UNIVERSITY OF COLORADO - BOULDER

ASEN 3112

STRUCTURES LAB 2

Shaker Lab

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November 30, 2020

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Contents

I. Experimental Results

The experimental data consisted of accelerometer data and displacement data of the system at multiple points. One sensor was placed on the connection between the shaker and the system to model the input acceleration. The remaining three sensors were placed at the tail nose and wing to measure the system's response. In order to determine the amplitude of response for each sensor, a fourier transform was used. Each set of accelerometer data was processed using a fourier transform, which was then scaled by the maximum amplitude response, so all data could be compared on the same scale. For this analysis, the only meaningful metric is the magnitude of the system's response, so only the absolute value was used to determine resonant frequencies.

Two plots were created to represent the experimental data. One displays the excitation frequency versus time, and the other displays the amplitude of the response versus the excitation frequency, which can be used to determine the resonant frequencies. The plots generated can be seen below. Figure 1 is a least squares fit of the frequency of the experimental data, which was performed to validate the linearity of the excitation frequency. The values for excitation frequency were computed by manually counting the number of times the accelerometer data crossed zero, and averaged over time spans of 0.3 seconds.

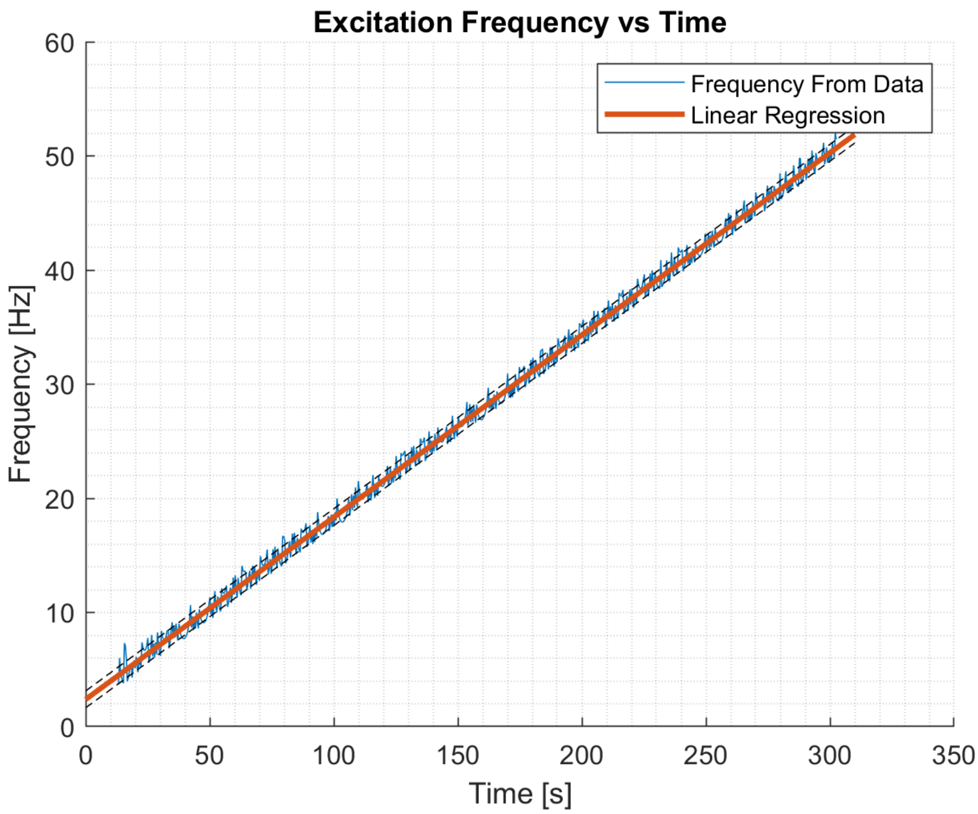

Figure 1: Excitation Frequency vs Time

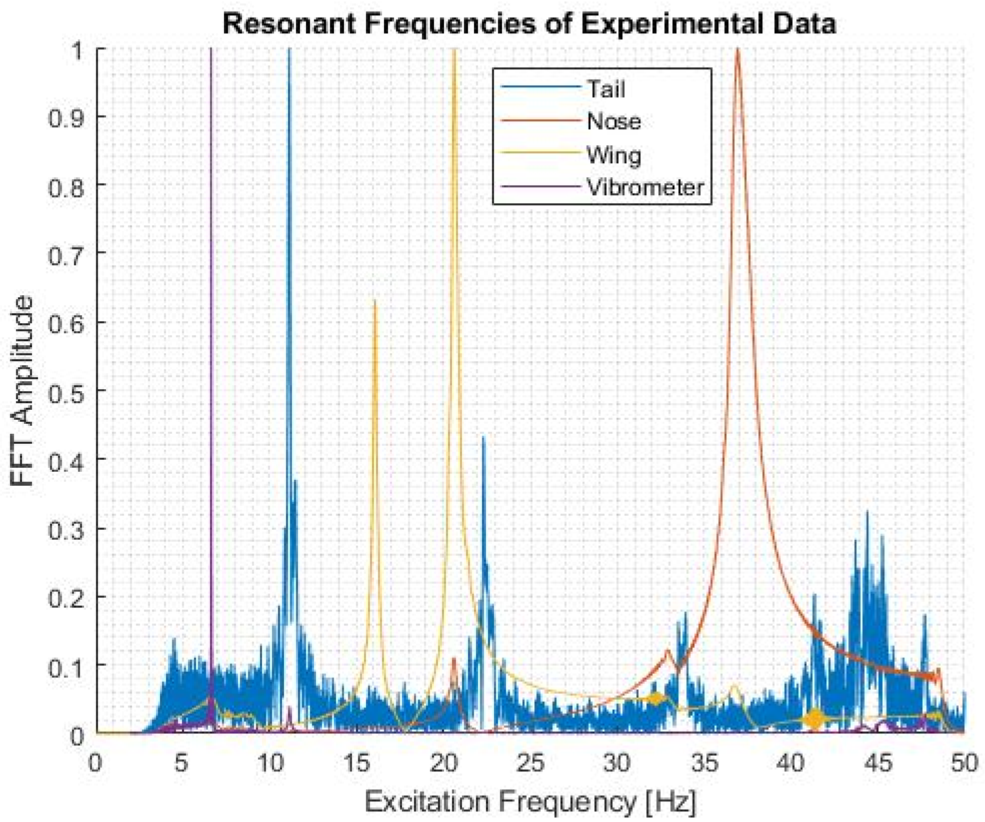

Figure 2: Amplitude of Response vs Excitation Frequency

The resonant frequencies for each section of the testing airplane can be seen in figure 2. The amplitude at each section approaches the value of one at a specific excitation frequency in the plot. When the amplitude achieves the value of 1, the section's response acceleration is equal to the input acceleration; thus, the section achieves resonant frequency. The resulting values can be seen in Table 1.

Location	Resonant Frequency
Vibrometer	6.64 Hz
Tail	11.14 Hz
Nose	36.92 Hz
Wing	20.65 Hz

Table 1: Resonant frequencies of different sensor locations.

The sensors used to capture resonance are the vibrometer, tail accelerometer, nose accelerometer, and wing accelerometer. To determine the shape of the model, experimental eigenmodes must be generated. Experimental eigenmodes require increased data from the accelerometers that were not utilized in the physical experiment. This additional sensor would need to be located somewhere on the midsection of the structure. The new accelerometer would provide insight into the eigenmode's actual shape, whereas the accelerometer located at the tail only tells us the eigenmode's end state. Each node in the FEM analysis could represent a physical accelerometer. The FEM analysis contains more nodes or accelerometers, which would allow for the understanding of the mid span's eigenmodes. In the FEM analysis, more nodes are used, which allows for greater data, but the experiment did not include additional accelerometer data to understand the mid spans movement. Thus, the two methods cannot be compared, and mode shapes cannot be determined from the experimental data provided.

II. FEM Results: Resonant Frequencies

Two different models were produced in the FEM analysis. The first was a two element model where the model was broken down into two sections between the shaker and the tail. Matrices were given for the mass and stiffness. The reduced system was an 4X4 matrix. Frequencies were obtained by first finding the eigenvalues from the following equation:

$$K_2 U = \omega^2 M_2 U \tag{1}$$

Then, the following equation yielded the frequencies:

$$f_i = \frac{\omega_i}{2\pi} \tag{2}$$

The three smallest frequencies were: 12.02, 51.02, and 202.5 Hz.

Compared to the experimental results some of the frequencies closely resembled those from experimental. The smallest frequency was 12.02 and this was close to the frequency located in the tail of the system (11.14 Hz). The other two frequencies were a lot higher than the experimental frequencies. The frequency of 51.02 Hz is near the resonant frequency value in the nose of the system (36.92 Hz).

The second model was a four element FEM model where the span between the shaker and the tail was broken down into four beams. The same process was followed to find the eigenvalues and the frequencies as in the two element model with the reduced matrix being reduced to an 8X8 matrix instead of the 4x4 matrix that the two element model was reduced to. From the analysis of this system, the three smallest frequencies were found to be 12.02 Hz, 51.05 Hz, 203.45 Hz. These frequencies were very similar to the frequencies of the two element model.

There are a few potential sources of error that highlight why the frequencies are different than the experimental results. First, both FEM models are significantly simplified, especially the two element model. The frequencies don't factor in the wings or the nose and it assumes the tail to have a point mass moment of inertia. It also only describes the vertical tail motion. The wings and nose could greatly affect how the whole system vibrates as they could act as a damper on the other parts of the system. Compared to the experimental results tail frequency, the two element model and the four element model were off by approx. 7.6%, at the first peak in the experimental data. These percentages were calculated using the following equation:

$$\frac{F_{FEM} - F_{exp}}{\frac{F_{FEM}+F_{exp}}{2}} * 100 = \%Error \tag{3}$$

This would suggest that the FEM model is a relatively good model but is clearly not extremely accurate due to assumptions and simplifications made to create the model.

Model	Mode 1	Mode 2	Mode 3
2-Element	12.03	51.03	202.51
4-Element	12.02	51.05	203.45

Table 2: 3 lowest resonant frequencies (**Hz**).

III. FEM Results: Mode Shapes

After obtaining the eigenvalues, the eigenvectors were plotted to yield the mode shapes. The following figure shows the mode shapes for the two and four element models at the aforementioned frequency ranges outline in the prior section. Note that the Mode Shape #3 is out of the frequency sweep range and will not be analyzed in depth and is mainly shown for completeness.

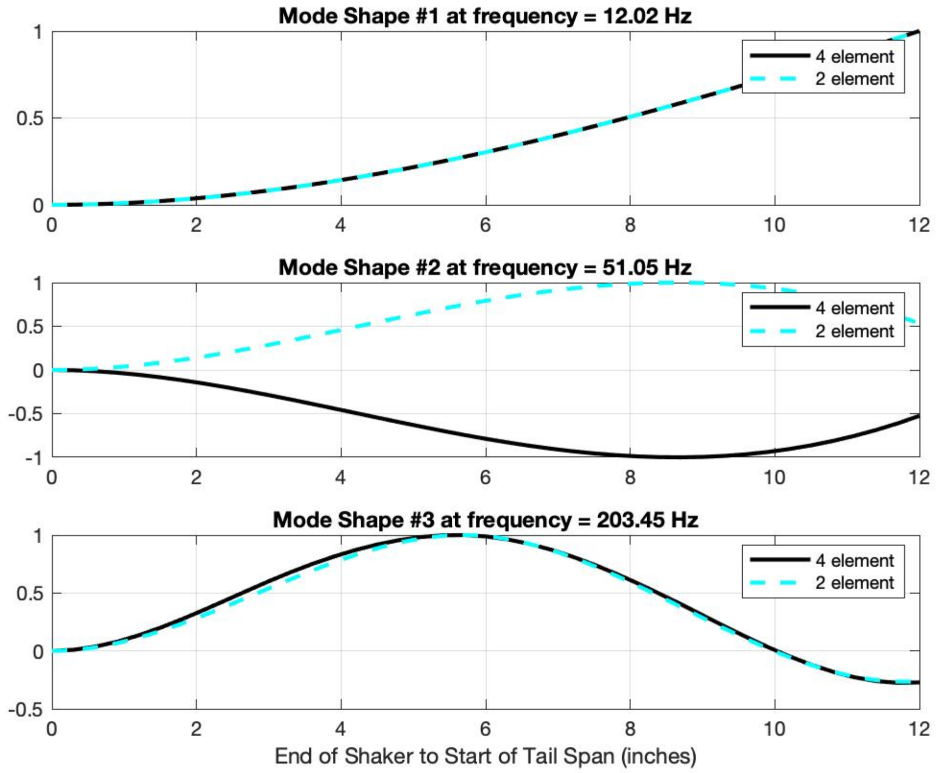

Figure 3: Mode shapes for the two and four element models (averaged).

Above we see that the modes between the two element model and the four element model behaviors render displacements across the section length. In this case, the section was from immediately behind the shaker to the tail, which was determined to be 12 inches. Plotting the modal behaviors allowed for the analysis of displacement at any given position within this section length. As it can be immediately noticed, all but the mode #2 plots match up. This was determined to likely be of no physical consequence, given that the magnitude is equal but deflecting in the opposite direction.

When looking at the magnitude of the displacements in mode #2, the maximum displacement fell at approximately 9 inches. The possible reason for the mirrored appearance may be due to the many intermediate step calculations that take place in the matrix math. Given the size of each of the matrices and the number of elements within them, this possibly caused the phase shift of approximately 90 degrees to occur. This is represented by a maximum and minimum value to occur at the same position in this sections length at the frequency range of 51.05 Hz.

The maximum displacements in the plots above occur at:

Model	Mode 1	Mode 2	Mode 3
2-Element Model	12.0	8.70	5.70
4-Element Model	12.0	8.70	5.70

Table 3: Location at max displacement (inches length).

Here, it can be seen that the position at which max deflection occurs is equal for both the two and four element models. This operation was performed within Matlab by finding the index at which the maximum displacement occurred and then relating it back to the length array. The significance of this proves that both the two element and four element models are viable methods to analyze the behavior of this structure. It should be noted that, though they are both equally useful, the amount of additional computations within the four element model make it a less likely candidate to further determine the behavior of this particular frame structure. This is strictly due to the comprehensive mathematics involved, but this will come at the cost of accuracy. Thus, in cases where the sensitivity analysis must be within a specified range, larger element models may be required so that more accurate results are produced, given models that closer resemble the real-world structure.

IV. References

[1] *ASEN 3112 Structures Lab 3 - (Fall 20), University of Colorado at Boulder, Boulder, CO, 2020.*
[2] *An Introduction to Error Analysis: The Studies of Uncertainties in Physical Measurements (2nd Edition)*, John R Taylor, University Science Books, 1997, Pages 102-103

V. Appendix A: Team Member Participation

The individual score of each team member is calculated by using a contribution factor varying from 0 to 100 and plugged into the following equation:

$$IndividualScore = GroupScorex\frac{100 + ContributionFactor}{200} \tag{4}$$

Every member of the group contributed to the writing of the report. Every member of the group contributed 100% effort and no one deserves a negative peer ranking. We all attended multiple out of class meetings and worked as a team.

Name	**Contribution Factor**	**Individual Score**	**Questions Worked On**
Bill Chabot	100	Group Score x 1	2,3
Andrew Thompson	100	Group Score x 1	1
Cole MacPherson	100	Group Score x 1	1,3
Lily Allen	100	Group Score x 1	2,3
Selmo Almeida	100	Group Score x 1	2,3
Nicolas Mayhan	100	Group Score x 1	1

VI. Appendix B: Matlab Code

A. Question 1 Code

```
%% ASEN 3112 Structures Lab #3 - main.m
%
%    Author: Andrew Thompson
%    Created: 11/22/20 Edited: 11/22/20

clc; clear all; close all;

%% Import & Clean Data
rawData = importdata("Lab3Data.txt");
data = rawData.data;
time = data(:, 1) - data(1, 1);
accCh0  = data(:,2);
accCh1  = data(:,3);
accCh2  = data(:,4);
accCh3  = data(:,5);
dispCh0 = data(:,6);
dispCh1 = data(:,7);
dispCh2 = data(:,8);
dispCh3 = data(:,9);
vibro   = data(:,10);

%% Excitation Frequency
% Sampling frequency
Fs = 2500;
% Sampling Period
T = 1/Fs;
% Sample length
L = length(time);

idx3 = 1;
% For every 1000 data points
for index = 1:1000:length(time)-1000
    idx1 = index;
    idx2 = index + 1000;
    % Compute the average frequency
    [f(idx3),t(idx3)] = computeFrequency(time(idx1:idx2),accCh0(idx1:idx2));
    idx3 = idx3 + 1;
end
% Linear portion
f = f(35:775);
t = t(35:775);
% Linear fit
[coefficients, error] = polyfit(t,f,1);
[frequency, uncertainty] = polyval(coefficients,time,error);
% Frequency for fourier transform
f = Fs.*(0:L/2)/L;

%% FFT Tail
accCh1fft = fft(accCh1);
P2ch1 = abs(accCh1fft/L);
P1ch1 = P2ch1(1:L/2+1);
P1ch1(2:end-1) = 2*P1ch1(2:end-1);
P1ch1 = P1ch1/max(P1ch1);

%% FFT Wing
accCh2fft = fft(accCh2);
```

```
P2ch2 = abs(accCh2fft/L);
P1ch2 = P2ch2(1:L/2+1);
P1ch2(2:end-1) = 2*P1ch2(2:end-1);
P1ch2 = P1ch2/max(P1ch2);

%% FFT Nose
accCh3fft = fft(accCh3);
P2ch3 = abs(accCh3fft/L);
P1ch3 = P2ch3(1:L/2+1);
P1ch3(2:end-1) = 2*P1ch3(2:end-1);
P1ch3 = P1ch3/max(P1ch3);

%% FFT Vibrometer
accVibrofft = fft(vibro);
P2vibro = abs(accVibrofft/L);
P1vibro = P2vibro(1:L/2+1);
P1vibro(2:end-1) = 2*P1vibro(2:end-1);
P1vibro = P1vibro/max(P1vibro);

%% Excitation Frequency vs Time Plot
figure; hold on; grid minor;

plot(t,f);
plot(time, frequency, 'LineWidth',2)
plot(time, frequency+uncertainty,'k--')
plot(time, frequency-uncertainty,'k--')

xlabel("Time [s]");
ylabel("Frequency [Hz]")
legend("Frequency From Data", "Linear Regression")
title ("Excitation Frequency vs Time")

%% FFT Amplitude vs Excitation Frequency Plot
figure; hold on; grid minor;

plot(f, P1ch1)
plot(f, P1ch2)
plot(f, P1ch3)
plot(f(636:end), P1vibro(636:end))

xlim([0 50]);
xlabel("Excitation Frequency [Hz]")
ylabel("FFT Amplitude")
legend("Tail", "Nose", "Wing", "Vibrometer")
title("Resonant Frequencies of Experimental Data")

%% ASEN 3112 Structures Lab 3 - computeFrequency.m
% Determines the frequency of a given data set. Also saves the indicies of
% the data set where frequency was computed, and the time values at those
% indices.
%
%   Author: Andrew Thompson
%   Created: 11/14/20 Edited: 11/14/20
%
%   Parameters:     time [N x 1] <double>
%                   data [N x 1] <double>
%   Returns:        frequency    <double>
%                   tOut         <double>

function [frequency, tOut] = computeFrequency(time, data)
```

```
    %%
    % Initialize full cycle counter
    wCount = 1;
    % Initialize half cycle counter
    crossCount = 0;
    % Initialize reference data point
    refPt = 0;
    % Initialize reference time
    time1 = time(1);
    % For all data points
    for index = 2:length(data)
        % Get data point 1
        pt1 = data(index-1);
        % Get data point 2
        pt2 = data(index);
        % If the points are on seperate sides of the reference
        if pt1 < refPt && pt2 > refPt || pt1 > refPt && pt2 < refPt
            % Increment the reference cross counter
            crossCount = crossCount + 1;
            % If it is the second time crossed
            if mod(crossCount, 2) == 0
                % Calculate the frequency
                frequency(wCount) = 1/(time(index) - time1);  %#ok<*AGROW>
                % Document current time
                tOut(wCount) = time1;
                % Increment the frequency count
                wCount = wCount + 1;
                % Set a new time
                time1 = time(index);
            end
        end
    end
    % Mean time of period
    tOut = mean(tOut);
    % Mean frequency of period
    frequency = mean(frequency);
end
```

B. Questions 2 & 3 Code

```
%% ASEN 3112 LAB 3 4-Element Model

% Housekeeping
clear;
close all;
clc;

%% Constants

% Longitudinal Dimensions
fuse_span = 22; % Fuselage Span [in]
wing_span = 18; % Wing Span [in]
L = 12; % End of Shaker to Start of Tail Span [in]
L_E = 4.5; % Elevator Span [in]
L_R = 5; % Rudder Span [in]

% Beam Dimansions
w = 1; % Width (every member) [in]
h = 1/8; % Thickness: fuselage & wing [in]
h_E = 1/4; % Thickness: elevator [in]
h_R = 1/25; % Thickness: rudder [in]

% Material Properties (Al 6063-T7 stock)
E = 10.175*(10^6); % Elastic modulus [psi]
rho = 2.505*(10^(-4)); % Density [lb-sec^2/in^4]

%% Calculated Constants

A = w*h; % Cross-sectional area of the fuselage [in^2]
A_E = w*h_E; % Cross-sectional area of the elevator [in^2]
A_G = w*h_R; % Cross-sectional area of the rudder [in^2]

c_M4 = (rho*A*L)/(806400); % M matrix coefficient
I_ZZ = (w*(h^3))/(12); % Moment of Inertia
c_K4 = (8*E*I_ZZ)/(L^3); % K matrix coefficient

M_T = 1.131*rho; % Mass of tail assembly
S_T = 0.5655*rho; % First mass-moment of tail assembly wrt B'
I_T = 23.124*rho; % Second mass-moment of tail assembly wrt B'

M_4 = (c_M4)*[77088 2916*L 23712 -1284*L 0 0 0 0 0 0;...
    2916*L 172*(L^2) 1284*L -73*(L^2) 0 0 0 0 0 0;...
    23712 1284*L 154176 0 23712 -1284*L 0 0 0 0;...
    -1284*L -73*(L^2) 0 344*(L^2) 1284*L -73*(L^2) 0 0 0 0;...
    0 0 23712 1284*L 154176 0 23712 -1284*L 0 0;...
    0 0 -1284*L -73*(L^2) 0 344*(L^2) 1284*L -73*(L^2) 0 0;...
    0 0 0 0 23712 1284*L 154176 0 23712 -1284*L;...
    0 0 0 0 -1284*L -73*(L^2) 0 344*(L^2) 1284*L -73*(L^2);...
    0 0 0 0 0 0 27312 1284*L 77088 -2916*L;...
    0 0 0 0 0 0 -1284*L -73*(L^2) -2916*L 172*(L^2)] + ...
    [0 0 0 0 0 0 0 0 0 0;...
    0 0 0 0 0 0 0 0 0 0;...
    0 0 0 0 0 0 0 0 0 0;...
    0 0 0 0 0 0 0 0 0 0;...
    0 0 0 0 0 0 0 0 0 0;...
    0 0 0 0 0 0 0 0 0 0;...
    0 0 0 0 0 0 0 0 0 0;...
```

```
    0 0 0 0 0 0 0 0 0  0;...
    0 0 0 0 0 0 0 0 M_T S_T;...
    0 0 0 0 0 0 0 0 S_T  I_T]; % M matrix

K_4 = (c_K4)*[96 12*L -96 12*L 0 0 0 0 0  0;...
    12*L 2*(L^2) -12*L L^2 0 0 0 0 0  0;...
    -96 -12*L 192 0 -96 12*L 0 0 0  0;...
    12*L L^2 0 4*(L^2) -12*L L^2 0 0 0  0;...
    0 0 -96 -12*L 192 0 -96 12*L 0  0;...
    0 0 12*L L^2 0 4*(L^2) -12*L L^2 0  0;...
    0 0 0 0 -96 -12*L 192 0 -96 12*L;...
    0 0 0 0 12*L L^2 0 4*(L^2) -12*L L^2;...
    0 0 0 0 0 0 -96 -12*L 96 -12*L;...
    0 0 0 0 0 0 12*L L^2 -12*L 2*(L^2)]; % K matrix

M_hat4 = M_4(3:10,3:10); % Reduced M matrix
K_hat4 = K_4(3:10,3:10); % Reduced K matrix

[U,omega_sq] = eig(K_hat4,M_hat4); % Eigen calculation

omega = sqrt(omega_sq); % Finding omega

freq = (omega)/(2*pi); % Finding frequency

% Eigenvector Normalization -- Unit Largest Entry:

new_U = [[0;0;U(:,8)]/max(abs(U(:,8))),[0;0;U(:,7)]/max(abs(U(:,7))),...
    [0;0;U(:,6)]/max(abs(U(:,6)))];
% ^^^^^^^^^^^^^^
% Needed to reverse the order of new_U to produce plots in the order of
% ascending frequencies from top (smallest) to bottom (largest)

%modes = U(:,6:8); % Modes

%% Plot Eigenvectors
for i = 1:3
    if i == 1
        col = 'k';
    elseif i == 2
        col = 'k';
    else
        col = 'k';
    end

    [final4_v(:,i),final4_x(:,i)] = ploteigenvector(L,new_U(:,i),4,10,50,col,i);
end
%% Determine where max displacmement occurs:
[~,idx4] = max(abs(final4_v));
maxDisp4 = zeros(1,length(idx4));
for l = 1:length(idx4)
    maxDisp4(l) = final4_x(idx4(l),l);
end

%% Two-element model

%%Variables (English units)
L = 12; %in - Length of back half of plane
L_E = 4.5; %in - Elevator span
L_R = 5; %in - Rudder span
```

```
w = 1; %in - Width
h = 1/8; %in - Thickness of fuselage
h_E = 1/4; %in - Thickness of elevator
h_R = 0.04; %in - Thickness of rudder
E = 10175000; %psi - Elastic modulus
rho = 0.0002505; %lb-sec^2/in^4 - density
M_T = 1.131*rho; %[rho]*in^3 - Mass of tail
S_T = 0.5655*rho; %[rho]*in^4 - First mass-moment of tail
I_T = 23.124*rho; %[rho]*in^5 - Second mass-moment of tail
A = w*h;
I_zz = w*h^3/12;
cm2 = rho*A*L/100800;
ck2 = 4*E*I_zz/L^3;
cm4 = rho*A*L/806400;
ck4 = 8*E*I_zz/L^3;


M2 = cm2*[38544,0,5928,-642*L;0,344*L^2,642*L,-73*L^2;...
    5928,642*L,19272,-1458*L;-642*L,-73*L^2,-1458*L,172*L^2]+ ...
    [0,0,0,0;0,0,0,0;0,0,M_T,S_T;0,0,S_T,I_T];
K2 = ck2*[48,0,-24,6*L;0,4*L^2,-6*L,L^2;-24,...
    -6*L,24,-6*L;6*L,L^2,-6*L,2*L^2];

[eigvec_raw,omegasq,eigvec2_raw2] = eig(K2,M2);
omega2 = sqrt(diag(omegasq));
freq2 = omega2./(2*pi);

freq2 = freq2(1:3);

%validate
% syms w2
% omega22 = double(solve(det(K2-w2^2*M2) == 0));

%Change the eigenvectors (add the two zeros)
eigvec2 = [zeros(2,3);eigvec_raw(:,1:3)];

for i = 1:3
    if i == 1
        col = '--c';
    elseif i ==2
        col = '--c';
    else
        col = '--c';
    end

    [final2_v(:,i),final2_x(:,i)] = ploteigenvector(L,eigvec2(:,i),2,40,1,col,i);

    if i == 1
        figure(1)
        legend('4 element','2 element')
    end
    if i == 2
        figure(1)
        legend('4 element','2 element')
    end
    if i == 3
        figure(1)
        legend('4 element','2 element')
    end
```

```
end
%% Determine where max displacmement occurs:
[~,idx2] = max(abs(final2_v));
maxDisp2 = zeros(1,length(idx2));
for l = 1:length(idx2)
    maxDisp2(l) = final2_x(idx2(l),l);
end
```

C. Plotting Function for Q2 & Q3

```
function [v,x] = ploteigenvector(L,ev,ne,nsub,scale,col,value)
    nv = ne*nsub+1;
    Le = L/ne;
    dx = Le/nsub;
    x = zeros(nv,1);
    v = x;
    k=1;
    for i = 1:ne
        xi = Le*(i-1);
        vi = ev(2*i-1);
        qi = ev(2*i);
        vj = ev(2*i+1);
        qj = ev(2*i+2);
        for n = 1:nsub
            xk = xi+dx*n;
            zeta = (2*n-nsub)/nsub;
            vk = scale*(.125*(4*(vi+vj)+2*(vi-vj)*((zeta^2)-3)*zeta+...
                Le*((zeta^2)-1)*(qj-qi+(qi+qj)*zeta)));
            k = k+1;
            x(k) = xk;
            v(k) = vk;
        end
    end
    v = v/max(abs(v)); % Normalizes data

    figure (1)
    if value == 1
        subplot(3,1,value)
        plot(x,v,col,'Linewidth',2)
        hold on
        title('Mode Shape #1 at frequency = 12.02 Hz')
        grid on
    elseif value == 2
        subplot(3,1,value)
        plot(x,v,col,'Linewidth',2)
        hold on
        title('Mode Shape #2 at frequency = 51.05 Hz')
        grid on
    else
        subplot(3,1,value)
        plot(x,v,col,'Linewidth',2)
        hold on
        title('Mode Shape #3 at frequency = 203.45 Hz')
        xlabel('End of Shaker to Start of Tail Span (inches)')
        grid on
    end
    saveas(gcf,'ModalBehavior.jpg','jpg')
end
```