Lab 2 Report

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Introduction:

This report is done to simulate the 4km vertical mine shaft at earth's equator using python coding. The mass being used will be a 1kg test mass which will be put under different conditions of which include: constant and variable gravitational fields, air resistance, rotational effects such as the Coriolis force, and more. The goal is to estimate the real time it will take for the mass to reach the bottom of a 4 km deep shaft at the mine. Additionally, this report will include further testing of the code to determine its accuracy by coding dropping the mass in conditions such as an infinitely deep mine shaft and extending the analysis to consider non-uniform density profiles of the Earth and a hypothetical mine shaft on the Moon. The main methods of achieving the times we require will be by using numerical and analytical methods and seeing if there are any large differences between them.

Calculation of fall times:

Time 1: Constant Gravity, No Drag

Using the classic equation for free fall, the analytic fall time for a 1kg mass in a 4km deep shaft with a constant gravity of -9.81 m/s^2 is 28.56 seconds. The numerical integration using the python equation “solve_ivp” also achieved the same answer, confirming its accuracy. (Refer to Figure 1)

Time 2: Variable Gravity

Using a variable formula for gravity ($g(r) = g_o \;\; r \; R_{\oplus}$) dependent on the radius of the earth and the height of the object, the same value as previously was obtained however over longer distances this value would be different to Time 1. (Refer to Figure 2)

Time 3: Variable Gravity and Drag

Now adding drag coefficient by assuming a terminal velocity of 50 m/s, which gives a coefficient $\alpha \approx 0.003924$ kg/m. Including both variable gravity and quadratic air resistance, the fall time increases significantly to 83.53 seconds. The drag force slows descent, especially as speed increases and drag scales with velocity squared making the fall longer. Due to all of this, the increased time of 83.53 seconds with drag is accurate. (Refer to Figure 2)

Feasibility of depth measurement approach:

No Drag

If the Earth’s rotation is taken into account while calculating the time then the Coriolis force is used. It moves the mass sideways during the fall which makes it more erratic. Without drag, the mass collides with the 5m mine wall at a depth of approximately 3739.80m at a time of 27.61s. (Refer to Figure 3)

Drag

When drag is included, the reduced velocity lessens the Coriolis acceleration. With drag, the mass still drifts sideways but no longer hits the wall; instead, it successfully reaches the bottom after approximately 74.35 seconds. (Refer to Figure 3)

Taking these two scenarios into consideration, we can see that drag is definitely a necessary and important calculation and by removing it, we will be less accurate by a large factor.

Crossing Times of trans-earth/moon

Using the formula for density $\rho(r) = \rho n(1 - r\ 2\ )^{\wedge}n$, we are able to figure out what the fall time would be with everything included and a non-uniform earth. This vastly depends on which n value we use. The ones used are 0, 1, 2, and 9. We can also apply all of this logic for when the mine shaft is instead at the moon (figure 8). For both Earth and Moon, the tunnel crossing times are interesting to note given the different gravities and densities of the planet. This can explain how the times differ so vastly which is fascinating to see.

Density

The plot for Normalized Density v/s normalized Radius has the 4 different iterations of n. When n is 0 the graph is at a constant 1 density but as we increase 9, the graph starts sloping downwards faster and faster. (Refer to Figure 5)

Force

The plot for Gravitational Acceleration v/s normalized Radius has the 4 different iterations of n. When n is 0 the graph is a small linear increase ending at 1. All subsequent graphs have a bell shape that starts at 0 and ends at 1. The higher the value of n is, the higher the bell shape will be with 2 reaching up to around 0.7 and 9 reaching over 1.75 (Refer to Figure 6)

Pos and Velocity

The plot for Position and Acceleration vs Velocity are very similar at the start and then deviate. For position the position keeps decreasing and the higher the n value is, the faster it decreases. Naturally the velocity one follows the same concept, the higher the n is, the lower the velocity is. (refer to figure 7)

Normalization constants (Rho):

n = 0 rho = 5495.125873970929 kg/m³
n = 1 rho = 13737.814684927325 kg/m³
n = 2 rho = 24041.175698622825 kg/m³
n = 9 rho = 135551.49711226675 kg/m³

Time and Speed at Center(s):

n = 0 Time to center = 0.00019869259154594592 s, Speed at center = 50434686351.20175 m/s
n = 1 Time to center = 0.00017191338087877315 s, Speed at center = 66704986547.96889 m/s
n = 2 Time to center = 0.00016229617913295446 s, Speed at center = 77683553251.87451 m/s
n = 9 Time to center = 0.00014790982435039952 s, Speed at center = 117573121223.81894 m/s

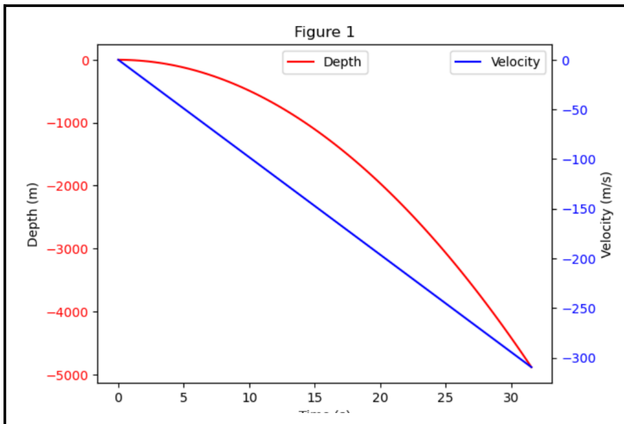
As for the earth and moon values:

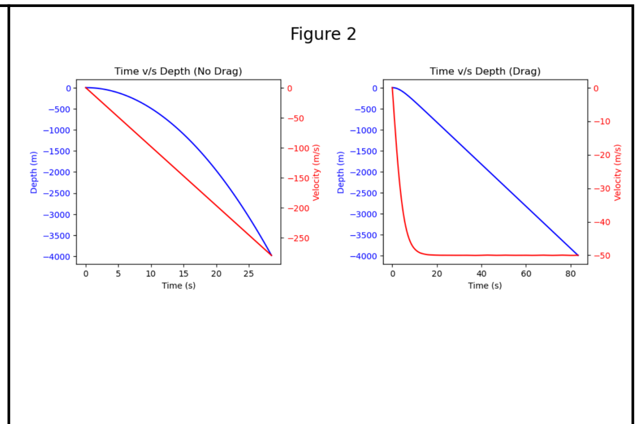
Earth density: 5510.0
Moon density: 3341.7538038703183
Moon is 60.64888936243772 % the density of Earth
Time to Moon center: 1625.1201867414677

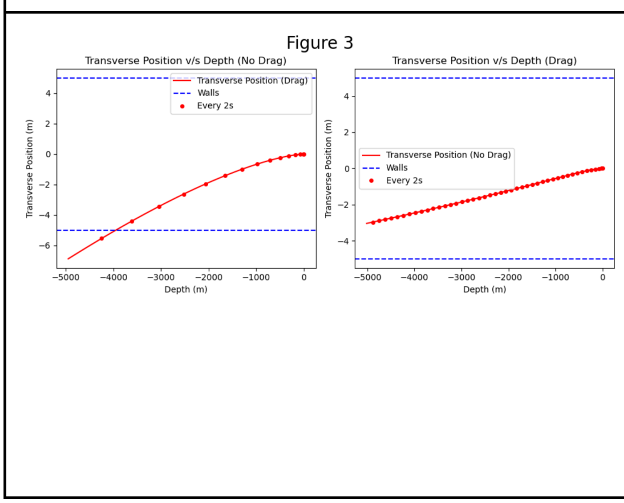
Speed at center: 1679.9994965766475
Scaled density factor 0.5 Time to center = 2298.2670085760788
Scaled density factor 1 Time to center = 1625.1201867414677
Scaled density factor 1.5 Time to center = 1326.9050760710352
Scaled density factor 2 Time to center = 1149.1335042880396

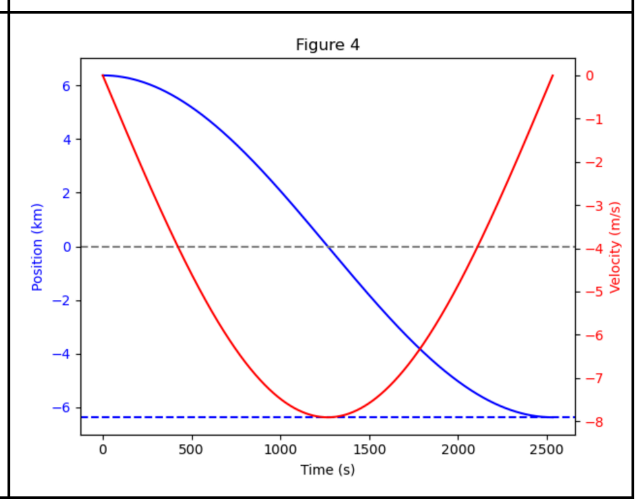
Conclusion

These numbers show how depending on what you account for in your calculation, the end result can differ massively. When you choose not to include drag or coriolis effect it can be simple if the calculation is a few meters but the farther it goes on, the more it will differ. There were still a few simplifications used such as gravity only being two decimal spaces but this could always be more and more accurate. If a more realistic approach is wanted then taking the weather and other environmental factors might be worth thinking about.









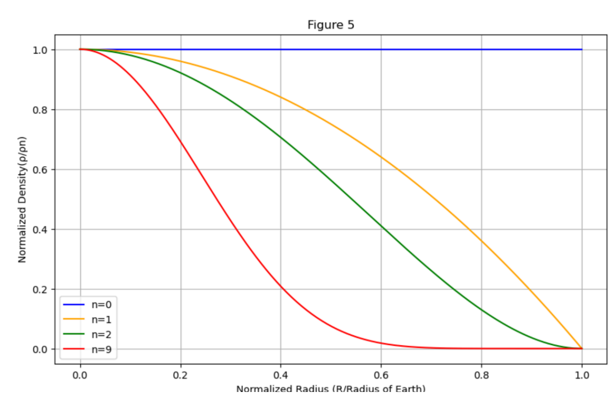


Figure 5
Normalized Density(ρ/ρn)
Normalized Radius (R/Radius of Earth)
n=0
n=1
n=2
n=9
0.0
0.2
0.4
0.6
0.8
1.0

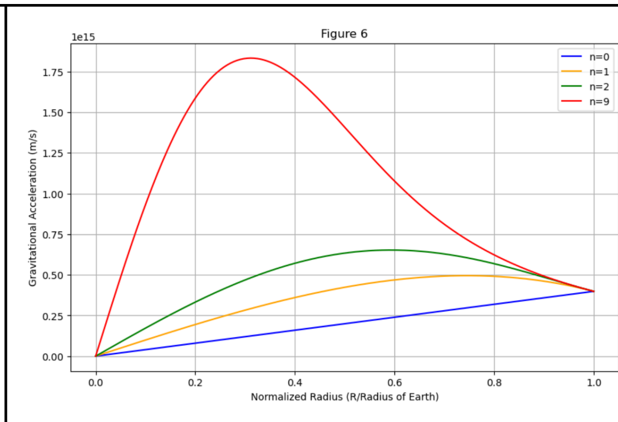


Figure 6
1e15
Gravitational Acceleration (m/s)
Normalized Radius (R/Radius of Earth)
n=0
n=1
n=2
n=9
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
0.0
0.2
0.4
0.6
0.8
1.0

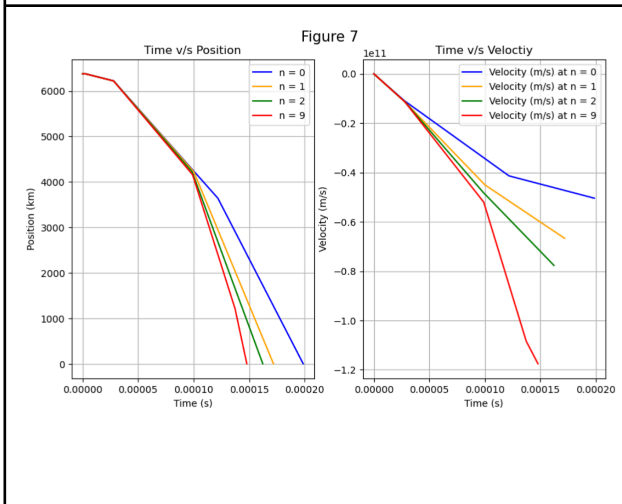


Figure 7
Time v/s Position
Position (km)
Time (s)
n = 0
n = 1
n = 2
n = 9
0
1000
2000
3000
4000
5000
6000
0.00000
0.00005
0.00010
0.00015
0.00020
Time v/s Velociy
1e11
Velocity (m/s)
Time (s)
Velocity (m/s) at n = 0
Velocity (m/s) at n = 1
Velocity (m/s) at n = 2
Velocity (m/s) at n = 9
0.0
−0.2
−0.4
−0.6
−0.8
−1.0
−1.2
0.00000
0.00005
0.00010
0.00015
0.00020

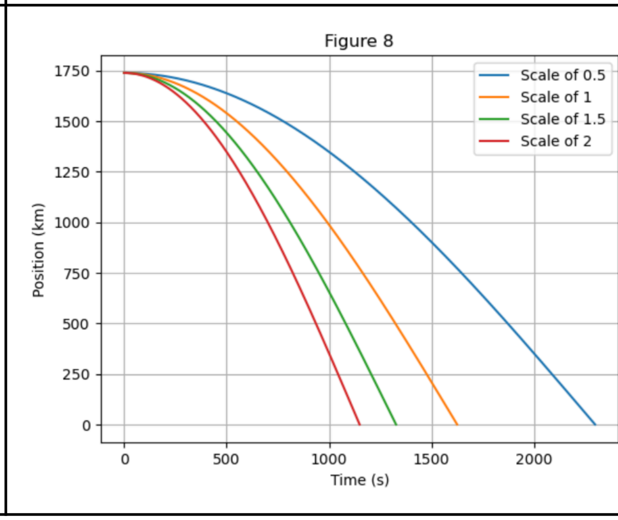


Figure 8
Position (km)
Time (s)
Scale of 0.5
Scale of 1
Scale of 1.5
Scale of 2
0
250
500
750
1000
1250
1500
1750
0
500
1000
1500
2000