UNIVERSITY OF COLORADO - BOULDER

ASEN 3200

LAB O-1

Orbital Lab 1

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Contents

In order to gain a better understanding of orbital mechanics and the principles that define it, a model of a satellite orbiting the Moon and the Earth was developed in the physics-based software, STK. Additionally, multiple MATLAB scripts were constructed that defined orbits and solved for Kepler's law to increase understanding of how software like STK function. Within this lab, plots produced from MATLAB as well as models from STK can be found that demonstrate an understanding of orbital mechanics. Specifically with regard to, two body orbit systems of a large mass and a satellite. Through this lab, a better understanding of orbit modeling software and the fundamentals of orbits was gained and can later be applied to work while in pursuit of our degree and in industry.

I. Introduction

The purpose of this lab is to study orbits around different central bodies, specifically the Earth and the Moon. This lab utilizes STK, a software utility that seamlessly models orbits with a set of initial parameters. Scenario I involves an orbit around Earth, which includes an STK model, as well as Matlab analysis of angular momentum and eccentricity. Scenario II focuses on an orbit around the Moon. This study on the Moon will build upon the theory and analyses of important characteristics of this orbit as well as calculate measurement windows using Kepler's equation and other orbit equations. STK is used for this lab because it allows for the experimentation of different orbit parameters that affect the shape, size, and duration of orbits. This experimentation is important for satellites that need to meet certain orbit requirements to complete their missions and measurements.

II. Scenario I

A. Question 1

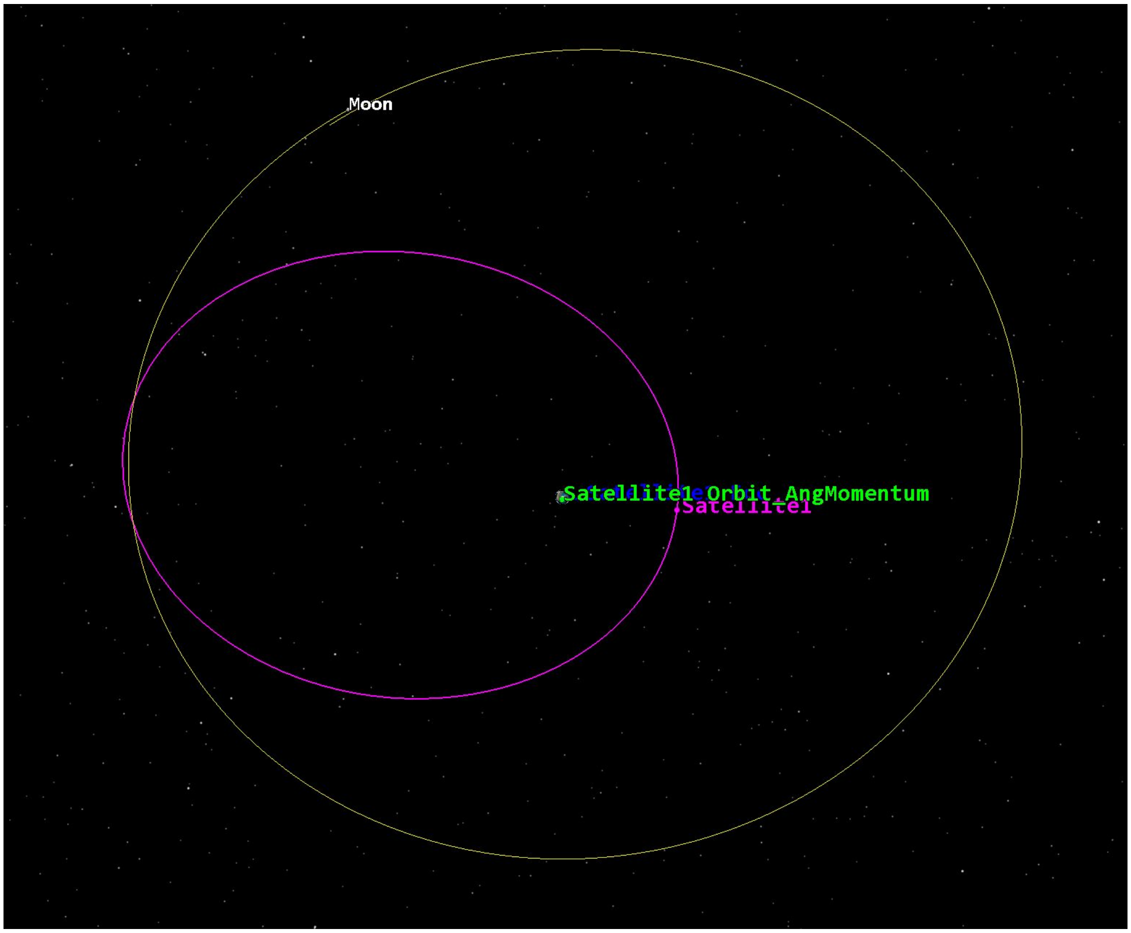

Figure 1. 3D Orbit from Orbital Plane

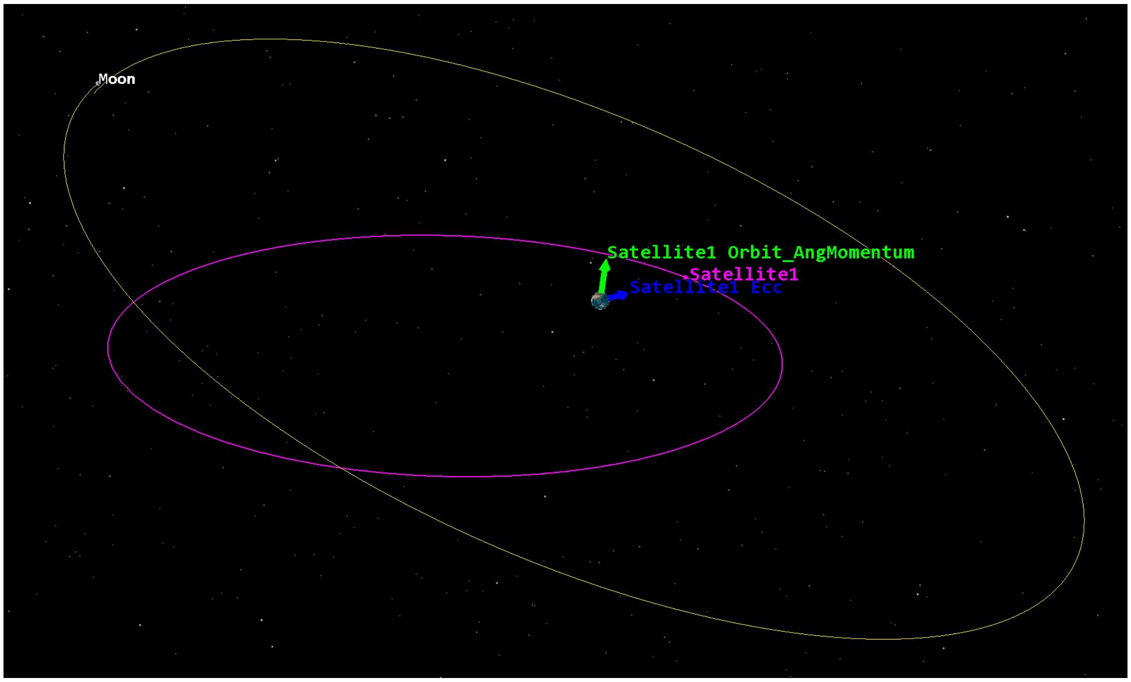

Figure 2. 3D Orbit of Moon and Satellite around Earth

The results of the orbit over one orbital period are shown above. Over the course of the orbit, the angular momentum and eccentricity vectors stay constant. This is in line with our expectations, since the orbit around a central body is governed by the conservative gravitational force. As a result, the angular momentum and eccentricity vectors should not change over the course of the orbit.

B. Question 2

A satellite in a two body model only experiences the force due to the large body it is orbiting but, in reality there are many other factors that can influence the orbit of a spacecraft. One of these influences is the addition of multiple massive body's whose gravitational forces can affect the satellites orbit. Bodies that could have this affect in the orbit above are the Sun and the Moon. The Sun is so massive that it's effects reach very far and a satellite would certainly feel some effects from the Sun. Similarly, the Moon would have an effect. Even if the effect of the Moon is small because it is not massive, the satellite's orbit does get close to the Moons orbit at one point. As such, the Moon could additionally cause the orbit to change due to it's gravitational force. Further, the Earth has a atmosphere that could cause atmospheric drag and result in the slowing down of the satellite. This would change the orbital period of the satellite and the velocity as well. If the satellite were to slow down enough, due to the drag, it could fall out of orbit. As space is not actually a two body system, it is important to realize that calculations based on two body systems are likely not to be the observed outcome when the satellite is in actual orbit.

C. Question 3

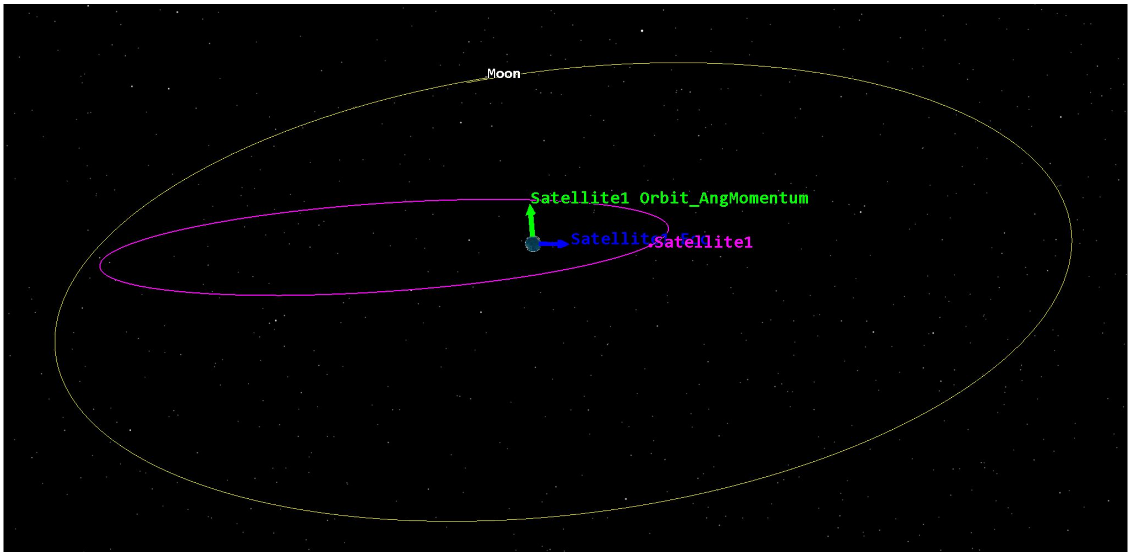

Figure 3. 3D Orbit of Moon and Satellite around Earth

The orbits of the satellite and the Moon do not intersect with each other. To see this clearly, refer to the figure above.

D. Question 4

In order to model an orbital period of the satellite, the ode45 function within Matlab was utilized. This method takes in an initial location vector, $\overrightarrow{r}_0$, and and initial velocity vector, $\overrightarrow{v}_0$, and uses the runga kutta method to numerically solve the following second order differential equation, modeled after the standard gravitational equation,

$$\ddot{r} = \frac{\mu \overrightarrow{r}}{|r|^3} \tag{1}$$

with $\mu = 3.9858\text{e}14\ m^3 s^{-2}$

With a coordinate system centered around the central body, this problem was modeled such that the initial conditions of the satellite were:

$$\vec{v}_0 = [r_p, 0]$$
$$\vec{r}_0 = [0, v_p]$$

After solving numerically via Matlab, the resulting orbit trajectory can be observed in Figure 4

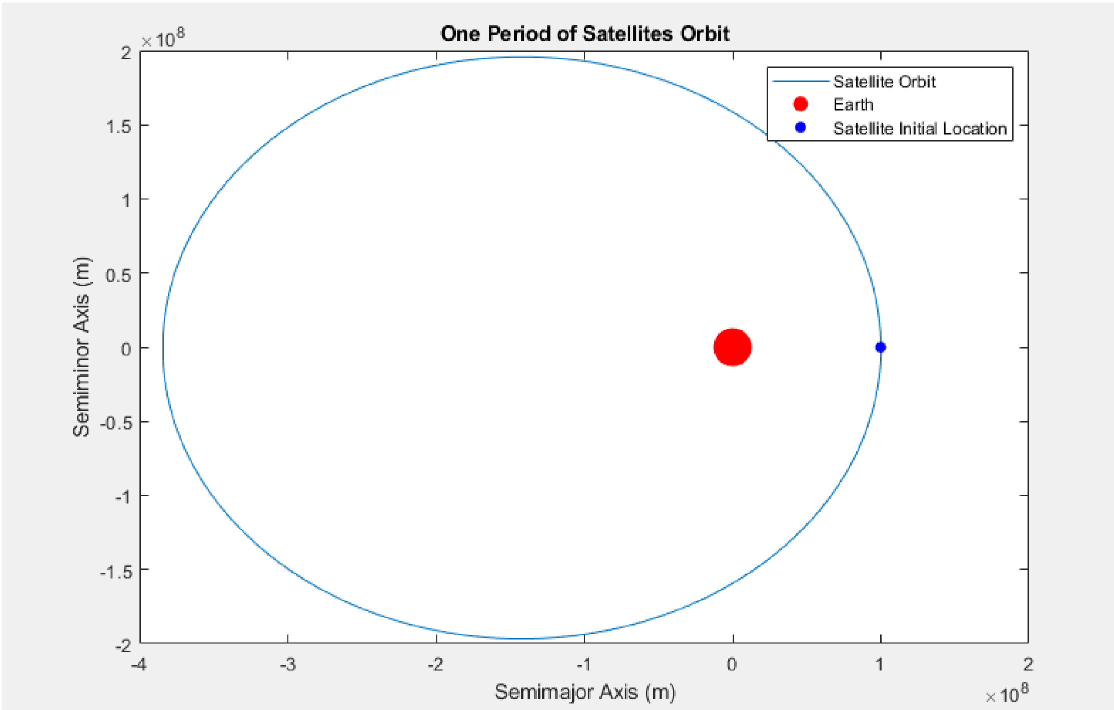

Figure 4. Trajectory of Elliptical Orbit Around Earth

In order to verify the integrity and accuracy of this orbit model, both angular momentum and eccentricity were calculated for all points in the orbit. Orbits surrounding a central body are governed by the conservative gravitational force and therefore both of these values should remain constant throughout the entire orbit. The following formulas were used to find angular momentum and eccentricity over this singular orbital period, and the results can be observed in the following plots.

$$\vec{h} = \vec{r} \times \vec{v} \tag{2}$$

$$\vec{e} = \frac{1}{\mu}(|\vec{v}|^2\vec{r} - (\vec{r} \cdot \vec{v})\vec{v}) - \frac{\vec{r}}{|\vec{r}|} \tag{3}$$

where

$$\vec{v} = [\dot{x}, \dot{y}]$$
$$\vec{r} = [\mathrm{x}, \mathrm{y}]$$

at any instantaneous location

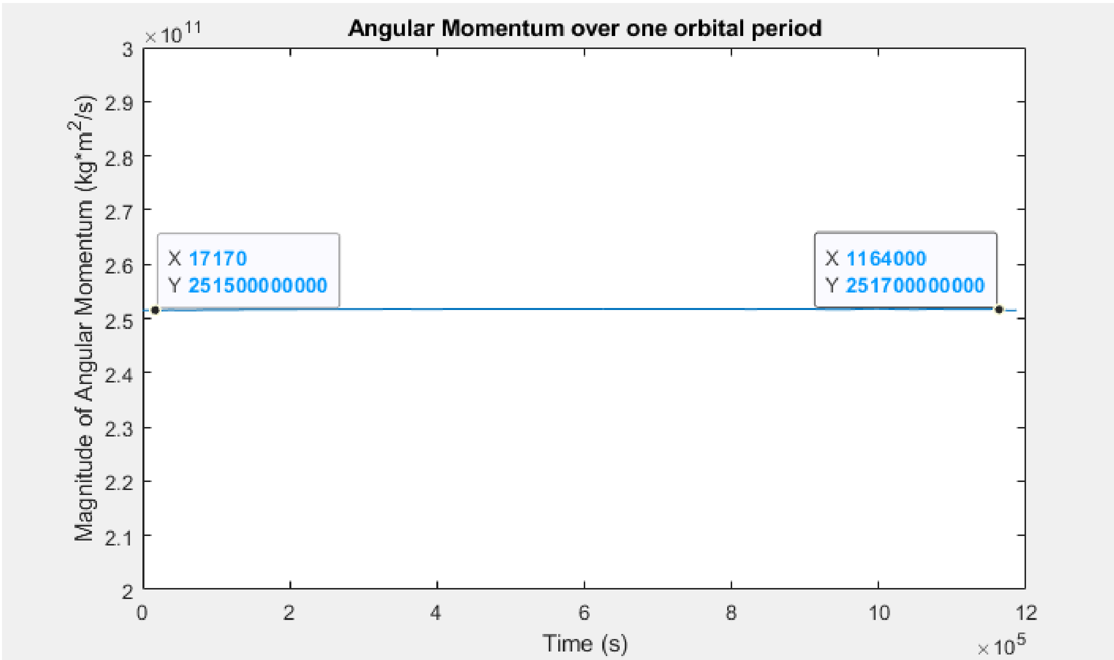

Figure 5. Angular Velocity of Elliptical Orbit Around Earth

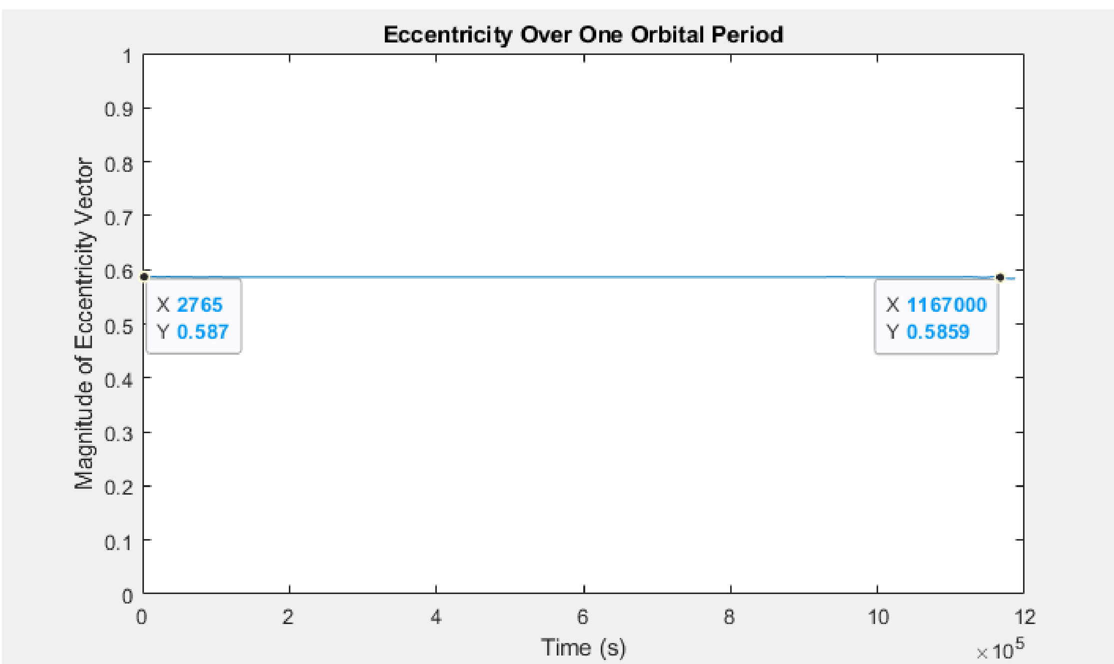

Figure 6. Eccentricity of Elliptical Orbit Around Earth

Both of these plots show almost entirely consistent results throughout the entire orbit, with slight degradation towards the end of the period. This slight variation at the end is most likely attributed to calculation and tolerance errors within ode45 in Matlab. Regardless, this model proves that a simple numerical solution of Newtons second order law of gravitation can be a powerful and accurate method of modeling two body orbits.

III. Scenario II

A. Question 5

Given the orbital period (P), periapsis radius (r_p), and the initial location of the spacecraft (r), we were able to calculate the values of the semimajor axis (a), eccentricity (e), and true anomaly (θ) of the orbit. The equations used to calculate these values are below.

$$P = 2\pi\sqrt{\frac{a^3}{\mu}} \tag{4}$$

where $P = 6.3$ hours or 22680 seconds, $r_p = 1800$ km, $r = 2400$ km, and $\mu = 4902.799\frac{km^3}{s^2}$. Rearranging for a, we get

$$a = \sqrt[3]{\frac{\mu P^2}{4\pi^2}} \tag{5}$$

After putting in all of the values, the result is that $a = 3997.52$ km.

Using that result, we can then calculate the eccentricity using the following equation

$$r_p = a(1 - e) \tag{6}$$

After rearranging, we get

$$e = 1 - \frac{r_p}{a} \tag{7}$$

After putting the required values into the equation, we get that $e = 0.5397$.

Now, with both of the above results, we can calculate the true anomaly of the orbit.

$$r = a\frac{1 - e^2}{1 + e\cos\theta} \tag{8}$$

After simplifying and rearranging, we get

$$\theta = \arccos\left(\frac{\frac{a}{r}(1 - e^2) - 1}{e}\right) \tag{9}$$

We then plug in the previous results and the given parameters and calculate that $\theta = 70.467$ deg or 1.229 radians.

B. Question 6

No deliverable asked for this question. North Pole indicator visible in Figure 7.

C. Question 7

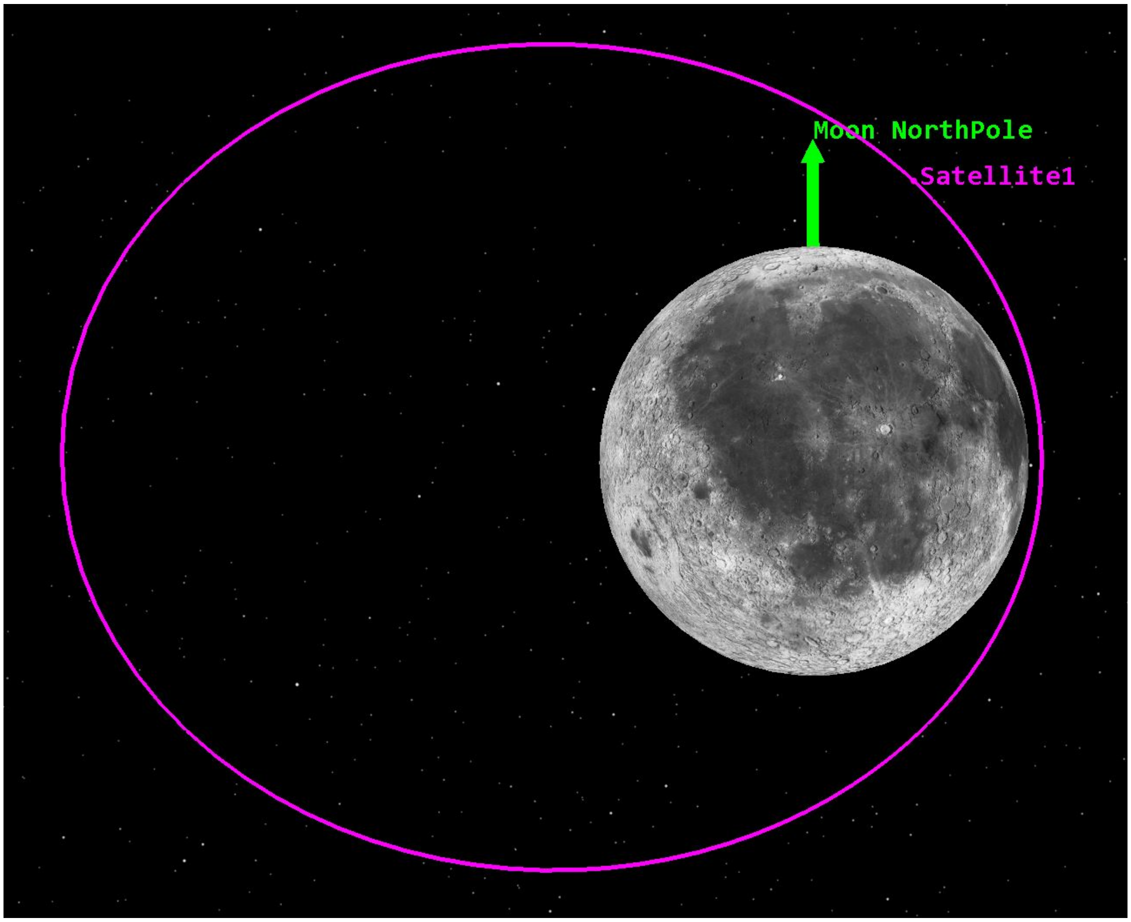

Figure 7. 3D Orbit of Satellite around Moon

As seen in Figure 7, the North Pole vector indicates the direction of the North Pole from the Moon.

D. Question 8

The spacecraft discussed above requires an altitude between $200km$ and $500km$ to make clear observations. Thus, it is important to know how long the spacecraft will take to reach the observation altitude. Using the initial conditions provided, the time it will take until the spacecraft is able to make it's first observation can be found through the following process. First, some key variables describing the orbit must be found, such as, the semi-major axis, a and the eccentricity, e. Since the orbital period, T, and the gravitational parameter, μ, were known the semi-major axis could be found using equation (10)

$$T = 2\pi\sqrt{\frac{a^3}{\mu_{moon}}} \longrightarrow a = \sqrt[3]{\left(\frac{T * \mu_{moon}}{2\pi}\right)^2} \tag{10}$$

When computing the semi-major axis with the given conditions, it was found to be 3997.57 km. To find the eccentricity of the orbit, equation (11) can be used as the radius of periapsis and, now, the semi-major axis are known values.

$$r_p = a\left(1 - e\right) \longrightarrow e = 1 - \frac{r_p}{a} \tag{11}$$

When the radius of periapsis and the semi-major axis values are plugged into the above equation, the eccentricity is calculated to be 0.5397. The next step in finding the time to observation is to find the eccentric anomaly, E, for both points in the orbit. First, for the current position of the spacecraft and additionally for the point where observations can be made, 500 km. To do this, equation (12) can be utilized which relates the spacecraft radius and orbit to the eccentric anomaly.

$$r = a\left(1 - e\cos(E)\right) \longrightarrow E = \cos^{-1}\left(\frac{1 - \frac{r}{a}}{e}\right) \tag{12}$$

Plugging in the the known values, the eccentric anomaly is found to be 0.737 radians at the initial radius of 2400 km and 5.667 radians at the first observation radius of 2237 km. 2237 km was found by taking the radius of the moon and adding the 500 km first observation point to it. Now that the eccentric anomaly has been found the mean anomaly, M, can be found using equation (13).

$$M = E - e\sin(E) \tag{13}$$

When solved for at the two radii, 2400 km and 2237 km, the mean anomaly is 0.928 radians and 5.355 radians, respectively. Finally, to calculate the time to the first opportunity for observation from the initial point. The time from periapsis to the two points can be found using equation (14).

$$M = t\sqrt{\frac{\mu_{moon}}{a^3}} \longrightarrow t = \frac{M}{\sqrt{\frac{\mu_{moon}}{a^3}}} \tag{14}$$

The time from periapsis to the two orbit radii, 2400 km and 2237 km, the times were 3350.94 seconds and 19329.05 seconds, respectively. Taking the initial position time and subtracting it from the first observation point time, produces the time from the initial condition location to the observation window. When computed, it is found that the time to the first observation point is 15978.109 seconds or 4.438 hours.

E. Question 9

Additionally, it is important to know how long the spacecrafts observation window is. To first determine this value, it is important to understand that the instrument will be both to high and to low, in terms of altitude, while orbiting the moon. This is important for how the problem should be approached as the observation period will be two discontinuous periods on the spacecrafts orbit. Since, the two observation periods will be the exact same, just on opposite sides of the orbit, it is possible to find one observation period and then double it to find the total observation period. To begin the calculations for the observation period, one can first find the true anomaly, θ, of the start and end altitudes. To find these anomalies, equation (15) can be used.

$$r = \frac{a\left(1 - e^2\right)}{1 + e\cos\theta} \longrightarrow \theta = \cos^{-1}\left(\frac{\frac{a\left(1-e^2\right)}{r} - 1}{e}\right) \tag{15}$$

When computed, with the given values and the values obtained through question 8, the true anomalies for 500 km and 200 km altitudes are 5.23 radians and 5.74 radians, respectively. Now, a similar process to question 8 can be used to solve for the observation period. The mean anomalies of the two altitudes needs to be found and to do this the eccentric anomalies of the two altitude need to be found. The eccentric anomalies can be found by using equation (12) for question 8. This calculation results in eccentric anomalies of 5.667 radians and 5.982 radians for 500 km and 200 km altitudes, respectively. Further, equation (13) from question 8 can be used to obtain the mean anomalies. These values for the mean anomalies were found to be 5.979 radians and 6.142 radians for 500 km and 200 km altitudes, respectively. To calculate the time from periapsis to each altitude, equation (14) can be used. This calculation finds the time to 500 km to be 359.7 minutes and 200 km to be 369.51 minutes. Subtracting these two values and you find that the observation time is 588.41 seconds.

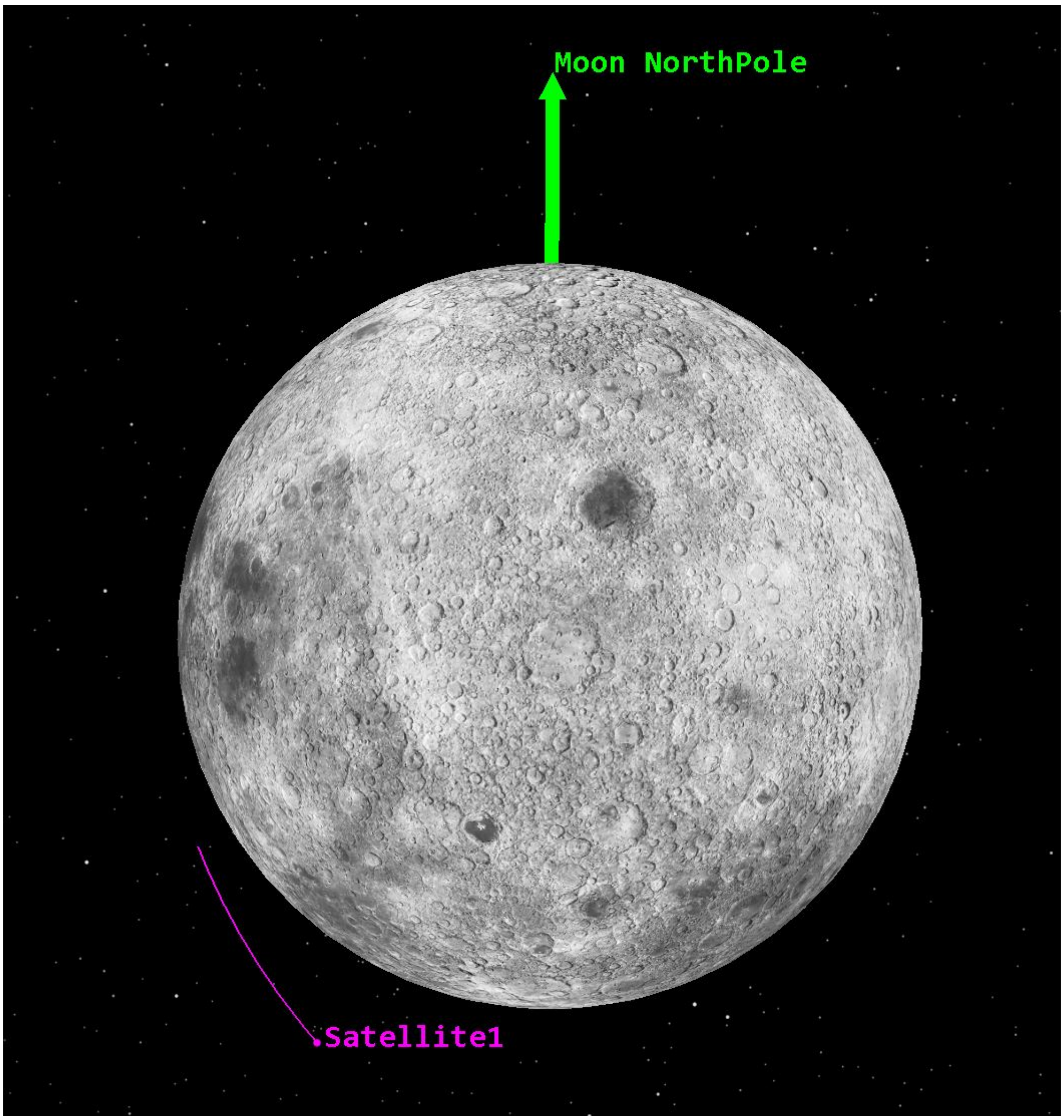

Figure 8. Orbit Observation Window

In Figure 8 the path of the satellite during the observation window can be seen. The satellite travels from 500 km to 200 km during a time window of about 588.41 seconds.

F. Question 10

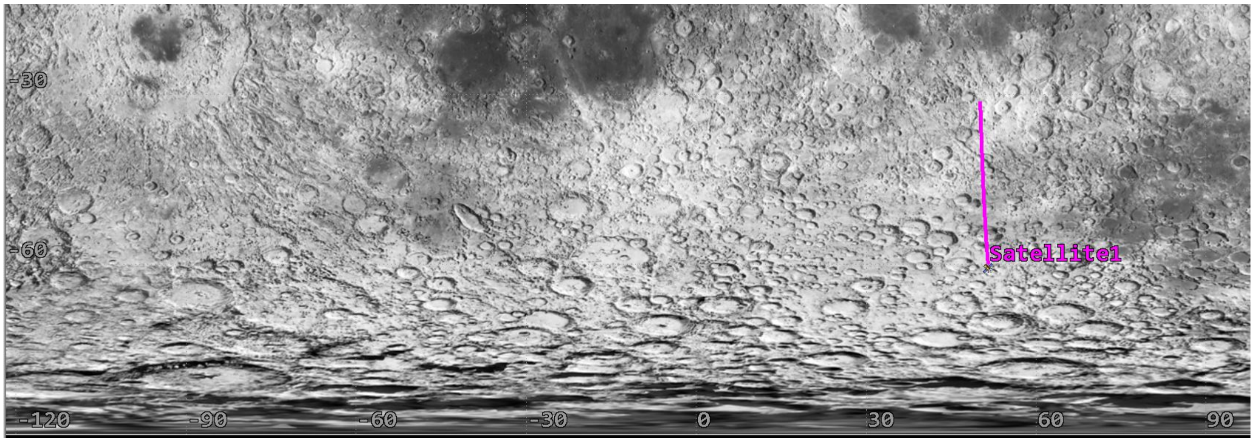

Figure 9. 2D Ground Track During Observation Window

The ground track (orbit projected onto Moon's surface) of the satellite during the observation can be seen above. The approximate range of longitude that the satellite crosses is between 50° and 52°. The approximate range of latitude that it crosses is between −65° and −30°. When comparing the ground track during the observation window to an image map of the Moon, we see that there are a few craters that are close to the ground track. Those craters are the Rosenberg crater and the Vallis crater. Furthermore, this area of the Moon is lighter than the rest, and that is also the same in the image map of the Moon.

G. Question 11

The Matlab script seen in the appendix was used to solve Kepler's equation to find radius of an satellite at any point within an orbit. Given that Kepler's equation as seen below is transient and has no simple solution, a method of iteration must be employed to solve it.

$$M = E - e\sin(E) \tag{16}$$

By starting with an initial guess for eccentric anomaly, E, such that

$$E_n = M \tag{17}$$

Kepler's equation can be iterated such that

$$E_{n+1} = M + e\sin(E_n) \tag{18}$$

This method can be used for any amount of iterations, or if a specific tolerance is required, until

$$|(E - e\sin(E)) - M| < tolerance \tag{19}$$

This should yield a final value for eccentric anomaly, where

$$M \approx E - e\sin(E) \tag{20}$$

H. Question 12

When analyzing an imaging satellites orbit, it's imperative to know its altitude at any given time in that orbit. In this case, the measurement window of the satellite is dependant on altitude and is thus of concern in order to understand when to take accurate measurements. Given the following initial conditions and orbit parameters, the methodology described above was used to determine the orbit altitude 13.5 minutes after periapsis and determine if the satellite was within the correct range for surface imaging.

Orbital Period (P) = 6.3 hours
Periapsis radius (r_p) = 1840 km
Time of interest (t) = 13.5 minutes after periapsis

These parameters yields values such that

$$M \cong E - e\sin(E) = 0.2244rad \tag{21}$$

and

$$E = 0.4678rad \tag{22}$$

The following formula was then used to solve for instantaneous radius of the orbit

$$r = a(1 - e\cos(E)) = 3997.8(1 - 0.5397\cos 0.4678) = 2071.8km \tag{23}$$

Now subtracting the radius of the moon

$$h = r - r_{moon} = 2071.8km - 1737.1km = 334.7km \tag{24}$$

The final altitude at 13.5 minutes past periapsis is 334.7 km, which is within the acceptable range for surface imaging.

I. Question 13

When considering a two body model of a satellite orbiting the Moon, the two most significant perturbations are the gravitational contributions of the Earth as well as the Sun. This is because the Moon orbits Earth and feels the effect of its gravity. Furthermore, the Moon moves around the Sun at the same time it moves around Earth. Therefore, it interacts with the additional gravitational contributions of both the Sun and Earth at the same time.

J. Question 14

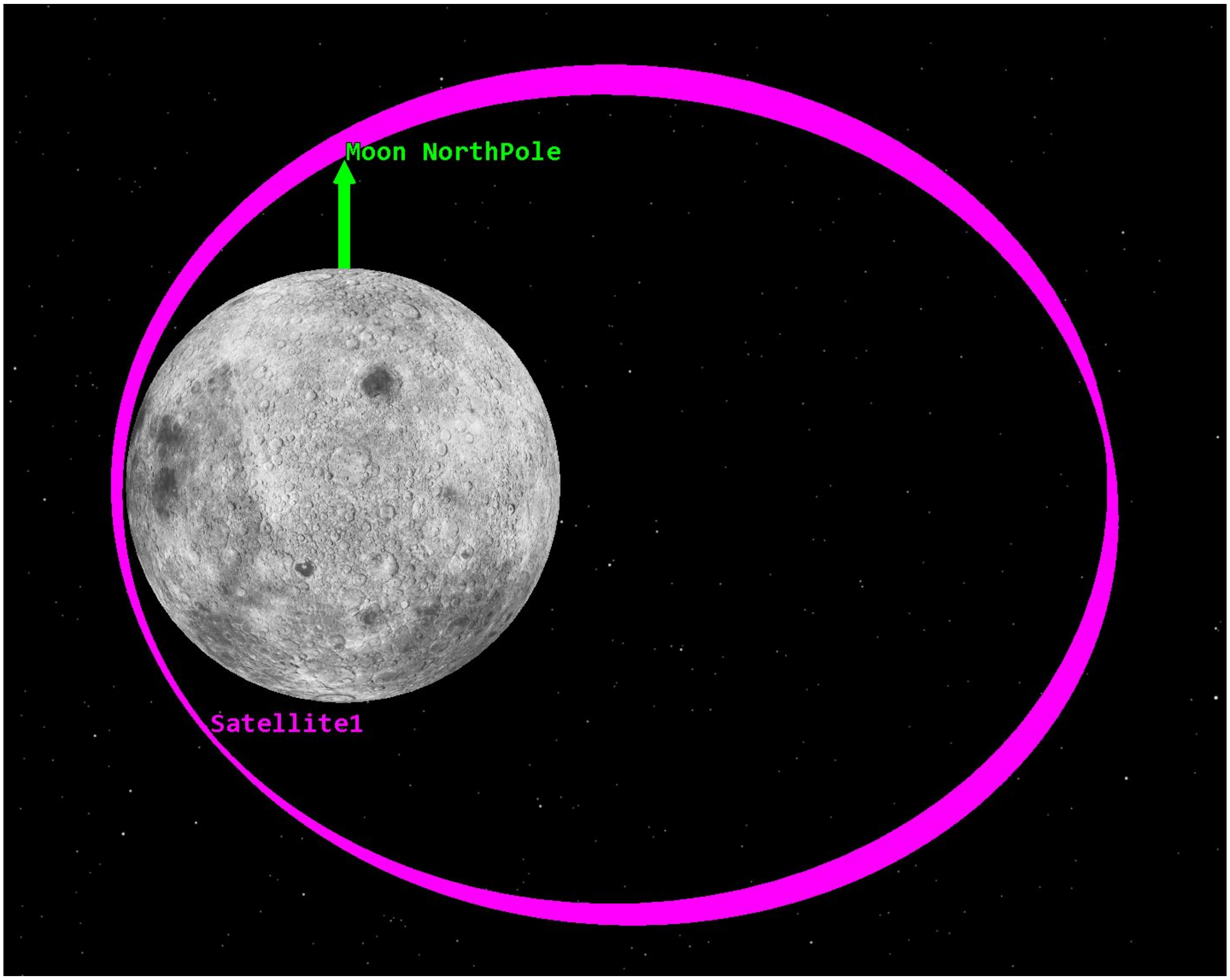

Figure 10. Orbit over 100 Orbital Periods Including Earth Perturbation

The perturbation that was applied was the effect of the Earth's gravity on the satellite's orbit around the moon. Over 100 orbital periods, a difference in shape was seen when compared to the original orbit. However, the difference was not very significant. The orbit stretched by a small amount for a number of orbits, but it was not significant when compared to the orbit without the Earth as a third body.

IV. Conclusion and Recommendations

In this lab, STK was used as a tool to model and visualize orbits with different parameters. This software can be difficult to navigate, but is excellent when the user already conceptually understands orbits and can quickly verify the results. This lab also allowed the team to become more familiar with applying the Conic Equation and Kepler's Equation to solve for orbit characteristics, both by hand and using Matlab. While STK is a good tool for quickly modeling two body problems, these sanity checks are very useful to ensure consistent results.

V. References

References

[1] Howard.D.Curtis, Orbital Mechanics for Engineering Student Third Edition

[2] ASEN 3200 Orbit Mechanics and Attitude Dynamics- Spring 2021 LABORATORY O-1 Write Up

[3] Image Map of the Moon

VI. MATLAB Code

```
%% Author: Tyler Candler
%% ASEN 3200 Orbital Mechanics
%% Lab O1
%% Written: 1/20/2021



%% Housekeeping
clear; close; clc;

%% ODE call

%Sets constants
constants = calculateConstants();

% Time
T = 2*pi*sqrt(constants.a^3/constants.mu);
tfin = T*1.001;
tspan = [0:tfin];
% Initial Conditions
% rx0     = constants.a*(1-constants.e^2)/(1+constants.e*cos(constants.theta0));
rx0 = constants.rp;
vy0 = sqrt(2*constants.mu/rx0-constants.mu/constants.a);
IC  = [rx0,0,0,0,vy0,0];
options = odeset('Reltol',1e-15,'AbsTol',1e-15);



%Calling ode45 for Phase1
[t,Phase1] = ode45(@(t,input) orbitalcalc(t,input,constants),tspan,IC);


x = Phase1(1:end,1);
y = Phase1(1:end,2);
z = Phase1(1:end,3);
x_dot = Phase1(1:end,4);
y_dot = Phase1(1:end,5);
z_dot = Phase1(1:end,6);
figure(1)
plot(x,y)
hold on
plot(1, 1, '.r', 'MarkerSize',69)
plot(x(1), y(1), '.b', 'MarkerSize',20)
% circles(0,0,500,'color','black')

title("One Period of Satellites Orbit")
xlabel("Semimajor Axis (m)")
ylabel("Semiminor Axis (m)")
legend("Satellite Orbit","Earth","Satellite Initial Location")


L = [];
```

```
m = 1000
p = [];
 for i = 1:length(x)
     r = [x(i),y(i),z(i)];
     v = [x_dot(i), y_dot(i), z_dot(i)];
     L(i) = norm(cross(r, v));
 end
figure(2)
plot(t,L)
title("Angular Momentum over one orbital period")
xlabel("Time (s)")
ylabel("Magnitude of Angular Momentum (kg*m^2/s)")
ylim([2e11 3e11])



e_vec = [];
for i = 1:length(x)
    r = [x(i),y(i),z(i)];
    v = [x_dot(i), y_dot(i), z_dot(i)];
    temp_vec = (1/constants.mu)*((norm(v)^2*r)-(dot(r,v)*v)) - (r/norm(r));
    e_vec(i) = norm(temp_vec);
end
figure(3)
plot(t,e_vec)
title("Eccentricity Over One Orbital Period")
xlabel("Time (s)")
ylabel("Magnitude of Eccentricity Vector")
ylim([0 1])



%% Orbit Function

function [results] = orbitalcalc(t,input,constants)

    %Calculate the current states
    rx = input(1);
    ry = input(2);
    rz = input(3);
    vx = input(4);
    vy = input(5);
    vz = input(6);

    %Magnitude of radius:
    r  = sqrt(rx.^2+ry.^2+rz.^2);

    x_vel = vx;
    y_vel = vy;
    z_vel = vz;
    x_accel = -constants.mu*rx/r.^3;
    y_accel = -constants.mu*ry/r.^3;
    z_accel = -constants.mu*rz/r.^3;

%     %Equate all ODES to one vector for ODE45
    results = [x_vel; y_vel; z_vel; x_accel; y_accel; z_accel];
end



%% Constants Structure
function constants = calculateConstants(constants)

constants.a = 37.9735 * 6378 *1000; % semimajor axis (m)
constants.e = 0.587; %orbit eccentricity

constants.ra = constants.a *(1+constants.e); %radius of apoapsis (m) (longest)
constants.rp = constants.a *(1-constants.e); %radius of periapsis (m) (shortest)
```

```
constants.me = 5.972e24 % (kg) (mass of earth)
constants.G  = 6.67408e-11; %universal gravitational constant
constants.mu = constants.G*constants.me; %mu of the earth

constants.i = 0; %(degrees) orbit plane inclination
constants.theta0 = 0;
end



%% Author: Tyler Candler
%% ASEN 3200 Orbital Mechanics
%% Lab O1
%% Written: 1/20/2021


%% Housekeeping
clear; close; clc;

%% Initial Conditions and constants
t = 13.5*60; %time of interest (s)
t0 = 0; %time of periapsis

rp = 1840*1000; %m
P = 6.3*3600; %seconds
r0 = 2400*1000; %m, moving away from periapsis

%% Solving for orbit parameters

G  = 6.67408e-11; %universal gravitational constant
Mm = 7.34767309e22; %mass of the moon (kg)
mu = G*Mm; %mu of the moon

n = 2*pi/P; %mean motion
a = (mu/n^2)^(1/3); % semimajor axis (m)

ra = 2*a-rp; %radius of apoapsis (m)

e = abs(ra-rp)/(ra+rp); %eccentricity

M = n*(t - t0);

%% solving for E

E = M
    for k = 1:100
        E = M + e*sin(E);
    end

%% Solving for location at t = 15 minutes after periapsis

r = a*(1-e*cos(E))

theta = sqrt((1+e)/(1-e))*tan(E/2);
theta = atan(theta)*2;

% theta = acos((a*(1-e^2)/(r*e))-(1/e))

theta = (theta/pi)*180;

r = r/1000;
h = r -1737.1


%% delta t = 0.5 P + 13.5 min
```