Sweep sin approach for system identification

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- theory: sinusoidal signals are perpendicular to each other if they are at different frequencies. sinusoidals at the same frequency are parallel to each other. Doing the inner product:

$$\begin{aligned}
&\int_{-\pi}^{\pi} A\sin(\omega t+\phi)\, e^{-j\omega t}dt \\
=&\int_{-\pi}^{\pi} A\frac{e^{j(\omega t+\phi)}-e^{-j(\omega t+\phi)}}{2j}e^{-j\omega t}dt \\
=&\int_{-\pi}^{\pi} A\frac{1}{2j}\left(e^{j\phi}-e^{-j(2\omega t+\phi)}\right)dt \\
=&\frac{A\pi}{j\omega}e^{j\phi}
\end{aligned}$$

gives the following procedure of sin-based system ID:

- considier the input and the output given by $A_1\sin(\omega t+\phi_1)$ and $A_2\sin(\omega t+\phi_2)$
- perform FFT to get $X_1=\frac{A_1\pi}{j\omega}e^{j\phi_1}$ and $X_2=\frac{A_2\pi}{j\omega}e^{j\phi_2}$
- extract the magnitude and phase information from the above to get $A_2/A_1=|X_2/X_1|$ and $\phi_2-\phi_1=\angle X_2-\angle X_1$