**Numerical Analysis and Optimization**
**Academic Year 2021-2022**

Project 5: Constrained Optimization

1. Consider the problem of minimizing the function $f(x_1, x_2) = (x_1 - 4)^2 + x_2^2$ subject to the following constraints:

$$x_1 + x_2 \leq 2, \quad x_1 \geq 0, \quad x_2 \geq 0.$$

   Explain why the problem has a unique solution. Write the Lagrangian and the KKT conditions for this problem. Discuss the complementarity conditions (8 cases in all) to find the unique solution to the KKT system and therefore to the minimization problem. What is the value of $f$ at the solution?

2. Do the same for the problem of minimizing the function $f(x_1, x_2) = 2x_2 - x_1^2$ under the constraints

$$x_1^2 + x_2^2 \leq 1, \quad x_1 \geq 0, \quad x_2 \geq 0.$$

3. For the two problems above, write down the Jacobians associated with the problem

$$\min f(\mathbf{x}) - \mu\, \mathbf{e}^T \log(\mathbf{z}) \quad s.t. \quad \mathbf{c}(\mathbf{x}) - \mathbf{z} = \mathbf{0}$$

   after rewriting the constraints in the form $\mathbf{c}(\mathbf{x}) \geq \mathbf{0}$.

4. Implement Newton's method to solve the corresponding problems, starting from an initial guess that is inside the feasible set and accepting new approximations only if they remain feasible. Feel free to experiment with inexact variants (simplified Jacobians), steplength strategies, different initial guesses, etc. Try using a small value of $\mu$, or variable $\mu$. Report your results using tables and plots.