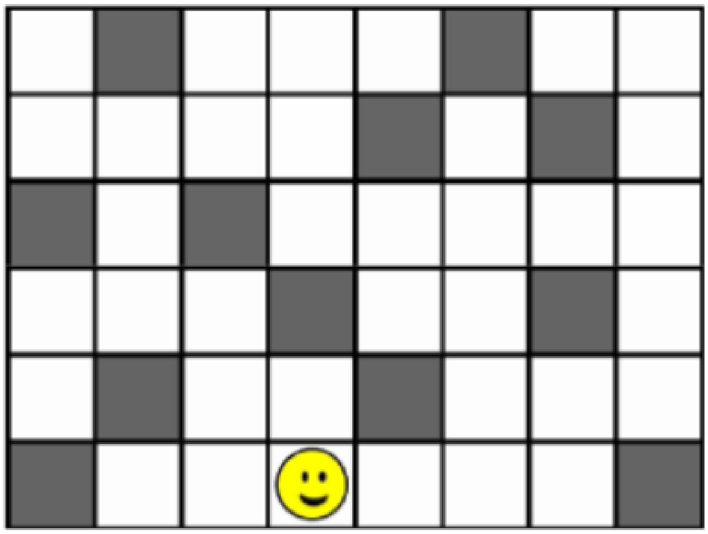


Physics in a simple universe:



Physics in a simple universe:

STEP 1: mathematical description of possible configurations of system



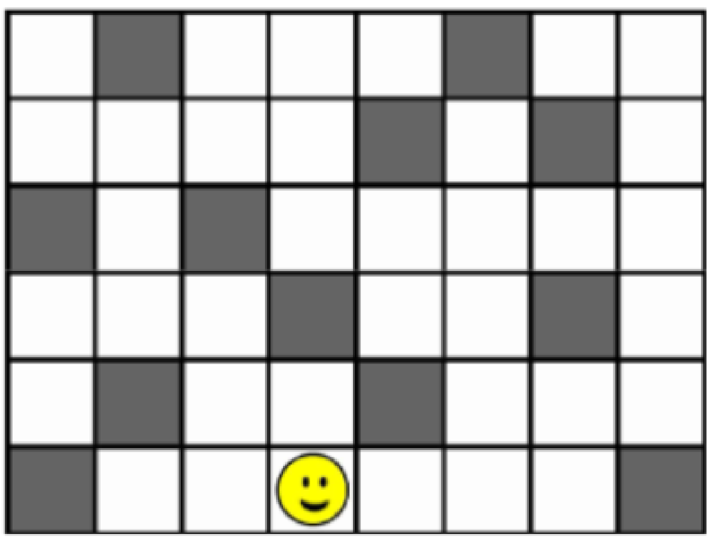
(4,6)

Physics in a simple universe:

STEP 1: mathematical description of possible configurations of system

= pair of integers $(x, y)$

$1 \leq x \leq 8 \quad 1 \leq y \leq 6$



(4,6)

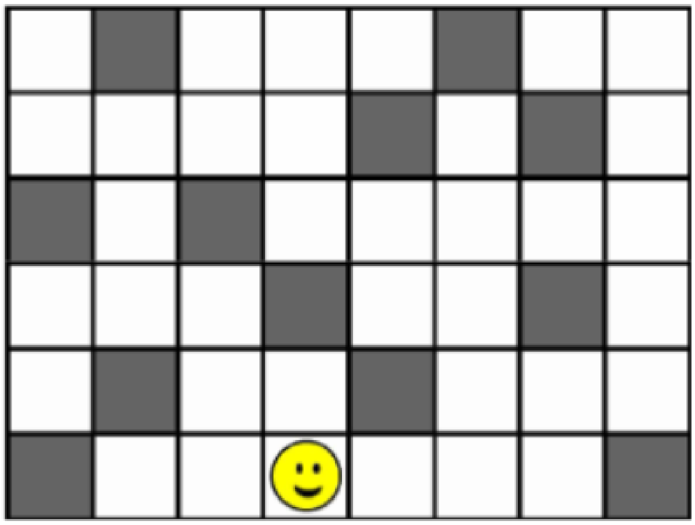
Physics in a simple universe:

STEP 1: mathematical description of possible configurations of system

= pair of integers $(x, y)$

$1 \leq x \leq 8 \quad 1 \leq y \leq 6$

Space in this universe is 2-dimensional, discrete, and periodic (torus)



$(4,6)$

Physics in a simple universe:

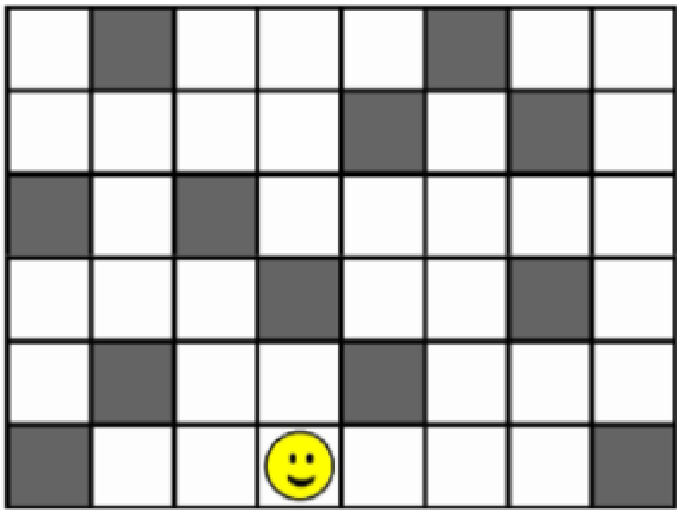
STEP 1: mathematical description of possible configurations of system

= pair of integers $(x, y)$

$1 \leq x \leq 8 \quad 1 \leq y \leq 6$

Configurations evolve with time (discrete)

Complete history of universe: ordered list of locations: $\ldots (4,6), (4,5), (4,4), (5,4), \ldots$



(4,6)

STEP 2: understand the rules for evolution

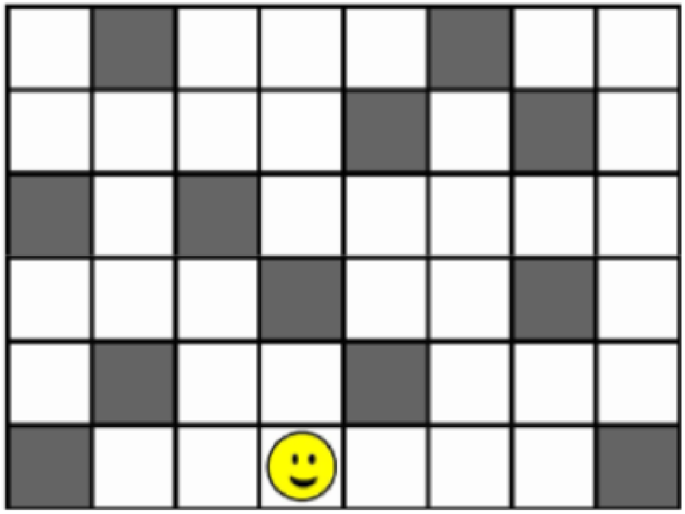
* Can predict all future configurations given:

- Initial location $(x_0, y_0)$
- Initial velocity

$\rightarrow (1,0)$ $\leftarrow (-1,0)$ $\uparrow (0,1)$ $\downarrow (0,-1)$

- Knowledge of environment

(where are the grey squares)



(4,6)

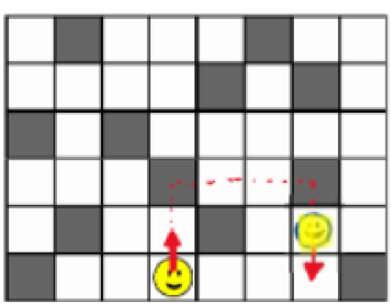
* Can predict all future + past configurations given:

- Initial location $(x_0, y_0)$
- Initial velocity
  ↑ → ↓ ←
- Knowledge of environment
  (where are the grey squares)

next position = current position + velocity

$$\text{next velocity} \begin{cases} \xrightarrow{\text{white}} \text{current velocity} \\ \xrightarrow{\text{grey}} \text{current velocity rotated clockwise} \end{cases}$$

Predicting the future with physics:



INPUT: current position + velocity
environment of object

$\Big\downarrow$ the rules = "laws of physics"

OUTPUT: position + velocity at some later time

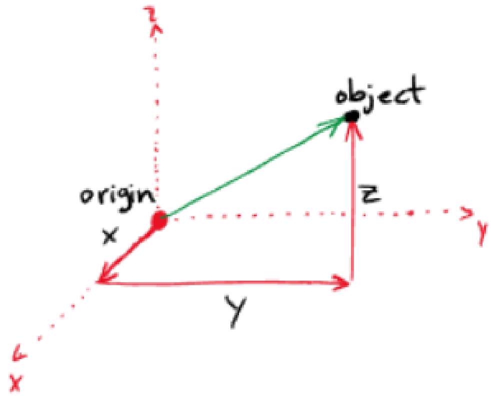
## Start Simple: Understand Motion of Solid Object

- Step 1: Mathematical description of possible configurations to evolution of system

- How can we mathematically represent the location and orientation of a solid object?

Simple case: pointlike object

Can describe location via 3 COORDINATES (real numbers)




① Choose origin

② Choose orthogonal $x, y, z$ directions

③ Any point can be reached in a unique way by moving distance
$x$ in the $x$ direction
$y$ in the $y$ direction
$z$ in the $z$ direction

$(x, y, z)$ depend on our choices

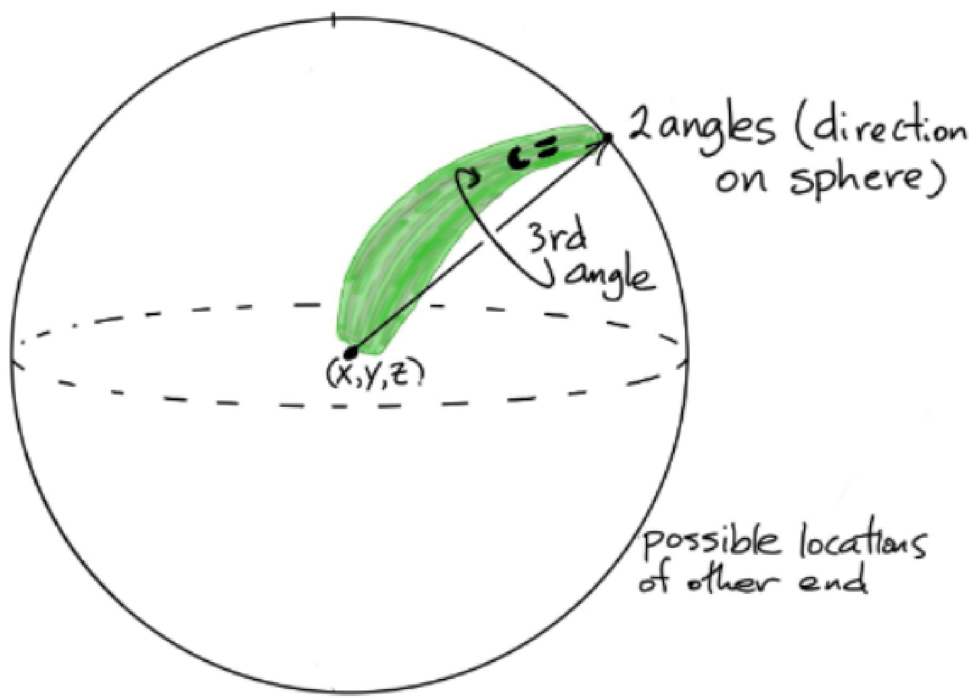
↳ 3 numbers ⟶ we live in 3 dimensions

ASIDE: construction assumed we live in flat (EUCLIDEAN) space

= space with coordinates $(x, y, z)$ with distances between points

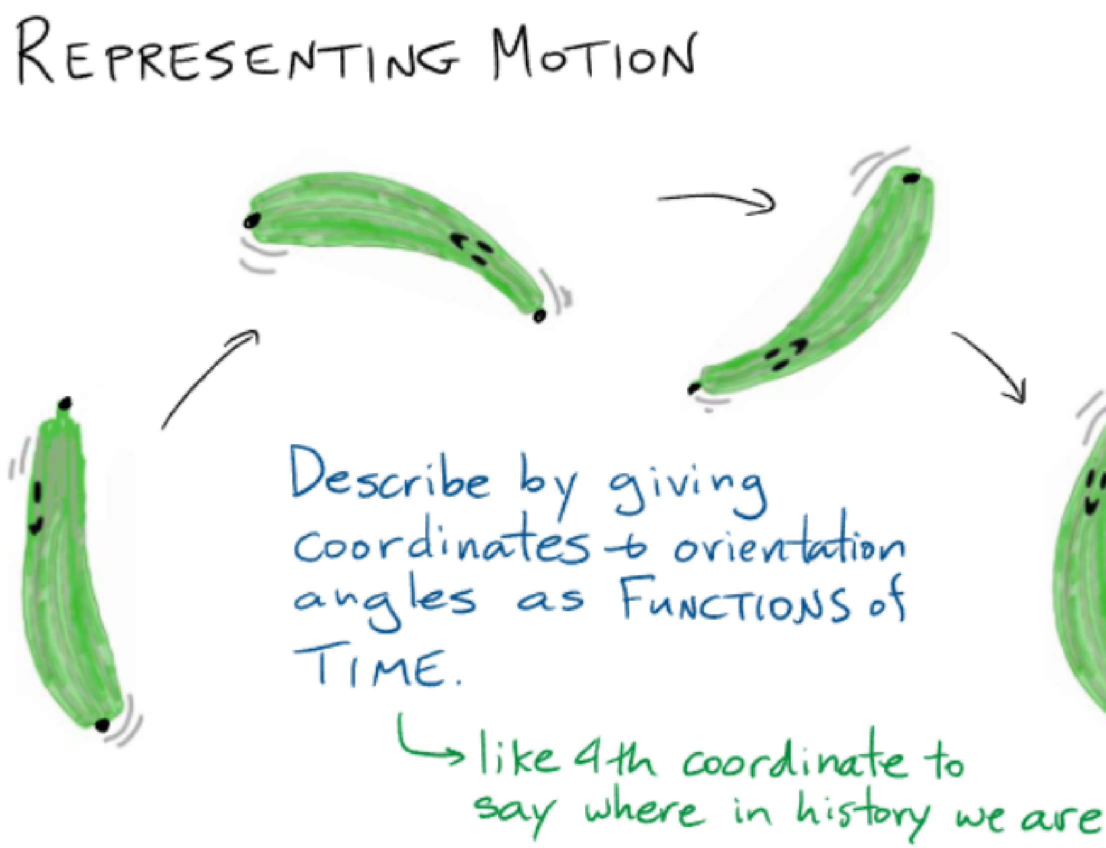
$$L = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$$

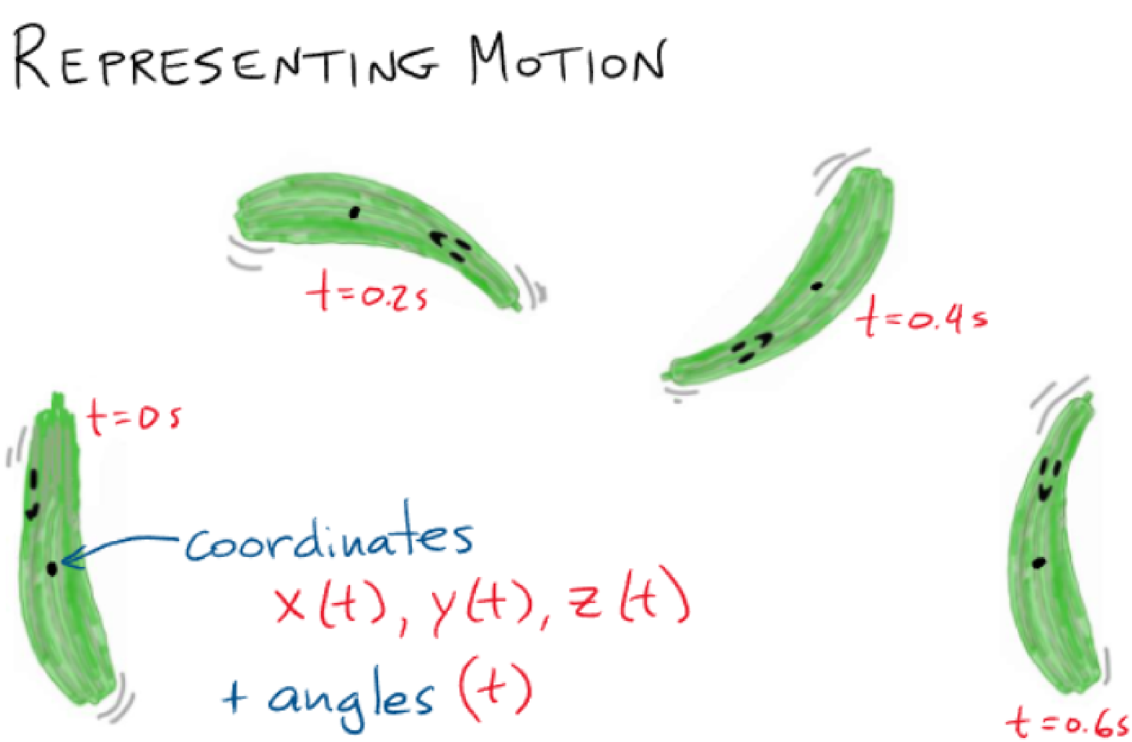
What about orientation?




Need 3 angles to describe orientation.

- Step 2: Represent motion

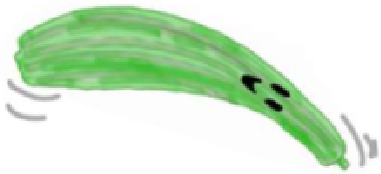


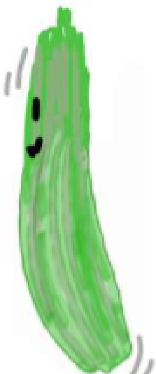




- This is only possible if there is some sort of standard time interval - dropping a ball from a certain height, pendulum/ SHM system etc.

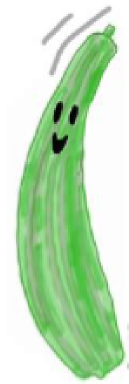
## Time, Velocity, and Acceleration **Kinematics** (Lecture 3) [ss]

# How do we quantify time?





Q: if you had to start from scratch without a clock, how would you assign specific numbers to different moments in time?



A: make use of a (perceived to be) PERIODIC = regularly repeating phenomenon to define equal time intervals.

1 second is the time taken for

9192631770 oscillations

of light of a specific wavelength

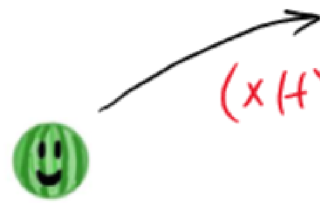
emitted by cesium-133 atoms.

1 meter is the length travelled by light

in $\frac{1}{299,792,458}$ seconds

Ignore rotations for now.

(Consider small non-rotating spherical zucchini)

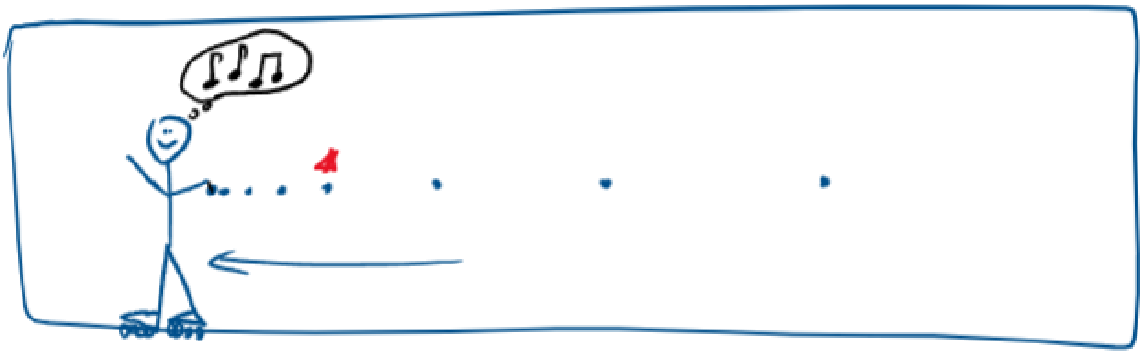


$(x(t), y(t), z(t))$

To predict future motion: need current position but also current VELOCITY

← how object is moving at present moment

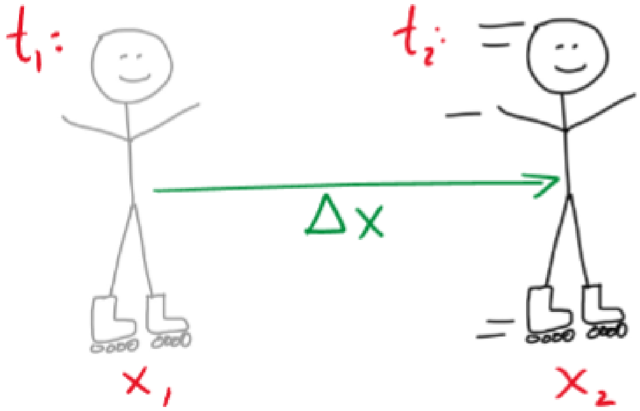
## Instantaneous Velocity, Velocity and Acceleration from Graphs (Lecture 4) [ss]



**FOOD FOR THOUGHT** How would you estimate the acceleration at ★?

We calculated AVERAGE VELOCITY over a time interval.

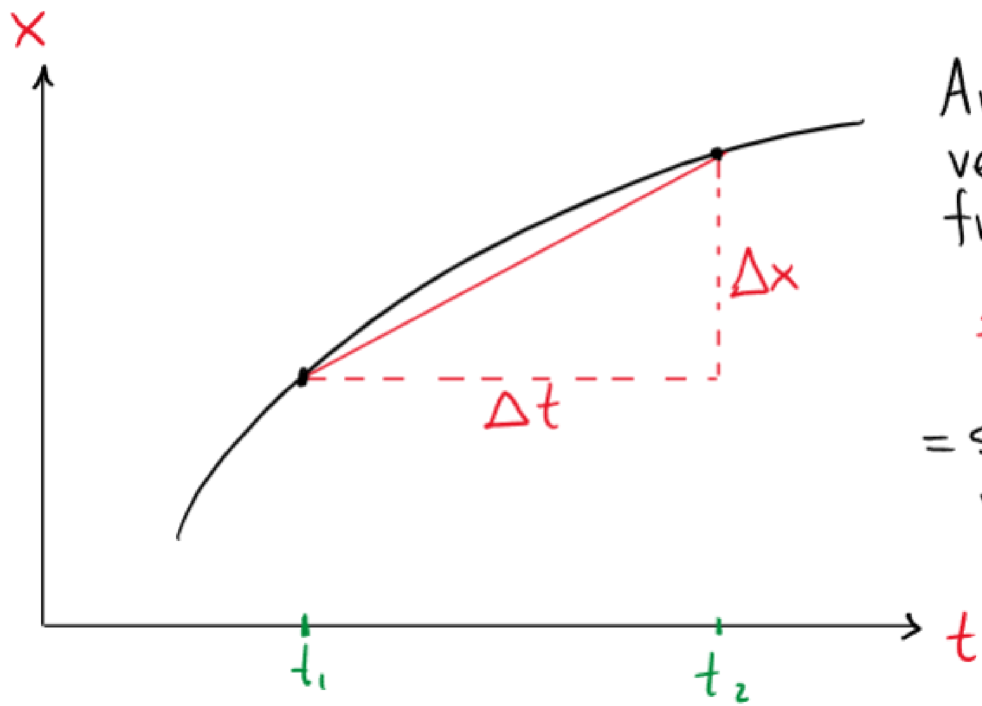
final minus initial




$$V_x^{(avg)} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}$$

(similar for $V_y$, $V_z$)

# Position vs time graphs

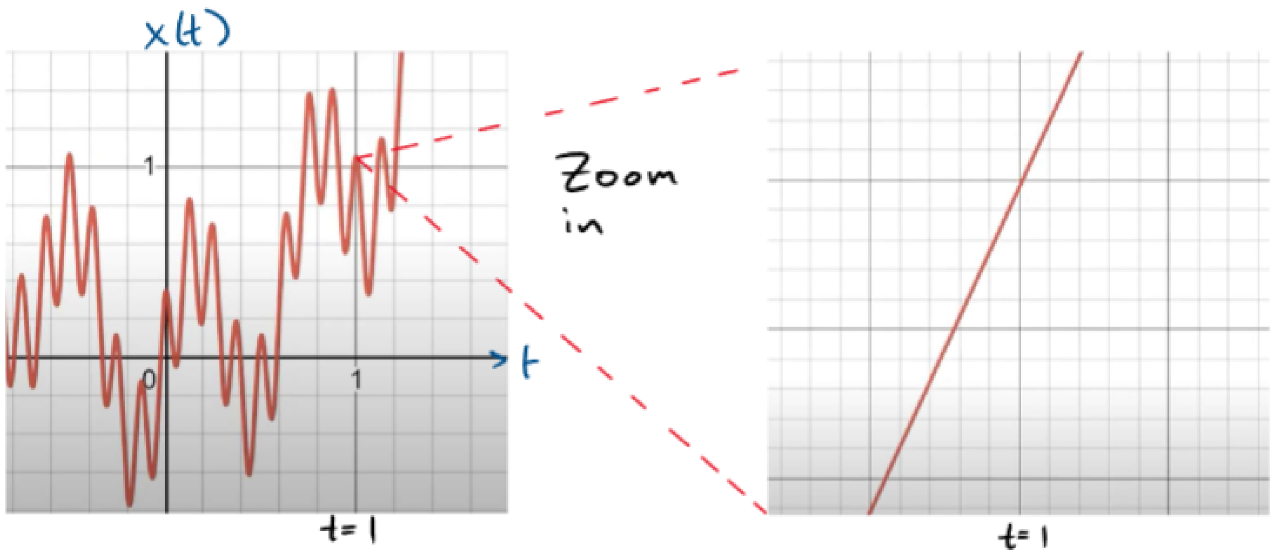



Average x velocity from $t_1$ to $t_2$:

$$\frac{\Delta x}{\Delta t}$$

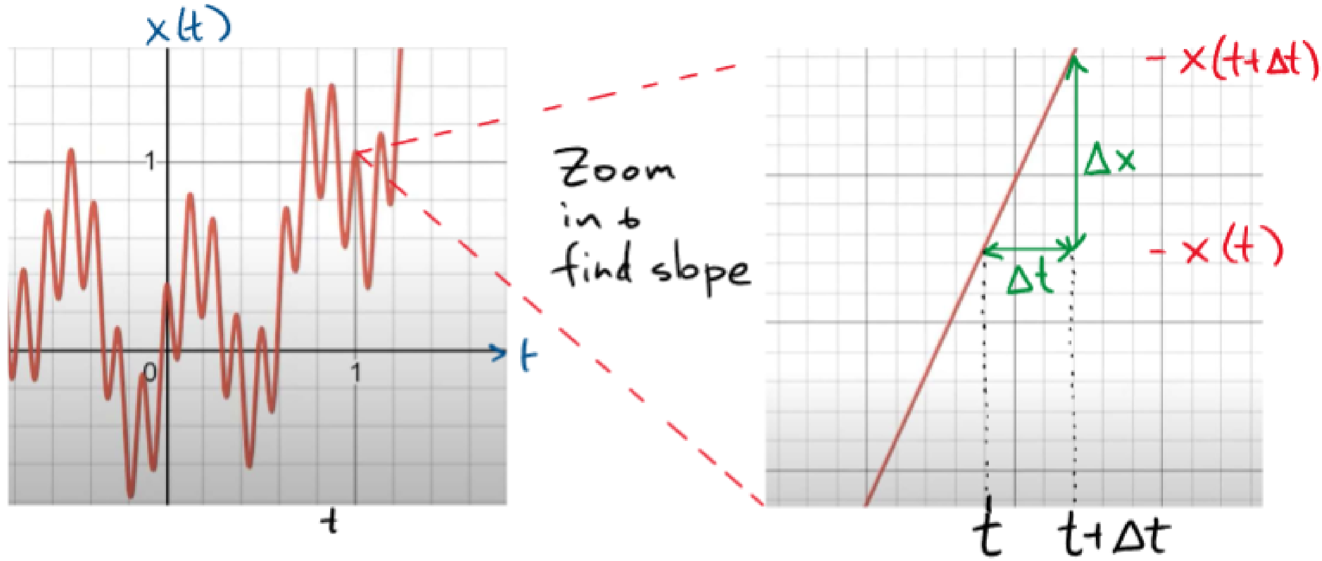
= slope of red line

Any smooth function looks like a straight line if we zoom in enough.

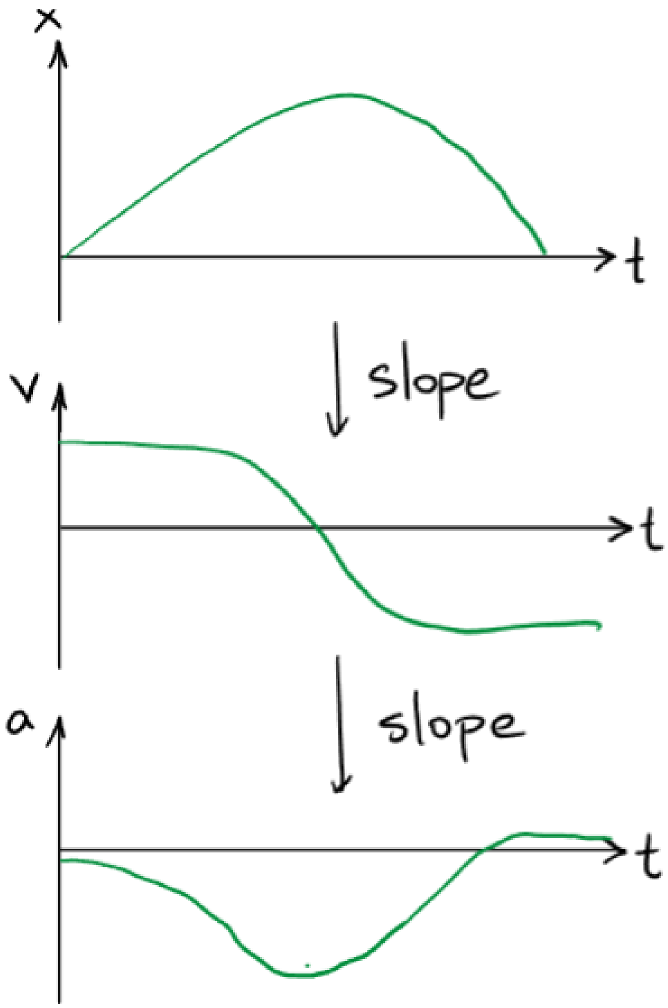



Instantaneous velocity at $t=1$: slope of this line

# INSTANTANEOUS VELOCITY




$$V_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \lim_{\Delta t \to 0} \frac{x(t+\Delta t) - x(t)}{\Delta t}$$




$$x(t)$$

$$\Big\downarrow \text{derivative}$$

$$v_x(t) = \frac{dx}{dt}$$

$$\Big\downarrow \text{derivative}$$

$$a_x(t) = \frac{dv_x}{dt}$$

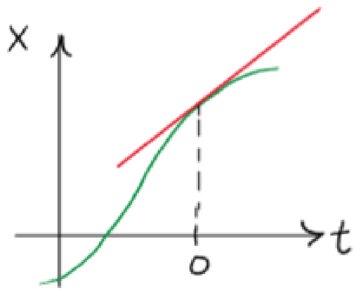
3 ways to find velocity:

1.

| t (s) | x (m) |
|---|---|
| 0 | 4.8 |
| 0.1 | 5.1 |
| 0.2 | 5.4 |
| 0.3 | 5.8 |

$$v(0) \approx \frac{x(0.1) - x(0)}{0.1}$$

2.




$v(0)$ = slope of curve at $t=0$

3.

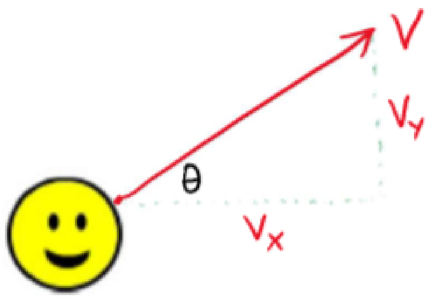
$$x(t) = t^2 + 3t$$

$v(0)$ = derivative $x'(t) = 2t+3$ evaluated at $t=0$

**Rules for Physics in Outer Space (Lecture 5) [ss]**

## VELOCITY IS A VECTOR → exists without reference to coordinates: can describe via SPEED + DIRECTION

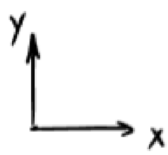
↳ also displacement, acceleration




- represent by COMPONENTS

$(V_x, V_y, V_z)$

once we choose coordinates (depends on choice)




e.g. $V_x = V \cdot \cos\theta$

$V_y = V \cdot \sin\theta$

SPEED:

$$V = \sqrt{V_x^2 + V_y^2 + V_z^2}$$

does not depend on our choice of coordinates.

## ROTATIONAL VELOCITY: can also visualize as a vector pointing along rotation axis, with length showing rate of rotation

Now, we're ready to understand the rules...

GOAL: Given information about an object & its environment at initial time

↓

Predict future motion.

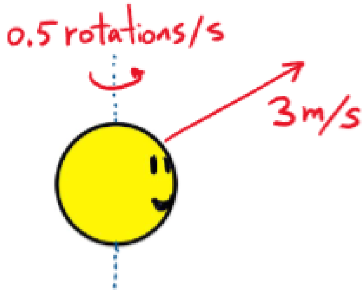
Start with simplest possible environment...

★DEEP SPACE★

The rules of physics in deep space

RULE #1: objects move at constant velocity

(This is Newton's 1st Law)

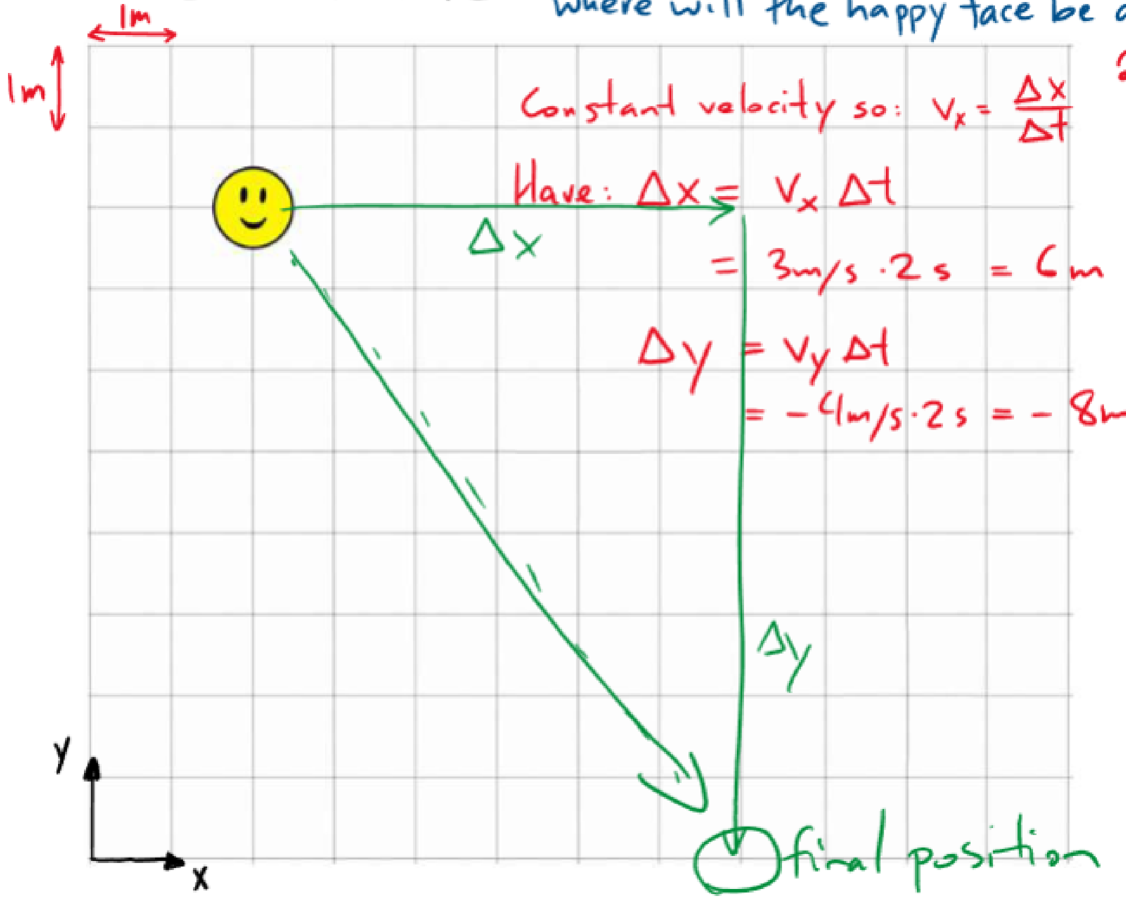



RULE #2: objects rotate about a fixed axis at a constant rate*

*assuming spherical*

# PREDICTING THE FUTURE:

Given $(v_x, v_y, v_z) = (3\,m/s, -4\,m/s, 0)$ where will the happy face be after $2\,s$?




Constant velocity so: $v_x = \frac{\Delta x}{\Delta t}$

Have: $\Delta x = v_x \Delta t$

$= 3\,m/s \cdot 2\,s = 6\,m$

$\Delta y = v_y \Delta t$

$= -4\,m/s \cdot 2\,s = -8\,m$

## Predicting the Future:

Given: initial location $(x_0, y_0, z_0) = \vec{r}_0$
initial velocity $(v_x, v_y, v_z) = \vec{v}_0$

Position after time $t$ is $\vec{r}_0 + \vec{v}_0 t$

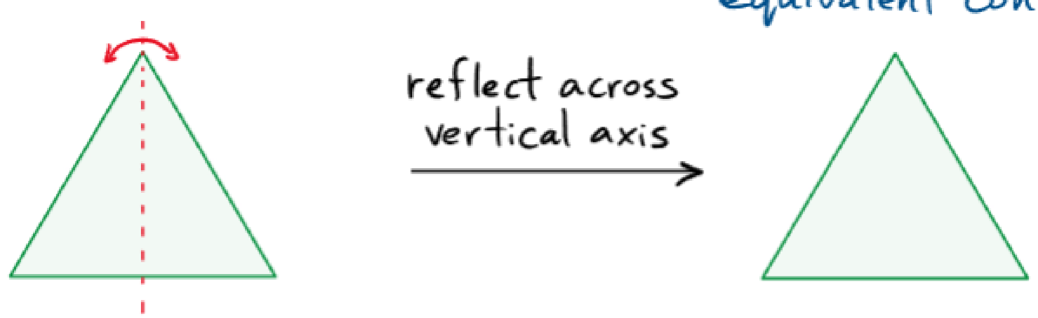
Velocity after time $t$ is $\vec{v}_0$

(similar rules for rotation)

## A detour: SYMMETRY in nature

What does it mean for something to have a symmetry?

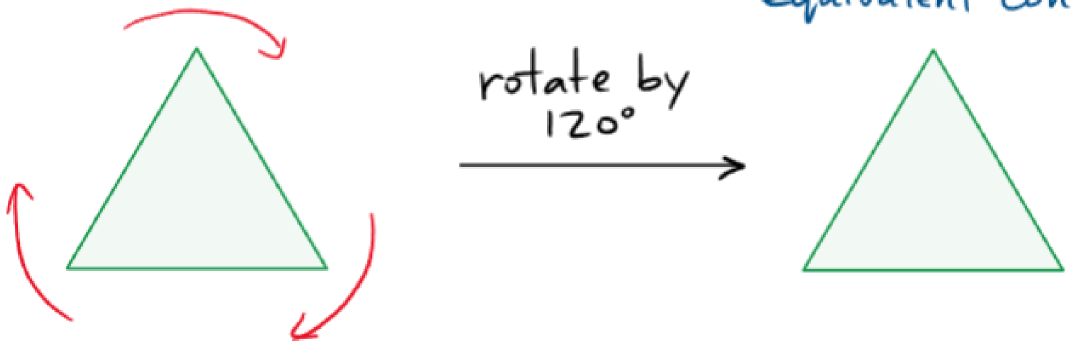
* we can perform some operation that brings us to an equivalent configuration*



# A detour: SYMMETRY in nature

What does it mean for something to have a symmetry?

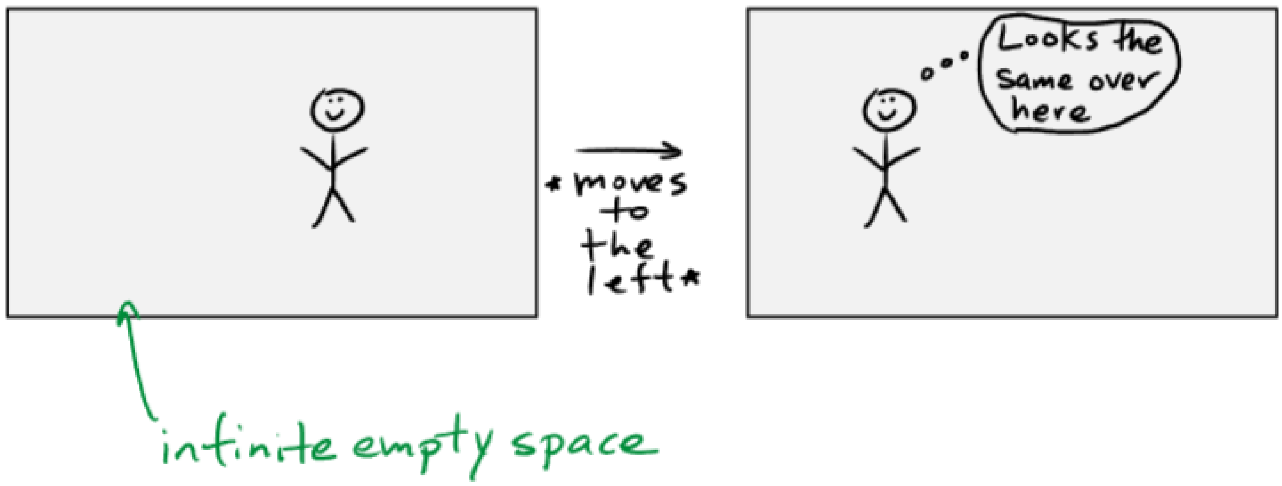
★ we can perform some operation that brings us to an equivalent configuration★



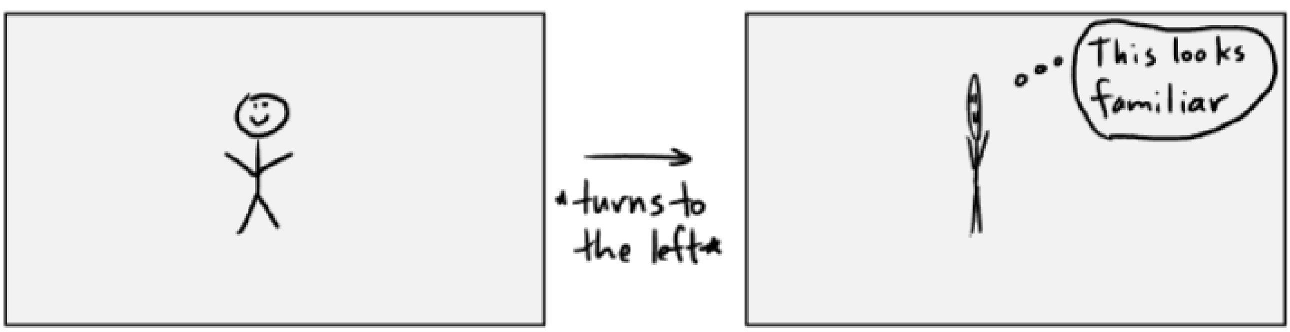

# SYMMETRY IN PHYSICS

The environment of a physical system has a symmetry if there is some operation we can perform on it that gives an equivalent environment.

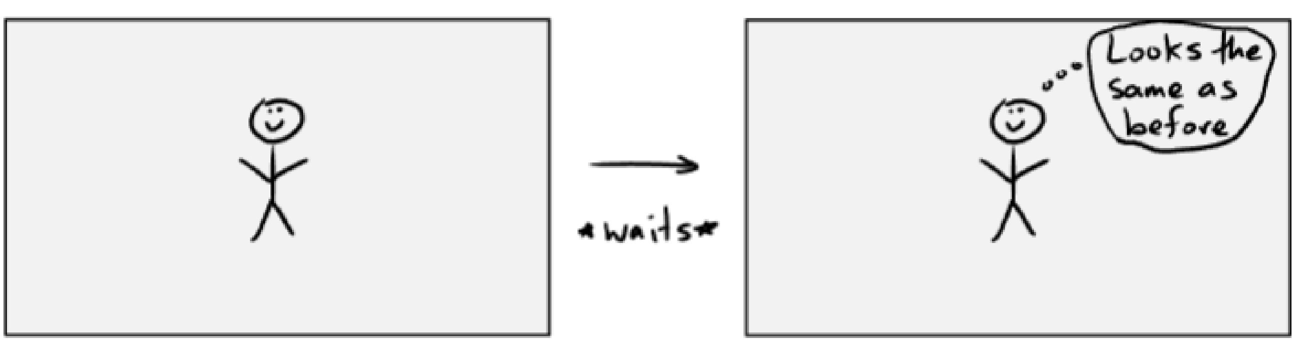
(easier: imagine being in that environment & performing the operation on ourself) example: a shift in some direction




example 2: a rotation about some axis




example 3: a shift in time



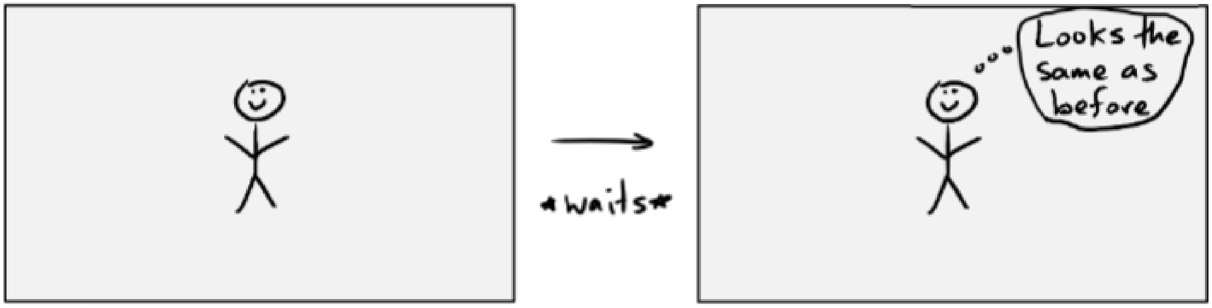
For each SYMMETRY in the environment of a physical system, there will be a CONSERVED QUANTITY, some physical entity that cannot change with time.

## Symmetry (physics) and Conservation Laws Momentum; Momentum Conservation (Lecture 6) [ss]

### SYMMETRY IN PHYSICS

The environment of a physical system has a symmetry if there is some operation we can perform on the system that gives us an equivalent configuration.

For an environment that is unchanging in TIME,

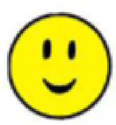



we have:

Conservation of energy

Can associate a quantity called ENERGY to a system, and this doesn't change with time

Einstein: The quantity we call MASS quantifies the energy of an object when it is stationary



$$E = mc^2$$

speed of light

Conservation of energy implies the mass of an isolated object cannot change

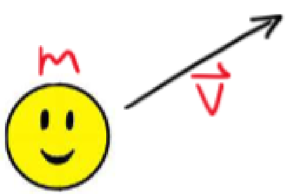
For an environment that unchanged if we shift in some direction


we have:

Conservation of momentum

in that direction

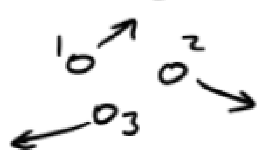
The MOMENTUM of an object is




$$\vec{p} \approx m\vec{v}$$ (if slow compared to light)

exact: $$\vec{p} = \frac{m\vec{v}}{\sqrt{1 - \frac{v_x^2 + v_y^2 + v_z^2}{c^2}}}$$

Momentum is additive: $\vec{p}_{TOT} = \vec{p}_1 + \ldots + \vec{p}_N$



For an environment w. translation symmetry in some direction, the component of total momentum in that direction is conserved.

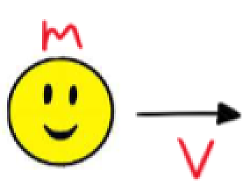
e.g. outer space: $\vec{p}$ constant for single object

$$p_x^{before} = p_x^{after}$$
$$p_y^{before} = p_y^{after}$$
$$p_z^{before} = p_z^{after}$$

for any interactions between objects

For an isolated object in outer space:

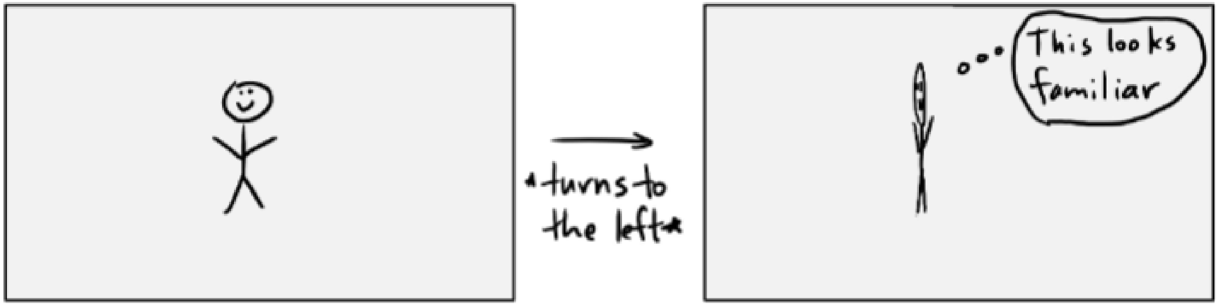
energy conservation
↓
Mass is constant: $m$ doesn't change




Momentum in any direction is constant
$m \cdot v$ doesn't change

So: velocity in any direction is constant
$v$ doesn't change

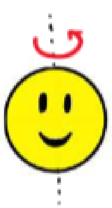
For an environment that unchanged if we rotate in some direction...




we have

Conservation of angular momentum ("spin")

The SPIN or ANGULAR MOMENTUM of an object is related to its mass, size, and period of rotation.



Conservation of angular momentum explains why isolated objects e.g. planets keep spinning at a constant rate.

# Physics of an isolated object from conservation laws:

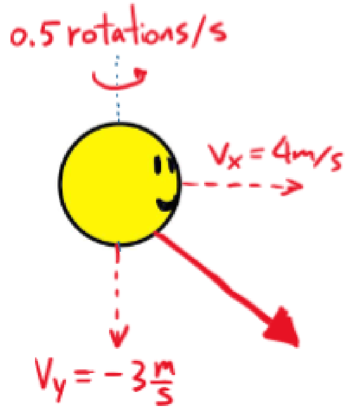
time shift symmetry → Conservation of energy → mass of the object remains constant

↓

translation symmetry → Conservation of momentum → V in each direction is constant

rotation symmetry → Conservation of angular momentum → rotational axis + period are constant

(for spherical object of fixed size)



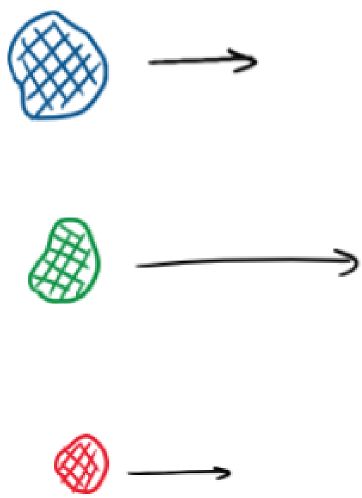
Another example:

In the 1st half of the 20th century, physicists were puzzled by certain kinds of nuclear decays in which a stationary nucleus decays into an electron and another nucleus, as shown:

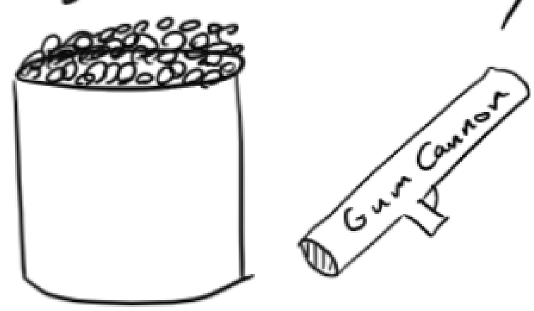

Why was such a decay puzzling? Can you think of a possible resolution to the puzzle?

## What is Mass? (Lecture 7) [ss]

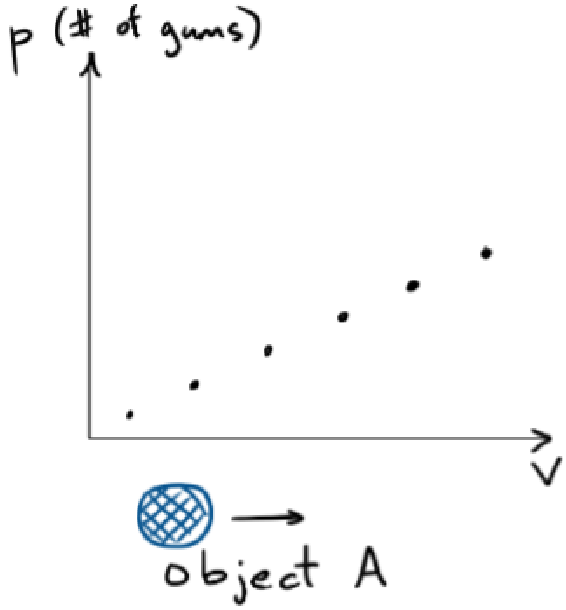
Key point: when we interact with an object (e.g. push/pull/collide) velocity can change. Different objects are more/less affected by the same interaction if their mass is more/less.

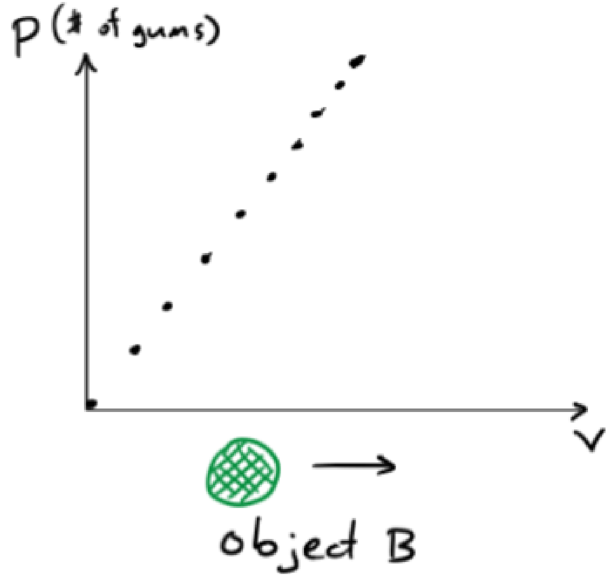


You have an unlimited supply of identical sticky chewed pieces of gum and a way to propel them at a standard velocity (1 m/s)




Devise a method by which you could quantify the momenta of other objects.



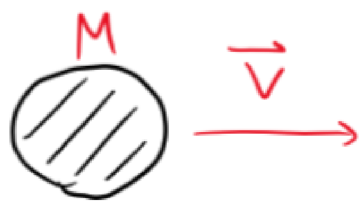

Find: momentum is proportional to $v$ for small $v$. Can <u>define</u> MASS to be the proportionality constant:

$$p = mv$$

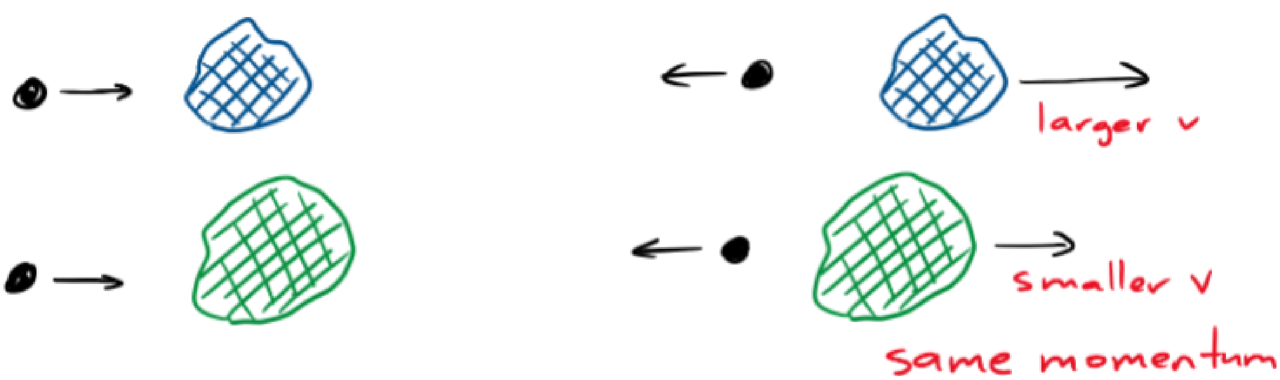
Momentum quantifies how hard it is to stop something from moving.

- proportional to mass
  double mass → double momentum

- proportional to velocity for small velocity.




$$\vec{p} = m\vec{v}$$




☆ two objects that experience the same external influence will have the same change in momentum

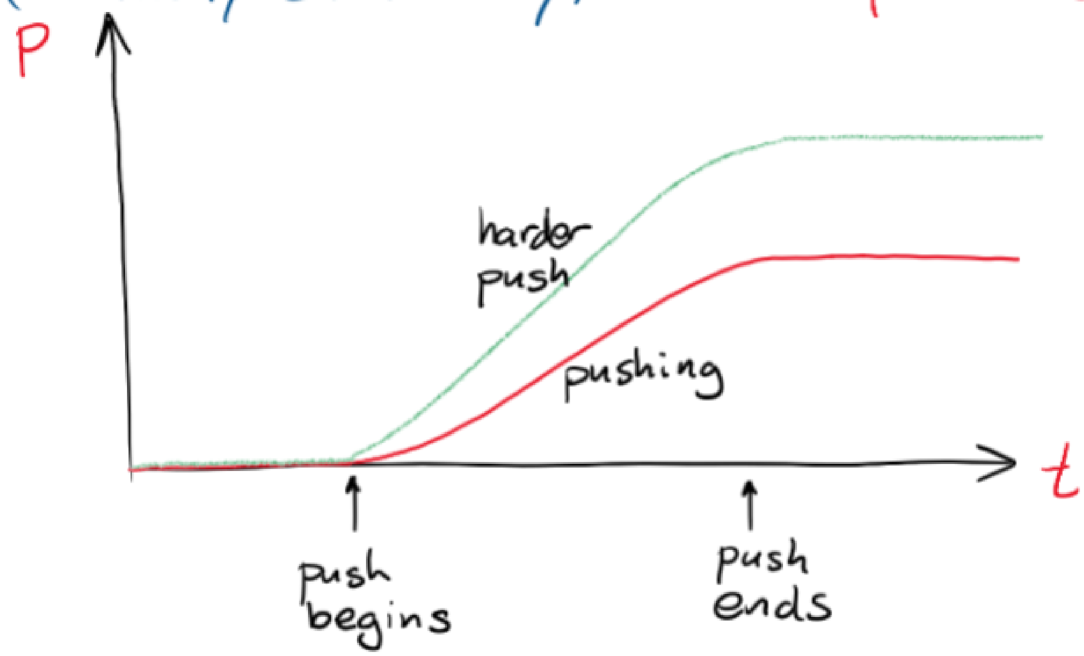
Have seen: can change momenta of objects by interacting with them.


These changes can happen at different rates.


NEXT: relate the rate of change to the strength of an external influence

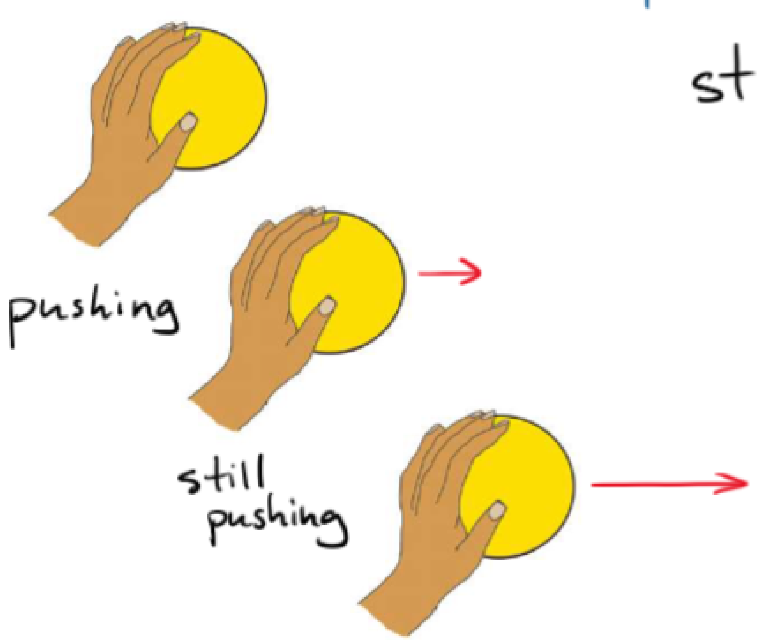
## Force, Newton's Second Law (Lecture 8) [ss]

Q: for the ball in the previous example (initially stationary) sketch $p$ vs $t$.




How would the graph change if we push harder for the same amount of time?

FORCE quantifies the instantaneous strength of a push or a pull

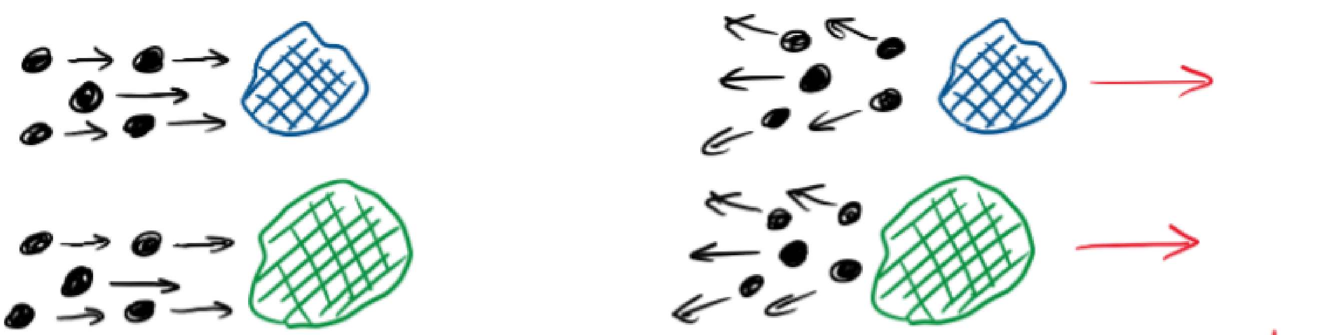



stronger force:
momentum changes faster

Could define force to be the rate of change of momentum that it produces on some standard object.

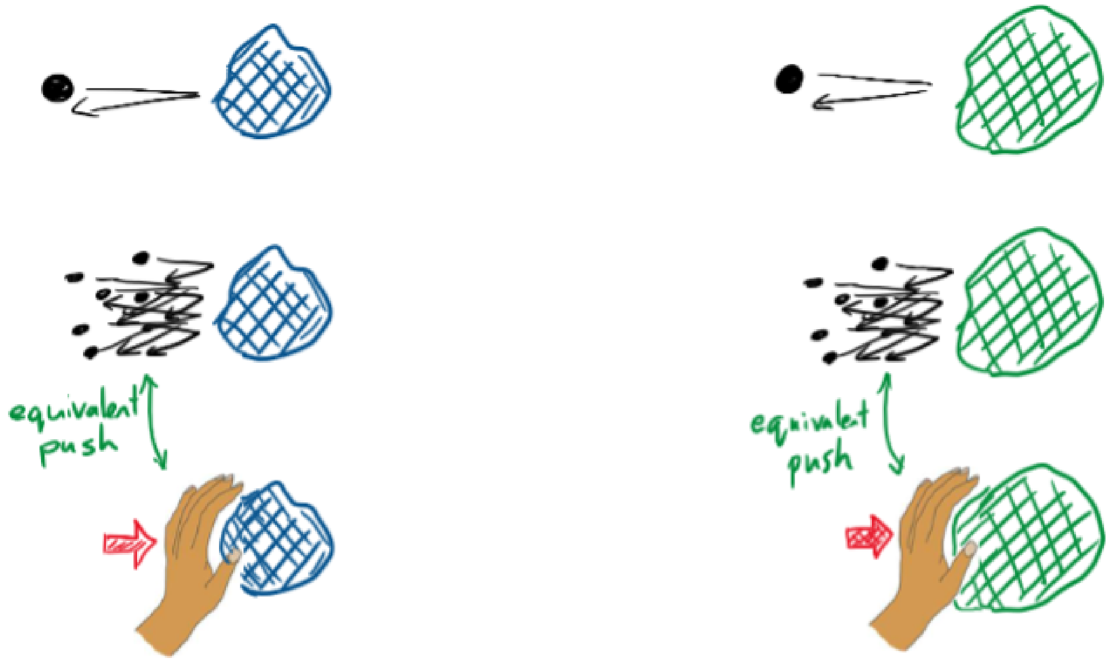

$F$ defined as $\dfrac{dp_{ball}}{dt}$

slope of $p$ vs $t$ graph

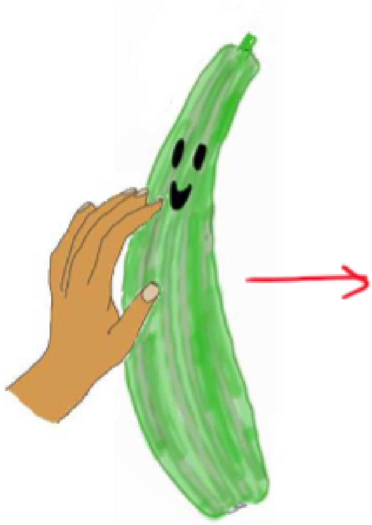
same influence ⇨ same momentum change

★ two objects that experience the same external influence will have the same change in momentu




☆ For two objects experiencing the same external influence, momentum will change in the same way. ☆

# Newton's Second Law



$$\frac{d\vec{p}}{dt} = \vec{F}_{NET}$$

ordinary velocities:

$$\frac{d\vec{p}}{dt} = m\frac{d\vec{v}}{dt} = m\vec{a}$$

$$\rightarrow \vec{F}_{NET} = m\vec{a}$$

Using Newton's Second Law to Predict the Future **Kinematics with drag** (Lecture 9)

Small Time Steps Method and Antidifferentiation Method **Terminal velocity** (Lecture 10)

**Antidifferentiation method for kinematics**, Constant Acceleration, **Normal and Friction Forces** (Lecture 11)

**Area method for predicting velocities** **Newton's 3rd Law** (Lecture 12)

**Mechanical equilibrium** (Lecture 13)

**Simple Harmonic Motion (SHM)** (Lecture 14)

**Energy Conservation & Work** (Lecture 15)

**Inertia**, (Lecture 16 )

**Time dilation & length contractions** (Lecture 17)

**Relativistic momentum**

**Conservation of angular momentum** (Lecture ...)

## Torque (Lecture ...)