UNIVERSITY OF COLORADO - BOULDER

ASEN 3112 : LAB 4

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Beam Buckling

Author:

Amanda NELSON[a]

Author:

Cole MACPHERSON[b]

Author:

Max MORGAN[c]

Author:

Thyme ZUSCHLAG[d]

Author:

Trey TAYLOR[e]

Author:

Zach LESAN[f]

Professor:

Professor MAUTE

Professor DOOSTAN

[a]109050655
[b]108521329
[c]106522203
[d]109221211
[e]109191082
[f]108625270

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College of Engineering & Applied Science

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I. Results

A. Question 1

In order to determine the experimental buckling loads for the beams, the provided data for the particular beam was parsed. All of the data points where there was buckling and zero lateral deflection were collected. Then the first value before the beam begins to deflect is found. Using this method, the experimental buckling loads for the rectangular and hollow square beams are 44.6705 and 202.5787 lbs respectively.

In the case of the Euler buckling of beam, the theoretical buckling load is calculated using equation (1) shown below. E is the Young's modulus, I is the moment of inertia of the beam, and L is its length.

$$P_{cr} = \frac{\pi^2 EI}{L^2} \tag{1}$$

Since both the Young's modulus and the length are known for each of the beams, the moment of inertia for each beam must be found. For the rectangular beam, the moment of inertia is found:

$$I_{rec} = \frac{1}{12}h^3 w = \frac{1}{12}(.125)^3(1) = .00016276 kgm^2 \tag{2}$$

Plugging this into equation (1), the corresponding theoretical buckling load for the rectangular beam is 129.2108 lbs.

For the hollow square beam, the moment of inertia is calculated below:

$$I_{sq} = \frac{1}{12}h^3 w - \frac{1}{12}(w-2t)(h-2t)^3 = \frac{1}{12}(.25)^3(.25) - \frac{1}{12}(.25-2*0.0625)(.25-2*0.0625)^3 = .00030518 kgm^2 \tag{3}$$

Plugging this value into equation (1), the corresponding theoretical buckling load for the rectangular beam is 265.5526 lbs.

For simplicity, the loads are shown in the table below.

	Experimental Buckling Load (lb)	Theoretical Buckling Load (lb)	Percent Error
Rectangular Beam	44.6705	129.2108	65.426
Hollow Square Beam	202.5787	265.5526	23.714

Table 1 Predicted and Experimentally Measured Buckling Loads

As seen in Table 1, both of the experimental critical load values were lower than their theoretical counterparts. The rectangular beam in particular had a significant discrepancy between experimental and theoretical critical load. Since buckling is defined as the point when the beam is no longer straight, the critical load is the maximum load while deflection is still zero. The theoretical model does not account for any initial imperfections in the beams which may cause a much lower load to cause buckling. Since the rectangular beam buckled at a much lower load relative to the predicted load, we can assume that its imperfections were more significant than those of the hollow square beam. If the beam was already slightly bent, the onset of buckling would begin at a much smaller load.

B. Question 2

In order to obtain the values of theoretical load associated with the lateral deflection of the beams, Equation 4 was utilized.

$$P = P_{cr}(1 + \frac{\pi^2}{8L^2} * \delta^2) \tag{4}$$

Using the previously calculated values of critical buckling load as well as the length of each of the beams, the values of applied load after buckling could be determined as a function of lateral deflection. From the experimental data it was observed that the test ran from zero to two inches of lateral deflection in increments of $\frac{1}{16}$ an inch. Creating a vector of lateral deflection values, spaced in the same manner as the experimental data, and substituting it along with other known values results in Figure 1 for the hollow square beam and Figure 2 for the rectangular beam. On these figures, for the theoretical values, a vertical line can also be seen at zero lateral deflection indicating the predicted load value at which post-buckling initiates. To find the value of lateral deflection where plastic post-buckling begins, the relationship between strain and curvature was used.

$$\epsilon = \kappa(x)y = v''(x)y \quad \text{where} \quad v(x) = \delta \sin \frac{\pi x}{L} \tag{5}$$

Rearranging and solving for δ yields the following.

$$\delta = \frac{\epsilon L^2}{\pi^2 y \sin \frac{\pi x}{L}} \tag{6}$$

The displacement where plastic deformation begins is associated with the yield strain. Given yield stress and Young's modulus, we can solve for this value using Hooke's law.

$$\epsilon_y = \frac{\sigma}{E} = \frac{35,000}{10,000,000} = 0.0035 \tag{7}$$

The maximum strain is associated with the point on the sample farthest away from the neutral axis, y_{max} which is defined as half the thickness of the beam's cross-section. Additionally, the x-value along the length of the beam that experience maximum deflecting is the midspan, $\frac{L}{2}$. Plugging in these values for each sample, we find the deflection of the square hollow beam to be 0.3218 in, and the deflection of the rectangular beam is 0.0882 in. These values were plotted on the Figure 1 and 2 as a vertical line to indicate the initiation of plastic post-buckling.

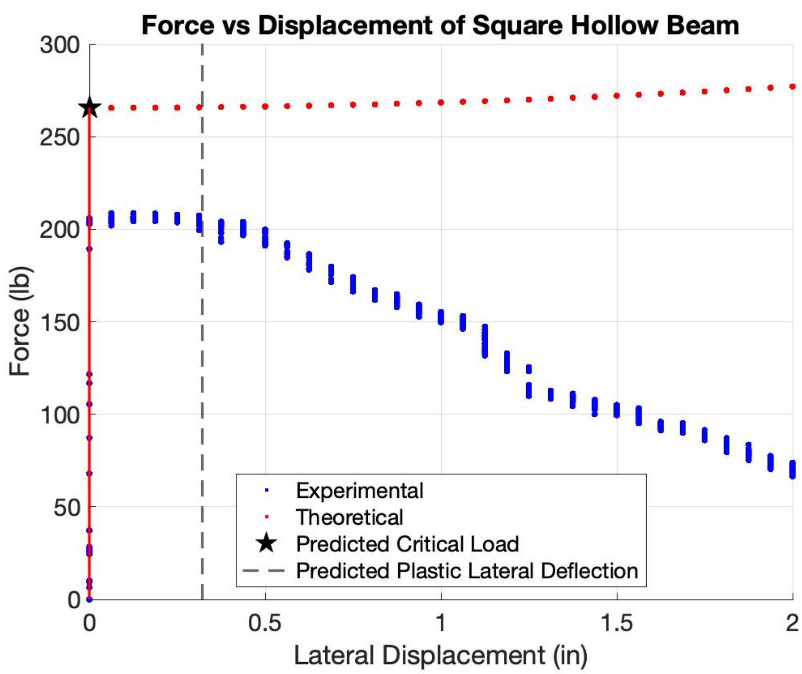

Fig. 1 Applied Load vs. Lateral Deflection - Hollow Square Beam

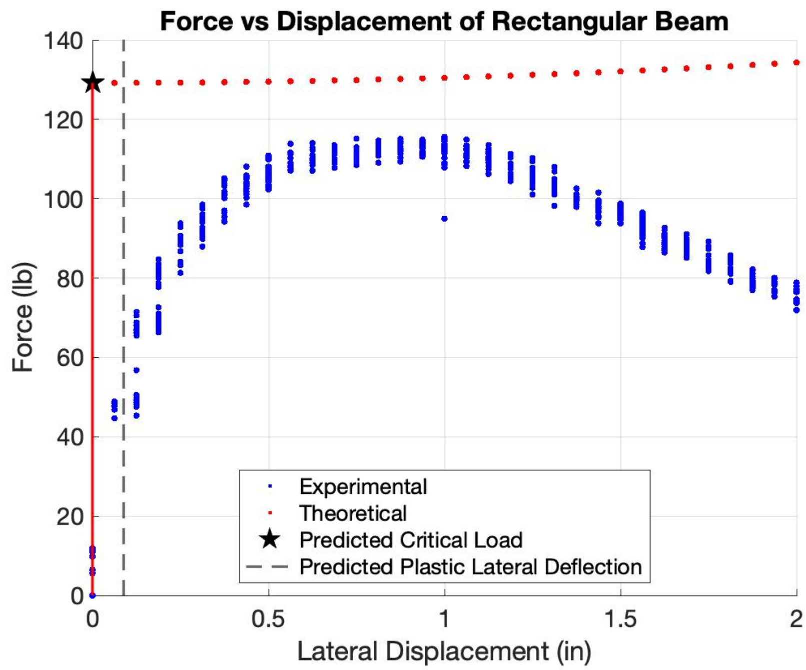

Fig. 2 Applied Load vs. Lateral Deflection - Rectangular Beam

From both the figures there are a couple things that can be seen. First is that the shape of the theoretical plots for both beams followed expectations as after the initial buckling, the load continued to rise at a slow rate as the lateral deflection increased. This followed the expected theoretical motion that was seen by the schematic in Fig.3 of the lab document. The theoretical results though were much different than those of the experimental. As already discussed in Question 1, the critical values at which buckling began were very different for both the rectangular and hollow square beams, with both theoretical values being greater than the actual values. In addition to this, for the hollow square beam, the experimental load drops instead of continuing to rise as in the theoretical case. The rectangular beam exhibits different behavior as its load initially rises at a very fast rate but after about one inch in lateral deflection the load begins to decrease as lateral deflection increases. This discrepancy is most likely due to the simplifications in the model. The values of load are related to the square of lateral deflection, and thus will continue to grow as lateral deflection increases ignoring real world effects on the beams, especially those present during plastic buckling. Another important observation concerning the experimental plots is that although they do not follow the trend of the theoretical load-displacement relationship, the predicted plastic post-buckling lateral displacement appears to accurately model the beginning of plastic buckling. For the square hollow beam, until it reaches the value of yielding, the load stays essentially constant for growing lateral deflection but after reaching the yielding deflection and entering the plastic buckling region the load drops as deflection increases. For the rectangular beam it follows the trend of a classic stress vs strain graph where there is about a linear load-displacement relationship until yielding. After yielding the relationship cannot easily be approximated by a simple curve but it can be seen that like the square beam the response eventually trends toward a decrease in load as displacement increases. These changes after plastic post-buckling are most definitely caused by the permanent deformations that are occurring to the beam.

C. Question 3

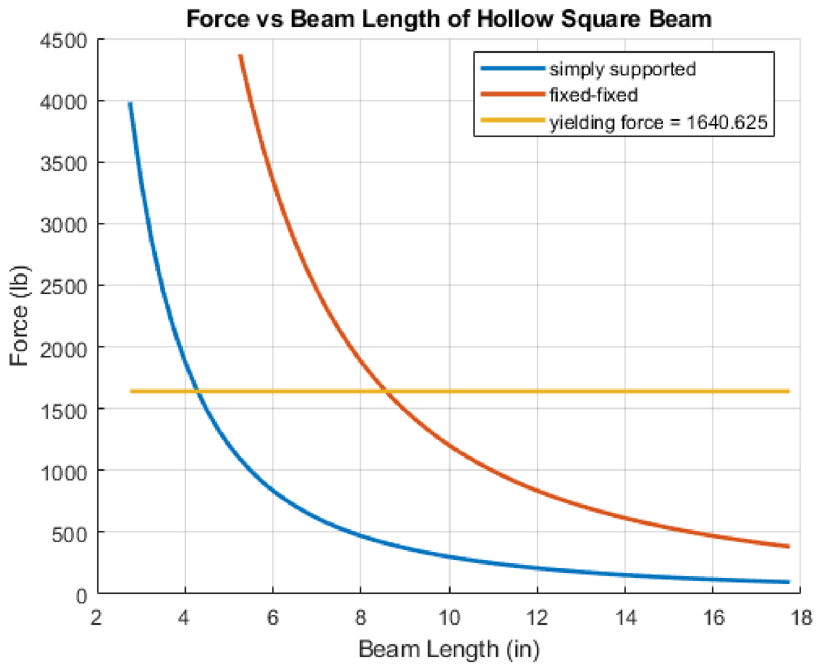

Fig. 3 Applied Load vs. Beam Length - Square Beam

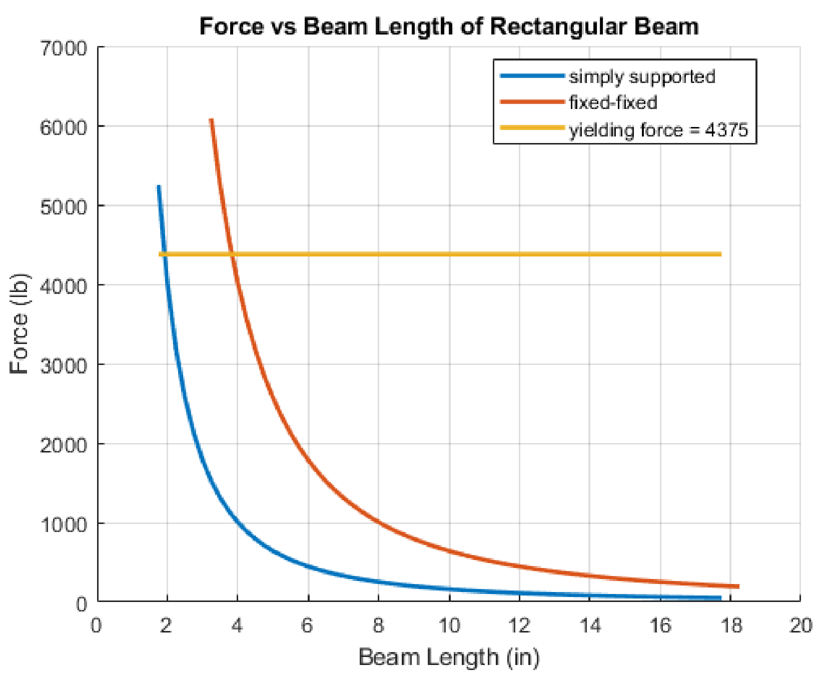

Fig. 4 Applied Load vs. Beam Length - Rectangular Beam

Figures 3 and 4 depict how the change in length of the beams affects the critical force at which buckling occurs at for both a simple support and a fixed-fixed support. Further, the force at which the material yields at under compression loads is plotted to give an idea as to how the beam will fail at certain lengths and loads. The yielding force of the hollow square beam and the rectangular beam was found to be 1640.625 lbs and 4375 lbs, respectively. This means that any applied force above those values will result in yielding. While that statement is true, the beam could buckle as a result of a certain applied load that has a value under the yielding force. For the hollow square beam, Fig. 3, it is seen that a simply supported beam and a fixed-fixed beam would buckle at lengths greater than about 4.25 inches and 8.5 inches, respectively, before they would yield. For lengths less then 4.25 inches and 8.5 inches, the hollow square beam would yield before it buckles. When looking at the rectangular beam, Fig. 4, it is seen that buckling would occur at much lower lengths. For the simply supported beam, buckling will occur at lengths greater than about 1.9 inches, and for the fixed-fixed beam, buckling would occur at lengths greater than about 3.85 inches. Thus at any length less than 1.9 inches and 3.85 inches, the simply supported beam and the fixed-fixed beam, respectively, would yield. The differences for the two support systems can be marked down to the effective area for each type of beam being different. The fixed support systems have an effective length of half of the total length and the effective length of the simple support system is the length of the beam. When these ideas are used to find the critical load from equation 1, it can be seen how the effective length would effect the critical load. Since the length value is in the denominator of the equation, this means that as the length gets smaller, the critical load value increases. Since the fixed support system has the smaller effective length, this means that for beams of the same lengths, and the same design, the fixed support system will always have higher critical load values when compared to a simple support system. Overall the results found that the rectangular beam had a higher yielding force and lower buckling forces for the same length beam. Thus, the opposite is true for the hollow square beam, it has a lower yielding force but a higher buckling force for the same length beam. When viewing the support type, it is clear that the fixed support will provide higher allowable applied forces. As far as applications for these beam go, it is best to use a rectangular beam with fixed supports in circumstance that require shorter beams of about 6 inches or less. For applications where the beam length will need to be longer than about 6 inches, it would be best to use a hollow square beam that has fixed supports. The length of 6 inches was determined to be the transition point as that is when the rectangular beam can no longer support a greater applied load than a hollow square beam of the same length can.

Participation Table

	Plan	Model	Experiment	Results	Report	Code	CF	ACK
Amanda Nelson	1	1	N/A	2	1	1	100	X
Cole MacPherson	1	1	N/A	1	1	1	100	X
Max Morgan	1	1	N/A	1	1	0	100	X
Thyme Zuschlag	2	1	N/A	1	1	1	100	X
Trey Taylor	1	2	N/A	1	2	1	100	X
Zach Lesan	1	1	N/A	1	1	1	100	X

2 = Lead, 1 = Participate, 0 = Not involved, for each element.

Appendix A: MATLAB Code

Code for questions 1 and 2:

```
clc
clear
close all

%% Q1 & 2

%Aluminum properties
E = 10000000;
sigyield = 35000;

%Properties of square beam
ts = 0.0625;
Ls = 10.25+0.4;
hs = 0.25;
ws = 0.25;

%Properties of rectangular beam
Lr = 10.75+0.4;
wr = 1;
```

```
hr = 0.125;

%Calculate moment of inertia
Is = 1/12*hs^3*ws - 1/12*(ws-2*ts)*(hs-2*ts)^3;
Ir = 1/12*hr^3*wr;

%Calculate buckling load
Pcrs = (pi^2*E*Is)/(Ls^2);
Pcrr = (pi^2*E*Ir)/(Lr^2);

%Load in new data
Thin = cell2mat(struct2cell(load("ThinBar_FA2020.mat")));
Square = cell2mat(struct2cell(load("Squarerod_FA2020.mat")));

Thin(:,2) = Thin(:,2).*1000/2.37;
Square(:,2) = Square(:,2).*1000/2.37;

%Find where the lateral deflection begins
Pcrre = Thin(max(find(Thin(:,3) == 0)),2);
Pcrse = Square(max(find(Square(:,3) == 0)),2);

% Solve for predicted lateral deflection to begin plastic region
deltar = (sigyield*Lr^2)/(pi^2*1/2*wr*E);
deltas = (sigyield*Ls^2)/(pi^2*1/2*ws*E);

%Set up predicted lateral deflection
delta = linspace(0,2,33);
Ps = Pcrs.*(1+pi.^2./(8.*Ls^2).*delta.^2);
Pr = Pcrr.*(1+pi.^2./(8.*Lr^2).*delta.^2);

%Plot the Experimental data
figure(1)
hold on
grid on
scatter(Thin(:,3),Thin(:,2),100,'b.');
```

```
scatter(delta,Pr,100,'r.')
plot(0,Pcrr,'pk','MarkerSize',15,'MarkerFaceColor','k')
xline(deltar,'--','LineWidth',1.6)
plot([0 0], [0 Pcrr],'r','LineWidth',1.6)
title('Force vs Displacement of Rectangular Beam')
set(gca,'fontsize', 15);
xlabel('Lateral Displacement (in)')
ylabel('Force (lb)')
legend('Experimental','Theoretical','Predicted Critical Load','Predicted Plastic
    Lateral Deflection','Location','best')
hold off

figure(2)
hold on
grid on
scatter(Square(:,3),Square(:,2),100,'b.');
scatter(delta,Ps,100,'r.')
plot(0,Pcrs,'pk','MarkerSize',15,'MarkerFaceColor','k')
xline(deltas,'--','LineWidth',1.6)
plot([0 0], [0 Pcrs],'r','LineWidth',1.6)
title('Force vs Displacement of Square Hollow Beam')
set(gca,'fontsize', 15);
xlabel('Lateral Displacement (in)')
ylabel('Force (lb)')
legend('Experimental','Theoretical','Predicted Critical Load','Predicted Plastic
    Lateral Deflection','Location','best')
hold off
```

Code for question 3:

```
clc
clear
close all

%% Q3

```

```
%Aluminum properties
E = 10000000;
sigyield = 35000;

%Define new lengths
Ldiff = 7.5;
Ldiff2 = 8.5;
Ldiff3 = 5;

%Properties of square beam
ts = 0.0625;
hs = 0.25;
ws = 0.25;
Ls = 10.25-Ldiff:0.25:10.25+Ldiff;
Ls2 = 10.25-Ldiff3:0.25:10.25+Ldiff;

%Properties of rectangular beam
wr = 1;
hr = 0.125;
Lr = 10.75-Ldiff:0.25:10.75+Ldiff;
Lr2 = 10.25-Ldiff2:0.25:10.25+Ldiff;

%Calculate moment of inertia
Is = 1/12*hs^3*ws - 1/12*(ws-2*ts)*(hs-2*ts)^3;
Ir = 1/12*hr^3*wr;

%Buckling load
Pcrs2_ss = (pi^2*E*Is)./(Ls.^2);
Pcrr2_ss = (pi^2*E*Ir)./(Lr2.^2);
Pcrs2_ff = (pi^2*E*Is)./((Ls2/2).^2);
Pcrr2_ff = (pi^2*E*Ir)./((Lr/2).^2);

%Yielding
As = 2*(hs*ts) + 2*((ws-(2*ts))*ts);
Ar = wr*hr;
```

```
sigmacrs = zeros(1,length(Ls))+sigyield*As;
sigmacrr = zeros(1,length(Lr2))+sigyield*Ar;

%Plot
figure(1)
hold on
grid on
plot(Ls,Pcrs2_ss,Ls2,Pcrs2_ff,Ls,sigmacrs,'linewidth',2)
title('Force vs Beam Length of Hollow Square Beam')
xlabel('Beam Length (in)')
ylabel('Force (lb)')
legend('simply supported','fixed-fixed',['yielding force = '
    num2str(sigmacrs(1))],'location','best')
hold off

figure(2)
hold on
grid on
plot(Lr2,Pcrr2_ss,Lr,Pcrr2_ff,Lr2,sigmacrr,'linewidth',2)
title('Force vs Beam Length of Rectangular Beam')
xlabel('Beam Length (in)')
ylabel('Force (lb)')
legend('simply supported','fixed-fixed',['yielding force = '
    num2str(sigmacrr(1))],'location','best')
hold off
```