ASEN 3113 Experimental Lab 1: Stirling Heat Engine

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A Stirling Engine is a powerful device that uses the thermodynamic properties that require the temperature of a gas to increase when the pressure is increased, and vice versa. They operate on a cycle, where when an initial temperature difference and net work are applied, the engine will continue to produce work until it is stopped, the temperature difference is altered, or the device experiences mechanical failure. For this lab, we analyzed a Gamma Stirling Engine. This particular engine has two pistons, a power piston, and a displacer piston. The displacer piston will move the air from the cold temperature area to the hot temperature area, however it does not create any seals so it wont change the volume inside the device. The power piston however, which is driven by the change in temperature, will expand and contract the volume inside, causing a change in pressure in the gas. Both of these pistons are connected to a flywheel to produce work out. For this lab, we preformed a thermodynamic analysis on this engine to see how it preformed with a 8°C, 9°C and 10°C temperature difference. From this analysis, we saw that for these temperatures, the engine produces a net work output of 1.21 W, 0.94 W and 0.82 W respectively, with efficiencies of 2.53%, 1.55%, and 1.23%.

I. Introduction

As mentioned in the abstract, Stirling Engines use temperate and pressure differences to produce a net amount of work. This begins with expansion, which will begin when most of the gas is on the side of the displacer piston with the heat source. The gas will heat up, causing the pressure of the gas to increase, which will push the power piston. the power piston will continue until its mechanical limit, and then theoretically stop there. At this time, isochoric heat removal will begin, since the chamber has more volume, decreasing the pressure, and thus the temperature.This will also move the displacer piston placing more of the gas in the cool area. When the gas has now cooled down, the piston will begin to contract and decrease its volume, which will also slightly increase the pressure and temperature. The piston will then pause there again, and since the displacer piston will have brought most of the gas back to the hot side again, it will heat up and continue back to the first part of the cycle.

Stirling Engines are incredibly powerful machines, due to their ability to produce continuous work using the same gas. Most engines in use today, such as the internal combustion engines which are used to power most vehicles, require intake and exhaust of gas. The Brayton and Otto engines were two of the first of these kind of engines. Both of these, unlike Stirling Engines, require some type of fuel in order to continue driving their work production.[2]

Unfortunately, Stirling Engines are not that practical in use for a few reasons. The first is that they require external heat differences as apposed to the required internal heat differences of internal combustion engines that are easier to implement. A Stirling Engine also had to be really big in order to produce any significant amount of power.[3] They are still relevant today however, largely for education purposes such as this lab. It is a good example of an engine which runs on the thermodynamic process' we have been learning about. The other big use for them is for powering submarines. They have the ability to extend the submersion period of the sub for a long time because it wont have to go up to recharge.[3] Besides those, while Stirling Engines have the capability to be used in most instances of energy transfer, there are often better options so it is not a common occurrence.

II. Experimental Procedure

The file named "Stirling Engine" should first be downloaded from the "Courses (Z:)\AES\Software\VIs\ASEN 3113\Lab 1 Stirling Engine\" directory. To ensure the correct sensor calibrations are being used, make sure the drop down box in VI labeled "Stirling Engine Number" matches the number of the engine being used.

Next, enter into the box labeled "Desired Temp Diff" a temperature differential between 7°C and 12°C . The resistive heater will work to maintain this differential. Start the VI by clicking the white arrow in the upper left corner and enter a logical location to save the data when prompted. Once the VI is running successfully, plug in the 48V power cable. A spike in the VI verifies that all is working. Wait until the the desired temperature differential has been reached which will be shown by the "Actual Engine Diff" box.

If the current does not begin oscillating in a square wave pattern at this point, there may be some error in the system. If this is the case, unplug the 48V power cable to thwart further damage.

Once you achieve the desired temperature differential, gently spin the fly wheel clockwise (when looking at the power piston) to begin the Stirling cycle. If rotation is not maintained, ask for help.

Check the ”Internal Change in Pressure” plot for a sine wave with amplitude 0.03 to 0.06 psi. If the pressure deviates from this, call an expert.

For the sake of our assumption that all heat from the heater is going into the cycle, wait until the temperatures ”Bottom of Bottom” and ”Top of Bottom” are within 1°C. Then let the engine run for at least an additional 2 minutes to achieve steady state which can be verified with a normal duty cycle shown on the current plot. If the current plot shows noise about zero instead of this duty cycle, make sure the 5V power supply is turned on.

Once all temperatures have converged, press ”Stream to File”. Allow about 5 cycles to occur before pressing the button again. This will save a .txt file with columns: time (sec), pressure (psi), Top of Top temp (°C), Top of Bottom temp (°C), Bottom of Bottom temp (°C), current to heater (Amps), and the optical switch data (logic 0 or 1).

Repeat this process, from setting the temperature differential to saving the text file, with two different temperature differentials to result in a total of three data files. When all data has been collected, stop the VI with the red ”STOP” button in the upper right corner.

III. Pre-Analysis

The procedure in the procedure section was performed three times using a desired temperature differentials of 8°, 9°, and 10°. Before beginning the analysis process the RPM and average temperature difference were obtained from the data. In addition an equation for the efficiency of an ideal Stirling Engine cycle was derived.

A. Temperature Difference

Three sensors were available to measure the temperature of the plates. The senors were located on the bottom of the top plate, the top of the bottom plate, and the bottom of the bottom plate. The recorded measurements of the top plate were averaged together to obtain one value for the top plate temperature. The same process was repeated for the bottom plate using only the measurements on the top of the plate, as the top of the plate is closest to the engine.

B. RPM Calculations

To calculate the RPM of the flywheel during testing, the data from testing was used. This data provided information from the optical sensor that was placed around the flywheel. There was a hole cut in the flywheel that when passed by the optical sensor would complete the sensor connection, via a laser, and record a one, instead of a zero which is recorded while the connection is not complete. Using this knowledge, the RPM can be calculated by recording the time it takes to go from each time the optical sensor outputs a one. Using this recorded time, it must be divided by 60 to change the time units into minutes and then divide 1 by the time to get the RPM of the flywheel. From this calculation process it was found that the average RPM for the eight, nine, and ten degree temperature differences are 81.16, 97.98, and 117.52, respectively.

C. Work Integration

In order to find the net work out of the system the integral of the PV diagram must be taken. This is because the area within the PV curve is the net work out of the system. using the trapz function in MATLAB this can be easily found by using the x and y coordinates of one cycle. To verify this, a circle with radius of 1 unit centered at 2,2 was tested using trapz. The result was the area of a circle with a radius of 1. This was further tested by using trapz with different starting points, this and the first test confirmed that trapz could be used to compute the area of the PV curve. These test and confirmation of there validity allowed for easy integration into the code to find the net work our.

D. Ideal Efficiency Equation

Using the four thermodynamic processes and the ideal gas equations, an expression for the ideal Stirling cycle efficiency. The ideal efficiency is given by equation 1 below and the derivation of the expression is shown in figure 1.

$$\eta_{IdealStirling} = \ln \frac{V_2}{V_2} \frac{T_H - T_L}{\frac{5}{2}(T_H - T_L) + T_H \ln(\frac{V_2}{V_1})} \tag{1}$$

1. Efficiency

$\eta_{th} = \frac{W_{net}}{Q_{in}} = \frac{W_2 + W_4}{Q_{in}} = \frac{W_2 + W_4}{Q_1 + Q_{out}}$

2. $Q = mc\Delta T$

$Q_1 = mc_p(T_H - T_L)$

where $c_p = \frac{R}{\gamma - 1} = \frac{5}{2}R$

$Q_1 = \frac{5}{2}Rm(T_H - T_L)$

3. 1st Law

$\Delta U = Q_{out} - W_{in}$

$\Delta U = 0 \rightarrow Q_{out} = W_{in}$

4. Integration of Work

$W = \int_{V_1}^{V_2} P\,dV$ where $PV = mRT$

$W = \int_{V_1}^{V_2} \frac{mRT}{V}\,dV \rightarrow mRT\int_{V_1}^{V_2} \frac{dV}{V}$

$W = mRT\ln\left(\frac{V_2}{V_1}\right)$

5. $Q_{out} \rightarrow T = T_H$

$Q_{out} = mRT_H\ln\left(\frac{V_2}{V_1}\right)$

6. $Q_{in} = Q_1 + Q_{out}$

$Q_1 = \frac{5}{2}Rm(T_H - T_L)$

$Q_{in} = \frac{5}{2}Rm(T_H - T_L) + mRT_H\ln\left(\frac{V_2}{V_1}\right)$

7. $W_2 = Q_{out} = mRT_H\ln\left(\frac{V_2}{V_1}\right)$

8. $W_4 = mRT_L\ln\left(\frac{V_1}{V_2}\right)$

9. Substitution

9. $\eta_{th} = \frac{W_2 + W_4}{Q_{in}}$ (eq. 1)

$\eta_{th} = \frac{mRT_H\ln\left(\frac{V_2}{V_1}\right) + mRT_L\ln\left(\frac{V_1}{V_2}\right)}{\frac{5}{2}Rm(T_H - T_L) + mRT_H\ln\left(\frac{V_2}{V_1}\right)}$

$\eta_{th} = \frac{T_H\ln\left(\frac{V_2}{V_1}\right) + T_L\ln\left(\frac{V_1}{V_2}\right)}{\frac{5}{2}(T_H - T_L) + T_H\ln\left(\frac{V_2}{V_1}\right)}$

$\boxed{\eta_{th} = \ln\left(\frac{V_2}{V_1}\right)\left[\frac{T_H - T_L}{\frac{5}{2}(T_H - T_L) + T_H\ln\left(\frac{V_2}{V_1}\right)}\right]}$

Figure 1: Derivation of the Ideal Efficiency for a Stirling Cycle

IV. Results

The data collected yielded many important values regarding the performance of the Stirling Engine. The internal pressure was recorded during the experiment and from the Solid Works file the volume was determined. As a result the P-V diagrams were plotted and the area under the curve was integrated as discussed previously in this report. The calculation of heat transfer and work, as well as the engine efficiencies is discussed below.

A. P-V Diagrams

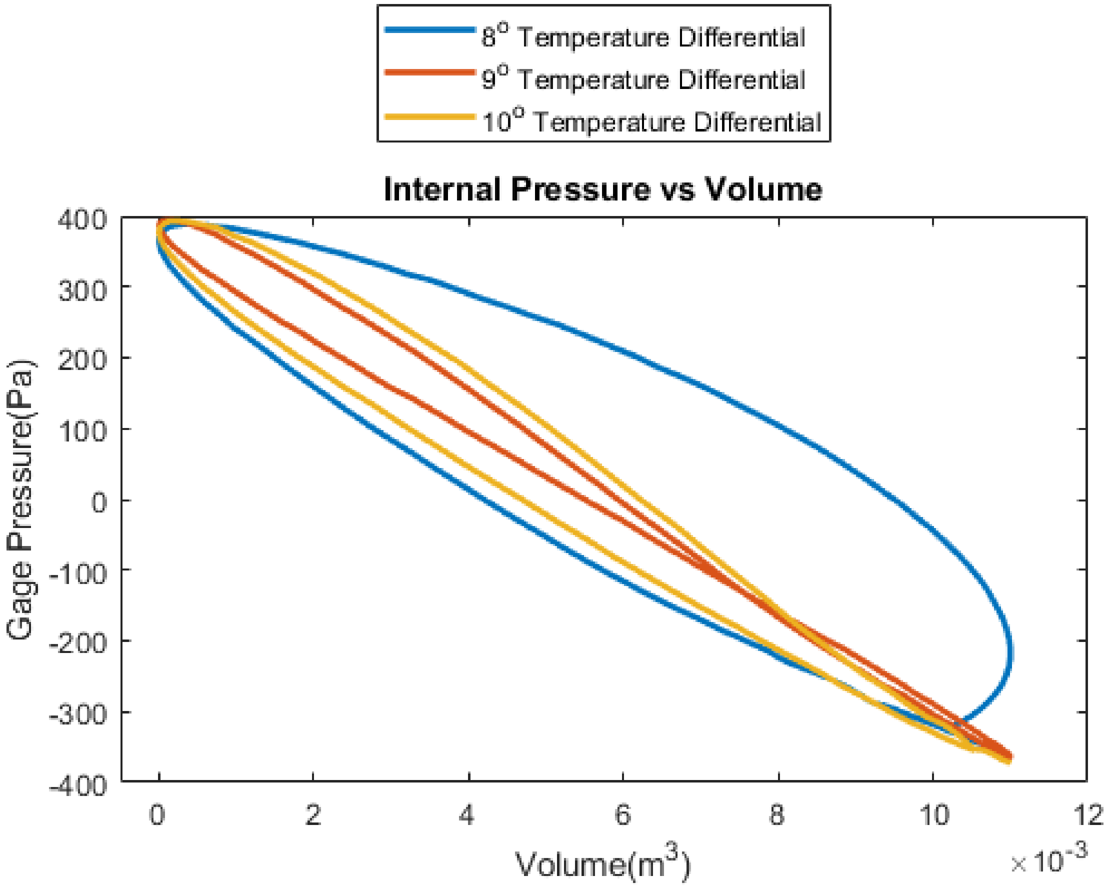

Figure 2: PV diagrams for an 8, 9, and 10 degree temperature difference

This diagram describes the relation between pressure and volume within the chamber due to the changing temperatures of the gas as well as the motion of the power and displacement pistons. We see that as the temperature differential increases, the data seems to represent more ideal behavior with the 10°C differential most closely resembling the ideal Stirling cycle. At a 9°C differential, we see that the cycle crosses over itself when the pressure is about -150 Pa. This could be the representation of some error in the data acquisition or an inefficiency in the specific cycle itself. With an 8°C differential, the cycle balloons out towards the low pressure side, again either indicating some error or a biased distribution of time or stored energy at the low pressure end.

B. Work and Heat Transfer Calculations

When finding the work in and out of the system and the heat transfer in and out of the system, it can be assumed that the work in to the system, from the power supply, is completely converted into heat and transferred into the system. Thus meaning that the heat transfer in is equal to the work in of the system. Using Ohm's Law to calculate the power inputted from the power supply, the work in to the system can be found by multiplying the power by the time elapsed. From this calculation, it is found that the work into the system and the heat transfer into the system was 95.77 J, 91.67 J, and 95.23 J for the eight, nine, and ten degree temperature difference, respectively. To find the work and heat transfer out of the system, equations (2) and (3) are used.

$$W_{out} = W_{net,out} + W_{in} \tag{2}$$

$$Q_{out} = (1 - \eta_{th})Q_{in} \tag{3}$$

From the equations (2), the work out of the system was found to be 98.19 J, 92.03 J, and 96.41 J for the eight, nine, and ten degree temperature difference, respectively. Using equation (3), the heat transfer out of the system was found to be 93.34 J, 91.31 J, and 94.06 J for the eight, nine, and ten degree temperature difference, respectively. The values for both work and heat transfer into and out of the system can be used to further help calculate the efficiency of the engine and described in the section below.

C. Efficiency Results

In order to find the actual efficiency of the Stirling Engine, it was important to begin this process by calculating the Carnot efficiency. The Carnot efficiency will provide an idea as to what the actual efficiency should be less than. Because the Carnot efficiency is an ideal cycle it should be expected that the actual cycle has a lower efficiency that the Carnot due to irreversibilities. To calculate the Carnot efficiency, the reversible Carnot thermal efficiency equation, (4), can be used.

$$\eta_{th,rev} = 1 - \frac{T_L}{T_H} \tag{4}$$

Utilizing this equation, the Carnot Thermal efficiencies for the Stirling engine were found to be 2.55%, 2.88%, and 3.29% for an eight, nine, and ten degree temperature difference, respectively. This was a good baseline as, moving forward it was known that the actual efficiencies needed to be lower than their respective Carnot efficiencies. Form here, the calculation of the actual efficiencies began. Since this cycle is not ideal or reversible the non-reversible thermal efficiency equation, (5), needs to be used.

$$\eta_{th} = \frac{W_{net,out}}{Q_{in}} \tag{5}$$

From this equation both the net work out of the system and the heat transfer into the system needed to be obtained. To find the net work out of the system, the area between the PV curve was found using the trapz function in MATLAB. Using the current values that were collected in the data and the 5V from the power supply, the heat transfer into the system was found with Ohm's Law, $P = IV$. Once the power was calculated, it was multiplied by the testing time of the engine to convert Joules per second into Joules, and the heat transfer in was found. Having both the net work out and the heat transfer in allowed for the thermal efficiency of the engines to be found. These calculations provided thermal efficiencies for an eight, nine, and ten degree temperature difference of 2.53%, 0.39%, and 1.23%, respectively. When considering these values reasonable, it was concluded that the values found for the tested Stirling engine were reasonable since they were less than the ideal efficiencies. The eight degree temperature difference thermal efficiency is very close to the ideal efficiency, but when viewing Figure 3, it is clear to see that an error most likely occurred while testing and as such the data was skewed to produce a high net work out which increased the calculated efficiency.

V. Final Analysis

A. Work and Q_{in} Analysis

The calculated work and heat transfer discussed above were the values for the whole experiment.Table 1 below presents the work and heat transfer values per revolution.

	8°	**9°**	**10°**
Q_{in}	95.77 J	91.67 J	95.23 J
$W_{net,out}$	2.42 J	0.36 J	1.17 J
Q_{out}	93.34 J	91.31 J	94.06 J

Table 1: Work and Heat Transfer Values for One Revolution

From the table above it can be seen that the work and heat transfer is highest for the engine with 8°temperature difference followed by 9°and lastly 10°. This trend is validated by the efficiencies calculated above. For an ideal engine the efficiency decreases as the temperature difference increases.

B. Comparison to Other Cycles

The Stirling Cycle is one of many types of thermodynamic cycles. Other examples of cycles include the Otto, Brayton, and Diesel cycles. The Otto cycle is an ideal cycle for a spark ignition engine, the Diesel cycle is the ideal cycle for a compression ignition reciprocating engine, and the Brayton cycle is the ideal cycle for a modern gas turbine.

A major difference between the cycles what the efficiency of the engine depends on. For the Carnot and Stirling cycle, the efficiency of the engine only depends on the temperature difference as seen in equation 4. The efficiency of the Otto and Brayton cycles depend on the compression ratio (r). The efficiency of the Diesel cycle is also dependant on the compression ratio and the cutoff ration r_c. The equations for the efficiency of the Otto, Brayton, and Diesel engines are represented by equations 6, 7, and 8 below respectively.

$$\eta_{Otto} = 1 - \frac{1}{r^{k-1}} \tag{6}$$

$$\eta_{Brayton} = 1 - \frac{1}{r_p^{\frac{k-1}{k}}} \tag{7}$$

$$\eta_{Diesel} = 1 - \frac{1}{k-1}[\frac{k_c^k - 1}{k(r_c - 1)}] \tag{8}$$

Although the efficiencies of each cycle are not dependent on the same variables they can still be compared to each other. For any given compression or temperature ratio the Stirling engine has a higher efficiency than all of the previously mentioned cycles.[4] As mentioned in the introduction Stirling engines are only practical in some situations, regardless they are more efficient than all of the cycles mentioned previously. A higher efficiency cycle results in more work produced from the cycle relative to the same work input to the cycle. Since more work is produced from the same amount of input, this makes Stirling Engines more cost effective than the other mentioned cycles.

C. Sources of Error

The efficiencies of these devices have been calculated using the concept of a Carnot cycle, which is not a very accurate way of calculating these values. Using the Carnot principles, it is assumed that the device is running with an ideal, fully internally reversible cycle. The discrepancies between the actual cycle and the Carnot cycle can be seen with their respective pressure-volume diagrams in Fig. 3. The Carnot efficiency is the maximum efficiency that the engine could reach if it was operating on a completely ideal cycle, however that is impossible to achieve in practice, so the actual efficiencies will be smaller than the ones we calculated.

VI. Discussion

A. Ideal vs. Actual P-V Diagram

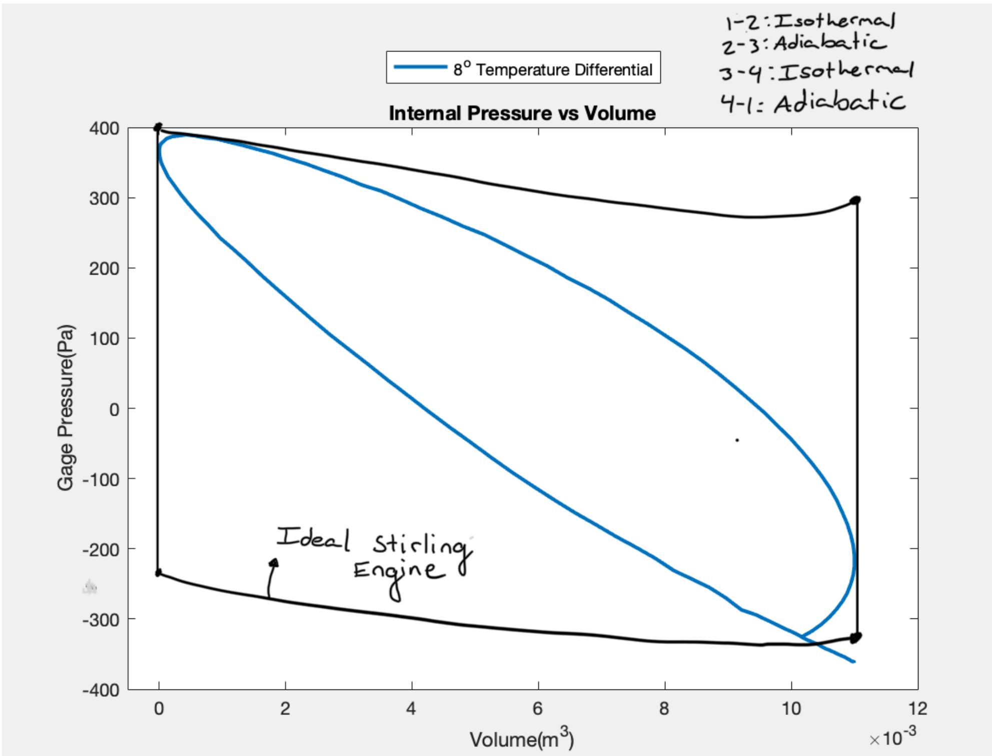

Figure 3: PV diagrams of a Cycle functioning on an 8 degree temperature difference vs Ideal Stirling Cycle

B. Justification of Work Output

The magnitude of work being produced by the real 8 degree difference cycle makes sense, though the pressure change is relatively large, the volume change of the air is small. It is also reasonable to believe that the ideal cycle produces more work than the real 8 degree cycle. As seen in Figure 3, the area of of the ideal Stirling cycle, representing the work output of the ideal system, incorporates the area of the entire real 8 degree cycle as well as additional area. This makes intuitive sense as the ideal cycle assumes a reversible process, witch the real process is not. The real process faces significant irreversible processes due to friction as well as heat transfer from the reservoirs, and because the processes are operating on the same heat input, the reversible process must produce more work.

C. Real World Application of the Stirling Engine

Applications of the Stirling Engine in the non-ideal world include the NASA SP-100 Program, which includes their 1050 K Stirling Space Engine. The Engine was part of a larger design for nuclear-powered spacecraft and produced $25kW$ of power while relying on temperature reservoirs of $1050K$ and $525K$. This Stirling Engine design showed a theoretical efficiency of 29% whereas many comparable engines show efficiencies of $6-7\%$. The disadvantage of the system is the energy requirement for thermal reservoirs of such high temperatures, hence the engine relying on nuclear power for fuel. The system conceptually compares to the Stirling Engine used in lab in that they share similar sources of error.

D. Sources of Error

Sources of error include accounted-for irreversible inefficiencies such as heat lost to friction as well as imperfections in the piston seals. These two sources of error result in added heat lost that are unaccounted for or can be confused for heat rejected by the system. When extra heat leaks from the piston, the system is no longer perfectly closed and adiabatic; as a result the pressure of the system would decrease because heat would be transferred out of the system via mass transfer. Furthermore, the system is also not perfectly insulated, thus heat can leave the system via the actual walls of the pistons and components, again lowering the pressure of the system given a certain volume.

E. Thermal Efficiency at Steady State

The efficiency of the system approaches zero as the reservoirs approach the same temperature because it is the temperature difference in the reservoirs that drives the cycle. When the reservoirs approach the same temperature, or all of their heat is depleted, then the system will no longer have any heat transfer to drive the cycle. The ideal Carnot cycle relies on two temperature reservoirs assuming the temperature remains constant of each reservoir, whereas the irreversible efficiency relies on heat transfer from two reservoirs. Both the reversible and irreversible efficiencies rely on the ratios of high and low temperature and heat reservoirs respectively (equations 4 and 5). As shown in the equations, as temperatures of the reservoirs approach the same value of heat transfer from the two reservoirs approach the same value, indicating stead-state, the system efficiency will approach zero.

References

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