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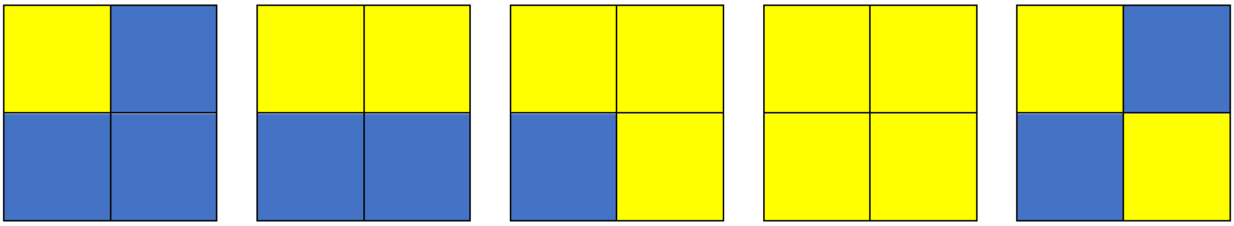
E4 – Coloured cubes

Problem:
Determine how many different 2x2x2 cube representations you can make using 8 smaller 1x1x1 cubes which are either yellow or blue, while also ensuring you have found them all. These representations must account for rotation and symmetry.

Workings:
We started by breaking the problem into a smaller one so it would be easier to work with – which in this case is 4 small cubes creating a 2x2x1 cuboid. Creating a smaller "cube" allows us to understand how rotations and symmetry work so we can apply it to the larger solution.
If we look at the square we can see that there are 5 unique colourings once rotation and symmetry are taken into account:



Now we know that there are 5 unique colouring of a "level" (2x2x1 square) and there are 2 "levels", there must be 5 different combinations of the bottom level for every 1 unique colouring of the top level (if you want to think of the 2x2x2 cube as two levels) which gives a total unique colouring of 25 options for a 2x2x2 cube (5 x 5).
We can be sure we have found all the options while taking into account rotation and symmetry because we have manually found all the combinations for the smaller problem (because it is small enough) taking into account those two requirements so increasing the size of the problem is just a matter of multiplication.
One real life object which assisted in solving this problem is the idea of a 4 digit pin code: each digit has a total of 10 possible unique numbers (numbers 0-9) so adding another digit results in 10*10 the number of unique possibilities and so forth. So each "level" in the cube can be thought of as a single digit in a 4 digit pin.