# Enhanced Control of a Vectored Thrust Aircraft

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**Abstract – The control of a simplified model of a vectored thrust aircraft is demonstrated in this report. The model is first controlled using a state feedback law and a PID controller tuned using Ziegler-Nichols Tuning. Additionally, Enhanced control using inverse based Youla-Kucera Parameterization is applied to the system to implement disturbance rejection.**

***Keywords:** Youla, Kucera, Parameterization, Control, Vectored Thrust aircraft, VTOL*

## 1. INTRODUCTION

With the continuous development in the field of aviation, one of the most sought over functions of a fighter jet is to increase maneuverability. The development of thrust vectoring started in the 1970s with the construction of the Harrier Jump jet. Thrust vectoring has several advantages such as making the need of long runways redundant. This provides a tactical advantage and the reduces cost of infrastructure. Additionally, vertical take-off and landings are possible, so the aircraft can be used on smaller aircraft carriers. There are some disadvantages to thrust vectoring. Thrust vectoring increases the design complexity of the aircraft. Additionally, vertical take-off provides no aerodynamic lift since there is no build-up velocity on a runway. Compensation must be made by directing thrust correctly around the aircraft body. This poses a dynamical and control problem.

The aim of the project is to analyze the dynamic system of a vertical take-off and landing (VTOL) aircraft and design a feedforward and feedback controller - with tools such as state feedback law and Youla-Kucera Parameterization - that enables good tracking and disturbance rejection.

## 2. PROBLEM FORMULATION

Firstly, the dynamics of the system are analyzed. The system model is very similar to the model used in *Hauser, Sastry and Meyer (1992)* [1]. The only difference in the model is an additional internal damping factor. The model description is given in [2].

### *A. Physics break-down*

To create the lift for take-off, the aircraft is designed to have thrust vectoring by using rotating nozzles to deflect the exhaust stream from the engines. For take-off, the aircraft must overcome the gravitational force acting on the plane and maintain its height controllably.

The aircraft has four thrust outputs – the exhaust from the main engine which assists in vertical motion, a fan (that operates using the turboshaft mechanism) for balancing vertical position, and one exhaust nozzle below each wing to assist with horizontal maneuverability – to stabilize hovering as shown in Fig. 1. It is evident that the total upward thrust must overcome the weight of the aircraft and the wing nozzles vary based on the horizontal motion requirement. For instance, if the desired displacement of the aircraft is in the positive direction on the $x$-axis, the thrust from the left-wing nozzle must be greater than the right-wing nozzle and vice-versa to move the aircraft in the negative direction on the $x$-axis.

Let $x$, $y$ and $\theta$ denote the position and orientation of the centre of mass of the aircraft. Table 1 shows the parameters used for the model. Let M = 10 Kg, J = 10 Kg-m$^2$, r = 1m, g = 9.81 m/s$^2$ and c = 1 N-s/m. From Newton's law, the equations for horizontal motion,

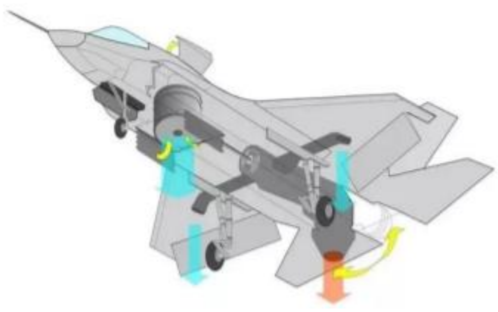

Fig. 1. Thrust outputs of a Vectored thrust Aircraft

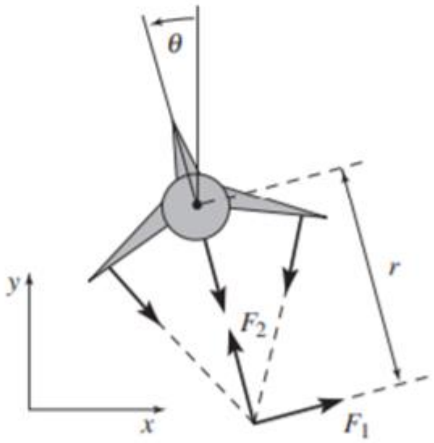


Fig. 2. Force Analysis of the given model

| Model Parameters | | |
|---|---|---|
| ***Notation*** | ***Parameter*** | ***Units*** |
| M | Mass of the Aircraft | Kg |
| J | Moment of Inertia | Kg-m$^2$ |
| r | Thrust offset | m |
| g | Acceleration due to gravity | m/s$^2$ |
| c | Damping Constant | N-s/m |
| x | Aircraft Horizontal Position | M |
| y | Aircraft Vertical Position | M |
| θ | Aircraft Roll Angle | Degrees |

Table 1. System Parameters

vertical motion, and roll motion, respectively, are

$$M\ddot{x} = F_1 cos\theta - F_2 sin\theta - c\dot{x} \tag{1}$$

$$M\ddot{y} = F_1 sin\theta + F_2 cos\theta - Mg - c\dot{y} \tag{2}$$

$$J\ddot{\theta} = rF_1 \tag{3}$$

*B. System Linearization*

The equations (1) to (3) are not linear. We now proceed to linearize around the equilibrium position for small perturbations. Let the state of the system be defined as

$$g = \left(x, y, \theta, \dot{x}, \dot{y}, \dot{\theta}\right)^T \tag{4}$$

From the equations above,

$$\dot{g} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \\ \frac{-c}{M}\dot{x} + \frac{F_1 Cos(\theta)}{M} - \frac{F_2 Sin(\theta)}{M} \\ \frac{-c}{M}\dot{y} - g + \frac{F_2 Cos(\theta)}{M} + \frac{F_1 Sin(\theta)}{M} \\ \frac{r}{J}F_1 \end{bmatrix}$$

$$\dot{g} = f(t) \tag{5}$$

The above system is in equilibrium when θ = $0^0$. This implies that $\ddot{\theta} = \dot{\theta} = 0$. After substituting $\ddot{\theta} = 0$, from eq. (3) we have, $F_1 = 0$ and $F_2 = Mg$.

The equilibrium state is given by $g_0 = \left(x_0, y_{0,} 0,0,0,0\right)^T$ with $\theta_0 = 0$ and the equilibrium input is given by $u_0 = [F_1, F_2] = [0, Mg]$. The state equation is given by

$$\dot{g} = Ag + Bu$$
$$y = Cg + Du \tag{6}$$

To linearize around the equilibrium point, *Jacobian Linearization* is used where the new state, $z = g - g_0$ and input, $u = u - u_0$.

The state space is now given by:

$$\dot{z} = A\,z + B\,u$$
$$y = Cg + Du \tag{7}$$

Where,

$$A \coloneqq \left[\frac{\delta f}{\delta g}\right]_{g=g_0, u=u_0}$$

$$B \coloneqq \left[\frac{\delta f}{\delta \mathrm{u}}\right]_{g=g_0, u=u_0} \tag{8}$$

Computing the partial derivative yields

$$A = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & -g & \frac{-c}{M} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{-c}{M} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{9}$$

$$B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \frac{1}{M} & 0 \\ 0 & \frac{1}{M} \\ 0 & 0 \end{bmatrix} \quad (10)$$

The output to be tracked is the horizontal and the vertical position of the aircraft without any initial feedforward input. Therefore, the matrices C and D are

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \quad (11)$$

With these matrices, the state space equation of the system is derived.

## 3. CONTROL PROBLEM

From the problem description and the dimensions of the state space matrices, it is evident that we have a MIMO system with 2 inputs and 2 outputs. The transfer function of the system from $F_1$ to $x(t)$ is

$$G_x(s) = \frac{0.1s^2 - 0.981}{s^4 + 0.1s^3} \quad (12)$$

And the transfer function from $F_2$ to $y(t)$ is

$$G_y(s) = \frac{0.1}{s^2 + 0.1s} \quad (13)$$

The transfer functions from $F_1$ to $y(t)$ and $F_2$ to $x(t)$ are zero since the system is linearized at the equilibrium position. Therefore, the inputs and outputs are decoupled, i.e., $F_1$ only affects horizontal displacement whereas input $F_2$ only affects the vertical displacement.

This report does not cover MIMO Youla-Kucera parameterization. Since the linearized system dynamics are decoupled, another way to proceed is to apply control to the systems separately by treating them as 2 different SISO systems. Since the dynamics for the x-direction have a non-minimum phase (NMP) zero and more unstable poles, inverse based Youla-Kucera (Y-K) parameterization is applied to that system as it is more challenging. The direction Y is controller using a simple PID controller tuned using Ziegler-Nichols Method.

### *A. Controlling X- Direction*

#### *I. State Feedback Control Law*

From equation (12), the zeros of the system are at $z = \pm 3.1321$ and the poles are $p = 0, 0, 0, -0.1$. The system has a NMP zero and 3 unstable poles. This makes it highly unstable. Therefore, it is difficult to control this system. It is difficult to design a baseline controller using only Y-K Parameterization. Thus, state feedback control law is used to initially stabilize the open loop continuous time system. The system is controllable and therefore, arbitrary closed-loop pole placement is possible. The state-feedback gains are calculated using MATLAB's 'lqr' command that outputs the gain for a minimized cost function. The control weights are Q = $I_{6x6}$ and R = 0.001. Using these parameters, the feedback gains are

$$K_{fb} = \begin{bmatrix} -31.62 & 0 & 371.69 & -55.31 & 0 & 149.71 \\ 0 & 31.62 & 0 & 0 & 39.41 & 0 \end{bmatrix} \quad (14)$$

Using the state feedback gain, the closed loop system is

$$\dot{X} = (A - BK_{fb})X + Bu \quad (15)$$

The stabilized system is converted to discrete time using a zero-order hold (ZOH) with sampling time of $T_s = 0.1$ seconds. The discrete time system now becomes the baseline plant, P, to which enhanced control is applied using inverse based Y-K Parameterization.

#### *II. Inverse based Youla-Kucera Parameterization*

This method is applied to implement disturbance rejection and a better system response. The all-stabilizing parameterization is given by

$$C_{all} = \frac{C + z^{-m}\hat{P}^{-1}Q}{1 - z^{-m}Q} \quad (16)$$

Where C is the baseline feedback controller, m is the relative degree of the plant, Q is the filter for disturbance rejection and $\hat{P}$ is an approximate of P. The discrete time plant is

$$P = \frac{10^{-4}(3.55z^5 + 10.8z^4 - 8.55z^3 - 2.46z^2 + 5.56z - 1.747)}{0.031z^6 - 0.15z^5 + 0.30z^4 - 0.3203z^3 + 0.1916z^2 - 0.06z + 0.008} \quad (17)$$

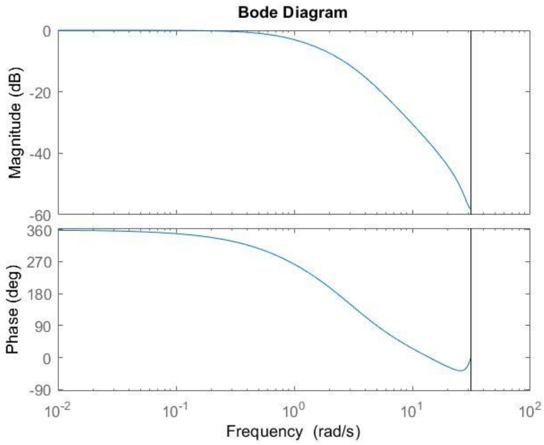


*Fig. 3. Bode plot of the plant P.*

The plant, P, is already stable due to the implementation of state feedback. Therefore, another controller, C, is not required. Thus, C = 0 for the rest of this report. From eq. (17), the relative degree of the plant is m = 1. The filter Q is designed such that it rejects any disturbance of a constant or low frequency. The kind of disturbance that a VTOL aircraft would face is are mostly constants (e.g. wind).

Currently, any disturbance acting on the system would act as a reference to the plant P. Fig. 3 shows the frequency response of plant P. From the magnitude plot, it is evident that an input of low frequency would cause the plant response to have a large magnitude. From Fig. 3, it can also be deduced that P is already good at rejecting high frequency disturbance. Therefore, Q should be designed to reject disturbance of lower magnitudes.

Let the disturbance signal be of the form

$$d(t) = cos(\omega_0 t) \qquad (18)$$

for all t > 0 with $\omega_0 = 0\ rad/s$. This gives a constant step-like signal of magnitude 1. For such a disturbance, the filter Q has the form

$$1 - z^{-m}Q = \frac{A(z^{-1})}{A(\alpha z^{-1})} \qquad (19)$$

Where,

$$A(z^{-1}) = 1 - 2cos(\omega_0)z^{-1} + z^{-2} \qquad (20)$$

such that $A(z^{-1}) = A_1(z^{-1})A_2(z^{-1})$ with $A_1(z^{-1})d_1(k) = 0$ and $A_2(z^{-1})d_2(k) = 0$.

*Proof 1(see [5] for further examples and discussions):*

Given disturbance signal $d(t) = cos(\omega_0 t)$, applying z-transform yields $d(k) = cos(\omega_o T_s k)$ (assume Ts = 1). This disturbance can be split into 2 parts using Euler's formula such that

$$cos(\omega_0 k) = \frac{e^{j\omega_0 k} + e^{-j\omega_0 k}}{2}$$

$$= \frac{e^{j\omega_0 k}}{2} + \frac{e^{-j\omega_0 k}}{2}$$

$$cos(\omega_0 k) = d_1(k) + d_2(k)$$

Need to find $A(z^{-1}) = A_1(z^{-1})A_2(z^{-1})$ with $A_1(z^{-1})d_{1(k)} = 0$ and $A_2(z^{-1})d_{2(k)} = 0$. Let $A_1(z^{-1}) = 1 - e^{j\omega_0}z^{-1}$ and $A_2(z^{-1}) = 1 - e^{-j\omega_0}z^{-1}$.

$$A_1(z^{-1})d_1(k) = (1 - e^{j\omega_0}z^{-1})e^{\{j\omega_0 k)}/2$$

$$= e^{j\omega_0 k} - e^{j\omega_0}e^{j\omega_0(k-1)}$$

$$= e^{j\omega_0 k} - e^{j\omega_0 k} = 0$$

Similar holds for $A_2(z^{-1})d_2(k)$. Thus $A(z^{-1}) = A_1(z^{-1})A_2(z^{-1}) = 1 - 2cos(\omega_0)z^{-1} + z^{-2}$.

From eq. (19),

$$Q(z^{-1}) = z\left[1 - \frac{A(z^{-1})}{A(\alpha z^{-1})}\right] \qquad (21)$$

Substituting from eq. (20), the result is

$$Q(z^{-1}) = \frac{(2-2\alpha)cos(\omega_0) + (\alpha^2 - 1)z^{-1}}{1 - 2\alpha cos(\omega_0)z^{-1} + \alpha^2 z^{-2}} \qquad (22)$$

The value of α is picked close to 1. This value determines the width of the notch in the frequency response of the filter. In this case, the value picked is $\alpha = 0.96$. Keep in mind that the Q filter was designed for a constant disturbance. Fig. 4 shows the bode plot of the internal model $A = 1 - z^{-m}Q$ for the designed Q. Notice the very low magnitude at zero/low frequencies. This internal model is transferred to the final sensitivity function.

Another requirement is to design the system inverse to be implemented in all stabilizing controller. Notice that the system transfer function has a non-minimum phase zero. As a result, direct inverse of the system is not possible due to arising unstable pole. Therefore, an inverse approximate must be carried out. The

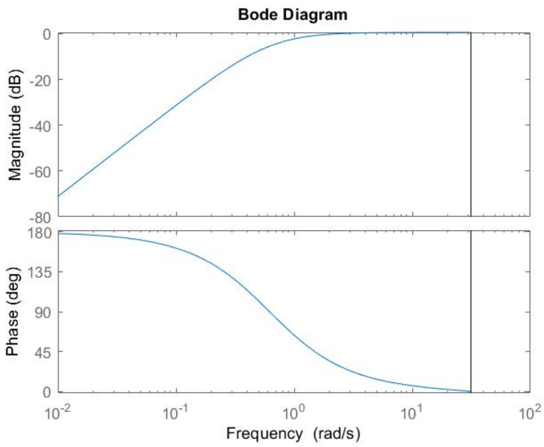


*Fig. 4 Bode plot of the internal model for disturbance rejection*

method used in this problem is called 'Zero Magnitude Error Cancellation', which involves the substitution of the non-minimum phase zero by it's stable counterpart such that the magnitude of the approximate system is the same as the original system. The stable zeros of P are [-0.7293 0.8993 0.7423 0.7311] and the unstable zero is 1.368. replace the zero at 1.368 by 1/1.3678 = 0.7310. Therefore, the approximate system, $\hat{P}$ has same poles of P and zeros with the substitution.

### *III. Desired Trajectory*

The desired trajectory undergoes sinusoidal acceleration of the form

$$a = A\sin(\omega t) \tag{23}$$

Integrating twice results in a trajectory of the form shown in Fig. 5. The trajectory starts at t = 5s with initial position equal to 0. The trajectory ends at t = 10s with a final position of 1.

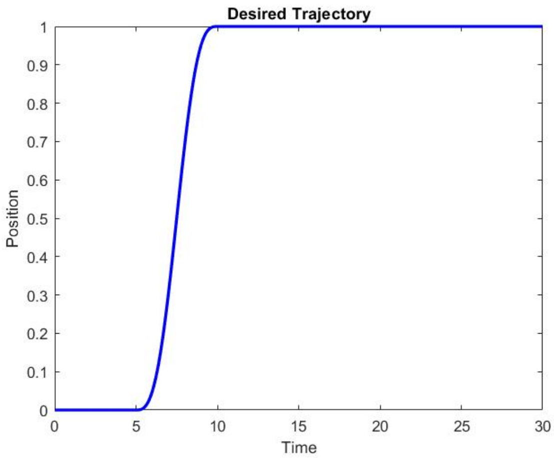


*Fig. 5. Desired Trajectory*

### *IV. Simulink Implementation*

The resulting system was implanted using Simulink for a reference trajectory and a constant disturbance. Fig. 6 shows the block diagram of the closed-loop system with the inverse based controller connected to the plant P. The all stabilizing controller, $C_{all}$ is given by the equation

$$C_{all} = \frac{C + z^{-m}\hat{P}^{-1}Q}{1 - z^{-m}Q} \tag{24}$$

This controller can be implemented in a direct feedback loop with the plant. Splitting it into the block diagram shown in Fig. 6 enables us to better understand the internal dynamics of them system. Moreover, Simulink implementation is easier.

Fig. 7 shows the system response to the desired trajectory. As expected, the NMP zero causes issues in the system response. Firstly, there is a delay in the response. Secondly, it causes the response to overshoot by a certain amount. The overall system response is acceptable as the inverse used is only an approximate of the plant. Moreover, the simulation is for a linearized version of the plant. The maximum absolute value of the error is 0.35. For precision tracking of NMP and nonlinear systems, the system internal dynamics must be solved using reduced order inversion methods as done in [3]. This method provides an exact inverse for non-minimum phase systems.

Fig. 8 shows the system response to a step disturbance signal with and without the implementation of inversion-based parameterization. With the implementation, the response to disturbance slowly goes to zero. When the Q-filter output is disconnected, the disturbance response settles at 1. This is because it acts as a reference input to the initial system with state feedback law implemented. The overall system response is satisfactory. Disturbance rejection plays a big role in practical applications. Enhanced control allows the system to reject disturbance automatically without any human interference.

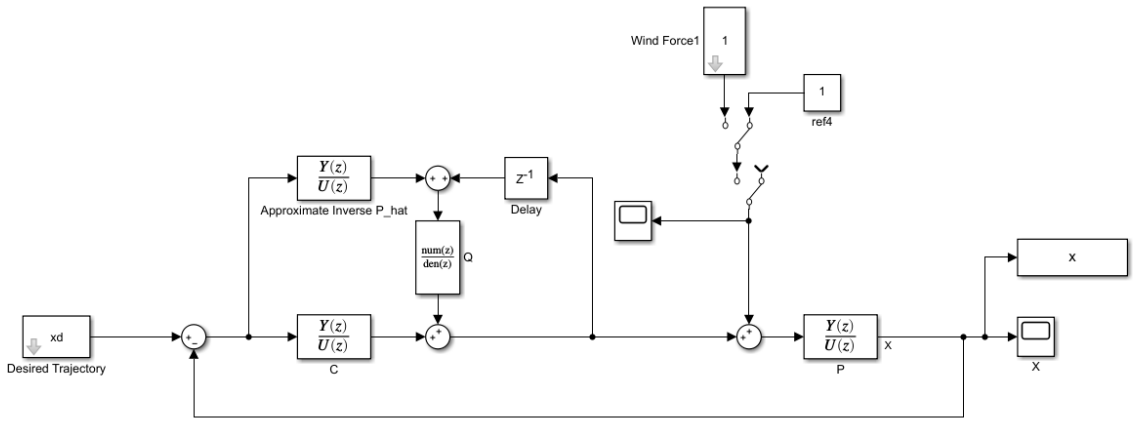


*Fig. 6. Block-diagram simplification of the inverse-based parameterization $C_{all}$.*

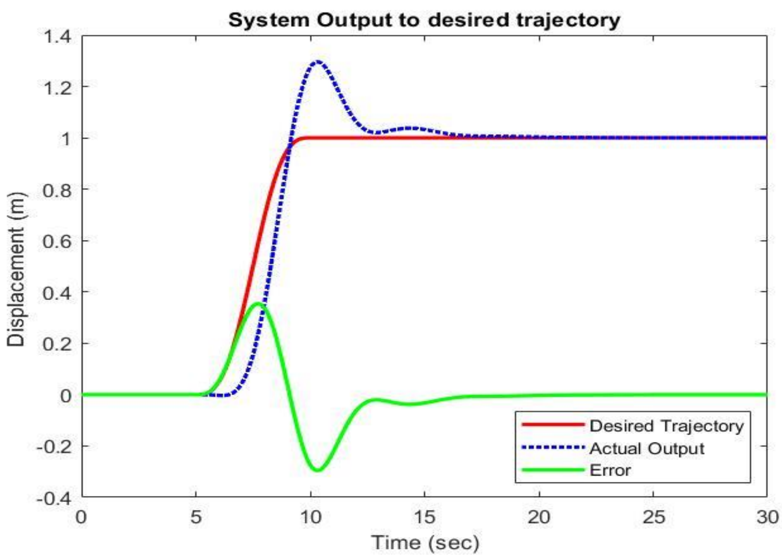


*Fig. 7. System response to the desired trajectory without disturbance.*

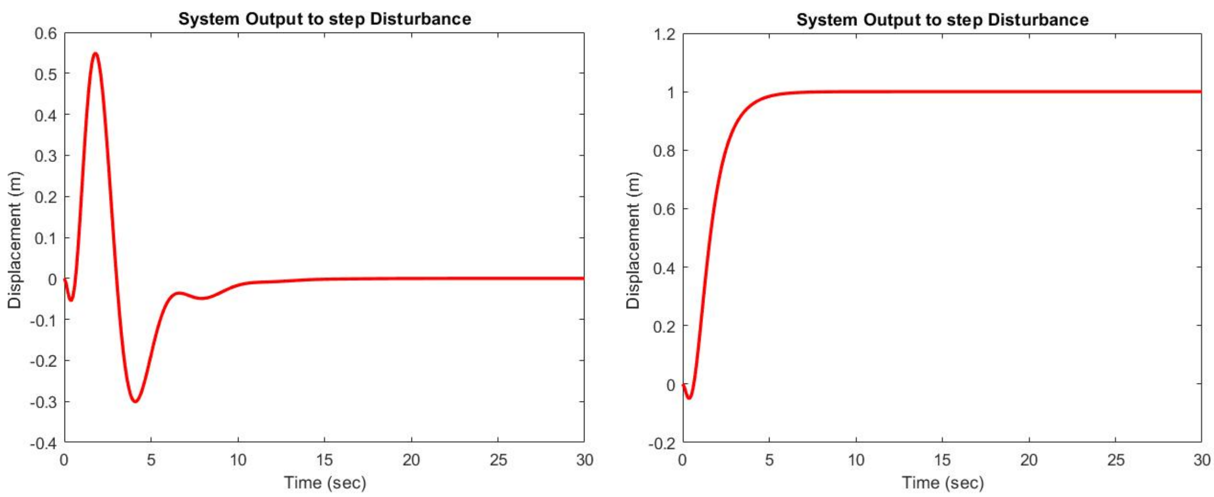


*Fig. 8. System response to a step disturbance with the implementation of inverse-based control (left) and system response to step disturbance without inverse-based control (right)*

*B. Controlling Y-Direction*

Controlling the Y-direction is much easier due to the absence of a NMP zero. The open-loop transfer function for the Y-direction is

$$G_y(s) = \frac{0.1}{s^2+0.1s} \tag{25}$$

This system is easily stabilized with a feedback and implementing a PID controller yields exceptional results. The PID controller used for this problem is designed using Ziegler-Nichols Tuning Method. Upon closing the loop, the system has 2 poles on the left half plane. Therefore, the system will be stable at all gains as the closed-loop poles never move to the RHP (based on Root locus Analysis). To cause oscillations, the feedback gain must be very high. This critical feedback gain was found out to be $K_{crit} = 10{,}000$. The period of oscillation at this gain was $P_{crit} = 0.2$ seconds. The PID controller has the form

$$C_{PID}(s) = K_p\left(1 + \frac{1}{K_I s} + K_d s\right) \tag{26}$$

Where,

$$K_P = K_{crit}$$

$$K_I = 0.5P_{crit}$$

$$K_D = 0.125P_{crit} \tag{27}$$

This yields a PID controller of the form

$$C_{PID}(s) = \frac{250s^2+10{,}000s+100{,}000}{s} \tag{28}$$

Applying this controller in a feedback loop with $G_y(s)$ yields a closed-loop transfer function of the form

$$G_{ycl}(s) = \frac{25s^2+1000s+10{,}000}{s^3+25.1s^2+1000s+10{,}000} \tag{29}$$

This system is simulated for a much faster desired trajectory that changes position from 0 to 1 in 2 seconds. The system response is shown in Fig.9. The PID controller provides excellent tracking. The response follows the desired trajectory very closely. Fig.10 shows the error in the response. The error is of the magnitude $10^{-3}$ which shows that the tracking precision is very high.

## 4. CONCLUSION

Initially, it was seen that the model to be worked on was highly unstable due to presence of multiple poles on the origin and a non-minimum phase zero. Linearizing the system at an equilibrium point showed that the system inputs and outputs can be decoupled at the equilibrium point. Using this fact, the system was treated as 2 separate SISO systems.

To control the X-Direction, the plant was stabilized using state feedback control law and then discretized using a zero-order hold. This provided a baseline plant to which enhanced control could be applied. The components for the inverse-based Youla-Kucera parameterization were designed based on requirements. Disturbance rejection was added to reject constant frequency disturbance signals such as wind. To control the Y-Direction, a simple PID controller tuned using the Ziegler-Nichols method proved effective. To summarize, the results achieved were satisfactory given that the system is inherently very unstable due to unstable poles and zero.

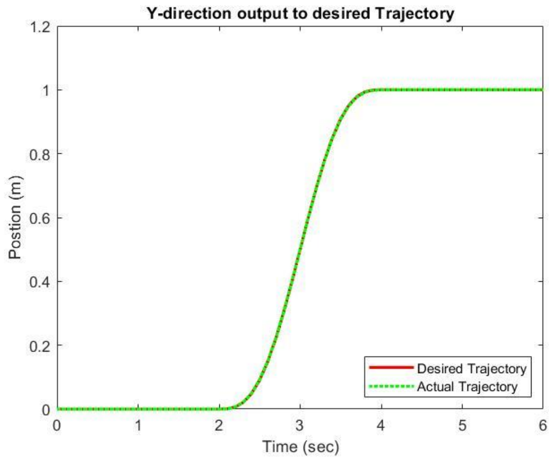


*Fig. 9. System Response to the desired trajectory*

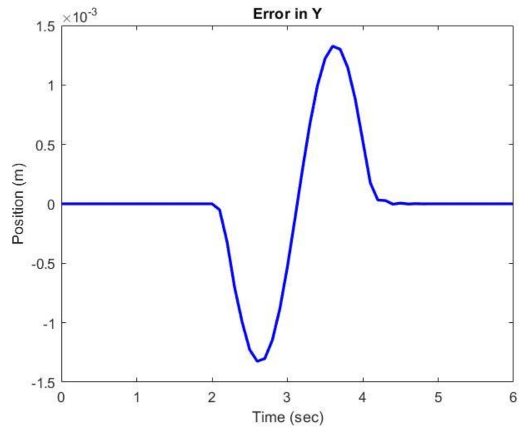


*Fig. 10. Tracking error*

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