ADVANCED TIME SERIES ANALYSIS

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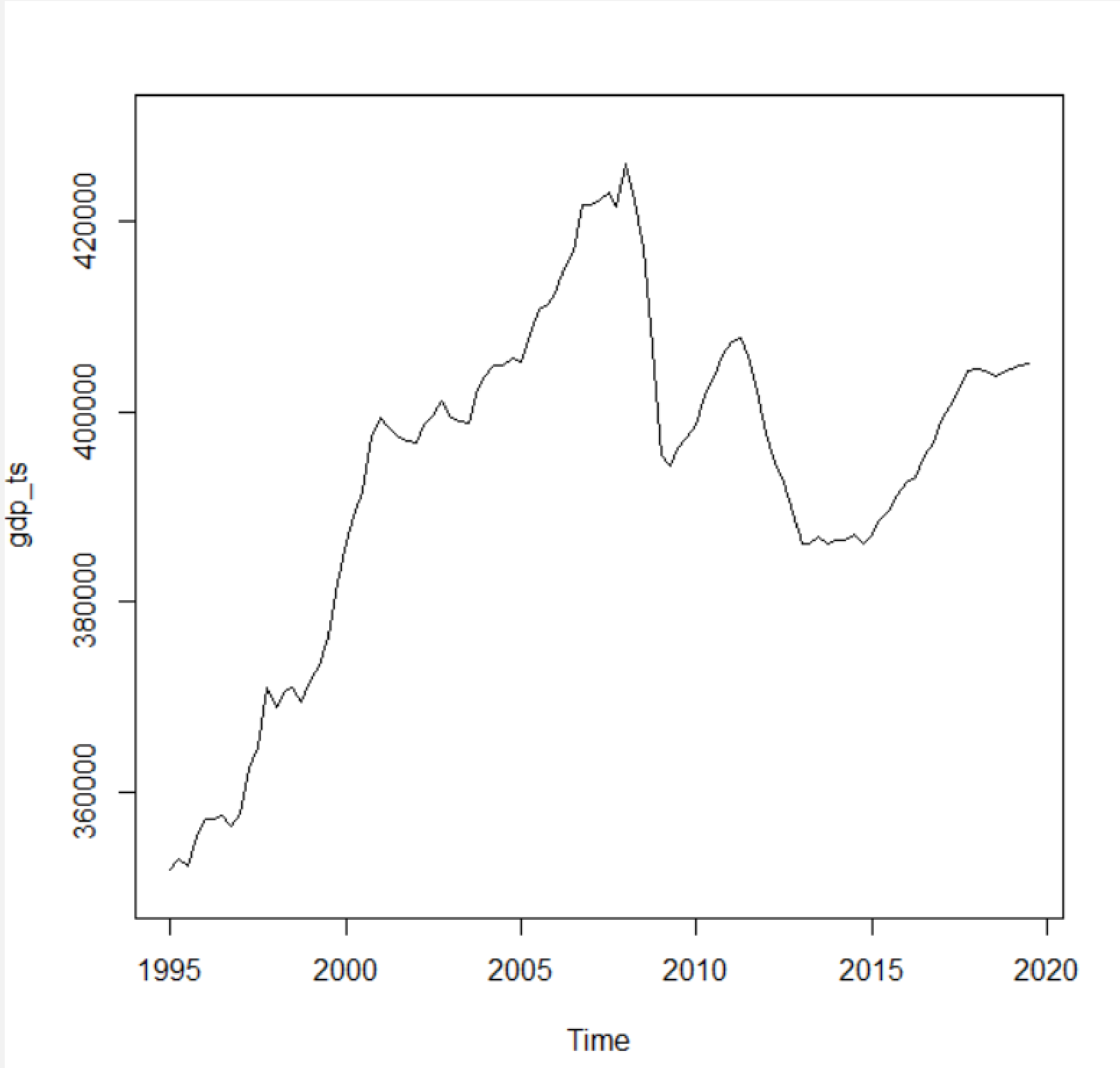
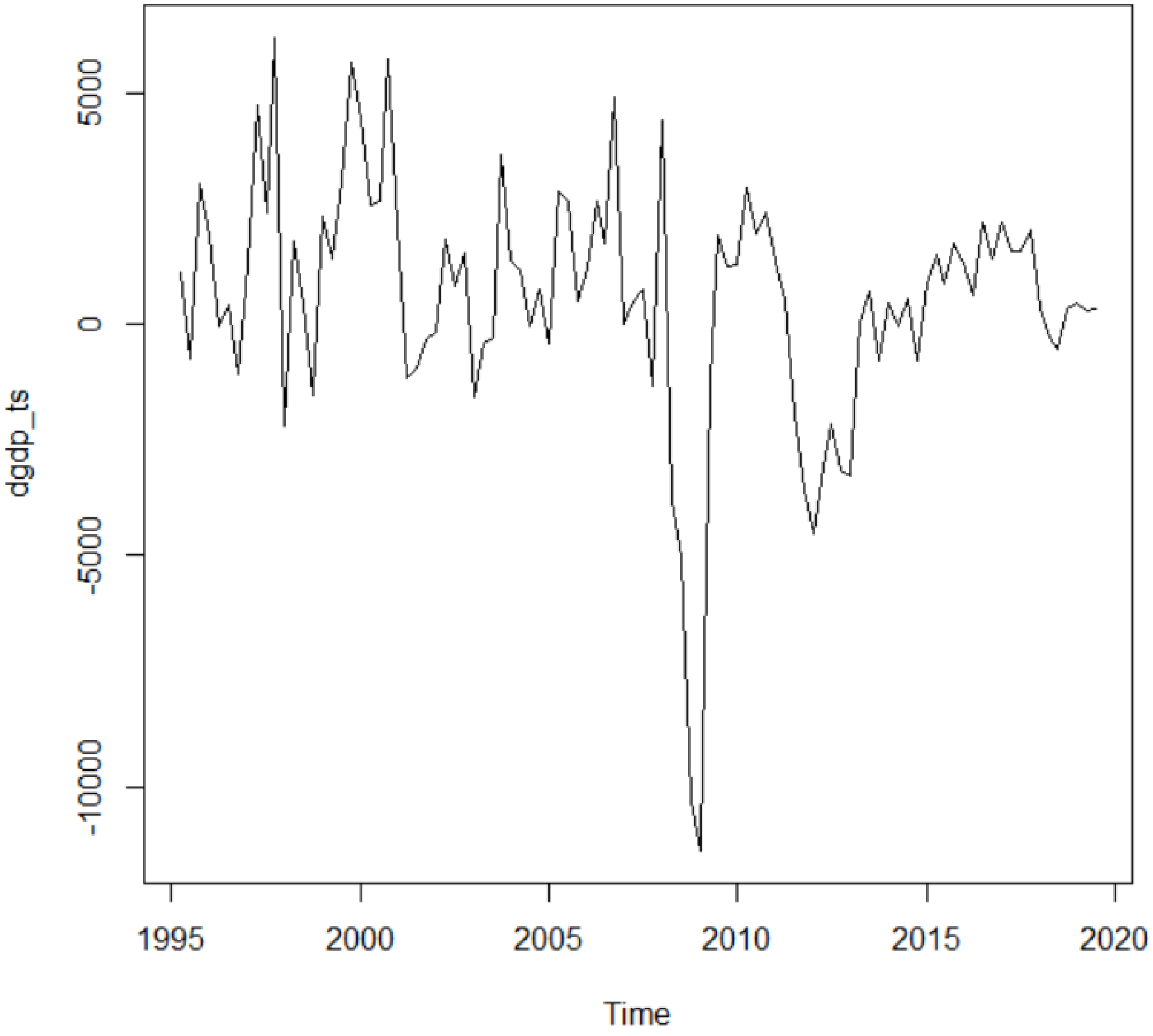
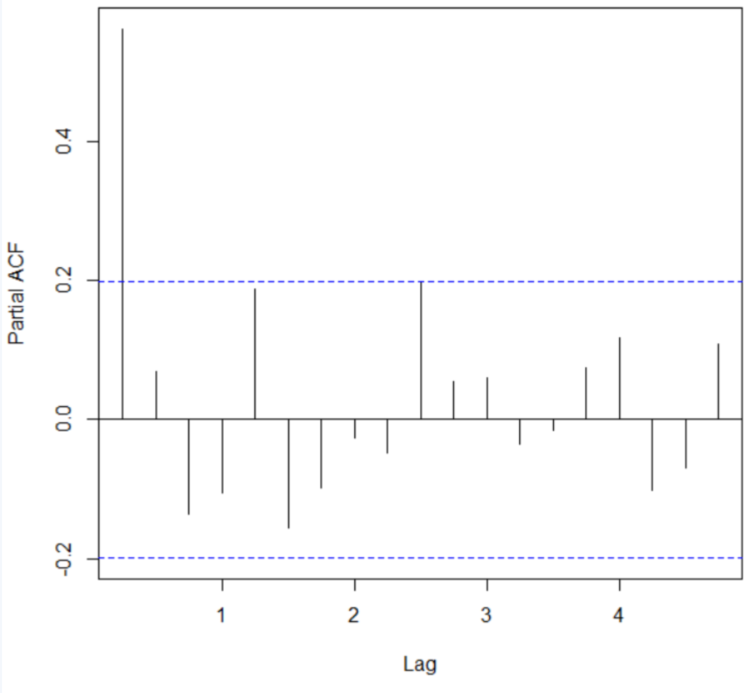
Italy's GDP and Germany's GDP

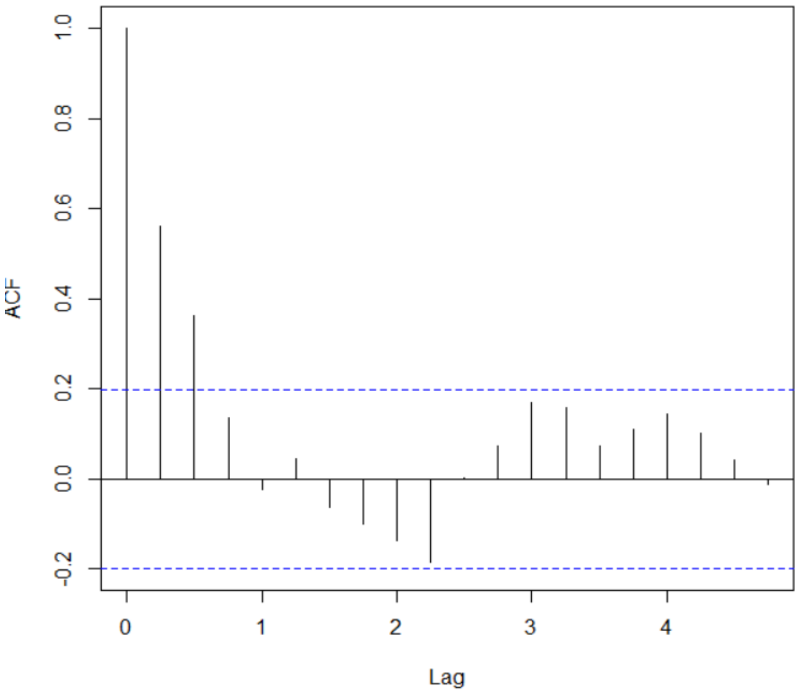
THE TIME SERIES CHOSEN TO BE ANALYZED FIRST IS THE ITALIAN GDP. THE SERIES CHOSEN IS A QUARTERLY SPECIFICATION OF DATA RANGING FROM THE FIRST QUARTER OF
1995 UP TO THE THIRD QUARTER OF 2019 IN MILLIONS OF CHAINED 2010 EUROS
AND SEASONALLY ADJUSTED. THE ANALYSIS STARTS WITH THE SPECIFICATION OF THE DATA OBJECT OF OUR INTEREST TO BE A TIME SERIES. THE GRAPH SHOWS THE LEVEL OF
ITALIAN GDP QUARTERLY AND IN LEVELS, WITHOUT ANY FURTHER MODIFICATION.



AS CAN BE EASILY DETECTED THERE EXISTS A TREND IN THE TIME SERIES, SO A UNIT ROOT TEST CAN BE PERFORMED IN ORDER TO SEE IF THE TREND IS STOCHASTIC OR NOT WITH MAX LAG=ROUND(SQRT(99))! SINCE WE HAVE 99 QUARTERLY OBSERVATIONS. THE TEST SHOWS THE PRESENCE OF UNIT ROOT, WE CONCLUDE THAT THE TIME SERIES IS NOT STATIONARY AND HAS A STOCHASTIC TREND, SO WE PROCEED TO THE ANALYSIS OF THE SERIES IN DIFFERENCE. THIS MEANS THAT THE UNIVARIATE ANALYSIS WILL CONCERN THE INCREASE OF THE GDP (I.E. THE DIFFERENCE BETWEEN TWO PERIODS T-(T-1)) THAT HAPPENS TO BE STATIONARY IF THE UNIT ROOT TEST IS PERFORMED.



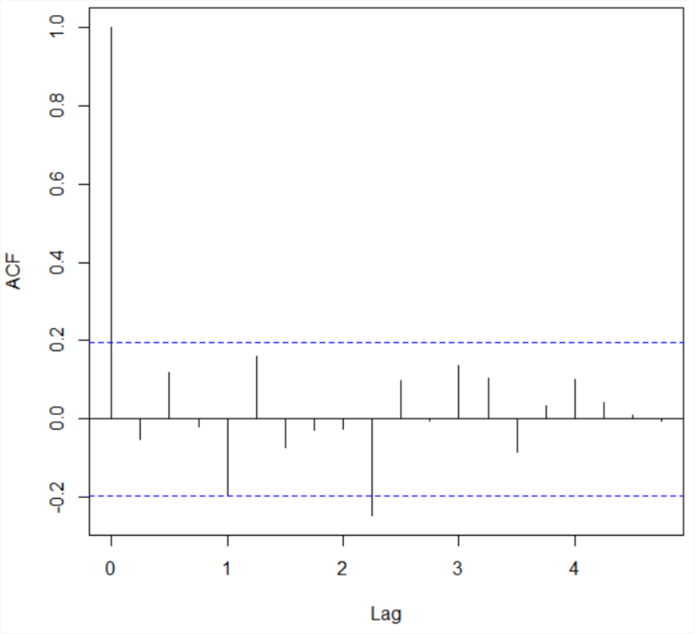




IN ORDER TO PROPOSE A MODEL IT IS USEFUL TO ANALYZE THE ACF AND THE PACF
GRAPHS THAT IN TURN SUGGEST EITHER AN MA(2) OR AN AR(1).
SUBSEQUENTLY SOME TESTS ON THE RESIDUALS TO VALIDATE THE MODELS ARE NEEDED (I.E. PLOTTING THE RESIDUALS OF THE SPECIFIED MODEL AND EVENTUALLY
LJUNG BOX)

STARTING FROM THE AR(1) MODEL:

Here the plotted residuals are not so convincing, thus a try with the Ljung-Box test is needed:

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Call:
arima(x = gdp_ts, order = c(1, 1, 0), seasonal = c(0, 0, 0))

Coefficients:
         ar1
      0.5729
s.e.  0.0816

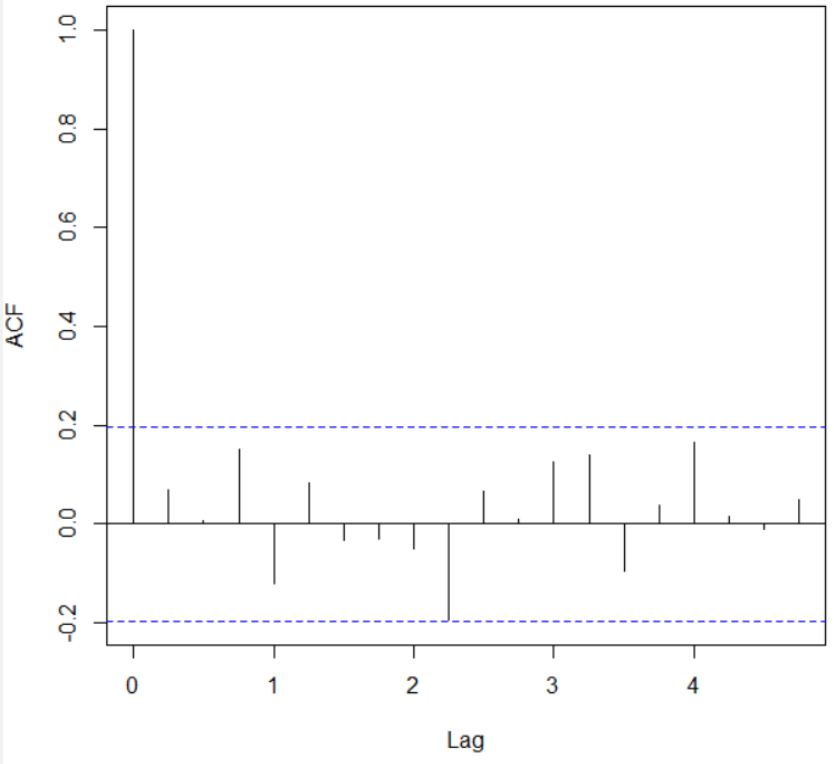
sigma^2 estimated as 4923142:  log likelihood = -894.32,  aic = 1792.64
> acf(fit_ar$res)
> Box.test(fit_ar$res, lag=max.lag, type="Ljung-Box")

        Box-Ljung test

data:  fit_ar$res
X-squared = 17.086, df = 10, p-value = 0.07249
```

And the Ljung Box barely validates the model even if the coefficient is statistically significant, hence the MA(2) could be the modeling choice in this case.

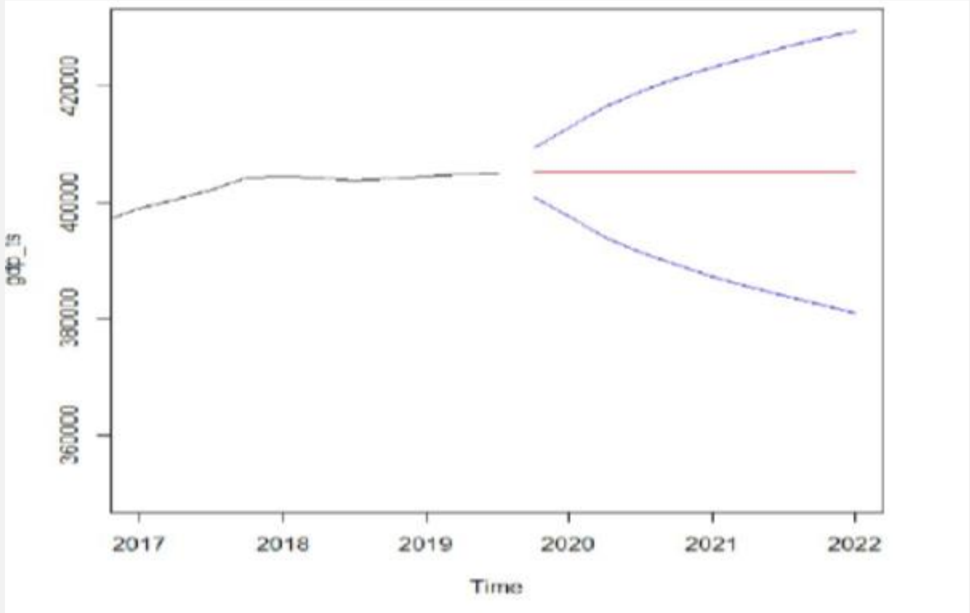
FOR WHAT ABOUT THE MA(2) MODEL, AFTER THE SPECIFICATION ON R, THE PLOT OF THE RESIDUALS GIVES THE FOLLOWING OUTPUT:



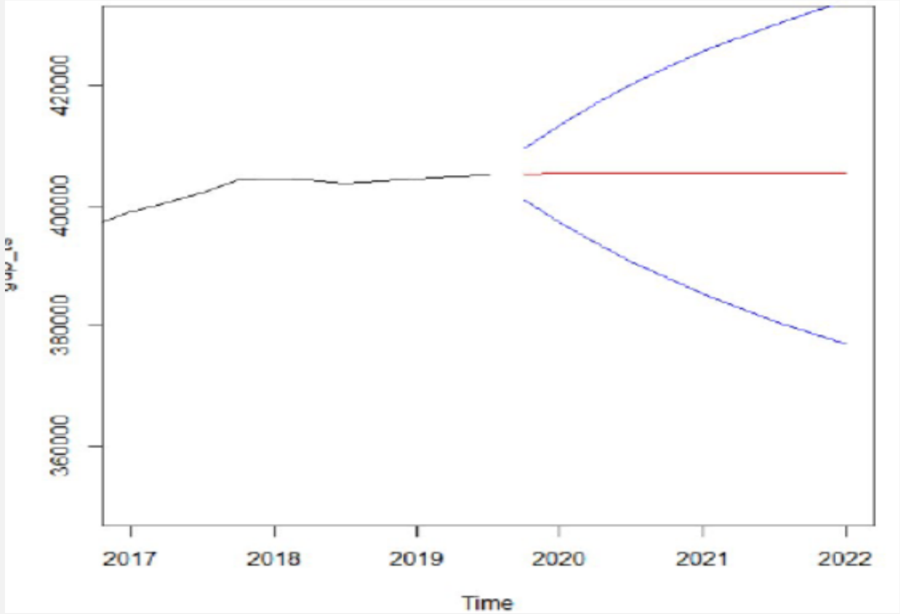
The parameters are statistically significant, and the correlation between lag 2 and 3 since it is borderline it is chosen to be ignored since the rest of the ACF behaves well. In any case the AR(1) model has not to be ignored but just set apart for the moment, if the ARIMA model is attempted to be fit the results are not satisfactory, thus it is put apart.

Having now found two possible models (even if not both have the same credibility) to fit the data at hand, forecasting can be performed.

For what about the MA(2) the forecasting graph is:



While for the AR(1) the graph for the forecast is:

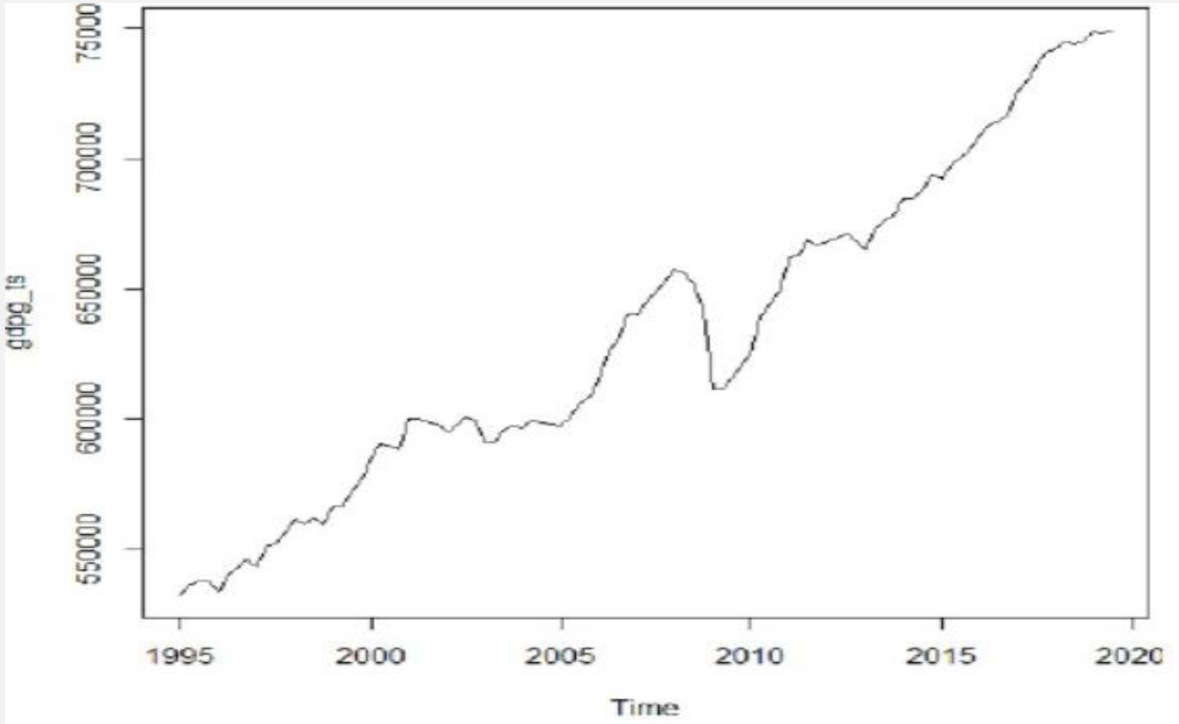


Summary tab about the characteristics of the two models

MA(2)	**AR(1)**
Good residuals ACF	'Acceptable' residuals ACF
L-Box test validates	L-Box test barely validates
AIC=1793.411 and BIC=1801.196	AIC=1792.637 and BIC=1796.954
MAE=787.4019	MAE=705.0877

THE CODES ARE QUALITATIVELY EQUIVALENT, SO THE DIFFERENCE LIES IN THE FORECAST OUTPUT. MA(2) AND AR(1) HAVE TWO SIMILAR OUTPUT, WHAT CAN BE DONE IS TO GENERATE OUT OF SAMPLE ERROR AND TO LOOK AT THEIR BEHAVIOUR (MAE) AND EVENTUALLY THE DIEBOLD MARIANO TEST WITH FORECAST HORIZON 1. THE P-VALUE IS NOT CLEAR (P-VAL.=0.0506), ANYWAY A RATIONAL DECISION COULD BE TO REJECT H0 IN THE TEST AND CONCLUDE THAT THE FORECASTING PERFORMANCES OF TWO MODELS ARE DIFFERENT ALTHOUGH NOT SIGNIFICANTLY TO A GREAT EXTENT. KEEP THE MA(2) INSTEAD OF THE AR(1) FOR FORECASTING SINCE MA GIVES A BETTER FORECAST THAT IS NEAR THE LAST OBSERVED VALUE, IT'S BETTER VALIDATED, AND THE PREDICTION INTERVAL HAS A MORE "REALISTIC" CONIC SHAPE EVEN IF AR(1) HAS LOWER MAE.

What remains to be investigated now is whether the GDP of Germany is capable to predict to some extent the Italian GDP. The German GDP in levels shows a trend so the usual unit root test must be performed. Although now the two series are analysed in log-differences to infer on the growth rate.



Subsequently Distributed Lag models are tried to be fitted regressing the two stationary time series without good results.

Trying out an Autoregressive DLM(3) something interesting is found:

```
Call:
lm(formula = dlgdp.0 ~ dlgdpg.0 + dlgdp.1 + dlgdpg.1 + dlgdp.2 +
    dlgdpg.2 + dlgdp.3 + dlgdpg.3)

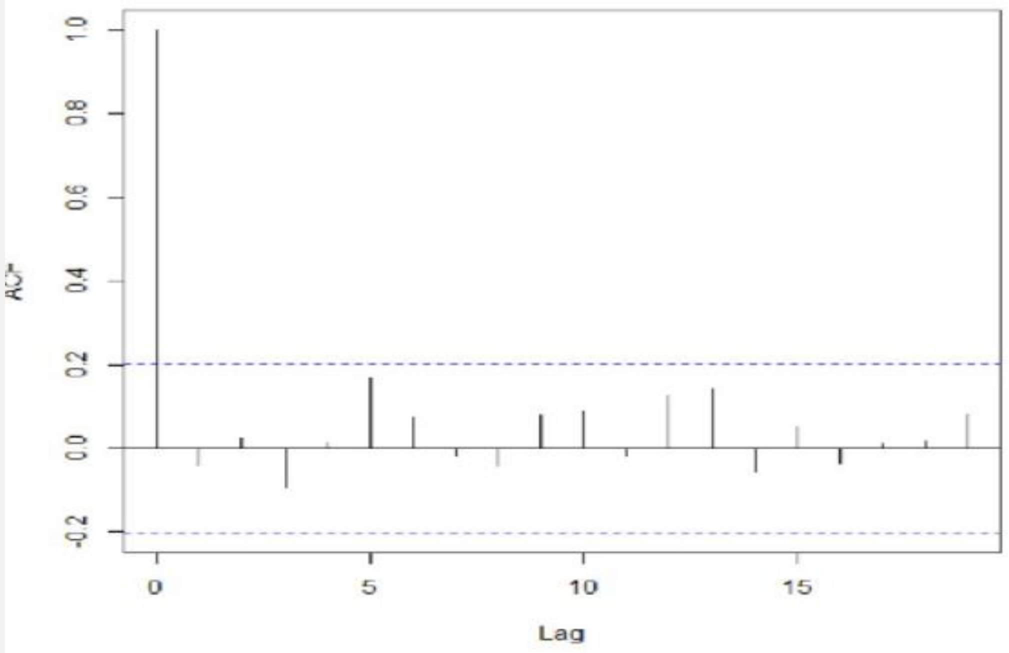
Residuals:
       Min         1Q     Median         3Q        Max
-0.0141349 -0.0021872  0.0000037  0.0020020  0.0143056

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)  5.237e-05  6.430e-04   0.081  0.93527
dlgdpg.0     3.800e-01  6.952e-02   5.466 4.33e-07 ***
dlgdp.1      3.102e-01  1.148e-01   2.703  0.00827 **
dlgdpg.1    -3.824e-02  8.022e-02  -0.477  0.63482
dlgdp.2      1.766e-01  1.172e-01   1.507  0.13535
dlgdpg.2    -6.403e-02  8.018e-02  -0.799  0.42674
dlgdp.3      2.094e-03  1.138e-01   0.018  0.98536
dlgdpg.3    -9.845e-02  7.823e-02  -1.259  0.21157
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.004924 on 87 degrees of freedom
Multiple R-squared:  0.5154,    Adjusted R-squared:  0.4764
F-statistic: 13.22 on 7 and 87 DF,  p-value: 1.722e-11
```

TESTING FOR NO GRANGER-CAUSALITY LEADS US TO ANALYSE THE TABLE OF VARIANCE OF TWO TEST EQUATIONS, ONE IS THE FULL AUTOREGRESSIVE DLM AND THE OTHER IS THE SAME BUT WITHOUT GERMAN GDP LAGS IN THIS CASE THE SIGNIFICANCE OF P-VALUE RH0 OF NO GRANGER CAUSALITY, THIS MEANS THAT GDP OF GERMANY HAS INCREMENTAL EXPLANATORY POWER ON THE GDP OF ITALY (IMAGE ON THE NEXT SLIDE)

Only the first two coefficients are significant but the F-stat RH0. The R squared indicates that the 0.5 percent of the variability in the growth of Italian GDP is explained. Moreover the plot of the residuals validates the model. It remains to investigate on Granger-causality



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> fit_adlm_nox <- lm(dlgdp.0 ~ dlgdp.1+dlgdp.2+dlgdp.3)
> anova(fit_adlm3,fit_adlm_nox)
Analysis of Variance Table

Model 1: dlgdp.0 ~ dlgdpg.0 + dlgdp.1 + dlgdpg.1 + dlgdp.2 + dlgdpg.2 +
    dlgdp.3 + dlgdpg.3
Model 2: dlgdp.0 ~ dlgdp.1 + dlgdp.2 + dlgdp.3
  Res.Df       RSS Df  Sum of Sq      F    Pr(>F)
1     87 0.0021090
2     91 0.0029013 -4 -0.00079219 8.1696 1.217e-05 ***
```
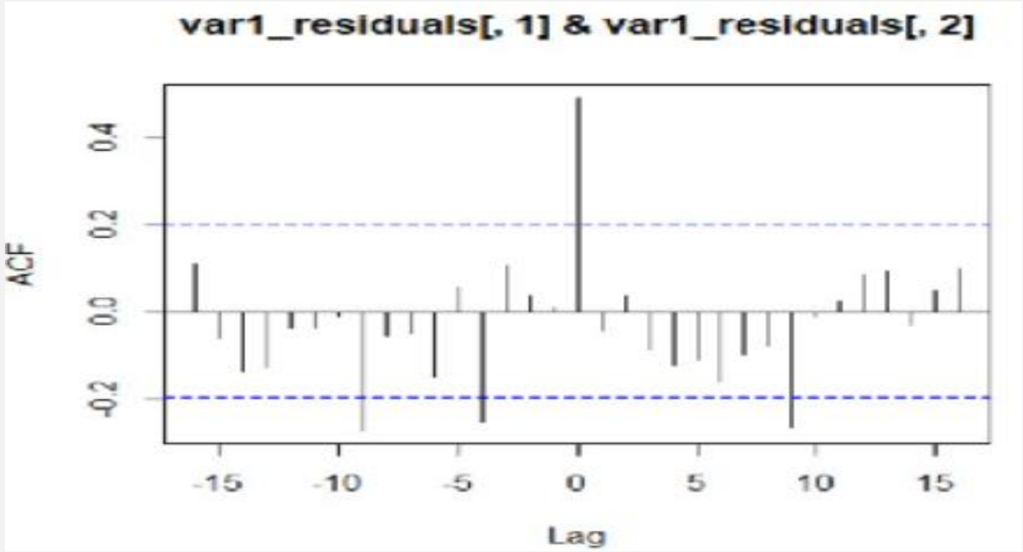
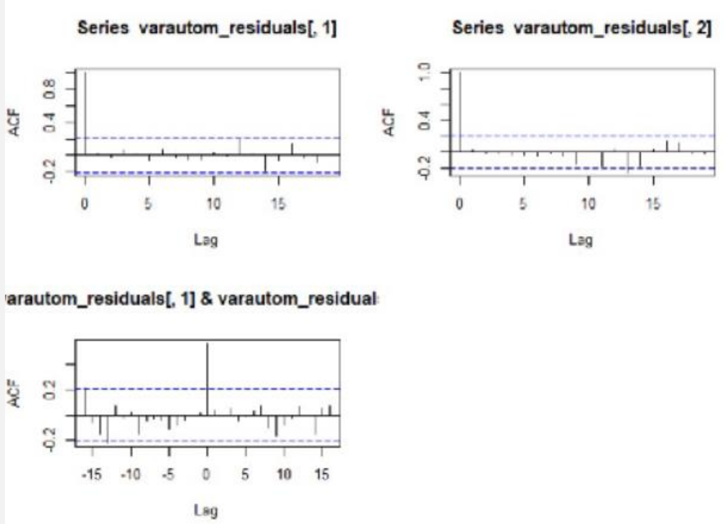

The Engle-Granger test is also performed and gives no evidence for cointegration if the two time series are regressed, thus no ECM is needed.

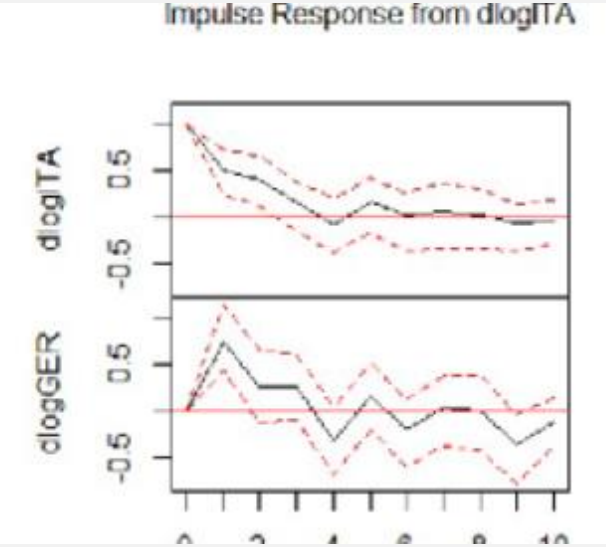
Even if some lags are significant these are far in time (i.e. lags -10 or -15), and indeed are also a bit borderline thus are chosen to be not considered as a reason to invalidate the model. The VAR(9) is kept even if not parsimonious. Next step involves impulse response functions. (image on the next slide)

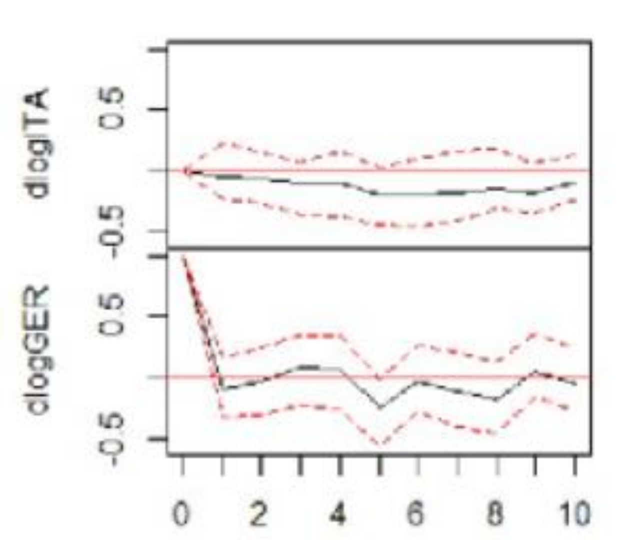
The object of interest now is to model the relationships among the two stationary time series and to forecast simultaneously all components of the model that is going to be specified with Italy and Germany's GDP. The first specification is a VAR(1) but this result is however not acceptable since the residuals for the var are not multivariate white noise, thus an automatic selection procedure is done that leads, not automatically, to the choice of a VAR(9) as a model, that is not parsimonious but is the only one that respects, after several analyses the conditions required .



THE IMPULSE RESPONSE FUNCTIONS AND THE PLOT FOR THE MULTIVARIATE RESIDUALS FOR THE VAR BUILT ARE SHOWN BELOW







The analysis of the Italian GDP time series led to the conclusion that the series can be modelled as an MA(2), looking at the test statistics' outcome and at the plots, when it comes to forecasting this model is the most suitable, even if it has not very low criteria, but it has better validations, the forecasting for this series give quite stable predictions of the future values. If compared to the German GDP time series in a multivariate analysis it can be seen that no reasonable DLM can be applied but an Autoregressive DLM fits the data, there is presence of granger causality (Germany's GDP provides additional explanatory power to the GDP of Italy) but no cointegration between the two series.

Multivariate granger-causality is analysed by fitting a vector autoregressive model that in turn shows how if one of the two series receives a shock, after one lag also the other has an impact in its level. The impulse response function is validated since the values are inside the confidence intervals. Forecasting for the var is performed too and also this gives quite stable-increasing results. Since the tests show no presence of cointegration neither the ECM nor the VECM are performed.

In the end some final remarks are due, the non perfect fit of the models here analyzed could be due to the relatively small data-set available, hence it could be possible that some test statistic could be biased and that the models chosen above are not in line with reality. For example the VAR(9) could not be parsimonious and in line with the dimension of the data-set but the criterion chosen for this analysis is to rely on R's output and on the tests (even if one's not always to rely on tests) hence some simplifications have been done for the sake of simplicity of this assignment.

The data has been collected from the federal reserve of Saint Louis site:
https://fred.stlouisfed.org/series/CLVMNACSCAB1GQIT (last visit 01/12/2019).

THANK YOU FOR YOUR ATTENTION!