Problem Set

#03

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[1] Perfectly competitive factor market

Let $Y = F(K, AL)$ be the production function. The profit, measured in Unit/Year, can be written as

$$F(K, AL) - (r + \delta)K - wL,$$

where r and w are real rental and real wage, respectively. δ is the depreciation rate.

a) Derive the first-order conditions for firms' profit maximization problem.
b) Suppose that F has constant returns to scale (CRS), i.e., $F(cK, cAL) = cF(K, AL)$ for any $c > 0$. Euler's homogeneous function theorem states that CRS function $f(x, y)$ satisfies $f(x, y) = x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y}$. Use this theorem and show the zero profit condition,

$$Y = (r + \delta)K + wL,$$

holds true. [Hint: By the chain rule, $\frac{\partial F}{\partial L} = \frac{\partial F}{\partial (AL)} \frac{\partial (AL)}{\partial L} = \frac{\partial F}{\partial (AL)} A$.]

[2] Kaldor's fact

Recall Kaldor's stylized facts.

1. Output per worker Y/L grows at a sustained rate.
2. Capital per worker K/L grows at a sustained rate.
3. Rental rate $r + \delta$ (gross) is constant.
4. Capital output ratio K/Y is constant.
5. Capital share $(r + \delta)K/Y$ and labor share wL/Y are constant, where w is the wage rate.
6. Among the fast growing countries of the world, there is an appreciable variation in the rate of growth.

Verify that two of the above facts are redundant by proving the following statements.

a) Facts 1 and 4 imply Fact 2.
b) Facts 3 and 4 imply Fact 5.