

Contents

- Desired velocity profile
- Vehicle Parameters
- PI controller (using Ziegler Nichols tuning)
- System in DT with disturbance filter
- MATLAB simulation: PI controller only
- Simulink
- Velocity profile

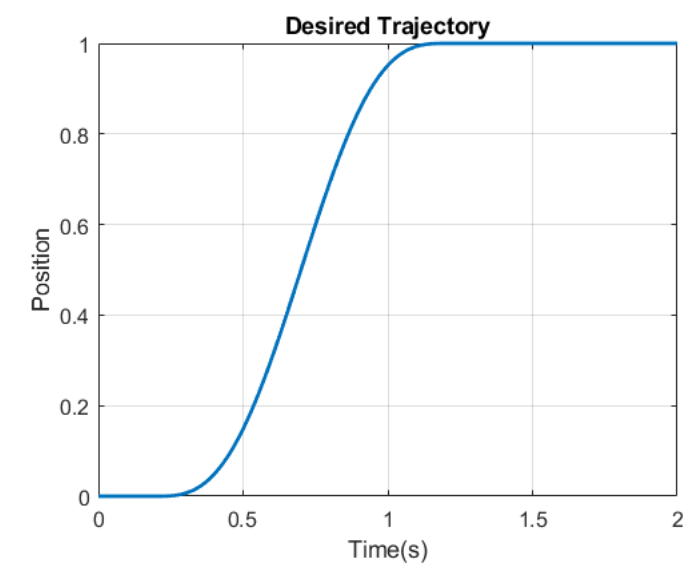
```
close all; clear all; clc
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Places that need manual modification will have the following
% Load vehicle_steering_YK.slx in the same path

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% Modify %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
% Instructions are provided
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% Final Project: inv YK Parameterization and disturbance rejection %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

Desired velocity profile

```
fs = 160; % sampling frequency, 10x times the max disturbance
Ts = 1/fs;
wth = 2; % linewidth for plotting

nfig = 0;
T = 1; % T is the time period in second
Npre = 0.2; Npost = 0.8; % pre and post actuation time
Tend = T+Npre+Npost; % Ending point for plotting
y_final = 1;
[yd,ydd,yd2d,t,nfig] = sinusoidal_yacc(Npre,(Npre+T),(Npre+T+Npost), 0, y_final, nfig,Tend);
```



Vehicle Parameters

```
a = 2.5; % length from rear wheels to center of mass
b = 5; % length of car
v_0 = 30; % m/s

% Open Loop system, linearized
A = [0 v_0; 0 0];
B = [a/b*v_0 v_0/b]';
C = [1 0; 0 1];
D = [0; 0];
ctrb(A,B);
s = tf('s');
[num, den] = ss2tf(A,B,C,D);

% Defining state names
states = {'theta' 'Y'};
inputs = {'delta'};
outputs = {'Y'; 'Theta'};
```

```

sys = ss(A,B,C,D,'statename',states,'inputname',inputs,'outputname',outputs);
sys_y = sys(1,1); % sigma to y
sys_th = sys(2,1); % sigma to theta
Kcr = 1; % ideal gain
[Ay By Cy Dy] = ssdata(sys_y); % Extract state space
[num den] = ss2tf(Ay,By,Cy,Dy);
Gy_ol = tf(num,den);
P = Gy_ol; % open loop plant

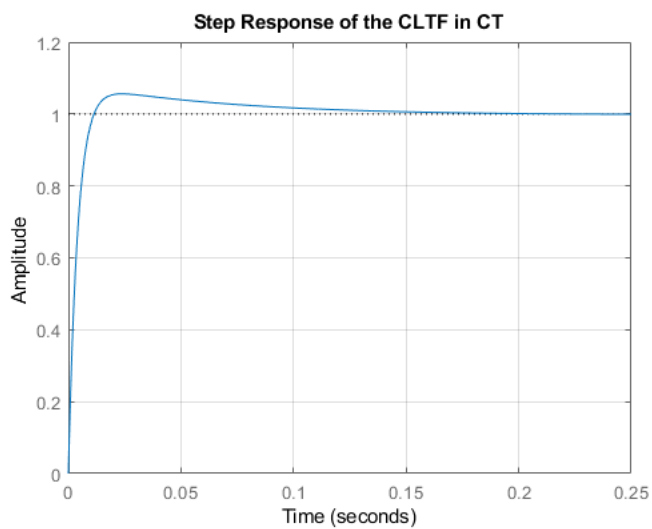
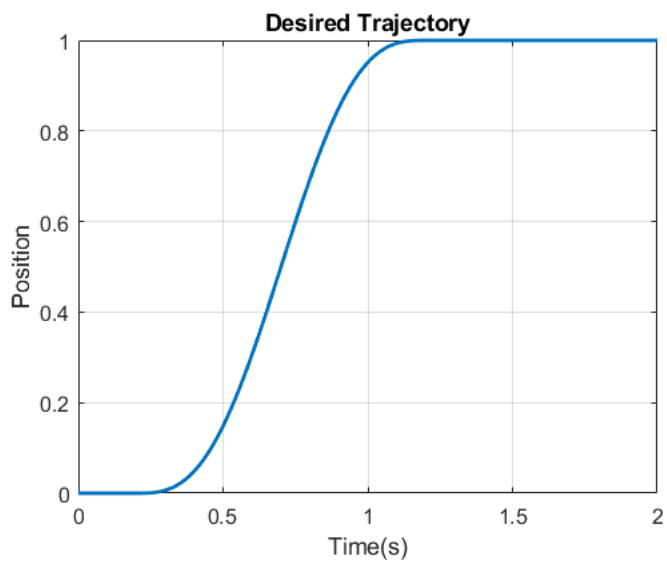
```

## PI controller (using Ziegler Nichols tuning)

```

L = 0.05;
T = 0.95;
Kp = 0.9*T/L;
Ti = L/0.3;
Td = 0;
C_ct = Kp*(1+1/(Ti*s)+Td*s); % PI controller in CT
Gy_cl = feedback(C_ct*Gy_ol,1); % CLTF system with PI controller
Gy_cl = minreal(Gy_cl); % Remove redundancy
% check stability
figure()
step(Gy_cl)
grid
title('Step Response of the CLTF in CT')

```



## System in DT with disturbance filter

```

%%%%%%%%%% Modify %%%%%%%%%%
% Uncomment one of the lines to save the MATLAB plots
% save = 1; % save == yes
save = 0; % save == no
%%%%%%%%%%

```

```

for i_a = 1:3
    param_a = i_a;
    if param_a == 1
        alpha = 0.2;
    elseif param_a == 2
        alpha = 0.99;
    else
        alpha = 0.6;
    end

    z = tf('z',Ts);
    m = 1; % relative degree
    Zm = z^-m;
    P_dt = c2d(P,Ts,'ZOH'); % convert plant to DT
    invP = inv(P_dt)*Zm; % invert plant to DT and make strictly proper
    wn = 80; % filtered out frequency
    w_dt = wn*Ts;

    % parameterized Q
    % mathematical solution
    Q_math = 2*cos(w_dt)-z^-1;
    figure()
    bode(Q_math)
    grid
    title('Mathematical Solution')

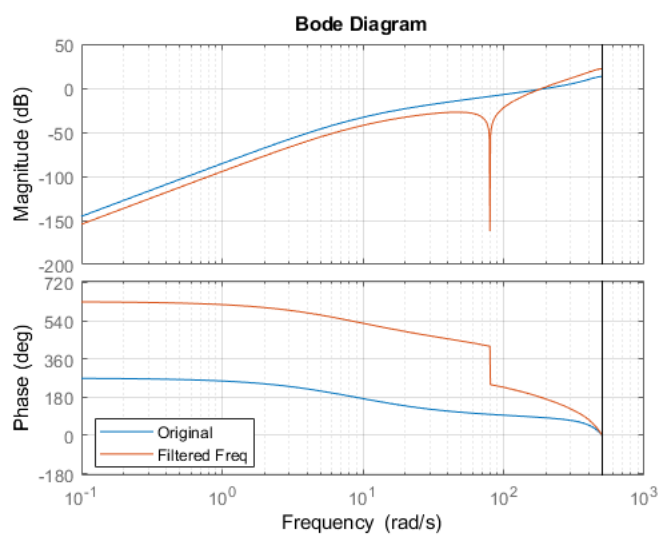
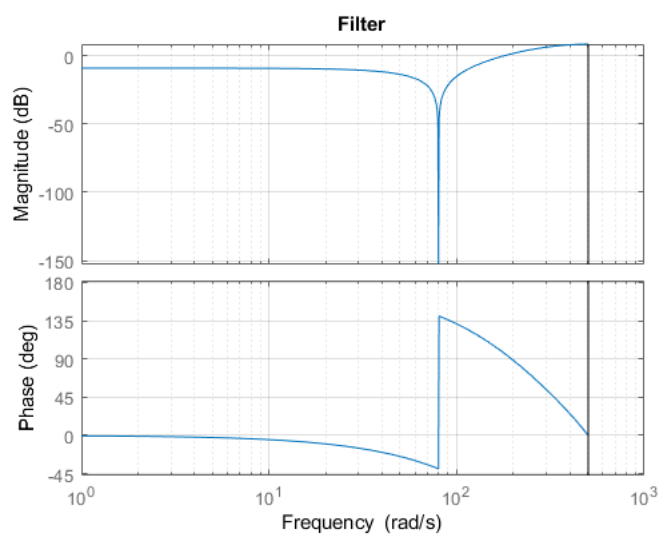
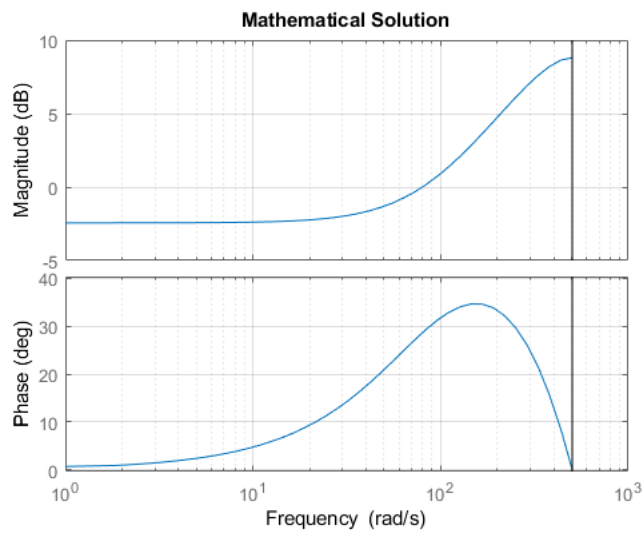
    Q = minreal(((2-2*alpha)*cos(w_dt)+(alpha^2-1)*z^-1)/(1-2*alpha*cos(w_dt)*z^-1+alpha^2*z^-2));
    A = 1-Zm*Q; % filtered out frequency
    figure()
    bode(A)
    title('Filter')
    grid on
    % Sensivity function
    C_dt = c2d(C_ct,Ts); % PI controller in DT
    S_dt = minreal(1/(1+P_dt*C_dt)); % original sensitivity
    figure()
    bode(S_dt)
    hold on
    S_dt_q = minreal(1/(1+P_dt*C_dt)*(1-z^-m*Q));
    bode(S_dt_q) % parameterized sensitivity function
    legend('Original','Filtered Freq','location','best')
    grid on
    hold off

    if save == 1 && param_a == 1
        title('Sensitivity Function, alpha = 0.2')
        saveas(gcf,'Sensitivity_S_alpha0.2.png')
    elseif save == 1 && param_a == 2
        title('Sensitivity Function, alpha = 0.99')
        saveas(gcf,'Sensitivity_S_alpha0.99.png')
    elseif save == 1 && param_a == 3
        title('Sensitivity Function, alpha = 0.6')
        saveas(gcf,'Sensitivity_S_alpha0.6.png')
    else
        save = 0;
    end

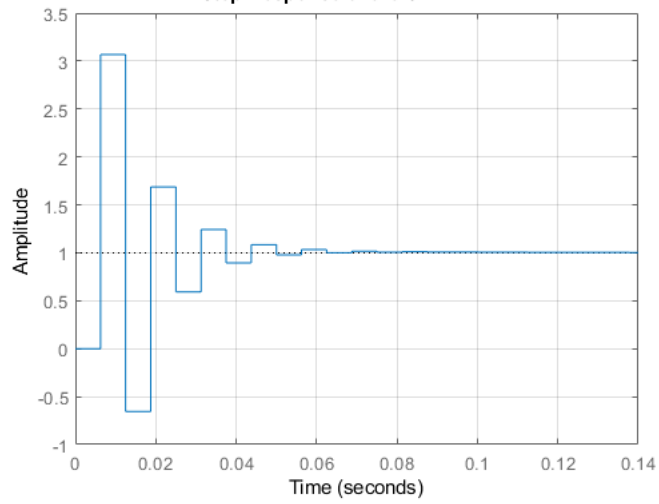
    % equivalent C_all for the parameterized controller
    C_all = minreal((C_dt+invP*Q)/(1-Zm*Q));
    G_cl = feedback(P_dt*C_all,1);
    G_cl = minreal(G_cl);

    % check DT is stable
    figure()
    step(G_cl)
    grid
    title('Step Response of the CLTF in DT')
    figure()
    pzmap(G_cl)
    title('P-Z Map of CLTF in DT')
end

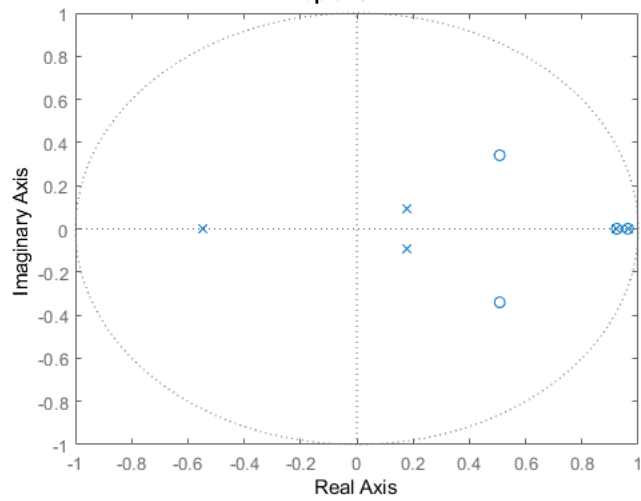
```



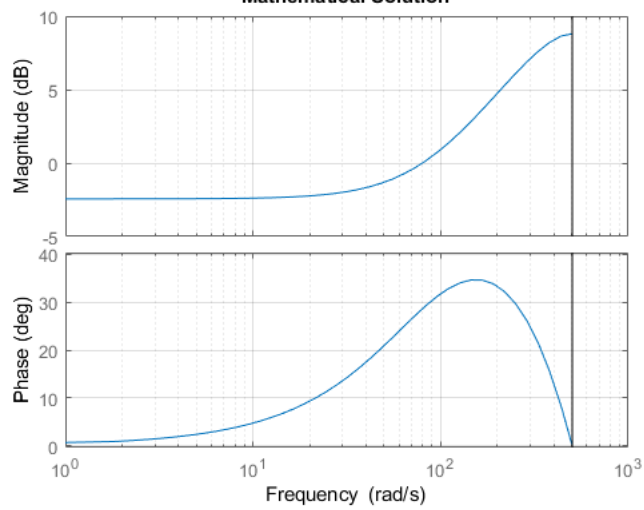
**Step Response of the CLTF in DT**

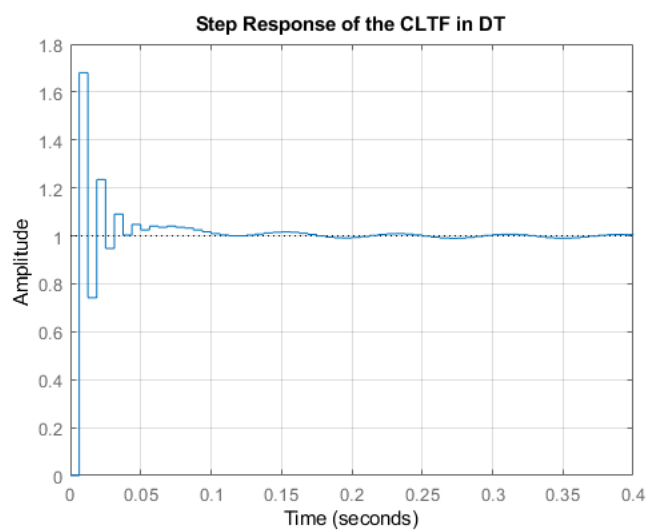
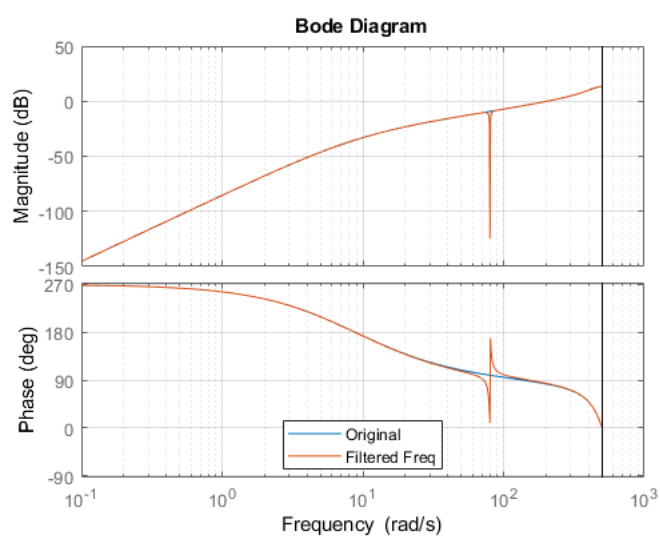
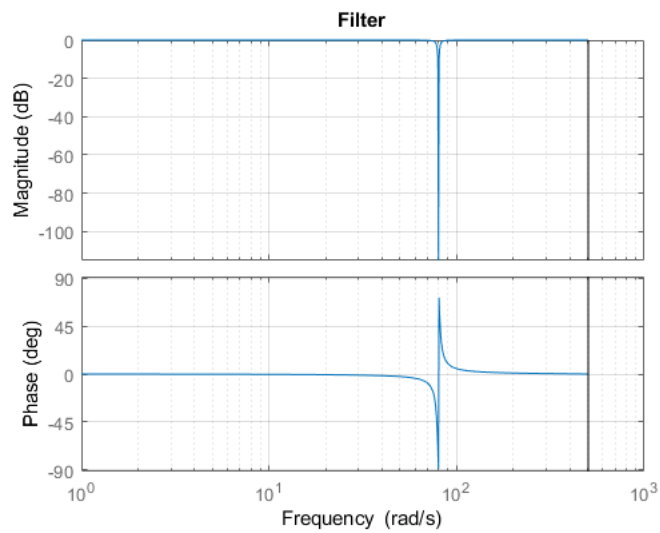


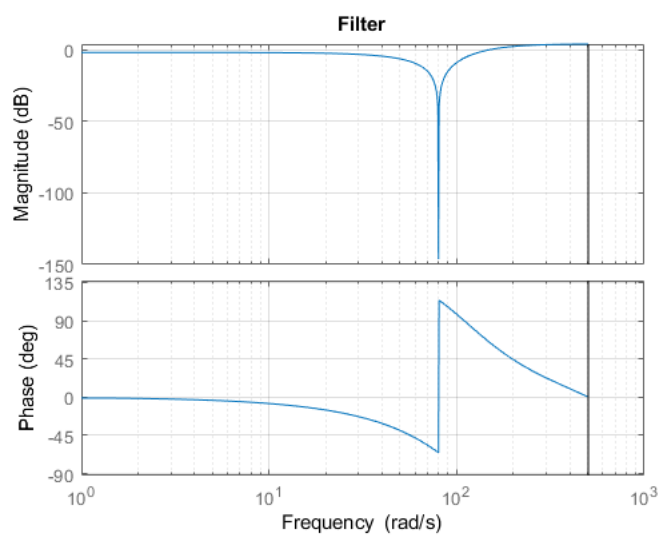
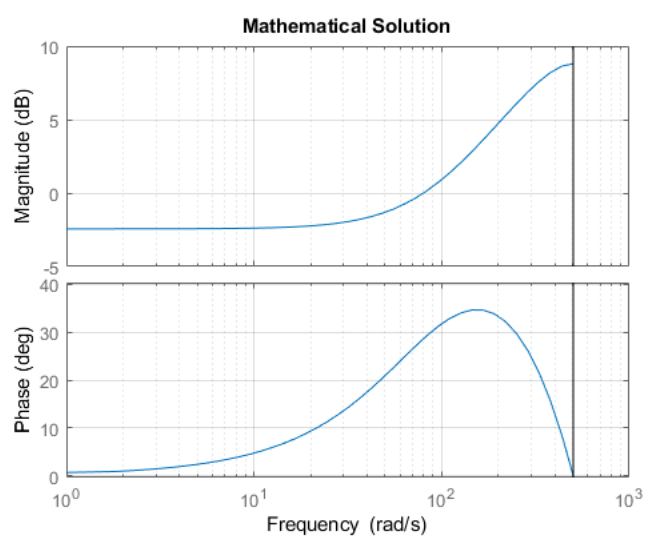
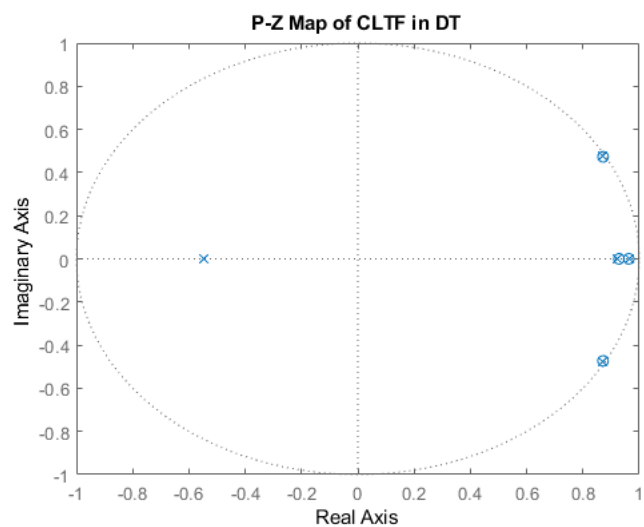
**P-Z Map of CLTF in DT**

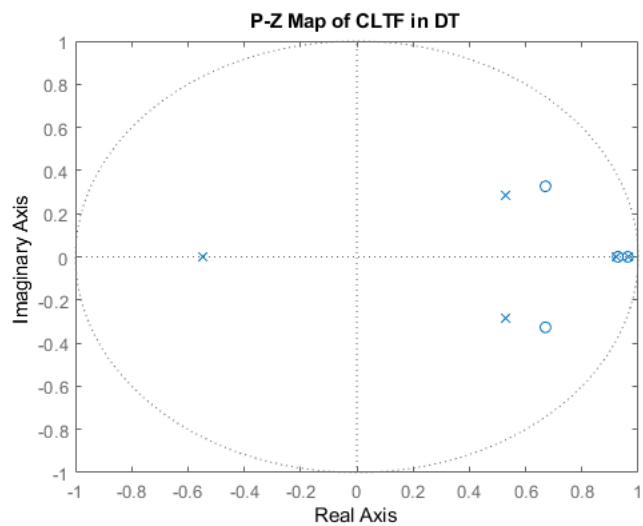
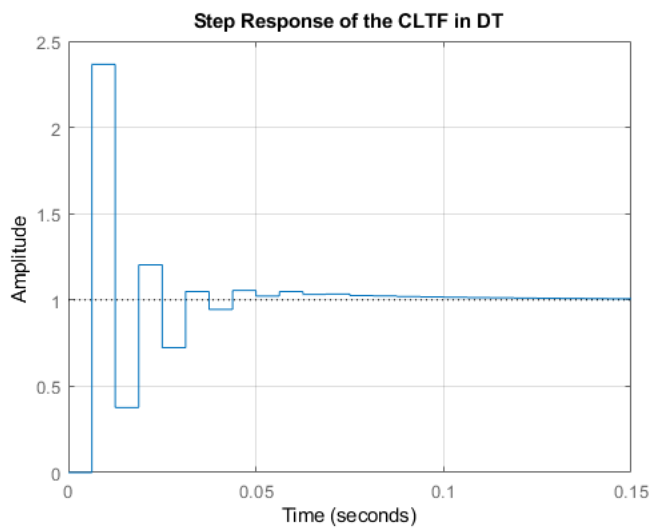
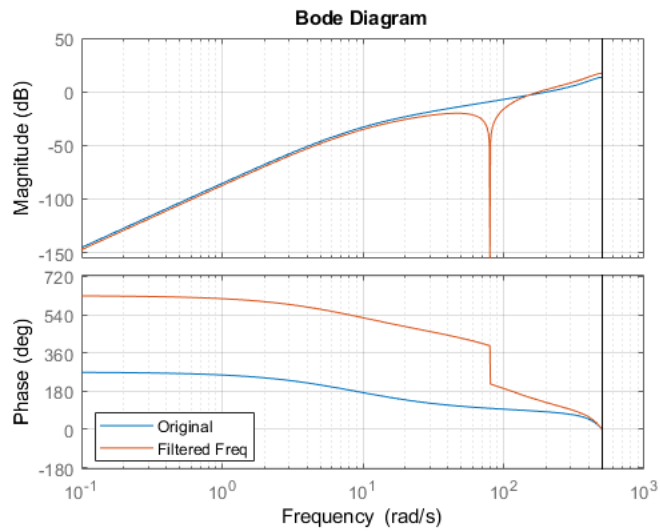


**Mathematical Solution**









#### MATLAB simulation: PI controller only

input to the controller: desired trajectory

```

%%%%%%%%% Modify %%%%%%%%%%
% Uncomment one of the lines to save the MATLAB plots
% save = 1; % save == yes
save = 0; % save == no

```



```

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

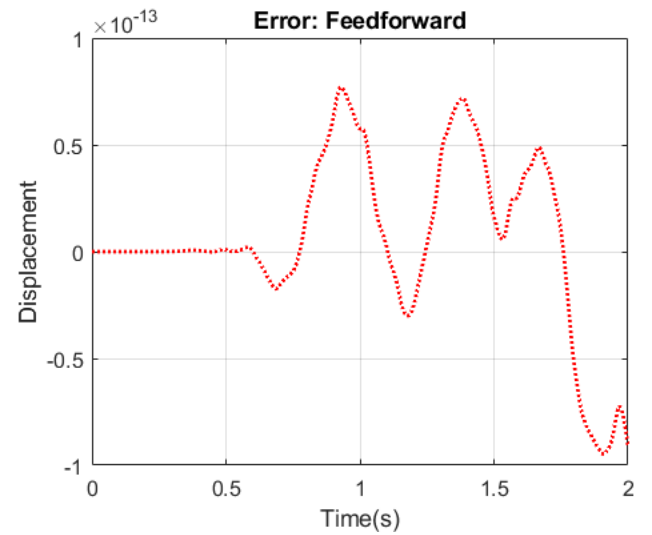
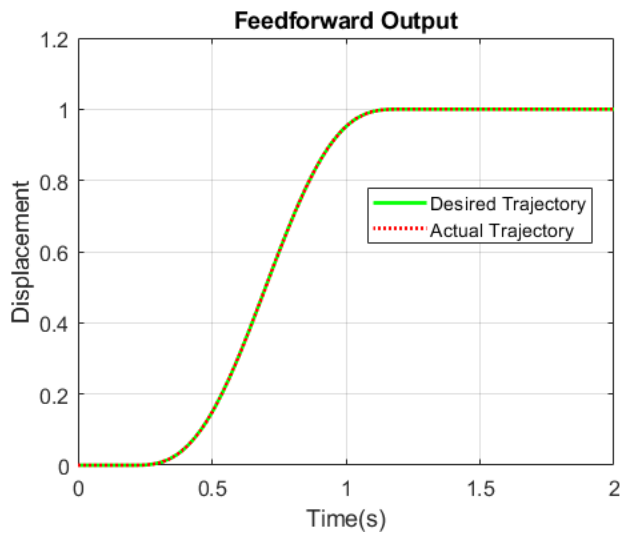
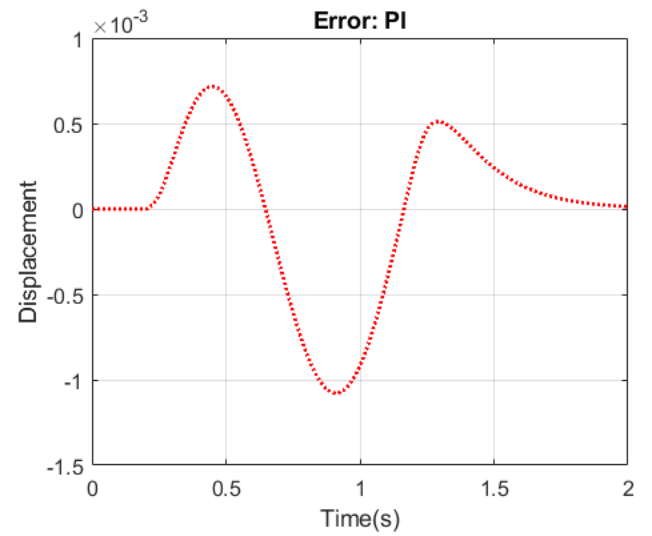
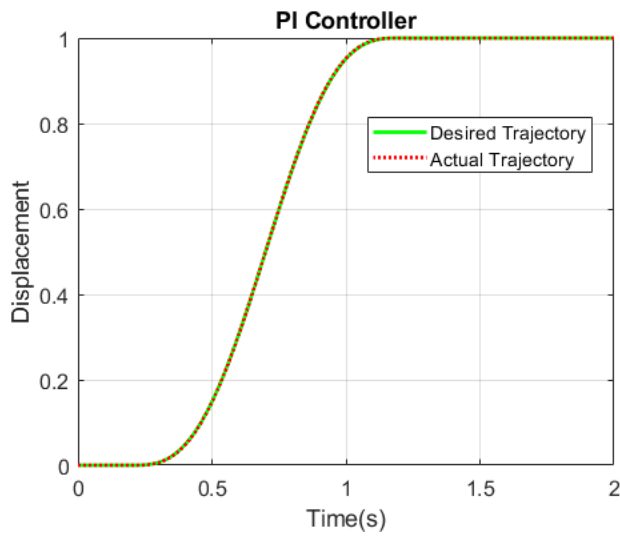
y_out1 = lsim(G_cl,yd,t);
y_out1 = y_out1';
% plotting
figure('Position', [100, 100, 1200, 400]);
subplot(1,2,1)
plot(t,yd,'g',t,y_out1,'r:','linewidth',wth) % output y
xlim([0 Tend])
xlabel('Time(s)'),ylabel('Displacement')
title('PI Controller')
legend('Desired Trajectory','Actual Trajectory','location','best')
set(gca,'FontSize',12), grid
error1 = yd-y_out1; % error
subplot(1,2,2)
plot(t,error1,'r:','linewidth',wth)
xlim([0 Tend])
title('Error: PI')
xlabel('Time(s)'),ylabel('Displacement')
set(gca,'FontSize',12), grid

% save file
if save == 1
    saveas(gcf,'MATLAB_PI.png')
else
    save = 0;
end

% Feedforward with PI controller: Inversion methd
invG = inv(G_cl/Zm); % inverse of the CL system with delay z^-1
yd_1k = yd(1,(1+m):end); % shift y(k+1) to reduce phase lag
end_k = length(yd_1k);
for i = 1:m % match the size of vector, holding final value
    yd_1k(1,(end_k+i)) = y_final;
end
uinv = lsim(invG,yd_1k,t); % U_inv for the closed loop system
uinv = uinv';
y_out2 = lsim(G_cl,uinv,t); % output y
y_out2 = y_out2';
% plotting y
figure('Position', [100, 100, 1200, 400]);
subplot(1,2,1)
plot(t,yd,'g',t,y_out2,'r:','linewidth',wth) % output y plotting
xlim([0 Tend])
legend('Desired Trajectory','Actual Trajectory','location','best')
title('Feedforward Output')
xlabel('Time(s)'),ylabel('Displacement')
set(gca,'FontSize',12), grid
%plotting error
subplot(1,2,2)
error2 = yd-y_out2;
plot(t,error2,'r:','linewidth',wth)
xlim([0 Tend])
title('Error: Feedforward')
xlabel('Time(s)'),ylabel('Displacement')
set(gca,'FontSize',12), grid

% save file
if save == 1
    saveas(gcf,'MATLAB_FF.png')
else
    save = 0;
end
end

```



## Simulink

Export input data for Simulink

```
ydes.signals.values = [yd'];
ydes.time = [t']; % y(k+1) equivalent
ydes.signals.dimensions = 1;
ydes1k.time = [t']; % y(k+1) equivalent
ydes1k.signals.values = [yd_1k'];
ydes1k.signals.dimensions = 1;
M_u = 1; % input magntidue

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Uncomment one of the lines to save the SIMULINK plots
% save = 1; % save == yes
% save = 0; % save == no
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Choose one of the alpha values
alpha = 0.2;
% alpha = 0.6;
% alpha = 0.99;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Parmeters to test the disturbance at that frequency
% param = 1; % no disturbance
% param = 2; % disturbance at 60 rad/s
% param = 3; % disturbance at 80 rad/s
% param = 4; % disturbance at 100 rad/s
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

Q = minreal(((2-2*alpha)*cos(w_dt)+(alpha^2-1)*z^-1)/(1-2*alpha*cos(w_dt)*z^-1+alpha^2*z^-2));
% equivalent C_all for the parameterized controller

for i = 1:4
    param = i;
```

```

% initiates the case to run
if param == 1
    M_dist = 0;
    w_dist = 0;
elseif param == 2
    M_dist = 1;
    w_dist = 60;
elseif param == 3
    M_dist = 1;
    w_dist = 80;
else
    M_dist = 1;
    w_dist = 100;
end

% Initiate simulation
% simout is the output that simulink returned
simout=sim('vehicle_steering_YK','StopTime','Tend', ...
    'SaveTime','on','TimeSaveName','timeout', ...
    'SaveOutput','on','OutputSaveName','yout');
t_sim = simout.timeout; % time used in Simulink
y_sim1 = simout.yout{1}.Values.Data; % parameterized Q system with no FF
y_sim2 = simout.yout{2}.Values.Data; % parameterized Q with FF inverse
y_sim3 = simout.yout{3}.Values.Data; % Control, controller + plant
e_sim1 = simout.yout{4}.Values.Data; % error of parameterized Q
e_sim2 = simout.yout{5}.Values.Data; % error of Q with FF
e_sim3 = simout.yout{6}.Values.Data; % error of control
u_sim1 = simout.yout{7}.Values.Data; % input for u(k)
u_sim2 = simout.yout{8}.Values.Data; % input for u(k+1)

% plotting the output and error
figure('Position', [100, 100, 1200, 400]);
subplot(1,2,1)
if param == 1
    plot(t,u_sim1,'g',t_sim,y_sim1,'r--',t_sim,y_sim2,'c:','linewidth',wth)
    title('Simulink: Output Y - No Disturbance')
    legend('Desired Trajectory','Y-K','Y-K with inverse','location','best')
    xlabel('Time(s)'),ylabel('Displacement'), xlim([0 Tend]), grid
    subplot(1,2,2)
    plot(t_sim,e_sim1,'r',t_sim,e_sim2,'c:','linewidth',wth)
    legend('Y-K','Y-K with inverse','location','best')
    set(gca,'FontSize',12), grid
    xlabel('Time(s)'),ylabel('Error'), xlim([0 Tend]), grid
    title('Simulink: Error')
    set(gca,'FontSize',12), grid
elseif param == 2
    plot(t,u_sim1,'g',t_sim,y_sim1,'r',t_sim,y_sim3,'c:','linewidth',wth)
    title('Simulink: Output Y - Disturbance at 60 rad/s')
    legend('Desired Trajectory','Y-K','Control','location','best')
    xlabel('Time(s)'),ylabel('Displacement'), xlim([0 Tend]), grid
    subplot(1,2,2)
    plot(t_sim,e_sim1,'r',t_sim,e_sim3,'c:','linewidth',3)
    legend('Y-K','Control','location','best')
    set(gca,'FontSize',12), grid
    xlabel('Time(s)'),ylabel('Error'), xlim([0 Tend]), grid
    title('Simulink: Error')
    set(gca,'FontSize',12), grid
elseif param == 3
    plot(t,u_sim1,'g',t_sim,y_sim1,'r--',t_sim,y_sim3,'c:','linewidth',wth)
    title('Simulink: Output Y - Disturbance at 80 rad/s')
    legend('Desired Trajectory','Y-K','Control','location','best')
    xlabel('Time(s)'),ylabel('Displacement'), xlim([0 Tend]), grid
    subplot(1,2,2)
    plot(t_sim,e_sim1,'r',t_sim,e_sim3,'c:','linewidth',3)
    legend('Y-K','Control','location','best')
    set(gca,'FontSize',12), grid
    xlabel('Time(s)'),ylabel('Error'), xlim([0 Tend]), grid
    title('Simulink: Error')
    set(gca,'FontSize',12), grid
else
    plot(t,u_sim1,'g',t_sim,y_sim1,'r',t_sim,y_sim3,'c:','linewidth',wth)
    title('Simulink: Output Y - Disturbance at 100 rad/s')
    legend('Desired Trajectory','Y-K','Control','location','best')
    xlabel('Time(s)'),ylabel('Displacement'), xlim([0 Tend]), grid
    subplot(1,2,2)
    plot(t_sim,e_sim1,'r',t_sim,e_sim3,'c:','linewidth',wth)
    legend('Y-K','Control','location','best')
    set(gca,'FontSize',12), grid
    xlabel('Time(s)'),ylabel('Error'), xlim([0 Tend]), grid
    title('Simulink: Error')
    set(gca,'FontSize',12), grid
end

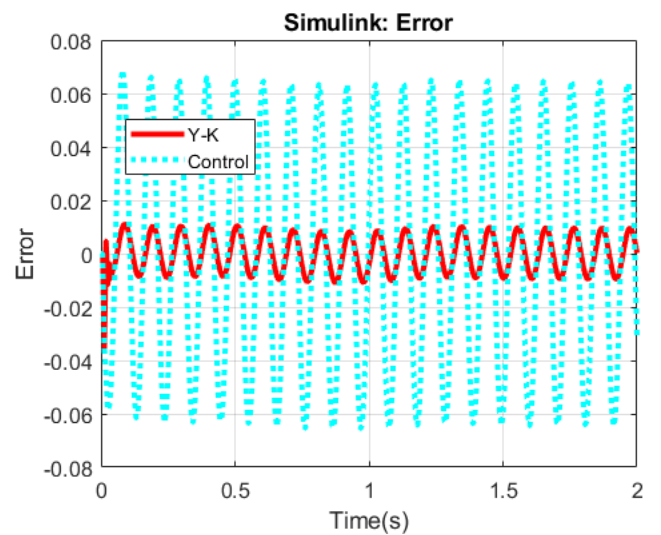
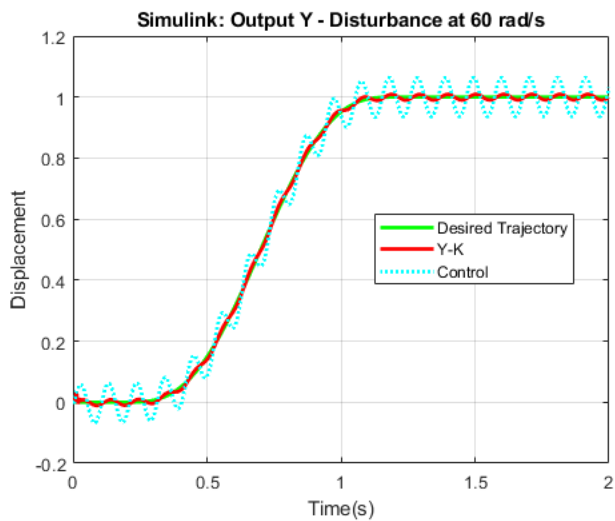
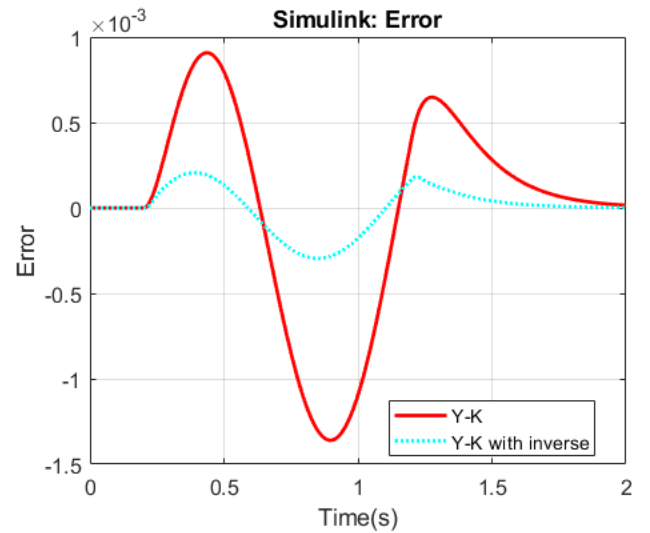
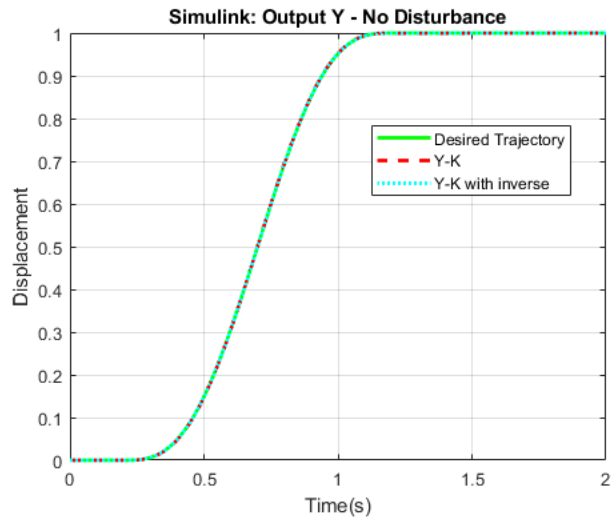
% save graphs
if alpha == 0.2
    if param == 1 && save == 1
        saveas(gcf,'no_dist_0.2.png')
    elseif param == 2 && save == 1
        saveas(gcf,'dist_60_0.2.png')
    elseif param == 3 && save == 1

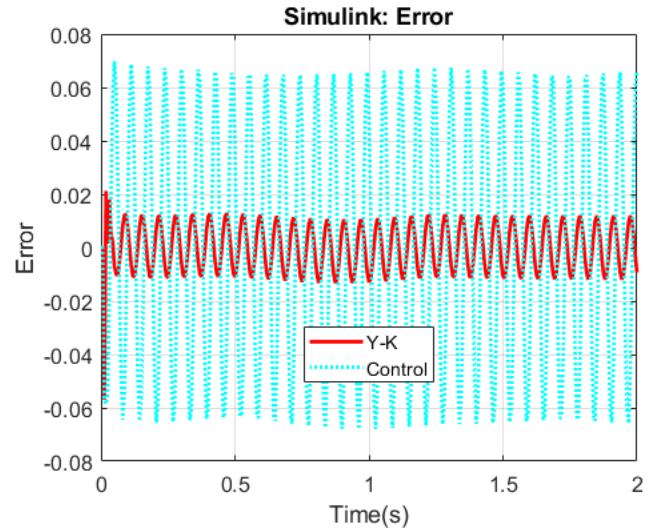
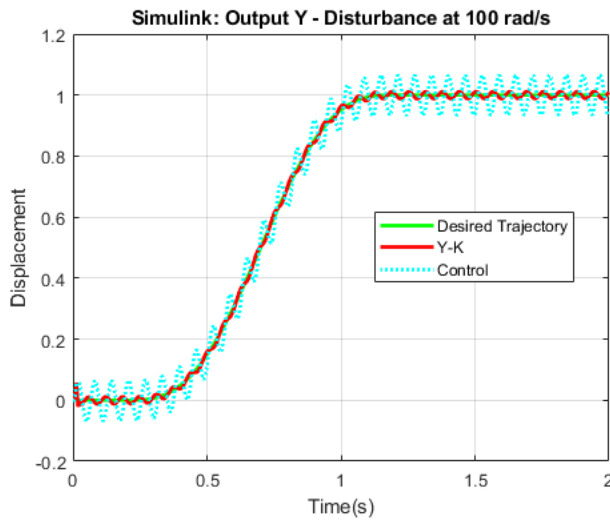
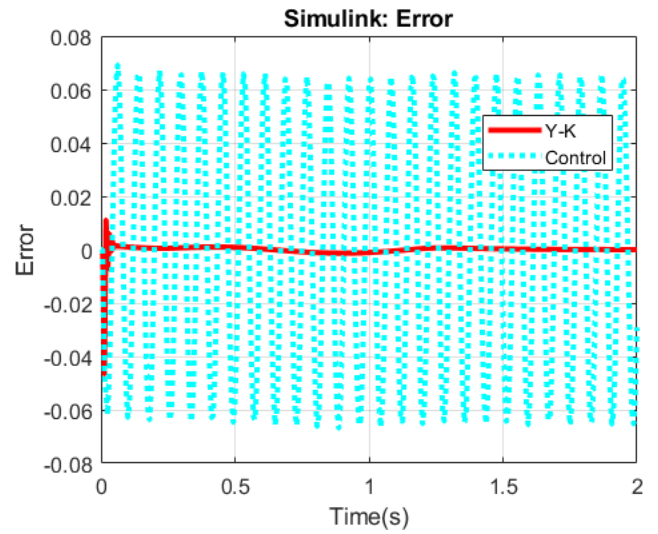
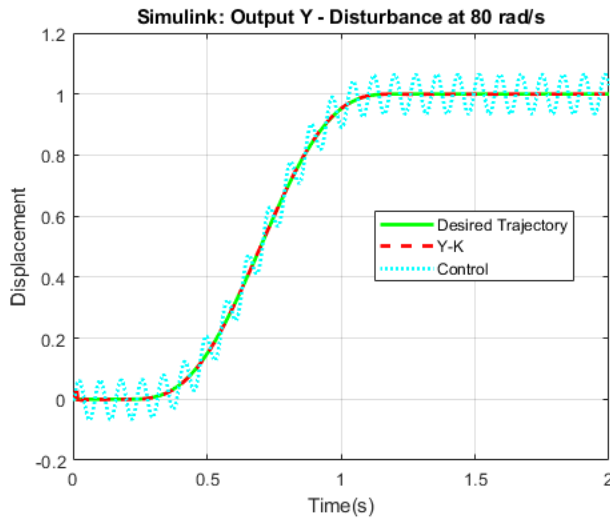
```

```

        saveas(gcf,'dist_80_0.2.png')
    elseif param == 4 && save == 1
        saveas(gcf,'dist_100_0.2.png')
    else
        save = 0;
    end % end save cases
elseif alpha == 0.6
    if param == 1 && save == 1
        saveas(gcf,'no_dist_0.6.png')
    elseif param == 2 && save == 1
        saveas(gcf,'dist_60_0.6.png')
    elseif param == 3 && save == 1
        saveas(gcf,'dist_80_0.6.png')
    elseif param == 4 && save == 1
        saveas(gcf,'dist_100_0.6.png')
    else
        save = 0;
    end % end save cases
else alpha == 0.99
    if param == 1 && save == 1
        saveas(gcf,'no_dist_0.99.png')
    elseif param == 2 && save == 1
        saveas(gcf,'dist_60_0.99.png')
    elseif param == 3 && save == 1
        saveas(gcf,'dist_80_0.99.png')
    elseif param == 4 && save == 1
        saveas(gcf,'dist_100_0.99.png')
    else
        save = 0;
    end % end save cases
end
end % end of freq disturb (4 loops)

```





## Velocity profile

```
function [xd,vd,ad,t,nfig] = sinusoidal_yacc(t_i,t_f,tmax,y_i,y_f,nfig,Tend);
T=(t_f-t_i); A=(y_f-y_i)*2*pi/(T*T); % this would make max yd =1
t = 0:0.00625:tmax;
ad = 0*t; % initialize desired acceleration;
vd = 0*t; % initialize desired velocity;
xd = 0*t; % initialize desired position;
for jj = 1:length(t)
    if t(jj)< t_i
        ad(jj) = 0;
        vd(jj) = 0;
        xd(jj) = y_i;
    elseif t(jj) < t_i+T
        tt = t(jj)-t_i;
        ad(jj) = A*sin(2*pi*tt/T);
        vd(jj) = (A*T/(2*pi))*(1 -cos(2*pi*tt/T));
        xd(jj) = y_i + (A*T/(2*pi))*tt - (A*T*T/(4*pi*pi))*sin(2*pi*tt/T);
    else
        ad(jj) = 0;
        vd(jj) = 0;
        xd(jj) = y_f; % (A*T/(2*pi))*T - (A*T*T/(4*pi*pi))*sin(2*pi*T/T);
    end
end

figure()
plot(t,xd,'LineWidth',2)
xlabel('Time(s)')
ylabel('Position')
title('Desired Trajectory')
xlim([0 Tend])
grid; set(gca,'FontSize',12)
end
```