UNIVERSITY OF COLORADO - BOULDER

ASEN 3200

LAB O-2

Orbital Lab 2

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Contents

Tasked with creating orbits for a Mars orbiter and a near Earth object, NEO, the orbital modeling software STK was utilized to visualize the many important orbital elements associated each orbit. Additionally, MATLAB was utilized to create a function that used Gibb's method to solve for the Keplerian elements of the NEO's orbit. Further, the NEO's orbit introduced the idea of asteroid groups and their respective orbits within our solar system. It was found that the NEO modelled in this lab belonged to the Atens NEO group. At the end of this lab, an increased understanding of orbits and NEOs was gained and will be important for use later in college and in industry.

I. Introduction

The purpose of this lab is to study the heliocentric orbits of the Earth and a near Earth object as well as a Mars orbiter. This lab utilizes STK, a software utility that seamlessly models orbits with a set of initial parameters. Scenario I involves an orbit around Mars, which includes an STK model, as well as Matlab analysis of the Keplerian orbital elements. Scenario II focuses on a heliocentric orbit with both the Earth and a NEO as the objects in orbit. This study on the heliocentric orbits will build upon the theory and analyses of important characteristics of this orbit as well as calculate Keplerian orbital elements from the use of Gibb's method. STK is used for this lab because it allows for the experimentation of different orbit parameters that affect the shape, size, and duration of orbits. This experimentation is important for satellites that need to meet certain orbit requirements to complete their missions and measurements.

II. Scenario I

A. Question 1

Considering a satellite orbitting Mars with the instantaneous elements such that

$$\vec{r} = \begin{bmatrix} 3424.7 & -47.5 & 1172 \end{bmatrix} \text{ km}$$
$$|\vec{v} \cdot \hat{x}| = 0.4250 km/s$$
$$\vec{v} \cdot \hat{y} = -3.333 km/s$$
$$\vec{h} = \begin{bmatrix} 3948.694 & 3556.357 & 11394.338 \end{bmatrix} km^2/s$$
$$\mu_{mars} = 42828 km^3/s^2$$

The z component of the spacecraft's velocity was solved for using the relation

$$\vec{h} = \vec{r} \times \vec{v} \tag{1}$$

Expanding this results in

$$\begin{bmatrix} h_1 \\ h_2 \\ h_3 \end{bmatrix} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ r_1 & r_2 & r_3 \\ v_1 & v_2 & v_3 \end{vmatrix} = \begin{bmatrix} r_2 r_3 - r_3 v_2 \\ r_1 v_3 - r_3 v_1 \\ r_1 v_2 - r_2 v_1 \end{bmatrix} \tag{2}$$

Solving for v_3

$$v_3 = \frac{h_1 + v_2 r_3}{r_2} = \frac{3948.694 + (-3.333)(1172)}{-47.5} = -0.8930 km/s \tag{3}$$

These complete radius and velocities can now be used to solve for Keplerian orbital elements. First inclination can be calculated:

$$i = \cos^{-1}\left(\frac{\overrightarrow{h}_k}{|\overrightarrow{h}|}\right) = 0.4364rads = 25.0035° \tag{4}$$

The Right Ascension of the Ascending Node, Ω can then be calculated

$$\Omega = cos^{-1}\left(\frac{\overrightarrow{n}}{|\overrightarrow{n}|}\right) \tag{5}$$

where

$$\overrightarrow{n} = \begin{bmatrix}0 & 0 & 1\end{bmatrix} \times \overrightarrow{h} \tag{6}$$

$$\Omega = 47.9925° \tag{7}$$

Performing a sign check on this value shows that it lies between 0 and 180 degrees

$$\overrightarrow{n} \cdot \hat{Y} > 0 \Rightarrow \Omega : [0, 180]° \tag{8}$$

Eccentricity of the orbit can now be solved using radius and angular momentum vectors

$$e = \left|\frac{\overrightarrow{v} \times \overrightarrow{h}}{\mu_{mars}} - \frac{\overrightarrow{r}}{|\overrightarrow{r}|}\right| = 0.0500 \tag{9}$$

This eccentricity value can now be used to calculate argument of periapsis, ω.

$$\omega = \cos^{-1}\left(\frac{\overrightarrow{n} \cdot \overrightarrow{e}}{|\overrightarrow{n}||\overrightarrow{e}|}\right) = 63.0503° \tag{10}$$

Performing a sign check on this value verifies that it does indeed lie between 0 and 180 degrees:

$$\overrightarrow{e} \cdot \hat{z} > 0 \Rightarrow \omega : [0, 180]° \tag{11}$$

In order to solve for the Period and the semimajor axis (a) of the orbit, first the energy of the orbit must be solved.

$$\epsilon = \frac{|\overrightarrow{v}|^2}{2} - \frac{\mu_{mars}}{|\overrightarrow{r}|} = -5.7874\frac{km^2}{s^2} \tag{12}$$

This value can then be used to compute the Period and semimajor axis:

$$a = \frac{-\mu_{mars}}{2\epsilon} = 3700.1km \tag{13}$$

$$\mathbb{P} = 2\pi\sqrt{\frac{a^3}{\mu_{mars}}} = 6833.3s \tag{14}$$

The true anomaly at time t_0 can be solved for using eccentricity, $\overrightarrow{e}$ and location, $\overrightarrow{r}$ Solve for θ^* magnitude and sign.

$$\overrightarrow{r} \cdot \overrightarrow{v} > 0 \Rightarrow \theta^* : [0, 180]° \tag{15}$$

$$\theta^* = cos^{-1}\left(\frac{\overrightarrow{r} \cdot \overrightarrow{e}}{|\overrightarrow{r}||\overrightarrow{e}|}\right) = 66.96° \tag{16}$$

B. Question 2

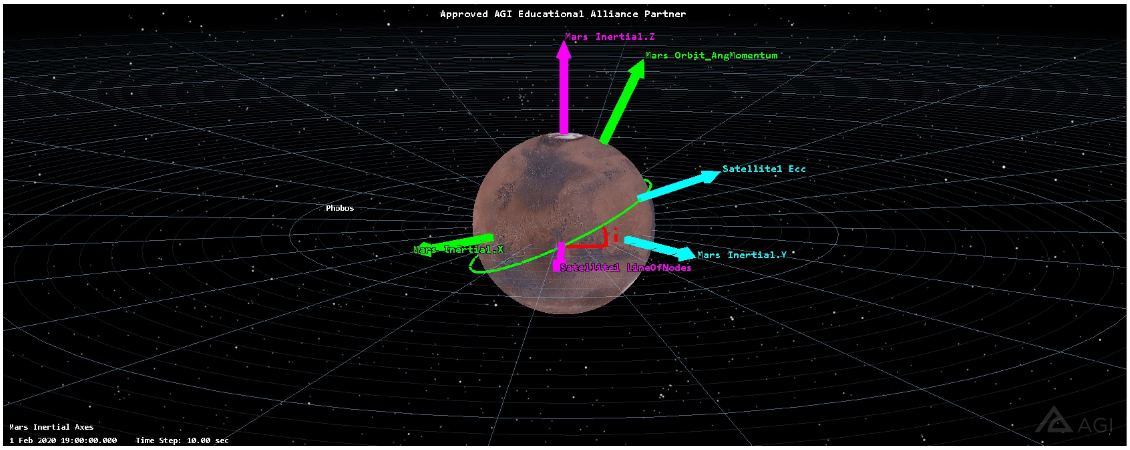

Figure 1. Inclination of the Mars Orbiter's Orbit

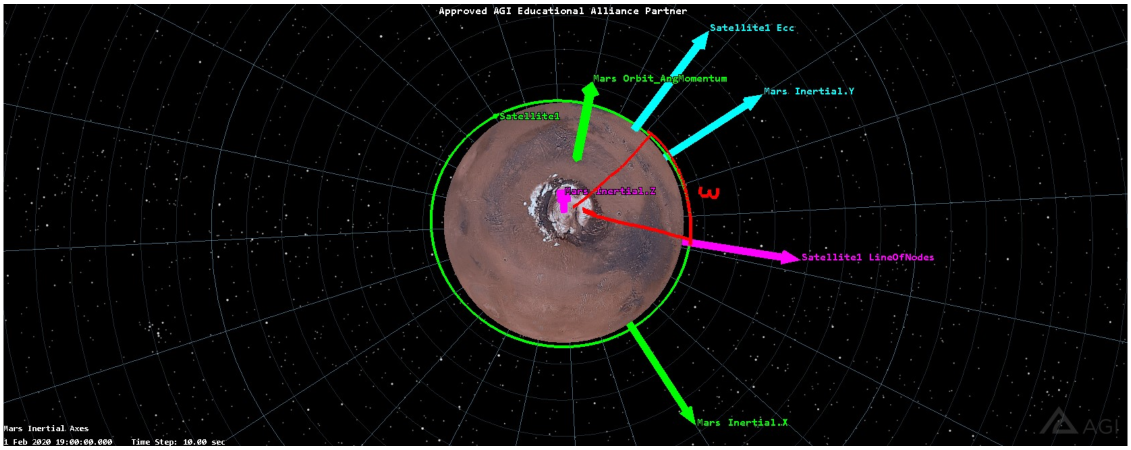

Figure 2. Argument of Periapsis of the Mars Orbiter's Orbit

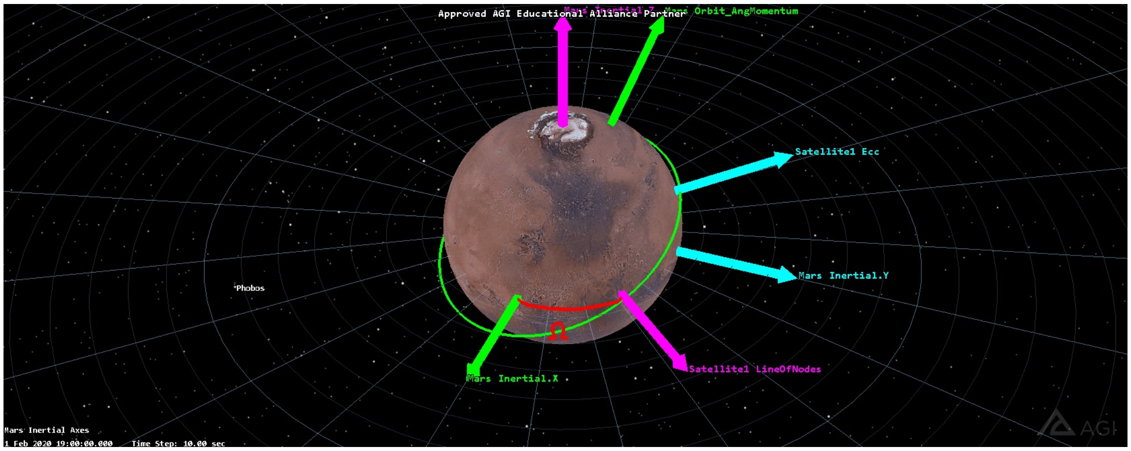

Figure 3. Right Ascension of the Ascending Node of the Mars Orbiter's Orbit

Based on the direction of the vectors, the inclination, argument of periapsis, and right ascension of the ascending node were all drawn onto the orbit. Visually, the angles make sense, since they are all drawn in accordance to the directions of the different vectors.

C. Question 3

```
State Vector in Coordinate System: Mars Inertial

Parameter Set Type:  Cartesian
        X:    -3424.6047767260106411 km              Vx:       -0.4250618960137187 km/sec
        Y:      -47.4826786239959731 km              Vy:       -3.3330712546789463 km/sec
        Z:     1171.9716855170458985 km              Vz:       -0.8929998088310547 km/sec

Parameter Set Type:  Keplerian
      sma:     3700.0000000000022737 km            RAAN:        47.99250000000004 deg
      ecc:        0.0500000000000003                  w:        63.05000000000074 deg
      inc:                   25.0035 deg             TA:        66.95609999999932 deg

Parameter Set Type:  Spherical
 Right Asc:         180.7943640060657 deg     Horiz. FPA:        2.583836056753723 deg
      Decl:         18.89033691331193 deg       Azimuth:        106.689920075399 deg
       |R|:    3619.9019480087563352 km             |V|:       3.4767068128880467 km/sec


Other Elliptic Orbit Parameters :
 Ecc. Anom:         64.34428832573826 deg       Mean Anom:         61.76193327837883 deg
 Long Peri:         111.0425000000008 deg       Arg. Lat:          130.0061000000001 deg
 True Long:         177.9986000000001 deg       Vert FPA:          87.41616394324627 deg
  Ang. Mom:         12572.54259873558 km^2/sec          p:     3690.7500000000013642 km
        C3:        -11.57523665972972 km^2/sec^2   Energy:        -5.787618329864861 km^2/sec^2
   Vel. RA:         262.7323793621065 deg       Vel. Decl:        -14.88336472428025 deg
 Rad. Peri:    3515.0000000000013642 km         Vel. Peri:        3.5768257748892105 km/sec
  Rad. Apo:    3885.0000000000036380 km         Vel. Apo:         3.236175701090236 km/sec
 Mean Mot.:        0.0526848621289765 deg/sec
    Period:         6833.082321041155 sec         Period:         113.8847053506859 min
    Period:         1.898078422511432 hr          Period:       0.07908660093797633 day
              Time Past Periapsis:          1172.289928882816 sec
         Time Past Ascending Node:          2273.741164061258 sec
   Beta Angle (Orbit plane to Sun):          28.3580176826064 deg
Mean Sidereal Greenwich Hour Angle:          56.4563929877859 deg
```

Figure 4. Final State of the Mars Orbiter's Orbit

The final state of this orbit can be verified with the inputted Keplerian elements to verify the integrity of the orbit after one period. These values reported for the position, velocity, and angular momentum from STK are slightly different than what was inputted at the creation of the scenario. This could be due to the fact that the values provided in the lab document were rounded and had slight errors associated with them. Because of this, STK would need to adjust the position, velocity, and angular momentum to have them all match up with each other using the precision that STK uses.

D. Question 4

In the case of one orbit around Mars, the ground track does not form a closed curve on the surface. This is because Mars is rotating and the satellite is rotating around Mars in the same direction but at a slower rate. The rate at which Mars rotates about it's axis is greater than the rate at which the spacecraft rotates around the same axis. As such, the spacecraft lags behind Mars slightly during one orbit and thus gives an appearance of an incomplete orbit on the ground track.

E. Question 5

During the Mars orbiters first revolution around its initial orbit, the orbiter did not pass over the location of the habitat. In order to, have the orbiter pass over the habitat in the orbiters first revolution an aspect of its orbit needed to be changed. In the end, the inclination of the orbit was changed to 85.6°. Figure 5 displays the ground track of the orbiter over one complete revolution of its orbit and how the changed inclination allows the orbiter to pass directly over the habitat.

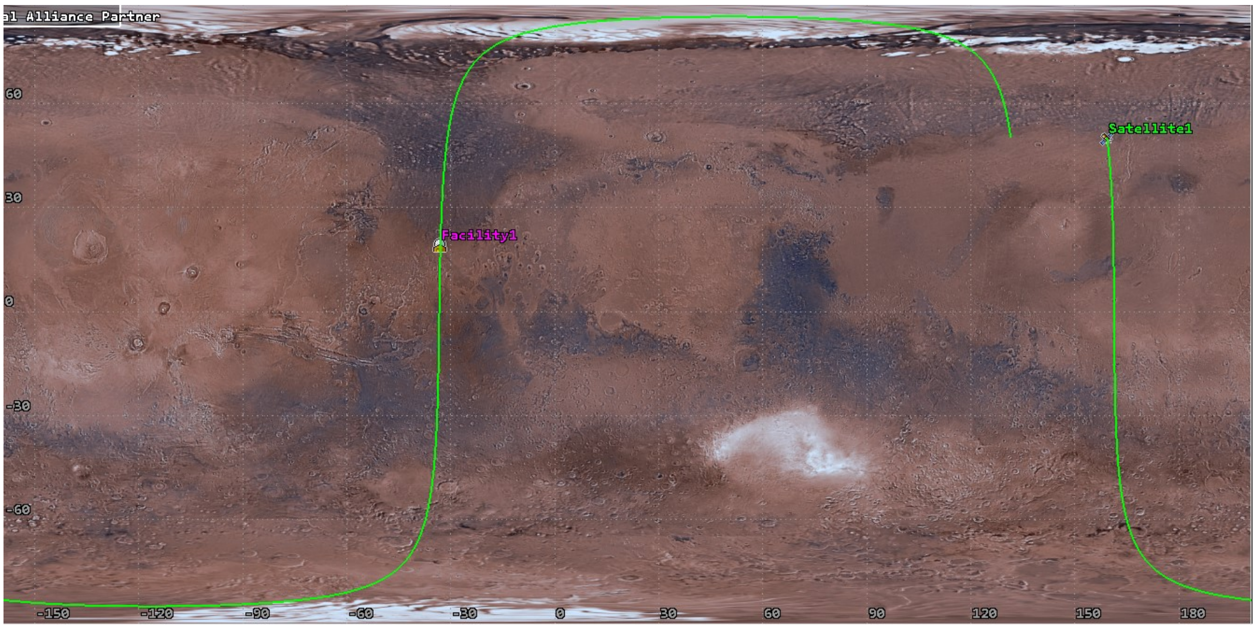

Figure 5. Ground Track of One Revolution of the Mars Orbiter with i = 85°

F. Question 6

With the new inclination of 85°, over one day the orbiter passes over the habitat 2 times. Figure 6 show the ground track for the orbiter for the entire day.

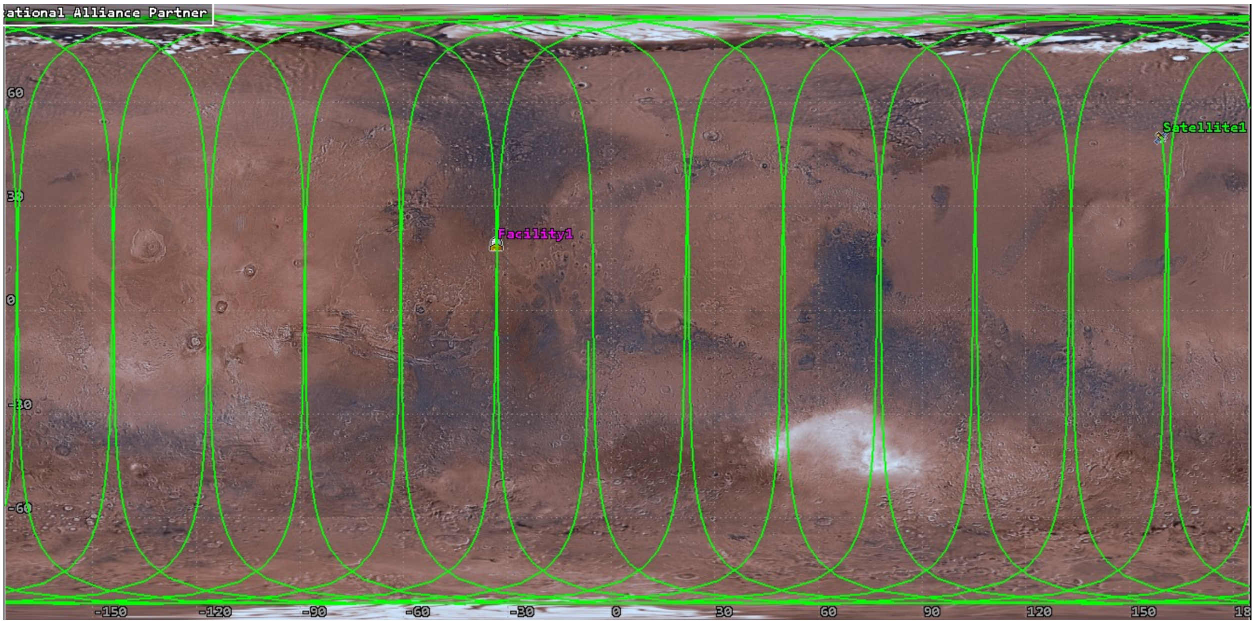

Figure 6. Ground Track for a Whole Day of the Mars Orbiter with $i = 85°$

III. Scenario II

A. Question 7

Given a body orbiting the Sun, it's of concern whether this will have an effect on Earth at all. In order to accurately model this object and determine whether it enters Earth's sphere of influence, three distinct, radius vectors are collected. Using these three sun centered coordinates, Gibb's method can be used to calculate the velocity of the object at any of these points. To do this, the following methodology was employed.

$$\vec{r}_1 = \begin{bmatrix} 88069520 & -33001760 & 3575440 \end{bmatrix} \text{ km}$$
$$\vec{r}_2 = \begin{bmatrix} 75203920 & 34243440 & 6522560 \end{bmatrix} \text{ km}$$
$$\vec{r}_3 = \begin{bmatrix} 48515280 & 68217600 & 6776880 \end{bmatrix} \text{ km}$$
$$\mu_{Sun} = 132712000000 \ \frac{km^3}{s_2}$$

Three intermediate vectors, $\vec{N}$, $\vec{D}$, and $\vec{S}$ were created as shown below.

$$\vec{N} = |\vec{r_1}| * (\vec{r_2} \times \vec{r_3}) + |\vec{r_2}| * (\vec{r_3} \times \vec{r_1}) + |\vec{r_3}| * (\vec{r_1} \times \vec{r_2}) \tag{17}$$
$$\vec{N} = \begin{bmatrix} -9.6414 * 10^{21} & -8.7518 * 10^{21} & 1.5749 * 10^{23} \end{bmatrix} km \tag{18}$$

$$\vec{D} = (\vec{r_1} \times \vec{r_2}) + (\vec{r_2} \times \vec{r_3}) + (\vec{r_3} \times \vec{r_1}) \tag{19}$$
$$\vec{D} = \begin{bmatrix} -8.3024 * 10^{13} & -7.5382 * 10^{13} & 1.5749 * 10^{15} \end{bmatrix} km \tag{20}$$

$$\overrightarrow{S} = \left(|\overrightarrow{r}_2| - |\overrightarrow{r}_3|\right) * \overrightarrow{r}_1 + \left(|\overrightarrow{r}_3| - |\overrightarrow{r}_1|\right) * \overrightarrow{r}_2 + \left(|\overrightarrow{r}_1| - |\overrightarrow{r}_2|\right) * \overrightarrow{r}_3 \tag{21}$$

$$\overrightarrow{S} = \begin{bmatrix} -3.1372 * 10^{14} & 4.5499 * 10^{14} & 6.0786 * 10^{12} \end{bmatrix} km \tag{22}$$

Using these three vectors, Gibb's method allows us to calculate a velocity vector at any three of these points, but only one is required to solve for Keplerian elements, so it was arbitrarily chosen to solve for the velocity at $\overrightarrow{r_2}$.

$$\overrightarrow{v_2} = \sqrt{\frac{\mu_{Sun}}{|\overrightarrow{N}||\overrightarrow{D}|}} * \left(\frac{\overrightarrow{D} \times \overrightarrow{r_2}}{|\overrightarrow{r_2}|} + \overrightarrow{S}\right) \tag{23}$$

$$\overrightarrow{v_2} = \begin{bmatrix} -21.8621 & 42.0417 & 0.9975 \end{bmatrix} \frac{km}{s} \tag{24}$$

The application of Gibb's method in Matlab can be seen in the Matlab Code appendix below.

B. Question 8

With a pair of position and velocity vectors at the same point in time, the Keplerian elements of this object orbiting the Sun can now be solved using the same methodology as described in Question 1.

Inclination (i) = 4.7376°
Right Ascension of the Ascening Node (Ω) = 47.8087°
Eccentricity (e) = 0.4057
Argument of Perigee (ω) = 82.3691°
Semi-major Axis (a) = $1.3887*10^8$ km
Orbital Period ($\mathbb{P}$) = 7840.3 hours
True Anomaly at $\overrightarrow{r_2}(\theta^*)$ = 10.022°

These Keplerian elements were used to model the orbit in STK to gain a better visualization of the object and to further analyze its potential interference with Earth.

C. Question 9

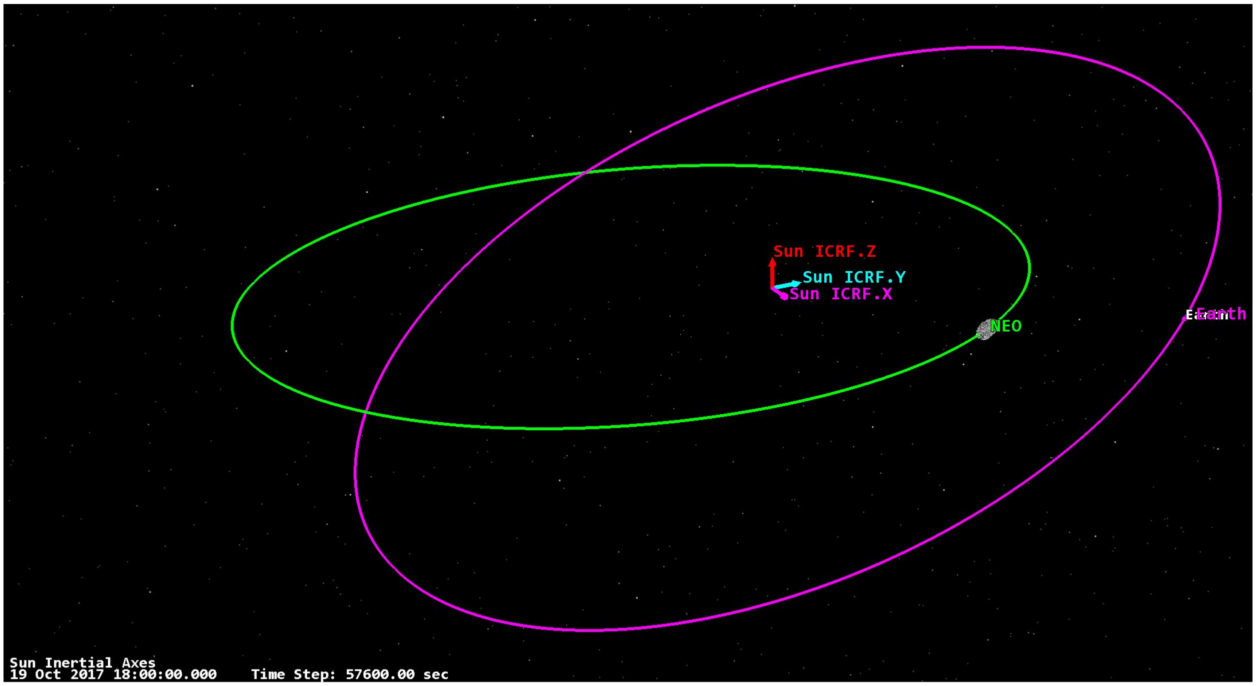

Figure 7. Heliocentric Orbit of Earth and NEO

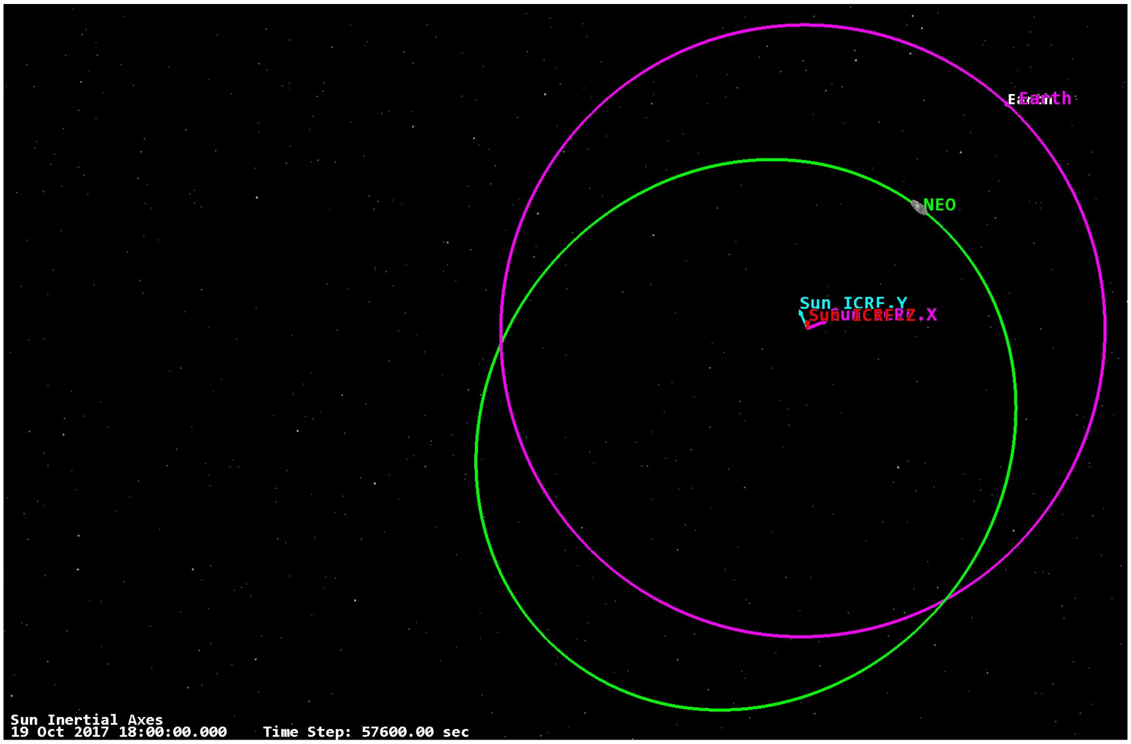

Figure 8. Top View of Earth and NEO Heliocentric Orbit

The NEO's orbit, as seen in the above figures, is an elliptical orbit with an eccentricity of 0.4057. It is not inclined very much, since it has a very small inclination. Furthermore, the orbit has a semi-major axis of about $1.38869 * 10^8$ km.

A perturbation that could alter the NEO's trajectory is the effect of the Earth's gravity. The addition of the Earth makes the two-body problem a three-body problem, and therefore the Earth's gravity must be accounted for. This force was chosen because besides the Sun, the Earth is the closest object to the NEO in this model.

D. Question 10

Based on the STK model of the NEO, the NEO does not intersect directly with the Earth's heliocentric orbit but it does cross over the orbit on two occasions. By assuming the NEO is a asteroid and using the NEO's orbit, it was determined that the NEO belongs to the Atens NEO group of asteroids. This was determined by considering the semi-major axis of the NEO's orbit and if the it's orbit crosses the orbit of the Earth. Since the orbit of the NEO has a semi-major axis less than 1 AU and it crosses Earth's orbit, it was determined that the NEO could only belong to the Atens NEO group as this was the only group that meet those two conditions.

E. Question 11

A close approach was defined as the point where the Earth and the NEO are both the closest to each other while on their respective trajectories. The close approach for this model was seen to happen around April 4, 2018.

IV. Conclusion and Recommendations

In this lab, STK was used as a tool to model and visualize orbits around Mars as well as the heliocentric orbits of the Earth and a NEO. This software can be difficult to navigate, but is excellent when the user already conceptually understands orbits and can quickly verify the results. This lab also allowed the team to become more familiar with applying the Conic Equation and Kepler's Equation to solve for orbit characteristics, as well as using Gibb's method to calculate preliminary orbit determination.

V. Acknowledgements

During the completion of this lab, the group sought out to complete lab tasks collaboratively, so all members good gather a good understanding of all content. This mainly included calculations for Keplerian elements as well as Gibb's Method. However, Tyler and Aneesh headed the STK modeling for Scenario I and Scenario II, respectively, while Cole worked on answering any technical questions in the lab report. The group as a whole then worked to fill out the lab report, mainly the introduction, abstract, and conclusion. We would like to thank the teaching and lab assistants for their assistance throughout the lab. We would also like to acknowledge Professor McMahon for his continued assistance and clarification during lecture.

VI. References

References

[1] Howard.D.Curtis, Orbital Mechanics for Engineering Student Third Edition

[2] ASEN 3200 Orbit Mechanics and Attitude Dynamics- Spring 2021 LABORATORY O-2 Write Up

[3] “NEO Basics.” NASA

VII. MATLAB Code

```
%% Author: Aneesh Balla, Tyler Candler, Cole Macpherson
%% ASEN 3200 Orbital Mechanics
%% Lab O-2
%% Written: 2/12/2021

%%
clc;
clear;
close all;

mu = 132712000000; % Sun gravitational parameter

% Gibbs method vectors
R1 = [0.5887, -0.2206, 0.0239]*1.496e8;
R2 = [0.5027, 0.2289, 0.0436]*1.496e8;
R3 = [0.3243, 0.4560, 0.0453]*1.496e8;

% Finding V2
[V2, ierr] = gibbs(R1,R2,R3,mu);

% Outputting orbital elements
[a, e, incl, w, Omega, theta, T] = OrbitalElements(R2,V2,mu);

% Ny < 0
Omega = 360 - Omega;

% r dot v < 0
theta = 360 - theta;



%% Author: Aneesh Balla, Tyler Candler, Cole Macpherson
%% ASEN 3200 Orbital Mechanics
%% Lab O-2
%% Written: 2/12/2021

%%
function [V2, ierr] = gibbs(R1,R2,R3,mu)
% Uses Gibbs method to calculate the N,D,S vectors and then find V2 vector

tol = 1e-4;
ierr = 0;

r1 = norm(R1);
r2 = norm(R2);
r3 = norm(R3);

c12 = cross(R1,R2);
c23 = cross(R2,R3);
c31 = cross(R3,R1);

if abs(dot(R1,c23)/r1/norm(c23)) > tol
    ierr = 1;
```

```
end

N = r1*c23 + r2*c31 + r3*c12;

D = c12 + c23 + c31;

S = R1*(r2 - r3) + R2*(r3 - r1) + R3*(r1 - r2);

V2 = sqrt(mu/norm(N)/norm(D))*(cross(D,R2)/r2 + S);
end
```

```
%% Author: Aneesh Balla, Tyler Candler, Cole Macpherson
%% ASEN 3200 Orbital Mechanics
%% Lab O-2
%% Written: 2/12/2021

%%
function [a, e, incl, w, Omega, theta, T] = OrbitalElements(R,V,mu)

r = norm(R);

Z = [0, 0, 1];

H = cross(R,V); % Angular momentum vector [km^2/s]
h = norm(H); % Angular momentum [km^2/s]

incl = acosd(H(3)/h); % [degrees]

N = cross(Z,H);
n = norm(N);

Omega = acosd(N(1)/n); % RAAN [degrees]

E = cross(V,H)/mu - R/r; % Eccentricity vector
e = norm(E); % Eccentricity

w = acosd(dot(N,E)/(n*e)); % [degrees]

theta = acosd(dot(E,R)/(e*r)); % [degrees]

p = h^2/mu; % Semiparameter [km]
a = p/(1 - e^2); % Semimajor axis [km]

T = 2*pi*sqrt(a^3/mu); % Orbital period [s]
end
```