Problem Set

#01

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Day 1

[1] Growth rates

The below table shows nominal GDP of the USA.

Year	GDP in bil. 2011 US$	Symbol
2011	15,591	Y_1
2012	15,978	Y_2
2013	16,274	Y_3
2014	16,705	Y_4

Table 1: GDP of the USA

For notational simplicity, let Y_1, Y_2, Y_3, Y_4 denote the GDP for years 2011, 2012, 2013 and 2014, respectively. The net annual growth rate between 2011 and 2012 is defined by

$$g_{2,1} = \frac{Y_2 - Y_1}{Y_1} = \frac{Y_2}{Y_1} - 1.$$

$g_{3,2}$ and $g_{4,3}$ are defined similarly.

1. Calculate the annual growth rates $g_{2,1}$, $g_{3,2}$ and $g_{4,3}$.

2. Compute compound annual growth rate between 2011 and 2014 defined by[1]

$$g_{4,1} = \left(\frac{Y_4}{Y_1} \right)^{\frac{1}{4-1}} - 1.$$

3. Observe that

$$\frac{\ln Y_4 - \ln Y_1}{4 - 1}$$

gives a nice approximation to the compound annual growth rate.

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[1]Note: Since $\frac{Y_4}{Y_1} = \frac{Y_4}{Y_3} \frac{Y_3}{Y_2} \frac{Y_2}{Y_1}$ holds, the compound annual growth rate is the **geometric average** of annual growth rates over multiple years.

[2] Effective interest rate.

Assume that a bank offers an annual, nominal interest rate of 6% **compounded monthly** and that you make a deposit of one thousand dollars ($1,000) at the bank today. Assume that there is no other engagement with the bank before and after that deposit.

1. How much do you expect to have in the bank account in one year from now?

2. How much will you have after 2 years, 3 years, and t years?

3. Compute the annual effective rate of interest.

4. How do the above results change if the interest is compounded daily?

5. How do the above results change if the interest is compounded continuously? That is, consider compounding N times per year and take the limit of $N \to \infty$.

6. Consider the continuous-time compounding again. If the nominal rate is r, then Y_0 dollars now will be $Y_t = Y_0 e^{gt}$ dollars in t years from now. Compute

$$\frac{\ln Y_t - \ln Y_0}{t}$$

   and interpret the result of Problem [1]-3.

[3] National Accounts Identity

Total product Y is decomposed into four components:

$$Y = C + I + G + NX$$

Explain what each symbol denotes.

Y $\underline{Y}$ields, i.e., GDP.

C

I

G

NX

Answer sheet. Please write your name and id number.