ASEN 3128 Aircraft Dynamics, Spring 2021, Lab 5
Posted: Friday 03/19/2021 on Canvas
Due: Friday 04/16/2021 at 10 am on Gradescope

For all assignments students will work in groups of two to four students. Groups are determined by the instructors. A single assignment should be submitted for the group.

Objectives

- Calculate and analyze longitudinal stability derivatives and linear dynamics models for a fixed-wing aircraft (Boeing B747-100);
- Design and analyze fixed-wing longitudinal stability augmentation using feedback control.

Background Information To compute non-dimensional longitudinal stability derivatives (i.e. stability coefficients) from dimensional stability derivatives, one must use the relations given in Table 4.4 in the textbook. As discussed in Chapter 5 of the textbook, these relations arise from considering the partial derivatives of the non-dimensional longitudinal aerodynamic force and moment equations with respect to relevant longitudinal variables,

$$
\begin{aligned}
C_x &= C_T + C_L\alpha_x - C_D, \\
C_z &= -(C_L + C_D\alpha_x), \\
C_m &= C_{m_{ac,wb}} + C_{L,wb}(h - h_{n,wb}) - V_H C_{L,t}.
\end{aligned}
$$

Note that the non-dimensional stability derivatives in Table 4.4 are defined with respect to non-dimensional state variables. For example, $C_{x_u} = \frac{\partial C_x}{\partial \hat{u}}|_0$, where $\hat{u}$ is the *non-dimensional velocity*. The required small disturbance small divisors for non-dimensionalization are provided in Table 4.1.

The modal response for the longitudinal dynamics of fixed-wing aircraft will be discussed further in lecture, but you should read Chapter 6.1-6.3 in the textbook to begin familiarizing yourself with the two characteristic response modes for longitudinal flight near trim: the *Short Period Mode* and the *Phugoid Mode*. Both modes are described by complex conjugate eigenvalues and eigenvectors. This means that they are physically associated to characteristic damped oscillations among the longitudinal state variables when they are in specific phase relationships with each other. It is often desirable to use feedback control in autopilot systems to damp out the phugoid mode, in order to improve longitudinal response characteristics for human pilots.

Part 1: Longitudinal Dynamics Modeling

1. Use the dimensional stability derivatives from Table E.3 in Appendix E, for case II of the B747-100 airplane in Table E.1, to construct the SI units version of table E.3 (longitudinal stability and control derivatives only).

2. Since Appendix E is for the case of the body frame shown in Figure E.1, to obtain corresponding derivatives for the case of the stability frame, where $\theta_0 = 0$ for horizontal flight, the stability frame must be rotated by the ξ angle about the body $\hat{y}$ axis from the initial alignment with the body frame (-6.8 deg in this case). With this in mind, do the following:

(a) Convert the derivative table from Question 1 into the stability frame, using the coordinate rotations in eqn. B.12,6 (also see the remarks at the beginning of Appendix E on how to do this).

(b) Use the dimensional results from part (a) to create a non-dimensional longitudinal stability derivative table with a format similar to Table 6.1.

3. Use the dimensional table from Question 2 to construct the 4×4 A_{long} matrix for the linearized longitudinal dynamics (eq. 4.9,18) in SI units.

4. Find the eigenvectors and eigenvalues of A_{long}. Identify eigenvectors and eigenvalues corresponding to the Short Period and Phugoid modes. What are the corresponding modal damping ratios and modal natural frequencies?

5. Compare the eigenvalues above of the Short Period mode to the approximation developed in class. Compare the oscillation period of the Phugoid mode above to the Lanchester approximation (see Ch. 6.3).

6. Simulate the linearized longitudinal dynamics from eq. 4.9,18 using ode45.

(a) Verify that the trim state is an equilibrium.

(b) Perturb the initial states, one at a time, as follows, and plot the responses. Discuss the results in reference to the modal behaviors found above:

(i) Initial $\Delta u^E = 10$ m/s;

(ii) Initial $\Delta w^E = 10$ m/s;

(iii) Initial $\Delta q = 0.1$ rad/s;

(iv) Initial $\Delta \theta = 0.1$ rad.

(c) Which initial deviations are good at exciting the Short Period mode? The Phugoid?

Part 2: Feedback Control for Stability Augmentation

7. Add elevator control derivatives to the linearized model for the B747, based on the non-dimensional values on p. 229 in the textbook, dimensionalizing according to table 7.1 (in SI units), using the case II flight conditions from Appendix E (ignore the rotation to stability frame for this part).

8. Design a proportional-derivative elevator feedback control law of the form

$$\Delta\delta_e = -k_2\Delta\theta - k_1\Delta q.$$

to augment damping in the phugoid mode as follows:

(a) First design for increased pitch stiffness in the short period mode by a variable scale factor k_s, ranging from 1 to 3, in steps of 0.01, while retaining the original (no control) damping ratio, using the short period approximation from class. Plot the resulting locus of these eigenvalues in the complex plane to verify increasing pitch stiffness and constant damping ratio in the 2nd order short period approximation as k_s varies.

(b) Implement the above range of control laws on the full linearized longitudinal dynamics and calculate the actual short period and phugoid mode eigenvalues. Plot the locus of these eigenvalues in the complex plane. Discuss the general trends in modal behavior as k_s varies.

(c) Simulate the closed loop behavior of the linearized longitudinal system, starting from trim except for a 0.1 rad deviation in $\Delta\theta$, for the case of $k_s = 1$ (no feedback) and for the case $k_s = 2$ (doubled pitch stiffness). Does this behavior make sense given the corresponding closed loop eigenvalues?

(d) Determine the reason for the effects on phugoid induced by the short period control. Is this primarily due to the natural coupling in the A matrix between the rotational and translational states, or due to the additional control effects in the B matrix (not modeled in the short period control design) of elevator actuation on the translational states?

Assignment

This assignment does not require a full lab report. Instead, submita single PDF file that includes answers to all questions with plots generated for the problems and the text of any code specifically requested for a problem. All code used for the assignment should be included as an appendix. Only a single example of the code is expected even if students made multiple versions (e.g. each student wrote their own).

Be sure to add titles to all figures that include both the problem number and a description of the plot, e.g. "2.c Output x versus Time". The assignment will be evaluated based on i.) correct answers; ii.) proper commenting and documenting of code; and iii.) the quality of the figures submitted (e.g. labeling, axis, etc).

All lab assignments should include the Team Participation table and should be completed and acknowledged by all team members. Description of the Team Participation table is provided in a separate document.