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Our group has been presented with a problem: there are four coins on a rotating board, hidden from our view, and we need to ensure that all the coins are showing the same side to complete the puzzle. During each check, we are able to look at two coins at a time, but the board is spun between these checks so we do not know what the overall state of the board is. Our task was to devise a way to approach this puzzle that guarantees a solution in a finite number of steps.
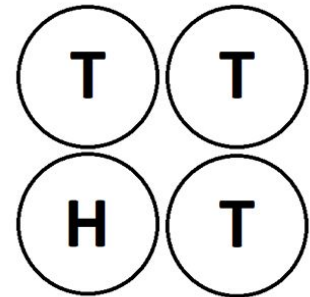
The approach we devised takes a maximum of five steps (with the example images to the right of the page being a worst case scenario):

1. We first choose two coins that are opposite to one another, and ensure that they are both showing the same side. For example, we can make any change necessary (change one, change both, or change neither) to have both coins showing heads. If the other two coins are already heads, then this action will solve the puzzle in a single step. If not, we know that at least one of the other two coins is showing tails.
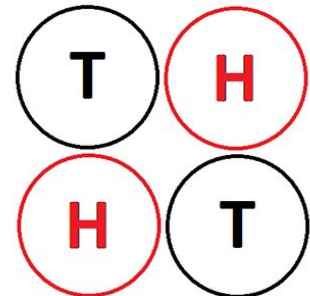
2. For our second step, we choose two coins that are beside each other. We know that these cannot be the same two as were checked in the first step, since those were opposites. We now ensure both of these coins are showing heads. One of them already will, as it will be one of the coins from the first step. If the other is showing tails, changing it to heads now could solve the puzzle in two steps. If both of these coins are showing heads, or changing the tails-showing coin to heads does not solve the puzzle, then we know that there is still one tails-showing coin on the board, with three showing heads.
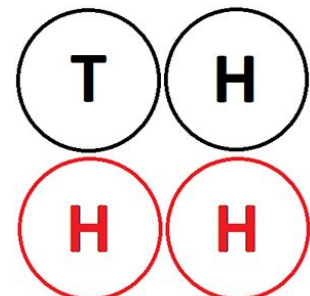
3. Next we take two opposite coins. If one is heads and one is tails then we know that changing the tails-showing coin to heads will make all four coins heads-showing, and the puzzle will be complete in four steps. If we find two heads-showing coins, then we know that to one side of both of them is a heads-showing coin, and to the other side is a tails-showing
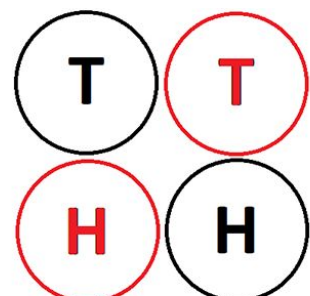
coin. If we now change one of these coins to show tails, this will guarantee that there are two tails coins beside each other, and two heads coins beside each other.

4. Now we take two coins beside each other again. If both are tails then changing them to heads will solve the puzzle. Likewise, if both are heads then changing them to tails will also solve the puzzle. But if we get one of each then we cannot solve the puzzle in four steps. Instead, we change (effectively swap) both coins so that we still have one of heads and one of tails, but now the overall layout will have two tails coins opposite one another, and two heads coins opposite one another.
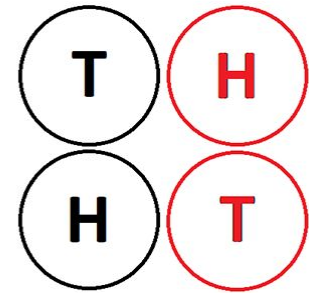
5. Because of the change at the end of the fourth step, we can now choose two opposite coins and be certain that they will either be both heads or both tails. Changing both coins now will solve the puzzle: if we find two tails coins then we know that the other two are heads and so changing the tails coins to heads now will make all four showing heads, while if we have two heads coins then we know that the other two are tails and changing these two to tails will also make four of the same.
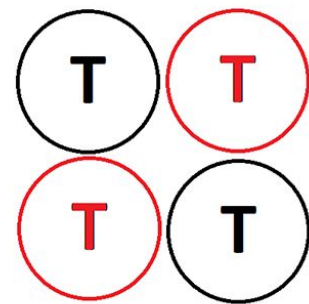
As described, these steps should work regardless of the starting conditions of the coins, with the only difference being whether it is solved in the full five steps or less. Assuming that the puzzle is not already solved at the beginning of the exercise, the minimum number of steps that it can be solved in is one, as described in our first step above. It is also possible to solve it in two, three, or four steps depending on the starting conditions. What we can guarantee is that, regardless of the starting conditions, these five steps will solve the puzzle.