UNIVERSITY OF COLORADO - BOULDER

ASEN 3200

LAB O-3

Orbital Lab 3

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Contents

In Lab 3, two different scenarios were modeled in order to solidify understanding of Sun-synchronous orbits as well as interplanetary orbit transfers. Matlab and STK were used to calculate orbital elements and then model a Sun-synchronous orbit. This orbit allowed the team to study the effects of J2 perterbations on an orbit's semimajor axis, RAAN, and argument of periapsis. Matlab was then used to calculate the different velocities needed for completing two different orbit transfers, a Hohmann transfer and a bielliptic transfer. After modeling these in STK, the team was able to analyze the different times of flight and velocity requirements between the two.

I. Introduction

The purpose of this lab is to create two different orbitting scenarios to model a sun-synchronous orbits and to better comprehend Hohmann and Bi-elliptic Transfers. Scenario I will take a look a satellite orbitting the Earth. The J2 perterbations will be taken into account and this will help facilitate the process of designing a sun-synchronous orbit. Scenario II will focus on orbit transfers from Earth to Mars in heliocentric orbit transfers. The maneruver velocities were calculated for these two different transfers and then compared and discussed.

II. Scenario I

The first scenario focused on designing a sun synchronous orbit of a satellite orbiting the Earth. Given a set of requirements a sun-synchronous orbit was constructed as follows. Since, a range of acceptable requirements were given for some variables, the average of that range was taken for these calculations. The specific requirements were

1) An orbital period of 1.62 hours
2) A perigee altitude between 500 km and 600 km
3) A perigee that regresses over time
4) After one orbital period, the ascending node should possess a longitude of zero with a tolerance of 0.1 degrees
5) An initial argument of perigee equal to 0 degrees

A. Question 1

To start, the semi-major axis of the orbit was found, using equation (1), since the period of the orbit and the gravitational constant of Earth are known.

$$a = \left(\mu \left(\frac{T}{2\pi}\right)^2\right)^{\frac{1}{3}} \tag{1}$$

The semi-major axis was found to be 7002.8 km. From this, the eccentricity of the orbit can be found using the averaged perigee altitude value of 550 km and the semi-major axis value just found in equation (2).

$$e = 1 - \frac{r_p}{a} \tag{2}$$

From that calculation the eccentricity of the orbit was found to be 0.0107. In order to make the calculation and visualization processes easier, the variable k was used to define the coefficient of the $\dot{\omega}$ and $\dot{\Omega}$.

$$\dot{\Omega} = -\left[\frac{3}{2}\frac{\sqrt{\mu}J_2R^2}{\left(1-e^2\right)^2 a^{\frac{7}{2}}}\right]cos\left(i\right) \tag{3}$$

$$\dot{\omega} = -\left[\frac{3}{2}\frac{\sqrt{\mu}J_2R^2}{\left(1-e^2\right)^2 a^{\frac{7}{2}}}\right]\left(\frac{5}{2}sin^2\left(i\right)-2\right) \tag{4}$$

$$k = -\left[\frac{3}{2}\frac{\sqrt{\mu}J_2R^2}{\left(1-e^2\right)^2 a^{\frac{7}{2}}}\right] \tag{5}$$

The next variable that can be calculated with the given data is the inclination of the orbit. This is computed using the $\dot{\Omega}$ value for a sun-synchronous orbit which is 0.9856 degrees per day or $1.99 * 10^{-7}$ radians per second. Using equation (6), the inclination can be computed.

$$i = cos^{-1}\left(\frac{\dot{\Omega}}{k}\right) \tag{6}$$

From this calculation the inclination was found to be 97.88 degrees. Using this new found value, $\dot{\omega}$ can now be computed using equation (7)

$$\dot{\omega} = k\left(\frac{5}{2}sin^2\left(i\right)-2\right) \tag{7}$$

From this equation, it was found that $\dot{\omega}$ is $-6.5756 * 10^{-7}$ degrees per second or -0.0568 degrees per day. It is now possible to determine the longitude, on Earth, the spacecraft ends at after one orbit. By utilizing the Earth's rotation rate, 15 degrees per hour or $7.292 * 10^{-5}$ radians per second, and the variables above in equation (8).

$$\Delta\lambda = -\left(\dot{\omega}_E - \dot{\Omega}\right)T \tag{8}$$

$\Delta\lambda$ was found to be -24.2996 degrees. Subtracting this from 360 degrees and we find the ending longitude after one orbit was 335.7 degrees. Finally it is possible to find the RAAN of the orbit by subtracting the ending longitude location from 360 and then subtracting the longitude location of the inertial x-axis which was estimated to be -81.55 degrees. Completing these calculations, it is found that the RAAN is 105.85 degrees.

B. Question 2

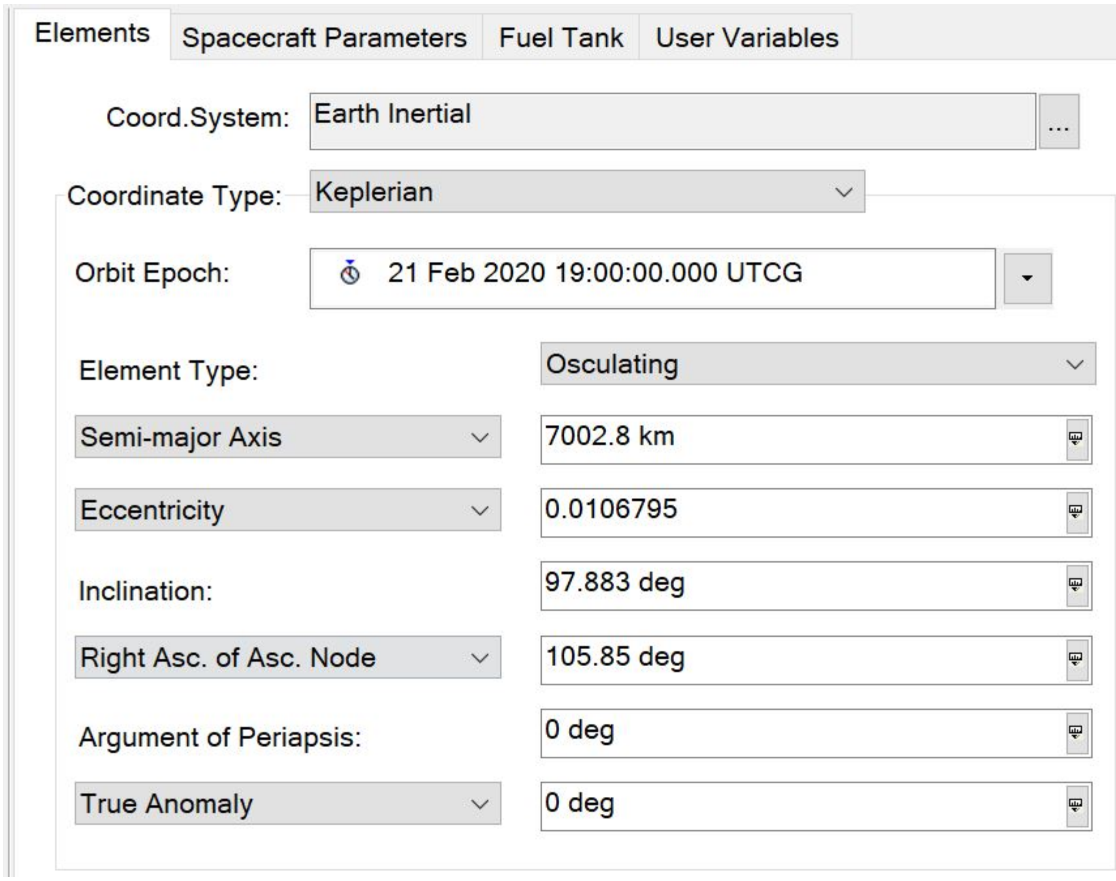

Figure 1. Orbital Elements and Initial Epoch

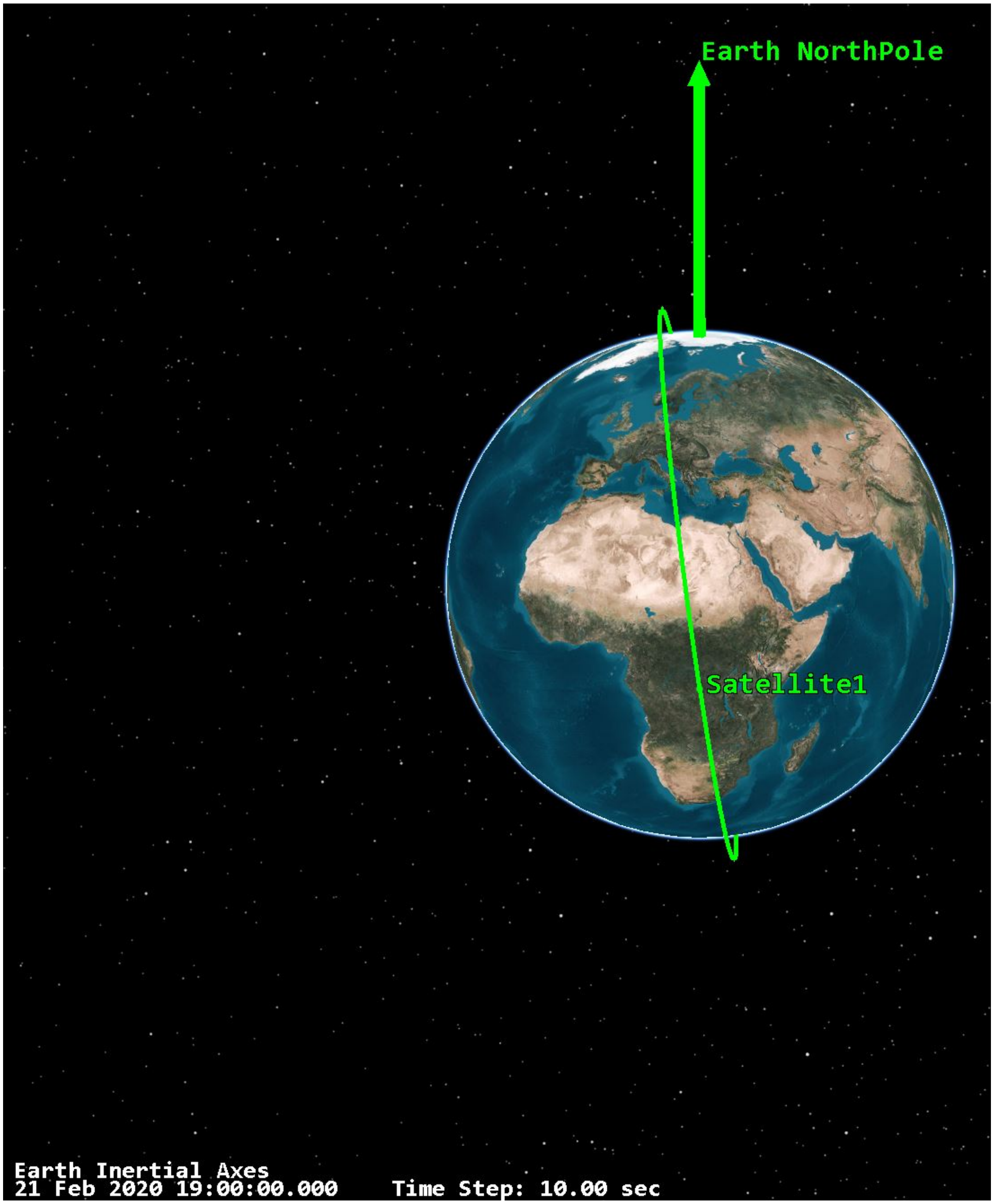

Figure 2. 3D Trajectory of Satellite Orbit with North Pole Vector

The orbital elements that were calculated for question 1 are used in the STK modeling as seen above in Figure 1. The resulting trajectory can be seen in Figure 2.

C. Question 3

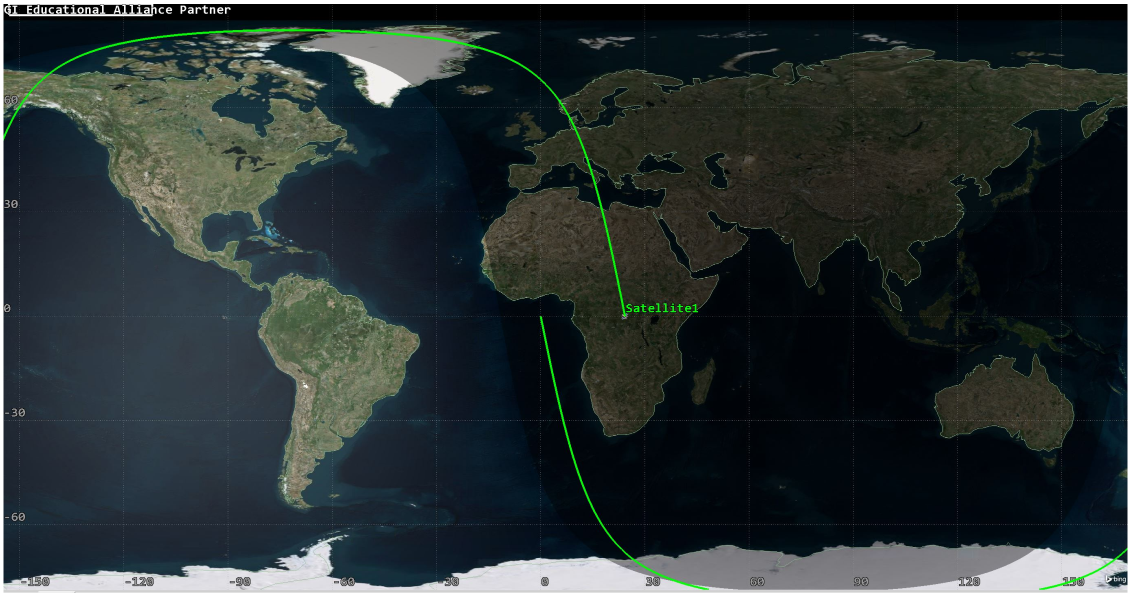

Figure 3. Ground Track for One Orbit

The resulting ground track for the orbit after one orbital period can be seen above in Figure 3.

D. Question 4

```
  Period:            5832.01356573415 sec           Period:         97.20022609556916 min
  Period:           1.620003768259486 hr            Period:       0.06750015701081191 day
                      Time Past Periapsis:        7.344170321200762 sec
                 Time Past Ascending Node:        4.588496608984723 sec
      Beta Angle (Orbit plane to Sun):             54.477631248845 deg
Mean Sidereal Greenwich Hour Angle:               100.535838859311 deg

Geodetic Parameters:
  Latitude:           0.2682926838199468 deg
 Longitude:         -0.04017127332467422 deg
  Altitude:         549.8798757056874820 km
```

Figure 4. Results of Calculated Orbital Elements

After propagating the orbit, a summary of the orbit and state characteristics can be obtained. The resulting characteristics are shown above in Figure 4. The orbital elements calculated in question 1 and the orbit itself satisfies the conditions that were required to be met. The period of the orbit was 1.62 hours like the requirement stated. The perigee altitude was 550 km, which is what was used in the calculations above and is between 500 km and 600 km. Furthermore, the longitude of the ascending node is about -0.04 degrees, which lies in the 0.1 degree tolerance that is required.

E. Question 5

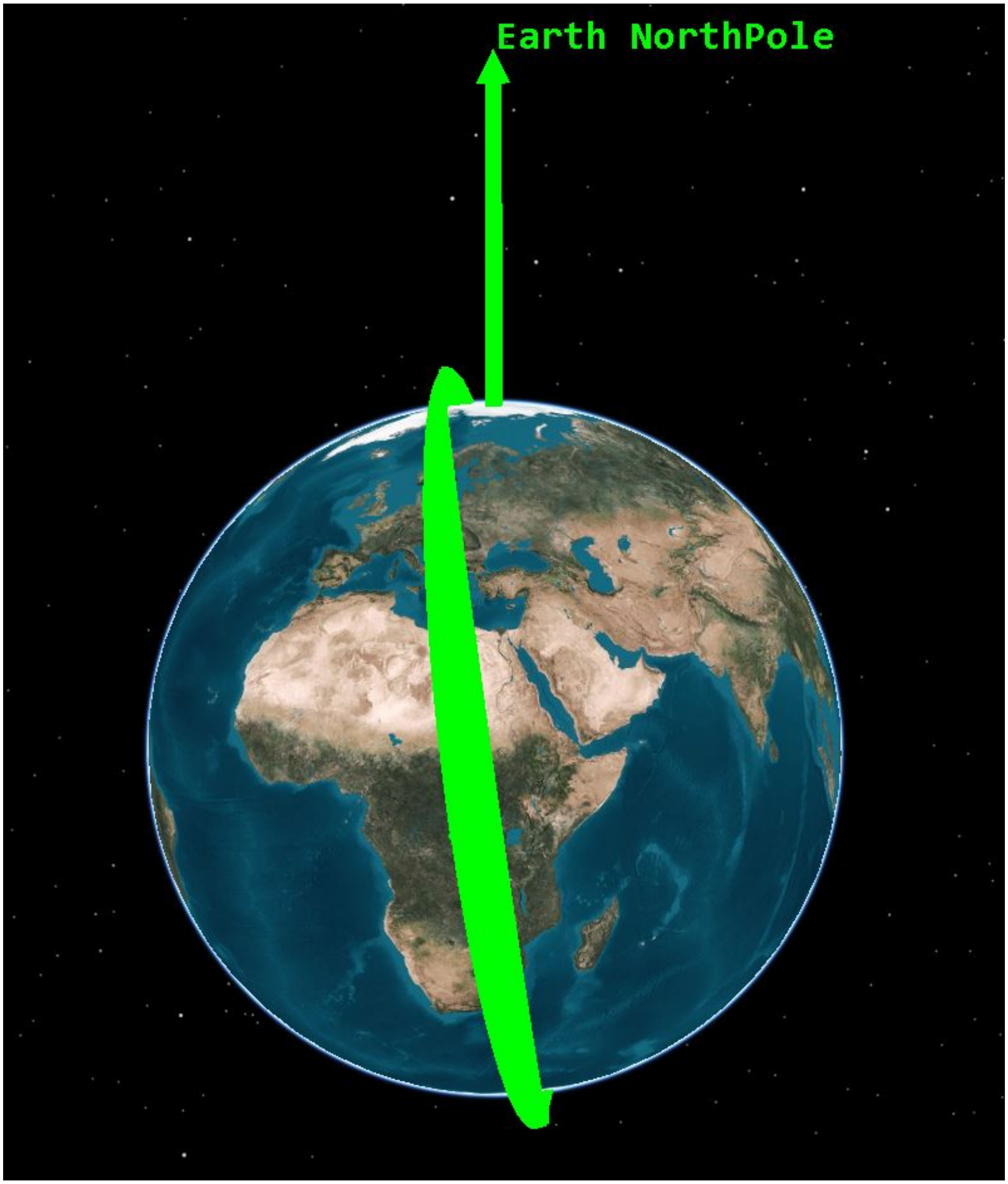

Figure 5. 3D Trajectory of Orbit for 100 Periods

The trajectory of the satellite over 100 orbital periods can be seen above in Figure 5. The trajectory is displayed in an Earth inertial coordinate system, specifically the Earth-Centered Inertial (ECI) coordinate system.

F. Question 6

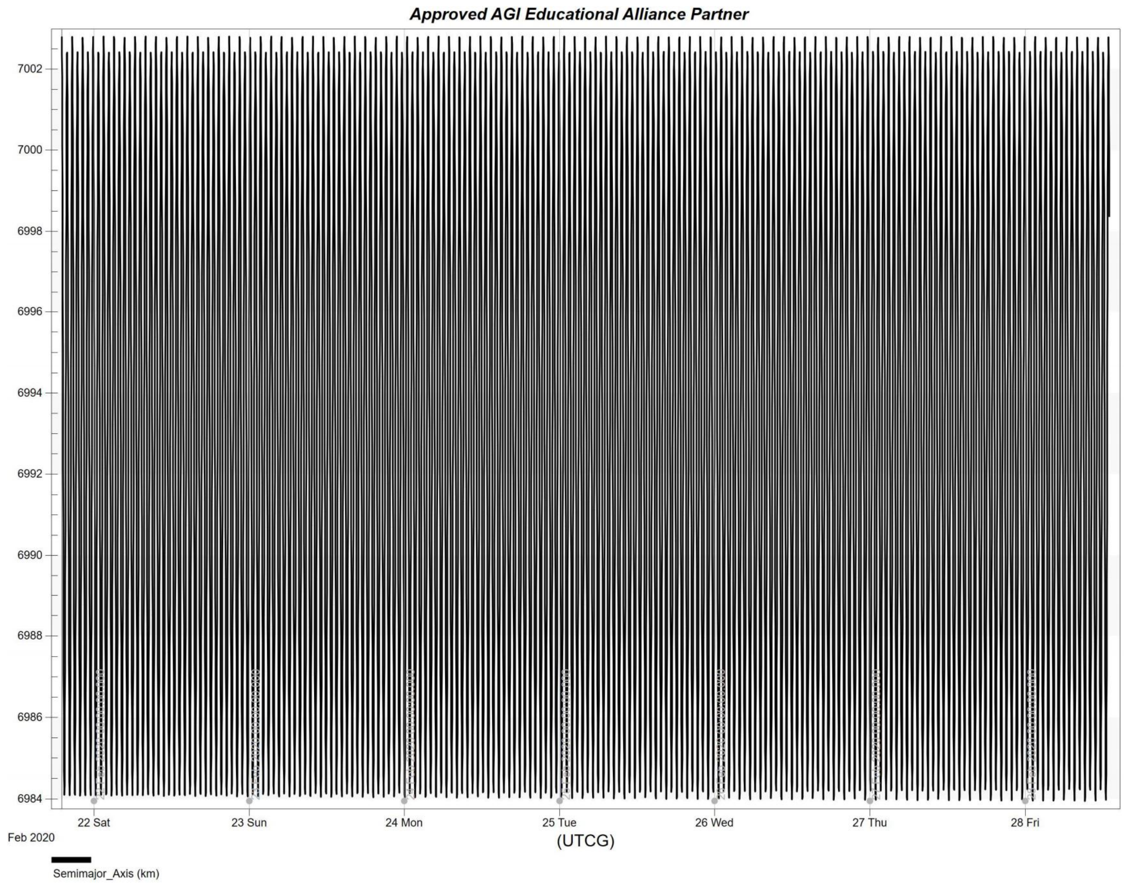

Figure 6. Semimajor Axis vs. Time for 100 Orbital Periods

The change in semimajor axis over time for 100 periods can be seen above in Figure 6. The semimajor axis appears to change a lot, however, the scales of the graph show that it only changes by a few kilometers. Furthermore, it also varies periodically over time with a fixed average value. This is consistent with what was presented in lecture and what was seen in the textbook, as it's expected that the semimajor axis should not have any secular changes due to J2 perturbations.

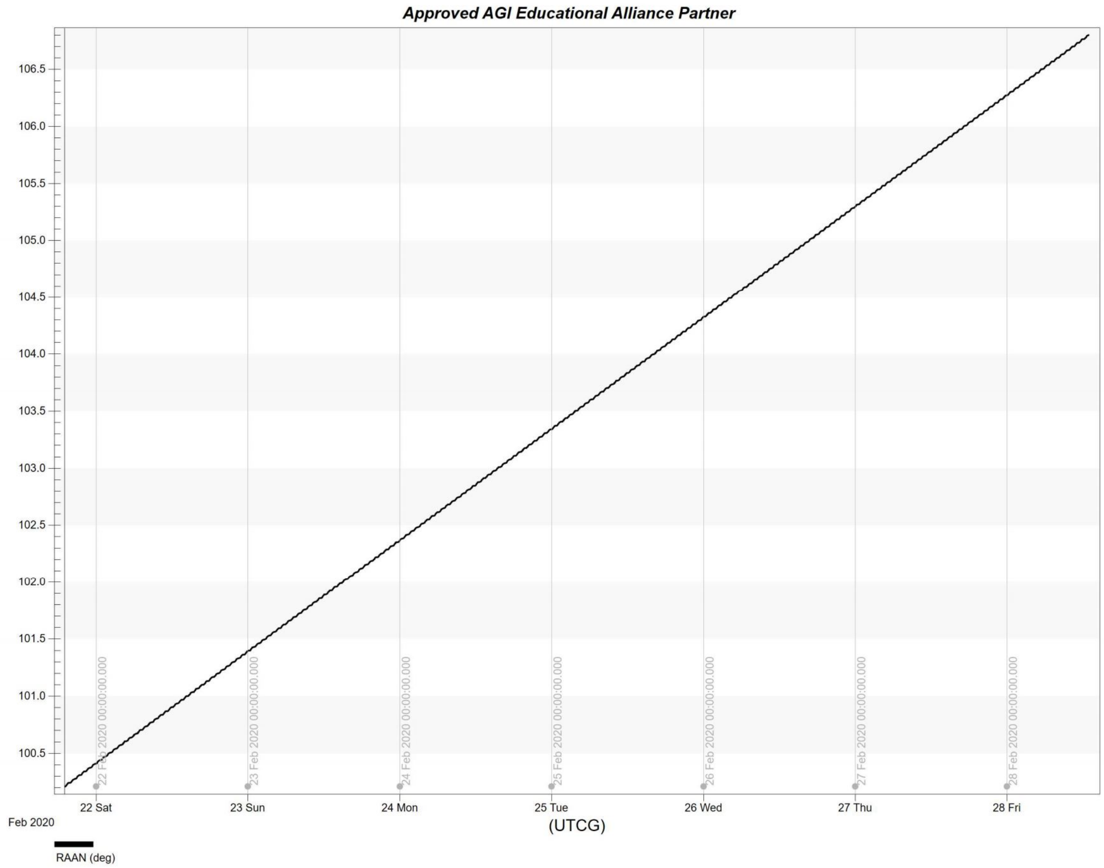

Figure 7. RAAN vs. Time for 100 Orbital Periods

The change in Right Ascension of Ascending Node (RAAN) over time can be seen above in Figure 7. The RAAN increases linearly with time over 100 orbital periods. This is consistent with what J2 perturbation is. A J2 perturbation is expected to move the RAAN over time at a constant rate, which is what is seen in the figure above.

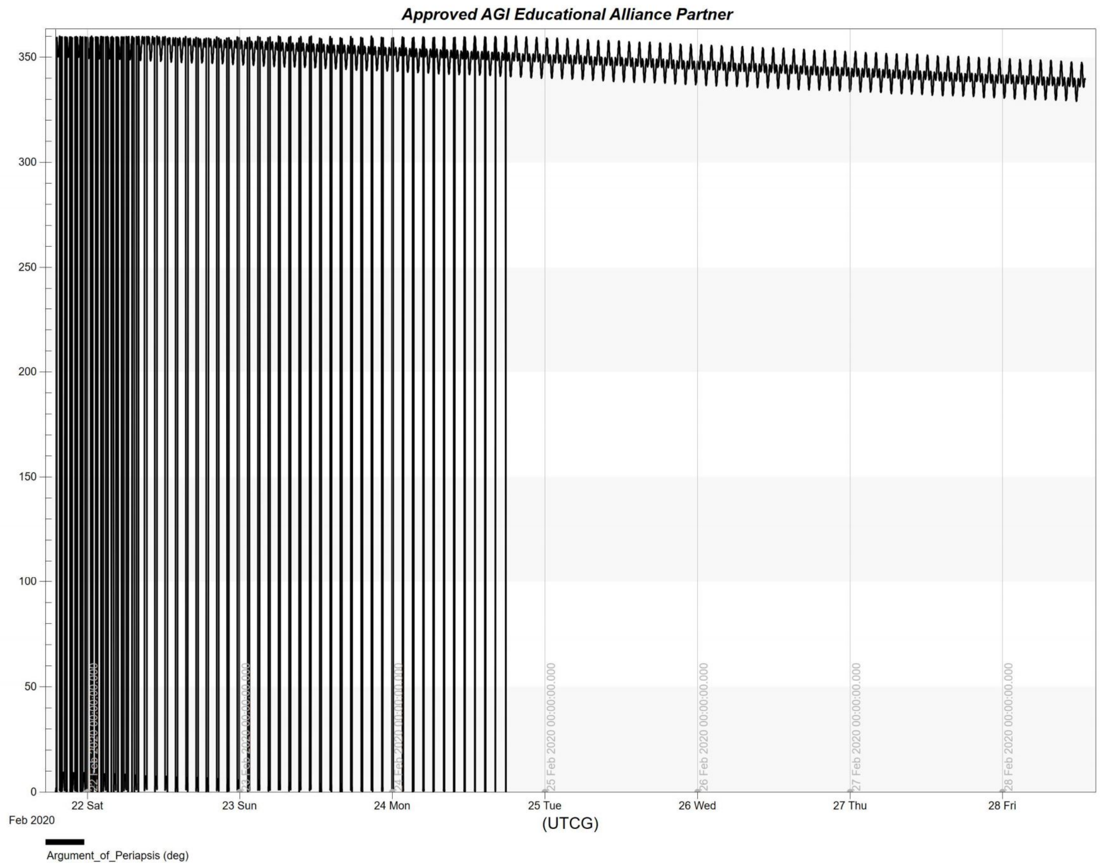

Figure 8. Argument of Periapse vs. Time for 100 Orbital Periods

The change in Argument of Periapsis over time can be seen above in Figure 8. The argument of periapsis varies dramatically over the roughly the first half 100 orbital periods. Over the rest of the orbital periods angle does not change as much. A dramatic change in the argument of periapsis is expected, since it is one of the variables that a J2 perturbation affects the most.

III. Scenario II

This scenario focused on Hohmann transfer from the Earth to Mars. The spacecraft leaves the Earth via an impulsive maneuver and is then placed on a heliocentric orbit that will then lead to the orbit of Mars with a few more maneuvers. A key assumption in this scenario is that Earth and Mars both travel along circular co planar orbits.

A. Question 7

To calculate the time of flight and the delta V's associated with a Hohmann transfer it is first important to define the Hohmann transfer orbit. To start, the semi-major axis was found by taking one half of the addition of the orbits radius of perigee and radius of apogee. This results in a semi-major axis value of $1.8875 * 10^8$ kilometers. From this the time of flight can be calculated by using equation (9).

$$TOF = \pi \sqrt{\frac{a^3}{\mu_{Sun}}} \tag{9}$$

From this equation the time of flight for the Hohmann transfer orbit was found to be $2.2363 * 10^7$ seconds or 258.828 days. From here the orbital velocities of the Earth and Mars needed to be found as well as the orbital velocities of the Hohmann transfer at perigee and apogee. To find the velocities of Earth and Mars, equation (10) can be used as their orbits are assumed to be circular. As for the Hohmann transfer orbit, the more general equation, equation (11), was used to determine the velocities at perigee and apogee.

$$V = \sqrt{\frac{\mu_{Sun}}{a}} \tag{10}$$

$$V = \sqrt{\frac{2\mu_{Sun}}{r} - \frac{\mu_{Sun}}{a}} \tag{11}$$

From thees equations the orbital velocities of Earth and Mars were found to be 29.78 $\frac{km}{s}$ and 24.13 $\frac{km}{s}$, respectively. When looking at the Hohmann transfer at perigee and apogee the velocities were found to be 32.73 $\frac{km}{s}$ and 21.48 $\frac{km}{s}$, respectively. To find the two required ΔVs, the velocity of the Earth needs to be subtracted from the Hohmann transfer perigee velocity, and the velocity at apogee of the Hohmann transfer needs to be subtracted from the velocity of Mars. Doing so results in a departure ΔV equal to 2.94 $\frac{km}{s}$ and an arrival ΔV equal to -2.65 $\frac{km}{s}$. The negative ΔV value indicates that the burn is to slow down where as a positive ΔV value indicates a burn to speed up. When calculating the total ΔV, the absolute value of all the ΔVs are taken and then all of those values are added together. For this case, it is found that the total ΔV is equal to 5.59 $\frac{km}{s}$. When designing a Hohmann transfer like the one above it is important to consider what was left out for simplification. In this case, the initial and final orbit of the spacecraft around the Earth and Mars were not considered, although these orbits would affect the required ΔV values because of the additional velocity the orbits provide. When considering the size of the total ΔV of this transfer, it seems to be around the total ΔV values seen throughout this class. For example the Galileo mission, had a total ΔV value of around 7 $\frac{km}{s}$, but of course Galileo was travelling to Jupiter using gravity assists which decrease the overall ΔV required. Figure 9, shows the Hohmann transfer for this problem as well as the ΔV burns and there directions.

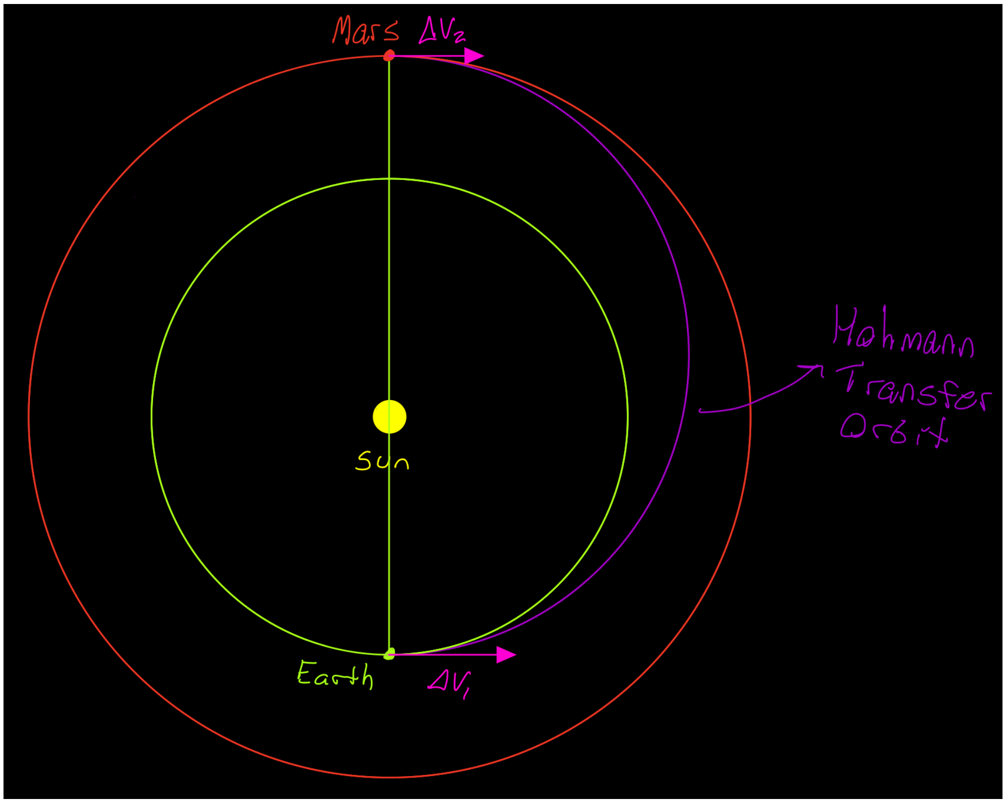

Figure 9. Diagram of the Hohmann Transfer Calculated in this Section

B. Question 8

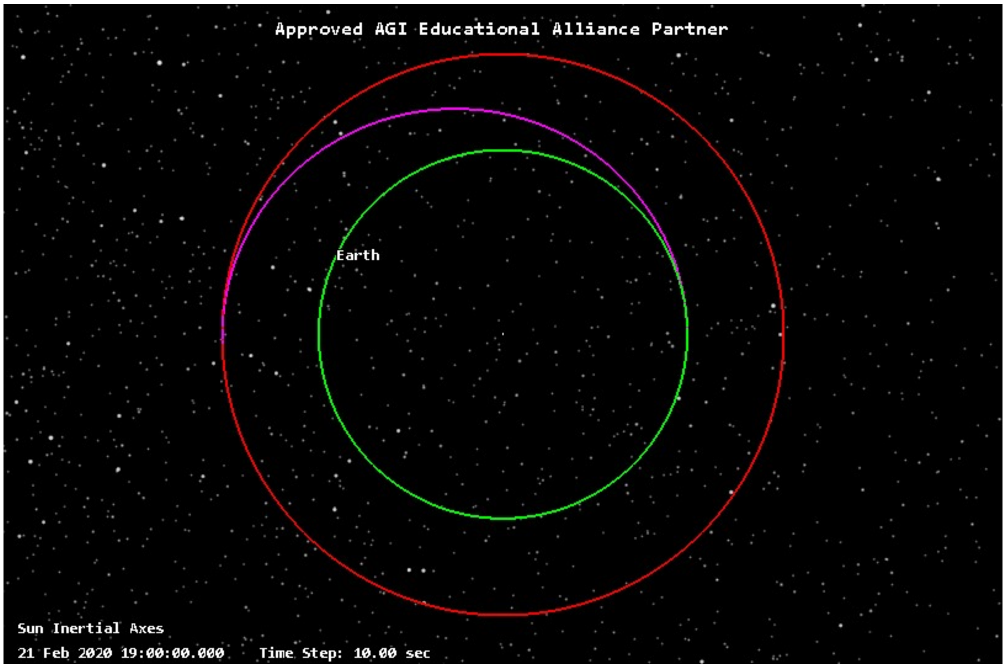

Figure 10. Hohmann Transfer from Earth to Mars Modelled in STK

C. Question 9

```
Satellite State at End of Segment:
----------------------------------

UTC Gregorian Date: 24 Sep 2023 20:56:40.000  UTC Julian Date: 2460212.37268519
Julian Ephemeris Date: 2460212.37348593
Time past epoch: 1.13277e+08 sec   (Epoch in UTC Gregorian Date: 21 Feb 2020 19:00:00.000)

State Vector in Coordinate System: Sun Inertial

Parameter Set Type:  Cartesian
         X:   -2.2789688146267474e+08 km              Vx:        0.0481348525774857 km/sec
         Y:  -450574.5521174150053412 km              Vy:      -24.1315082502324145 km/sec
         Z:        0.0000000000000000 km              Vz:        0.0000000000000000 km/sec

Parameter Set Type:  Keplerian
       sma:    2.2789641297048438e+08 km            RAAN:                         0 deg
       ecc:        0.0000180418226381                  w:         282.9567791177474 deg
       inc:                         0 deg             TA:         257.1565001267475 deg

Parameter Set Type:  Spherical
 Right Asc:         180.1132792444949 deg     Horiz. FPA:      -0.00100786155156513 deg
      Decl:                         0 deg        Azimuth:                        90 deg
       |R|:    2.2789732687734500e+08 km             |V|:        24.1315562572136635 km/sec


Other Elliptic Orbit Parameters :
 Ecc. Anom:         257.1575079862782 deg       Mean Anom:         257.1585158478297 deg
 Long Peri:         282.9567791177474 deg        Arg. Lat:         180.1132792444949 deg
 True Long:         180.1132792444949 deg        Vert FPA:         90.00100786155156 deg
  Ang. Mom:         5499517163.558414 km^2/sec          p:    2.2789641289630240e+08 km
        C3:        -582.3366779323877 km^2/sec^2   Energy:        -291.1683389661939 km^2/sec^2
   Vel. RA:         270.1142871060464 deg       Vel. Decl:                         0 deg
 Rad. Peri:    2.2789230130382171e+08 km        Vel. Peri:       24.1320884123530028 km/sec
  Rad. Apo:    2.2790052463714707e+08 km         Vel. Apo:         24.13121765434502 km/sec
 Mean Mot.:     6.066975136809239e-06 deg/sec
    Period:         59337642.21577678 sec          Period:         988960.7035962796 min
    Period:         16482.67839327133 hr           Period:         686.7782663863053 day
               Time Past Periapsis:          42386611.12810744 sec
```

Figure 11. Final Propagate Segment Values for the Hohmann Transfer

The data displayed above depicts the final state of the spacecraft after it completed the Hohmann transfer. When we compare it to the calculated value of what we except, the values produced by STK in figure 11, are very similar. The important values such as the semi-major axis should be the semi-major axis of Mars and this is true based on the data. Additionally, the eccentricity is nearly zero, which means the spacecraft is in a circular orbit as desired. Lastly, the velocity of the spacecraft at this point is close to the predicted velocity once in the orbit of Mars. Based on this, it can be concluded that the assumption of circular and co-planar orbits for the Earth and Mars is a reasonable assumption as the values are as expected.

D. Question 10

Following the same process as described in section III part A, with the addition of a few more ΔVs along the way, the ΔVs from Earth orbit to the first transfer orbit, from the first transfer orbit to the second transfer orbit, and from the second transfer orbit to the final Mars orbit were found. These ΔV values were found to be 6.1975 $\frac{km}{s}$, 2.069 $\frac{km}{s}$, and -3.1551 $\frac{km}{s}$, respectively. When looking a the signs associated with each ΔV value, it is seen that every burn, except the burn for getting into Mars orbit from the second transfer orbit, is a burn to increase the velocity. The burn to get into Mars orbit from the second transfer orbit is to slow down the spacecraft. From the ΔV values calculated above the total ΔV can be found by taking the absolute value of all the ΔVs and adding them together. When this is done, it is found that the total ΔV required for the bi-elliptic transfer is 11.4216 $\frac{km}{s}$.

Additionally, the time of flight for the bi-elliptic was found to be $8.8126 * 10^7$ seconds or 1020 days.

E. Question 11

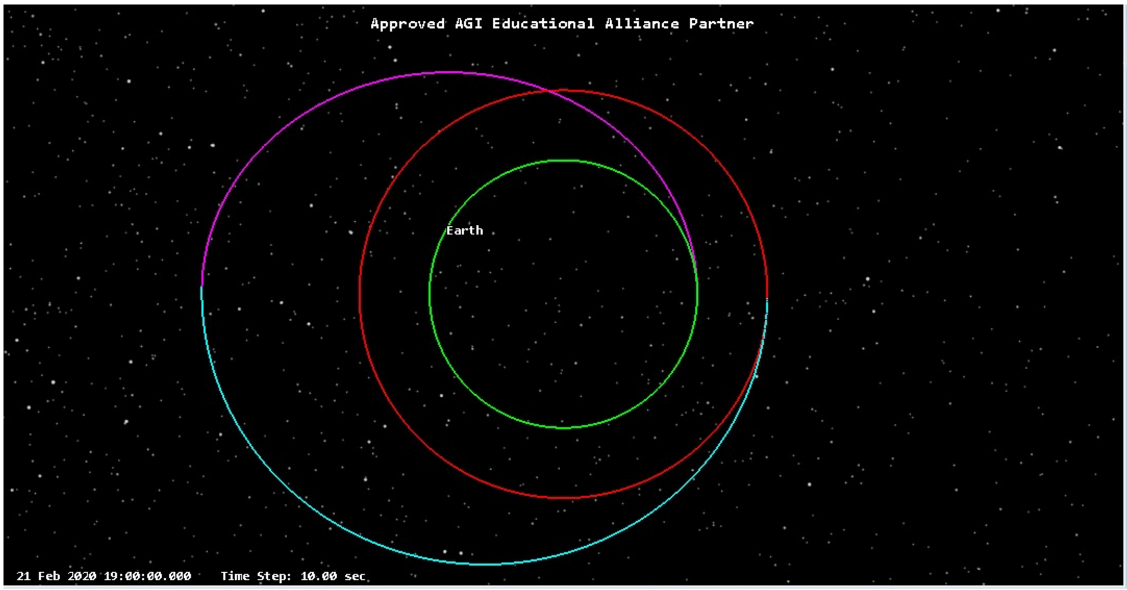

Figure 12. Bi-elliptic Transfer from Earth to Mars Modelled in STK

F. Question 12

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---------------------------------
Satellite State at End of Segment:
---------------------------------

UTC Gregorian Date: 25 Oct 2025 00:20:00.000  UTC Julian Date: 2460973.51388889
Julian Ephemeris Date: 2460973.51468963
Time past epoch: 1.7904e+08 sec   (Epoch in UTC Gregorian Date: 21 Feb 2020 19:00:00.000)

State Vector in Coordinate System: Sun Inertial

Parameter Set Type:  Cartesian
         X:    2.2789622978383008e+08 km             Vx:       -0.0475336405487058 km/sec
         Y:   456674.4453881461522542 km             Vy:       24.1316407837626414 km/sec
         Z:        0.0000000000000000 km             Vz:        0.0000000000000000 km/sec

Parameter Set Type:  Keplerian
       sma:    2.2789761465574360e+08 km           RAAN:                         0 deg
       ecc:        0.0000343477179356                 w:         276.9163680806788 deg
       inc:                         0 deg            TA:         83.19844506622852 deg

Parameter Set Type:  Spherical
 Right Asc:        0.1148131469073237 deg     Horiz. FPA:      0.001954121265088409 deg
      Decl:                         0 deg       Azimuth:                        90 deg
       |R|:    2.2789668734150863e+08 km            |V|:       24.1316875987474262 km/sec


Other Elliptic Orbit Parameters :
 Ecc. Anom:         83.19649094098777 deg       Mean Anom:         83.19453681972308 deg
 Long Peri:         276.9163680806788 deg       Arg. Lat:         0.1148131469072931 deg
 True Long:        0.1148131469072931 deg       Vert FPA:          89.99804587873491 deg
  Ang. Mom:         5499531660.516151 km^2/sec          p:    2.2789761438687780e+08 km
        C3:        -582.3336073192844 km^2/sec^2   Energy:        -291.1668036596422 km^2/sec^2
   Vel. RA:         90.11285902564224 deg       Vel. Decl:                         0 deg
 Rad. Peri:    2.2788978689275721e+08 km       Vel. Peri:       24.1324182864946941 km/sec
  Rad. Apo:    2.2790544241873002e+08 km        Vel. Apo:         24.13076055644112 km/sec
 Mean Mot.:     6.066927150885888e-06 deg/sec
    Period:         59338111.54258431 sec          Period:         988968.5257097385 min
```

Figure 13. Final Propagate Segment Values for the Bi-Elliptic Transfer

Again, as seen with the Hohmann transfer the values for the semi-major axis, eccentricity, and velocity are all as expected and extremely close to the calculated values. Thus, for the bi-elliptic transfer the assumption of circular and co-planar orbits for the Earth and Mars is a reasonable assumption as well. Although, since the bi-elliptic transfer orbits have an apogee radius of 2.7 AU, it is possible that the spacecraft could encounter other celestial bodies. This is because the asteroid belt lies between the orbit of Mars and the orbit of Jupiter, which is where the spacecraft flies at it's apogee radius. The asteroid belt is is found between 2.2 and 3.2 AU and since the spacecraft reaches apogee at 2.7 AU, this is well within the range of the asteroid belt. As such it is very likely that the spacecraft would encounter at least one other celestial body along the way if no many more.

G. Question 13

When comparing the Hohmann transfer and bi-elliptic transfer, it is clear that the Hohmann transfer is better for this transfer than the bi-elliptic orbit. When comparing the two total ΔVs for each transfer orbit the ΔV value associated with the bi-elliptic orbit is about two times larger than that of the Hohmann transfer. Additionally, the time of flight for the bi-elliptic orbit is almost four times greater than the time of flight for the Hohmann transfer. With all of this said, it is clear to see that the Hohmann transfer is the better transfer orbit for this situation as it is cheaper in terms of fuel budget and it takes much less time than the bi-elliptic transfer.

However, because Earth and Mars are so close, the inefficiencies of the bi-elliptic transfer are greatly exaggerated. As the ratio of final to initial semi-major axis increases, so does the feasibility and efficiency of a bi-elliptic transfer, where in some cases it can rival a Hohmann transfer.

H. Question 14

There are many factors that would help determine which orbit transfer is employed for a mission. Time-sensitive payloads, like humans, will require the shortest time of flight. Overall budget will effect the available ΔV available for a mission, and total mass will determine how much fuel is needed to produce the velocity changes, further constraining the transfer. The positioning of Earth and Mars in relation to each other, in addition to how long the team is willing to wait for a launch, will determine when the optimal time of launch is based on which transfers are available.

IV. Conclusion and Recommendations

In this lab, there were two primary concepts to study. The first was designing and modeling sun-synchronous orbits. The design was completed by using knowledge of orbital elements of perturbations to calculate how both Argument of Perigee and Right Ascension of the Ascending Node changed over time. The design was modeled in STK and verified to ensure all requirements were fulfilled. The second concept was orbit transfers. The ΔV requirements were calculated for both a Bi-elliptic and Hohmann transfer using Matlab. These were then implemented into STK and Keplerian Elements for the final orbit were verified to ensure the spacecraft ended in a stable orbit were it was intended to.

Acknowledgements

During the completion of this lab, the group sought out to complete lab tasks collaboratively, so all members good gather a good understanding of all content. This mainly included calculations for Keplerian elements as well as the Hohmann transfer. However, Aneesh headed the STK modeling for Scenario I, while Tyler and Cole worked Scenario II. The group as a whole then worked to fill out the lab report.

We would like to thank the teaching and lab assistants for their assistance throughout the lab. We would also like to acknowledge Professor McMahon for his continued assistance and clarification during lecture.

References

[1] Howard.D.Curtis, Orbital Mechanics for Engineering Student Third Edition

[2] ASEN 3200 Orbit Mechanics and Attitude Dynamics- Spring 2021 LABORATORY O-3 Write Up

MATLAB Code

Part 1 Calculations

```
%% Housekeeping

clc;
clear;
close all;

%% Constant declaration
J2 = 1.08263e-3;
T = 5832;
r_p1 = 6878+50;
r_p2 = 6978;
omega_0 = 0;
lambda_f = 0;
RAAN_dot = 0.9856*(pi/180)*(1/(24*60*60));
mu = 398600;
R = 6378;
omega_e = 7.292e-5;

a = (mu*(T/(2*pi))^2)^(1/3);

e1 = (1-(r_p1/a));
e2 = (1-(r_p2/a));

k1 = -(3/2)*((sqrt(mu)*J2*R^2)/(a^(7/2)*(1-e1^2)^2));
k2 = -(3/2)*((sqrt(mu)*J2*R^2)/(a^(7/2)*(1-e2^2)^2));

i1 = acosd(RAAN_dot/k1);
i2 = acosd(RAAN_dot/k2);

omega_dot1 = k1*((5/2)*sind(i1)^2-2);
omega_dot2 = k1*((5/2)*sind(i2)^2-2);

delta_lambda = -(omega_e-RAAN_dot)*T;
lambda_0 = 360+(delta_lambda*(180/pi));
RAAN = 360 - lambda_0 - (-81.55);
```

Part 2 Calculations

```
%% Housekeeping
clc;
clear;
close all;

%% Constants
mu_sun = 132712000000;
a_E = 149.6e6;
a_M = 227.9e6;
r_p = a_E;
r_a = a_M;

%% Question 7
a = (r_p+r_a)/2;
TOF_tot1 = (1/2)*(2*pi*sqrt(a^3/mu_sun));
```

```
V_E = sqrt(mu_sun/a_E);
V_M = sqrt(mu_sun/a_M);
V_p = sqrt((2*mu_sun/r_p)-(mu_sun/a));
V_a = sqrt((2*mu_sun/r_a)-(mu_sun/a));

deltaV_p = abs(V_p-V_E);
deltaV_a = abs(V_a-V_M);
deltaV_tot = abs(V_p-V_E) + abs(V_a-V_M);

%% Question 10
r_a1 = 2.7*149597871;
a1 = (r_p+r_a1)/2;
V_p1 = sqrt((2*mu_sun/r_p)-(mu_sun/a1));
deltaV_p1 = abs(V_p1-V_E);
TOF1 = (1/2)*(2*pi*sqrt(a1^3/mu_sun));
V_a1 = sqrt((2*mu_sun/r_a1)-(mu_sun/a1));
a2 = (r_a+r_a1)/2;
TOF2 = (1/2)*(2*pi*sqrt(a2^3/mu_sun));
V_a2 = sqrt((2*mu_sun/r_a1)-(mu_sun/a2));
deltaV_a1 = abs(V_a1-V_a2);
V_p2 = sqrt((2*mu_sun/r_a)-(mu_sun/a2));
deltaV_p2 = abs(V_p2-V_M);
deltaV_tot2 = deltaV_p1 + deltaV_p2 + deltaV_a1;
TOF_tot2 = TOF1 + TOF2;
```