Online Knapsack Problem

Term Paper - Theoretical Computer Science

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1 Preamble

Online problems and algorithms are problems where the inputs for the algorithm are not known at the beginning, but appear one by one. This could be a job-scheduler in an operating system where you have to decide whether to do a job immediatly after it appears or wait for other, maybe shorter jobs. It could also be the decision whether to buy new ski-gear at the beginning of the season or rent it every day, when you do not know, how the weather will be like during the season. Every day, it could snow, rain or be a perfect powder-day, but should you buy skis on one sunny day, when you do not know if there will be another nice day to go to the mountains?

It would be nice to have some information about the future, something like an omniscient oracle, that gives us a glimpse of whats coming next.

We introduce such an oracle for online problems and try to find out, how much information do we need from the oracle to get an optimal solution.

In this interactive paper, we deal with the so called online knapsack problem, where we have knapsack that we want to fill with a maximum amount of value but respect the maximum capacity of it.

1.1 About this paper

This paper is written as *literate CoffeeScript*-source-code¹ of a set of experiments with these online problem running on *Meteor*².

It is compiled using *pandoc*³, which enables you to compile Markdown and many other formats to latex+pdf.

A live version of the experiments is available at: http://online-knapsack.macrozone.ch, the source-code is available on github: https://github.com/macrozone/seminar_np.

¹See http://coffeescript.org/#literate

²https://www.meteor.com/

³http://pandoc.org/

2 The knapsack problem

Consider a knapsack with a certain capacity of weight (or volume) and a set of items, each with a value and a weight.

Which subset of these items would you put into the knapsack to get the maximum possible total value respecting the capacity of the knapsack?

This question is the so called knapsack problem.

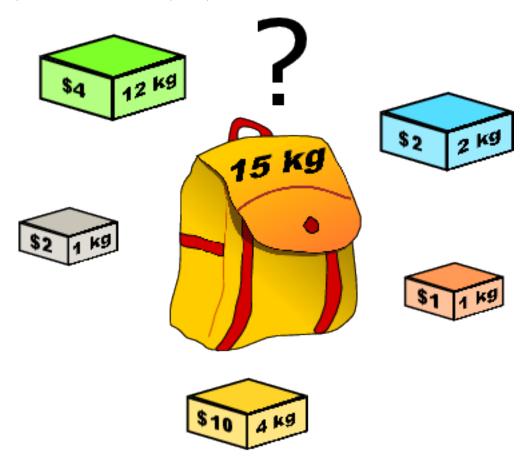


Figure 1: The knapsack problem (Source: wikipedia)

2.1 The simple-knapsack problem

In this paper, we only consider the so called *simple-knapsack problem* where the value of one item is the same as its weight and where the knapsack has always a capacity of 1.

We call the total value of all items in the knapsack as the gain.

Let's define such a knapsack:

```
Knapsack = class
2
       constructor: ->
            @size = 1
3
            @dep = new Tracker.Dependency
            @reset()
       fits: (item) ->
            @gain() + item.value <= @size</pre>
9
       addItem: (item) ->
10
            if Ofits item
11
                @items.push item
12
                @dep.changed()
13
15
       gain: ->
           roundValue _.reduce @getItems(), ((total, item) -> total+item.
                value), 0
17
       getItems: ->
18
            @dep.depend()
19
            @items
20
21
       reset: ->
22
            @items = []
23
            @dep.changed()
```

2.2 The Online-Knapsack-Problem

In the former *offline*-knapsack problem, we know all items that we want to put in the knapsack. In the *online*-version of this problem, we do not know every item, but get the items one by one. We therefore have to decide after every item, whether we put the item in the knapsack or not.

We create a base-algorithm for that:

```
Algorithm = class
       constructor: ->
2
           @act = new ReactiveVar
3
           @_knapsack = new Knapsack
       knapsack: -> @_knapsack
5
       handle: (item) ->
           if {\tt Odecide} item
8
                @_knapsack.addItem item
                return yes
10
           else
11
                return no
12
       decide: (item) ->
13
           # implement me and return yes or no
14
15
       reset: ->
         @_knapsack.reset()
```

What maximum gain would can we achieve and how would an online-algorithm perform in comparison with an optimal offline-algorithm, which would know every item?

Let's try out.

```
experiments = []
```

Lets start with the greedy aproach. Here, we just take every item we get, if it fits:

```
decideGreedy = (item) -> if @knapsack().fits item then yes else no
```

and we define an algorithm with it:

```
Greedy = class extends Algorithm
decide: decideGreedy
```

The gain of this algorithm is at least $1-\beta$, where β is the size of the item with the highest value (weight). The proof is simple: if we get this item with value β , the gain is certainly higher than β . If this item does not fit anymore in the knapsack, we will have at least $1-\beta$ gain.

Lets do some experiments with it to verify this:

```
experiments.push
       name: -> "Greedy G"
2
       description: -> "G archieves at least 1-beta, where beta is here
3
          #{@beta}"
       beta: 0.5
       Algorithm: Greedy
5
6
   experiments.push
7
       name: -> "Greedy G"
8
       description: -> "G archieves at least 1-beta, where beta is here
           #{@beta}"
       beta: 0.2
10
       Algorithm: Greedy
11
12
   experiments.push
13
       name: -> "Greedy G"
14
       description: \rightarrow "G archieves at least 1-beta, where beta is here
15
          #{@beta}"
       beta: 0.8
16
       Algorithm: Greedy
```

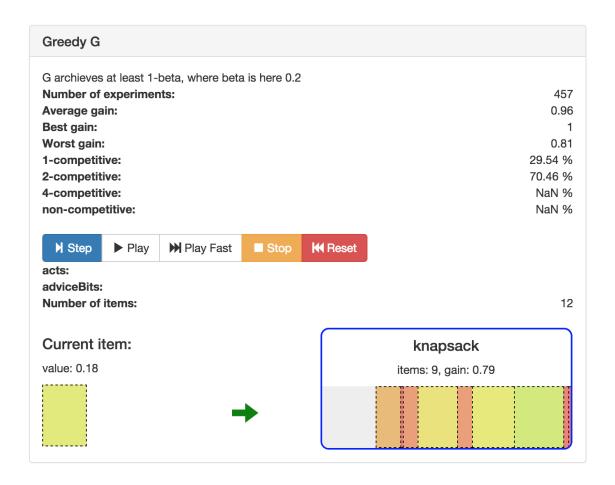


Figure 2: The greedy approach will at least gain $1-\beta$

3 Online-Algorithm with advice

Imaging you had an oracle, that would know all items that will come. How many bits of information from this oracle would you need to get an optimal solution? And for a given amount of these advice bits, how good would your algorithm perform?

We define such an algorithm as online algorithm with advice.

Let I be an input of such an online algorithm A and Φ an (infinite) sequence of bits (1 or 0), called *advice bits. The online-algorithm can read a finit prefix of this sequence.

The gain of this Algorithm is $gain(A^{\Phi}(I))$.

If we have n items in a solution and have read s(n) advice-bits while computing this solution in the algorithm we call s(n) the advice-complexity.

If we compare the *gain* of this algorithm with the gain of an optimal offline algorithm OPT, we can define its *competitiveness*:

```
gain(A^{\Phi}(I))* \ge \frac{1}{c}*gain(OPT(I)) - \alpha
```

where α is a constant and we call this algorithm *c-competitive*. If $\alpha = 0$, A is *strictly c-competitive*.

Let's implement a base class for such an algorithm:

```
AlgorithmWithAdvice = class extends Algorithm
       constructor: ->
           @adviceBits = new ReactiveVar
3
           super
       askOracle: (items) ->
           if @oracle?
               @adviceBits.set @oracle items
       oracle: (items) ->
           # implement me and return an array of advice-bits
9
       readAdviceBit: (index) ->
10
           @adviceBits.get()?[index]
11
       reset: ->
12
           super
13
           @adviceBits.set null
```

3.1 Optimal online algorithm with advice

Let's go back to the first question with the first question: how many advice bits do we have to read to get an optimal solution?

Consider an algorithm with an oracle, that would give us a bit for every item coming with

- · value 1 if the item is part of the solution
- · value 0 if the item does not belong to the solution

Obviously, we need n bits of advice for that, or n-1, because for the last item, we can assume that it is part of the optimal solution.

We now define an algorithm for that.

Note: The items are prepared in a way, that some are allready marked as solution. That makes it easier to define the oracle here:

```
TotalInformation = class extends AlgorithmWithAdvice
oracle: (items) ->
bits = []
```

```
for item in items
bits[item.index] = if item.isPartOfSolution then 1 else 0
# we do not need the last bit
bits.pop()
return bits
```

The decision is now easy. If we have a bit (yes / no), we use it:

```
decide: (item) ->
adviceBit = @readAdviceBit item.index
if adviceBit? then adviceBit else yes
```

Lets do an experiment with it:

```
experimentsWithAdvice = []
experimentsWithAdvice.push
name: -> "Total Information"
beta: 0.4
Algorithm: TotalInformation
```

As (Böckenhauer et al.) states, any algorithm for the online simple knapsack problem needs at least n-1 bits to be optimal.

3.2 1 Advice bit

What's the best gain if we had only 1 advice bit?

Let's do an experiment where we have an oracle that gives us one bit:

```
AONE = class extends AlgorithmWithAdvice
oracle: (allItems) -> [ _.some allItems, (item) -> item.value >
0.5 ] # array with one bit
```

The bit tells us:

- 1: There exists an item with a size > 0.5
- 0: There is no such item

If the bit is 0, the algorithm acts greedy (like before). If the bit is 1, the algorithm waits until the item with size > 0.5 appears and will start acting greedyly:

```
decide: (item)->
            adviceBit = @readAdviceBit item.index
2
            if adviceBit? # existance
3
                if adviceBit is false then @doAct "greedy" else @doAct "
            if {\tt Qacts} "greedy" then {\tt decideGreedy.call} {\tt Q}, item else {\tt Qwait}
                item
       wait: (item) ->
            if item?.value > 0.5
                @doAct "greedy"
                decideGreedy.call @, item
10
            else
11
12
```

This algorithm is 2-competitive:

- If there is no item with weight > 1/2, the gain is at least 1/2 as we have already seen in the greedy approach.
- On the other hand if such an item exists, the algorithm will wait for it and put it in, so it will get a gain of at least 1/2

We do an experiment with a max size of one item of 0.55 to verify this:

```
experimentsWithAdvice.push
name: "AONE - with one advice bit"
description: "AONE is 2-competitive"
beta: 0.55
Algorithm: AONE
```

This one single bit gives us an competitive-ratio of 2, but what happens if we increase the amount of bits? Can we achieve a better ratio?

Unfortunatly, more advice bits does not give us a better competitive-ratio, at least for a sub-logarithmic amount s(n) of advice bits. Figure 3 shows the number of bits compared with the achieved competitive-ratio.

There is a second jump at *SLOG*-bits, where competitiveness is $1 + \varepsilon$. The proof for these intervals is found in the source (Böckenhauer et al.).

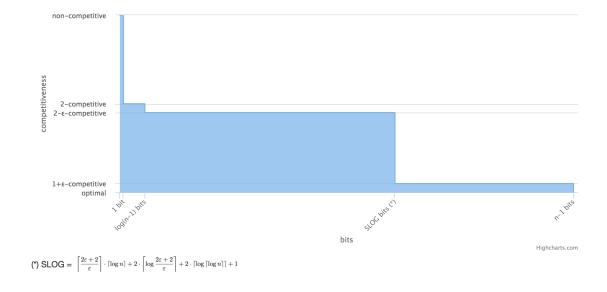


Figure 3: Number of bits VS competitiveness

4 Randomized online algorithms

Obviously in real online problems, we do not have an omniscient oracle. But we can use the idea of the oracle and just guess the advice bits *randomly*.

We can then estimate the competitiveness of this randomized online-algorithm.

4.1 RONE - AONE with random advice bit

Let's start with AONE from the previous experiment, but guess the adviceBit randomly:

```
RONE = class extends AONE
oracle: ->
[Math.random() < 0.5]
```

If we guess wrong, we might get a lower gain then 0.5 or even 0, if the adviceBit is 1 and we have no item with size > 0.5.

So while we have a 2-competivenes in AONE, we have here a 4-competitivenes in expectation (in 50% of the cases, we are wrong).

```
randomExperiments = []
randomExperiments.push
name: "RONE - one random bit"
description: "Is 4-competitive in expectation"
beta: 0.55
Algorithm: RONE
```

The experiment does not show this directly, because the items are prepared in a way, that not all possible cases are evenly distributed. We expected that in 50% of the cases, the algorithm would guess wrongly and we would gain nothing, but in the experiment this probability is lower.

4.2 2-competivenes with 1 advice bit

The competitive-ratio of 4 is somewhat obvious, but suprisingly, we can also achieve a ratio of 2 with only 1 advice bit.

Consider an algorithm that choses randomly between two algorithms A1 and A2. A1 is the greedy approach we already know:

```
1 A1 = Greedy
```

A2 internaly simulates A1 at the beginning:

```
A2 = class extends Algorithm
reset: ->
super
da1 = new A1
ddoAct "simulateA1"
```

To decide wheter it will use the item or not, it first offers it to the simulated A1-Algorithm. As soon as A1 won't take the item anymore (A1' knapsack is full), A2 starts to act greedyly:

```
decide: (item) ->
    if @acts "simulateA1"
    if @a1.handle item
        return no
    else
        @doAct "greedy"
        return @decide item
    else if @acts "greedy"
        return decideGreedy.call @, item
```

We now compose an algorithm "RONE2", that choses randomly between A1 and A2:

```
RONE2 = class extends AlgorithmWithAdvice
constructor: ->

@a1 = new A1
@a2 = new A2
super
```

5 WHATS NEXT 4.3 The limit

```
oracle: -> [Math.random() < 0.5]</pre>
       reset: ->
           super
           @a1.reset()
10
           @a2.reset()
       knapsack: -> @algorithm().knapsack()
11
       \# handle decides and put the item in the knapsack
12
       handle: (item) ->
13
           adviceBit = @readAdviceBit item.index
14
           if adviceBit? # existance of the first bit
15
               if adviceBit then @doAct "A1" else @doAct "A2"
16
           @algorithm().handle item
17
       algorithm: ->
           if @acts "A1" then @a1 else @a2
```

We do now an experiment with it:

```
randomExperiments.push
name: "RONE2 - one random bit"
description: "Is 2-competitive in expectation"
beta: 0.55
Algorithm: RONE2
```

To show that this algorithm is 2-competitive in expectation, we consider two cases:

- If the sum of all items is less than the knapsack's capacity, A1 is optimal, while A2 gains 0. Because we chose randomly between the two algorithm, we have a 50% chance to get an optimal gain (or to get 0).
- If the sum is greater, the total gain of A1 and A2 is at least 1. Because we chose randomly between the two, we get a 0.5 gain in expecation.

Considering both cases, we get a gain of 0.5 in expecation, so the algorithm is 2-competitive.

4.3 The limit

While we can achieve different levels of competitivenesses by increasing the number of bits in online algorithms with advice, this is not the case in randomized online algorithms.

As (Böckenhauer et al.) states, there is no algorithm that performs better than 2-competitive in expecation. So 2-competiveness with 1 bit is the best we can achieve.

5 Whats next

Resource augmentation: If we allow the online algorithm to pack a little bit more (δ) in the knapsack than allowed, we can achieve up to $(2-\delta)$ -competitiveness.

The weighted case: In this paper, we only considered items, where the value is equal to the weight of the item. If we introduce a different weight of each item, we will see, that online algorithms for this weighted knapsack problem is only competitive for at least a logarithmic amount of advice bits.

Randomized online algorithms for the weighted knapsack: If we create a randomized online algorithm for the weighted case, we see that these algorithms are not competitive at all, with and without resource-augmentation.

Further details and proof for these extensions can be found in the source (Böckenhauer et al.) .

6 Setup

The following code sets the experiments up. First, define some constants and helpers:

```
Constants =

SCALE: 300

roundValue = (value) -> Math.round(value*100)/100
```

6.1 Create items

The creation of items is done here. The items are prepared in a way, so that we now which elements are part of the solution (for experiment "Total information").

```
createItems = ({beta, maxSize}) ->
2
       items = []
3
       beta ?= 0.5
       maxSize ?= 1
5
       totalSize = 0
       100p
7
           randomValue = -> roundValue Math.random()*beta
8
           value = randomValue()
9
           if totalSize+value < maxSize
10
                totalSize += value
11
                items.push {value, isPartOfSolution: yes}
12
           else
13
                # add one that fits exactly
14
                items.push
15
                    value: roundValue maxSize - totalSize
16
                    isPartOfSolution: yes
17
                # add the one that does not fit
18
                items.push {value}
19
                break
20
21
       items = _.shuffle items
22
       for item, index in items
           item.index = index
       # we later pop the elements out (from the end) because it is
           faster. So we reverse here:
       return items.reverse()
```

6.2 Experiment templates

Add the experiments to the template:

```
Template.experiments.helpers
experiments: -> experiments
experimentsWithAdvice: -> experimentsWithAdvice
randomExperiments: -> randomExperiments
```

Initialize it. We use some ReactiveVars to store the state of the experiment on the template instance.

Lets define a history, where we can read some stats about the experiments from:

```
@gainHistory =
           history: []
2
            dep: new Tracker.Dependency
3
            add: (gainValue) ->
                if gainValue > 0
                     @worstGain = Math.min @worstGain ? gainValue,
                        gainValue
                @bestGain = Math.max @bestGain ? gainValue, gainValue
                @history.push gainValue
                @dep.changed()
            size: ->
10
                @dep.depend()
11
                @history.length
12
            worst: ->
13
                @dep.depend()
14
                @worstGain
15
            best: ->
16
                @dep.depend()
17
                @bestGain
            competitiveCount: ->
                @dep.depend()
20
                _.countBy @history, (value) ->
21
                     if value is 1
22
                         "1-competitive"
23
                     else if 0.5 \le value \le 1
24
                         "2-competitive"
25
                     else if 0.25 \le value \le 0.5
26
                         "4-competitive"
27
                     else
                         "non-competitive"
29
30
            competitivePercentage: (cGroup)->
31
                @dep.depend()
32
                if @history.length > 0
33
                    roundValue 100 * @competitiveCount()[cGroup] /
34
                        @history.length
            avg: ->
35
                @dep.depend()
36
                if @history.length > 0
37
                    roundValue (_.reduce @history, (total, value) -> total
                        +value)/@history.length
            reset: ->
39
                @history = []
40
                @bestGain = null
41
                @worstGain = null
42
                @dep.changed()
43
```

6.3 Initialize and reset experiment

```
resetExperiment = =>

@items = createItems beta: @data.beta

@algorithm.reset?()

@algorithm.askOracle? @items

@numberOfItems.set @items.length

@currentItem.set @items.pop()

do reset = =>
```

```
geainHistory.reset()
resetExperiment()
```

6.4 Running the experiment

The Ticker is a package, that can run a callback in a loop. We can run it step-by-step or fast.

Expose the state to the template:

```
Template.Experiment.helpers
       adviceBits: -> Template.instance().algorithm.adviceBits?.get()
2
       act: -> Template.instance().algorithm.act.get()
3
       knapsack: -> Template.instance().algorithm.knapsack()
4
       ticker: -> Template.instance().ticker
       currentItem: ->Template.instance().currentItem?.get()
       gainHistory: -> Template.instance().gainHistory
       numberOfItems: -> Template.instance().numberOfItems.get()
       willMatch: ->
           ctx = Template.instance()
10
           ctx.currentItem?.get()?.value + ctx.algorithm.knapsack().gain
               () <= ctx.algorithm.knapsack().size
12
13
  Template.Knapsack.helpers
       totalWidth: ->
14
           @size * Constants.SCALE + 2
15
       items: ->
16
           @getItems()
17
18
19
  Template.KnapsackItem.helpers
20
       width: ->
21
           @value * Constants.SCALE
       color: ->
22
           hue = @value*360
23
           "hsl(#{hue}, 73%, 69%)"
```

6.5 Chart

The chart from 3 is created by this code:

```
Template.competitivenessChart.helpers
chartObject: ->
legend: enabled: false
title: text: ""
yAxis:
title: text: "competitiveness"
tickPositioner: -> [1,1.1,1.9,2,3]
```

6 SETUP 6.6 GUI for the Ticker

```
labels:
                    formatter: ->
                         switch @value
10
                             when 1 then "optimal"
11
                             when 1.1 then "1+eps-competitive"
12
                             when 1.9 then "2-eps-competitive"
13
                             when 2 then "2-competitive"
14
                             when 3 then "non-competitive"
15
16
           xAxis:
17
                title: text: "bits"
18
19
                tickPositioner: -> [0,1,7,77,127]
20
                labels:
                    rotation: -45
21
                    formatter: ->
22
                         switch @value
23
                             #when 0 then "0 bits"
24
                             when 1 then "1 bit"
25
                             when 7 then "log(n-1) bits"
26
                             when 77 then "SLOG bits (*)"
27
                             when 127 then "n-1 bits"
28
            series: [
29
                type: "area"
30
                step: "left"
32
                data: [
                    (x: 0, y: 3, name: "non-competitive")
33
                    (x: 1, y: 2, name: "2-competitive")
34
                    (x: 7, y: 1.9, name: "2-eps-competitive")
35
                    (x: 77, y: 1.1, name: "1+eps-competitive")
36
                    (x: 127, y: 1, name: "optimal")
37
                ]
38
           ]
39
```

6.6 GUI for the Ticker

```
Template. TickerGui. helpers
       counter: -> @ticker.getCounter()
2
3
  Template.TickerGui.events
       'click .btn-step': -> @ticker.step()
       'click .btn-play': ->
6
           @ticker.setTimeout 100
7
           @ticker.play()
8
       'click .btn-play-fast': ->
9
           @ticker.setTimeout 0
10
           @ticker.play()
11
       'click .btn-stop': -> @ticker.stop()
12
       'click .btn-reset': -> @ticker.reset()
```

A Additional source files

A.1 client/app.jade (template)

```
title Advice Complexity of the Online-Knapsack-Problem
3
   body
5
       .container
            .page-header
6
                h1 Advice Complexity of the Online-Knapsack-Problem
7
                img(src="knapsack.svg" style="display:block; margin: 0
8
                    auto")
            +experiments
10
11
   template(name="experiments")
14
       each experiments
15
           +Experiment
16
       h2 Online algorithms with advice
17
       each experimentsWithAdvice
18
           +Experiment
19
       +competitivenessChart
20
       h2 Randomized online algorithm
21
       each randomExperiments
           +Experiment
24
25
26
   template(name="Experiment")
27
       .panel.panel-default
28
            .panel-heading
29
                h2.panel-title {{name}}
30
            .panel-body
31
                {{description}}
32
33
                     .col-xs-12.col-sm-5
35
                         table.table
36
                             tr
                                  th Number of experiments:
37
                                  td {{gainHistory.size}}
38
39
                             tr
                                  th Average gain:
40
                                  td {{gainHistory.avg}}
41
                             tr
42
                                  th Best gain:
43
                                  td {{gainHistory.best}}
44
45
                                  th Worst gain:
46
                                  td {{gainHistory.worst}}
47
                             t.r
48
                                  th 1-competitive:
49
                                  td {{gainHistory.competitivePercentage "1-
50
                                      competitive"}} %
                             tr
51
                                  th 2-competitive:
52
                                  td {{gainHistory.competitivePercentage "2-
                                      competitive"}} %
```

```
tr
54
                                   th 4-competitive:
55
                                   td {{gainHistory.competitivePercentage "4-
                                       competitive"}} %
                              t.r
57
                                   th non-competitive:
58
                                   td {{gainHistory.competitivePercentage "
59
                                       non-competitive"}}  %
                 .row
60
                      .col-xs-12
61
                          +TickerGui ticker=ticker
62
63
                      .col-xs-12.col-sm-5
                          table.table
                              tr
                                   th acts:
67
                                   td {{act}}
68
                              tr
69
                                   th adviceBits:
70
                                   td.adviceBits=adviceBits
71
72
                                   th Number of items:
73
                                   td {{numberOfItems}}
75
76
                 .row
                      .currentItemContainer.col-xs-6
77
                          h4 Current item:
78
79
                          if currentItem
80
                              p value: {{currentItem.value}}
81
82
                              +KnapsackItem(currentItem)
83
                               if willMatch
84
                                    .matches.yes
                                        . \verb| glyphicon.glyphicon-arrow-right|\\
87
                               else
                                   .matches.no
88
                                        .glyphicon.glyphicon-remove
89
90
                      .knapsackContainer.col-xs-6
91
                          +Knapsack knapsack
92
93
94
   template(name="Knapsack")
95
        .knapsack(style="width: {{totalWidth}}px")
97
            h4 knapsack
            p items: {{items.length}}, gain: {{gain}}
98
            .items
qq
100
                 each items
101
                     +KnapsackItem
102
103
104
   template(name="KnapsackItem")
105
        .knapsack-item(style="width: {{width}}px; background-color: {{
106
            color}}")
107
   template(name="competitivenessChart")
108
109
110
      .panel.panel-default
111
```

```
.panel-heading
112
                 h2.panel-title Bits VS competitiveness
113
             .panel-body
                 +highchartsHelper chartId="competitivenessChart"
                      chartWidth="100%" chartHeight="400px" chartObject=
                      chartObject
                 p (*) SLOG =
116
                      img(src="slog.png" height="40px")
117
118
   template(name="TickerGui")
119
        .ticker
120
121
             .btn-group
122
                 button.btn.btn-step.btn-primary
                      . \verb|glyphicon.glyphicon-step-forward|\\
                      | Step
                 \verb|button.btn.btn-play.btn-default|
125
                      .glyphicon.glyphicon-play
126
                      | Play
127
                 button.btn.btn-play-fast.btn-default
128
                      . \verb|glyphicon.glyphicon-fast-forward|\\
129
                      | Play Fast
130
                 button.btn.btn-stop.btn-warning
131
                      . \verb|glyphicon.glyphicon-stop|\\
132
                      | Stop
134
                 \verb|button.btn.btn-reset.btn-danger|
                      . \, {\tt glyphicon.glyphicon-fast-backward}
135
                      | Reset
```

A.2 client/app.styl (stylesheet)

```
body
2
       background-image: url("bg.png")
3
   .knapsack
      border: 2px solid blue
      box-sizing: content-box
      border-radius: 10px
      overflow: hidden
      text-align: center
11
      .items
           background-color: #eee
12
           height: 75px
13
           .knapsack-item
14
                float: right;
15
                border-right: none
16
   .knapsack-item
17
       border: 1px dashed black height: 75px
18
19
       box-sizing: border-box
   . \ {\tt currentItemContainer}
      position: relative
23
       .matches
24
           position: absolute
25
           right: 60px
26
           bottom: 16px
27
           border-radius: 10px
28
           font-size: 32px
29
           &.yes
30
                color: green
           &.no
33
               color: red
34
  .adviceBits
35
       font-family: monospace
36
37
   .table
38
39
           text-align: right
40
41
           text-align: left
42
   .panel
       margin-bottom: 200px
```

References

Böckenhauer, Hans-Joachim, Dennis Komm, Richard Kralovic, and Peter Rossmanith. "The Online Knapsack Problem: Advice and Randomization."