Camera Projective Geometry

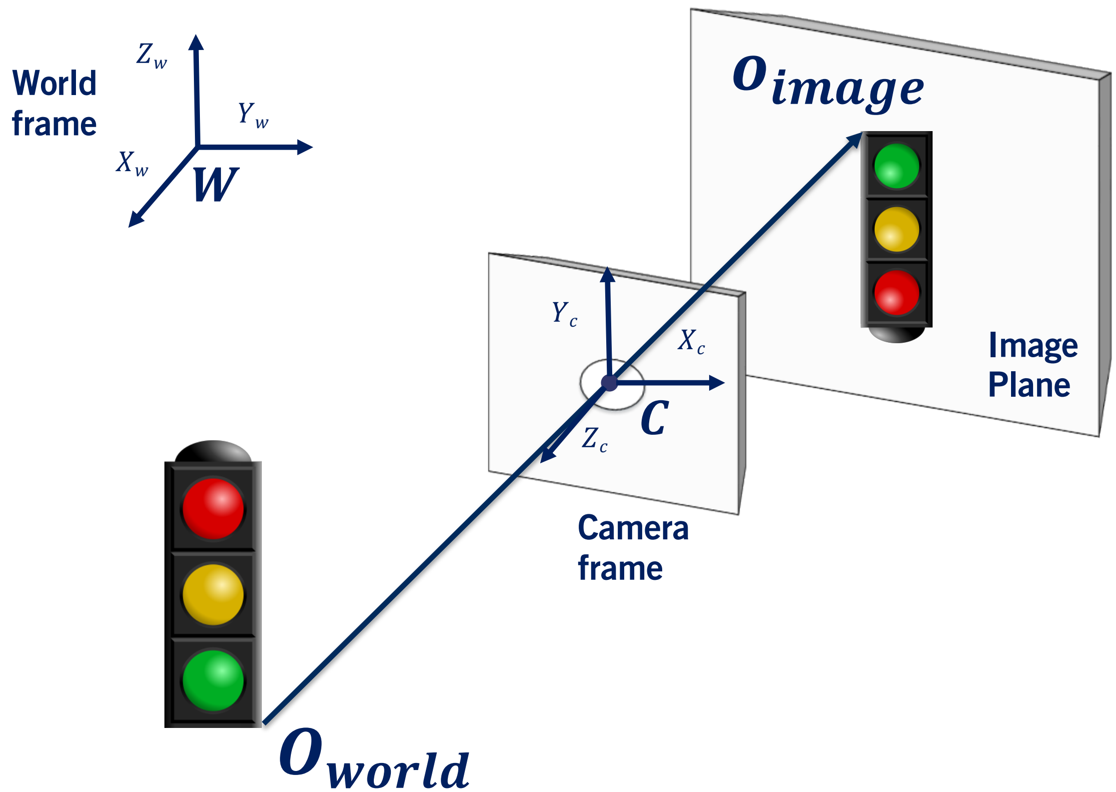
Course 3, Module 1, Lesson 1 – Part 2

UNIVERSITY OF TORONTO
FACULTY OF APPLIED SCIENCE & ENGINEERING

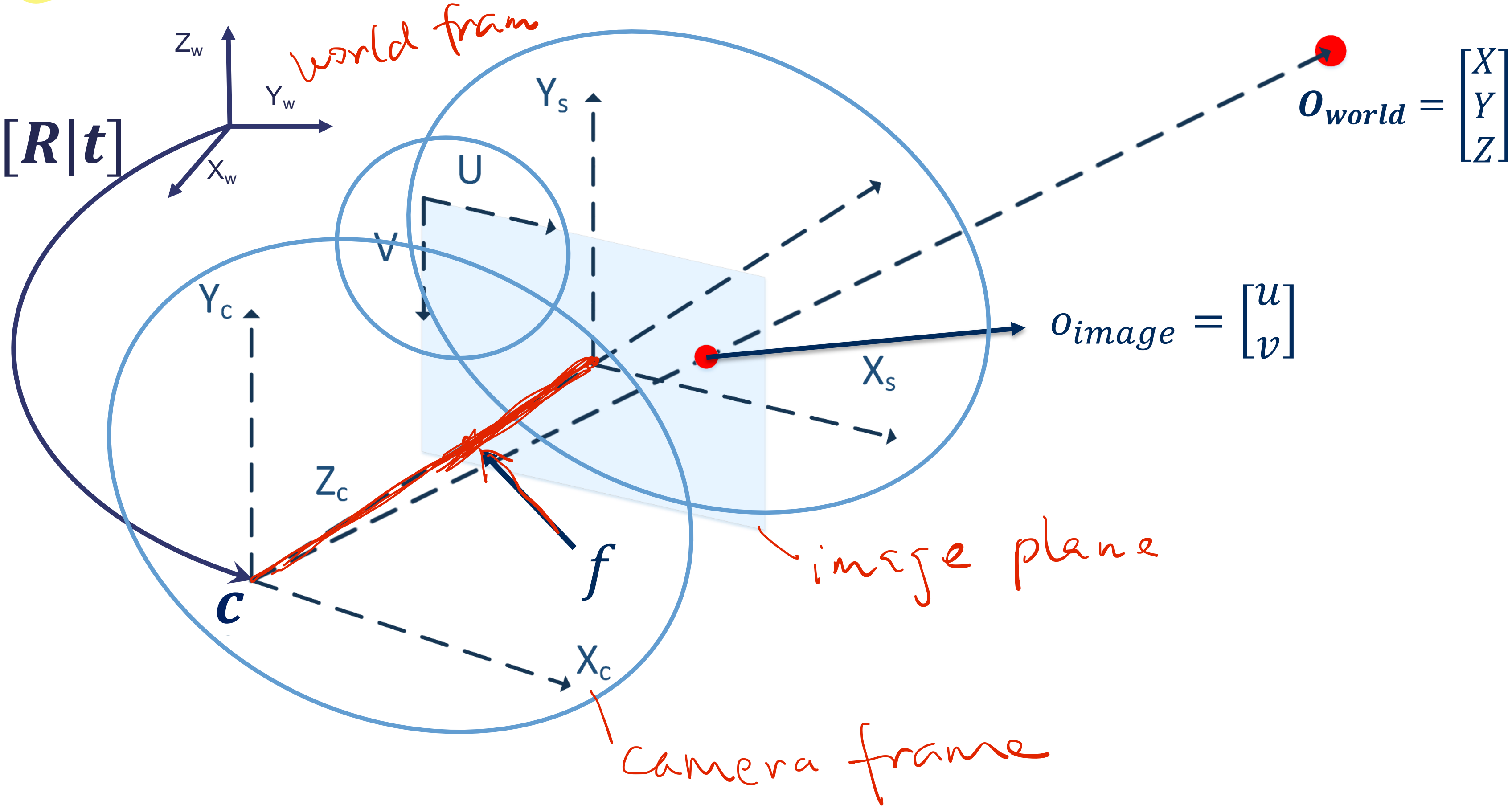
Learning objectives

- Learn how to model the camera's projective geometry through **coordinate system transformation**
- Learn how to model these transformations using matrix algebra and apply them to a 3D point to get its 2D projection on the image plane
- Learn how a digital image is represented in software

Projection: World → Image (Real Camera)

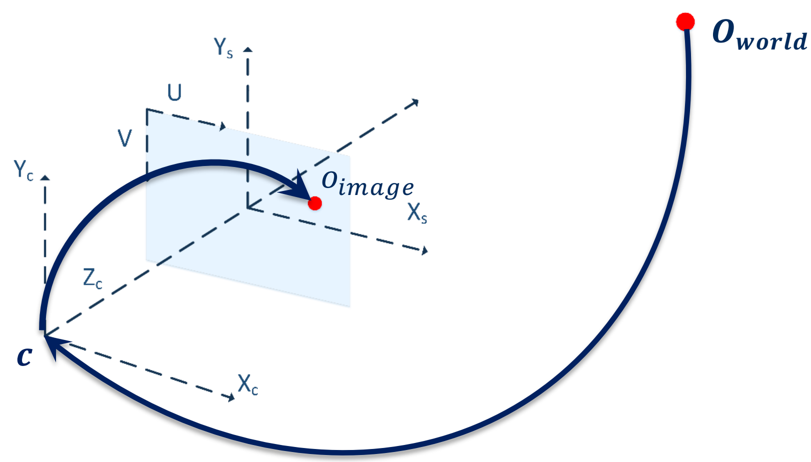


Projection: World → Image (Simplified Camera)



Computing the Projection

- Projection form World coordinates → Image coordinates:
  1. Project from World coordinates→ Camera coordinates
  2. Project from Camera coordinates to Image coordinates

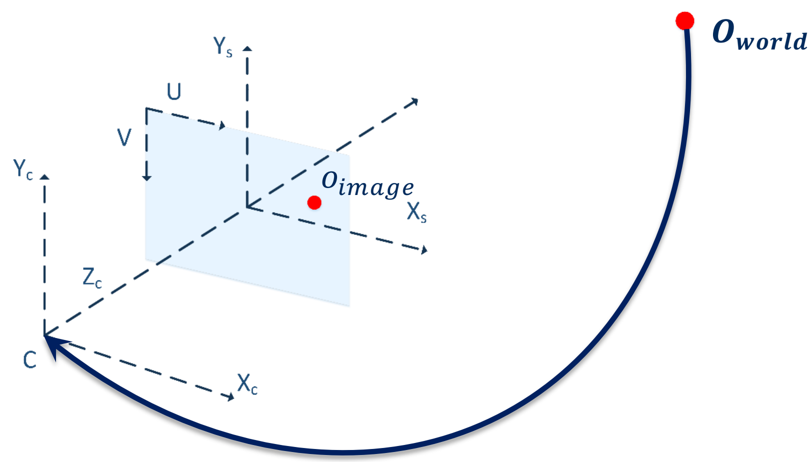


Computing the Projection

- World → Camera:

$$o_{camera} = [R|t]O_{world}$$

rigid body transformation matrix T

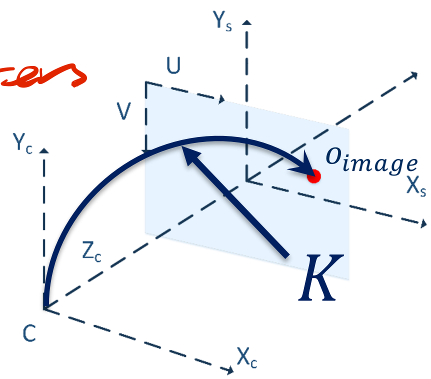


Computing the Projection

- Camera → Image:

$$o_{image} = \begin{bmatrix} f & 0 & u_0 \\ 0 & f & v_0 \\ 0 & 0 & 1 \end{bmatrix} o_{camera} = K o_{camera}$$

Camera intrinsic parameters

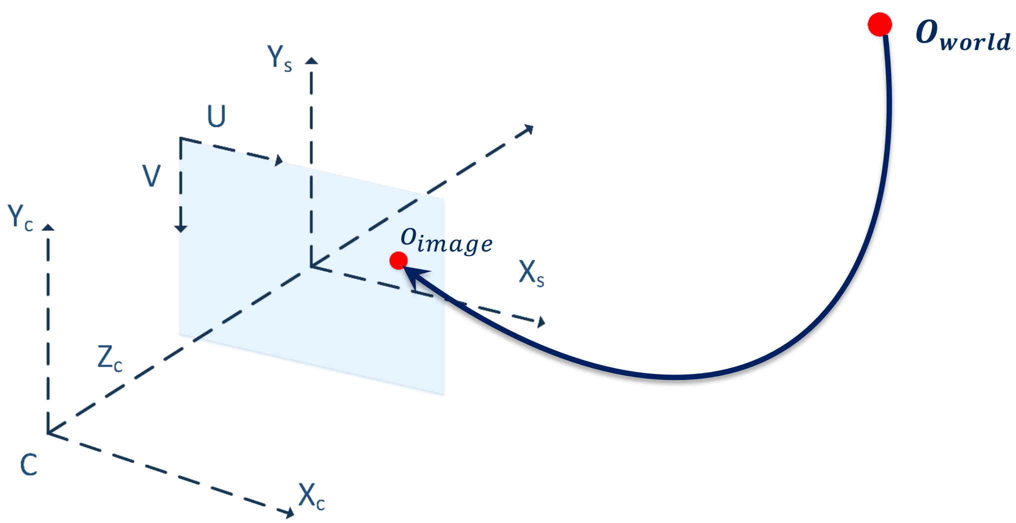


o_{world}

Computing the Projection

- World → Image:

$$P = K\,[R|t]$$



Computing the Projection

- Projection from World Coordinates → Image Coordinates:

$$o_{image} = PO_{world} = \underbrace{\underset{3x3}{K}\,\underset{3x4}{[R|t]}}_{3x4}\,\underset{4x1}{O_{world}} \qquad O_{world} = \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

3x1 → 4x1

Computing the Projection

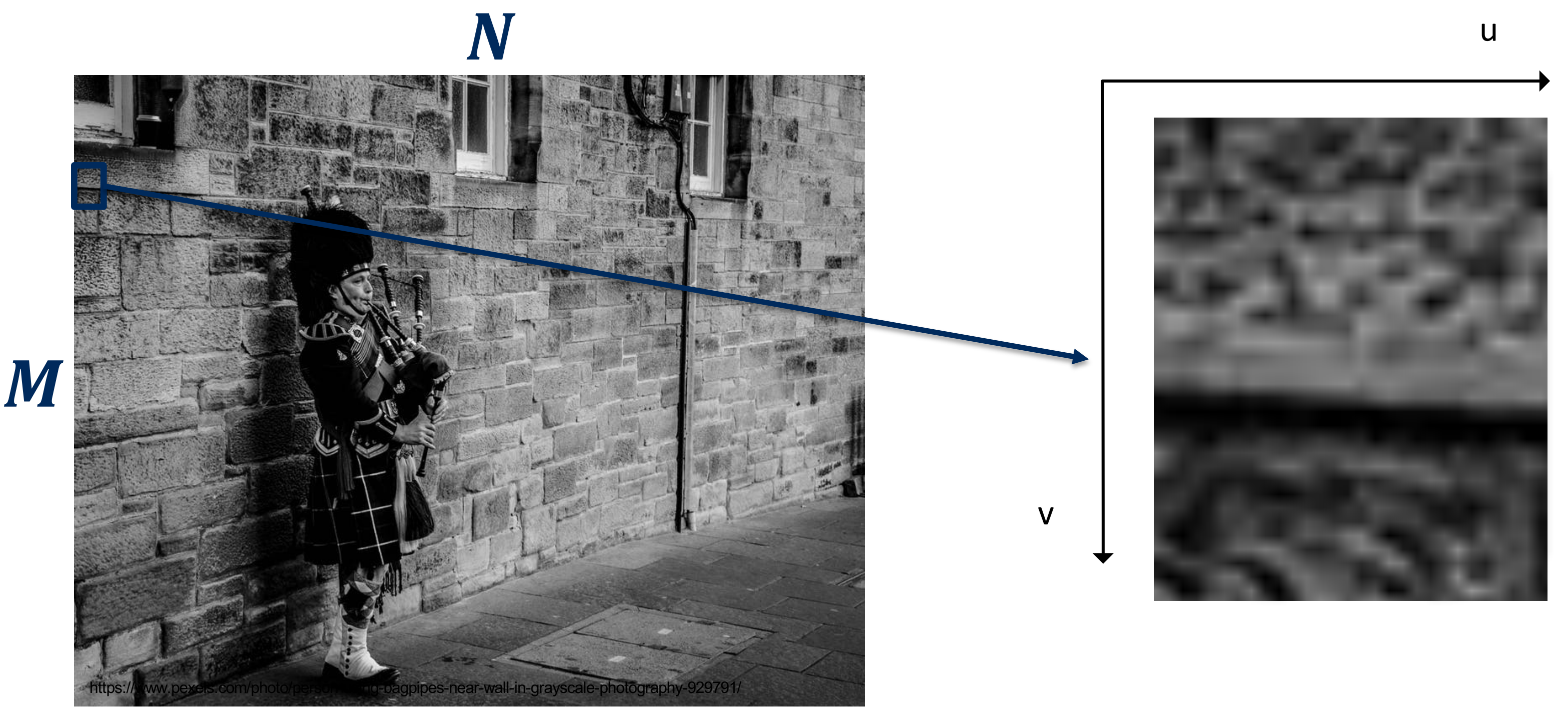
- **World** coordinates to **Image** coordinates:

$$o_{image} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = K[R|t]\begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

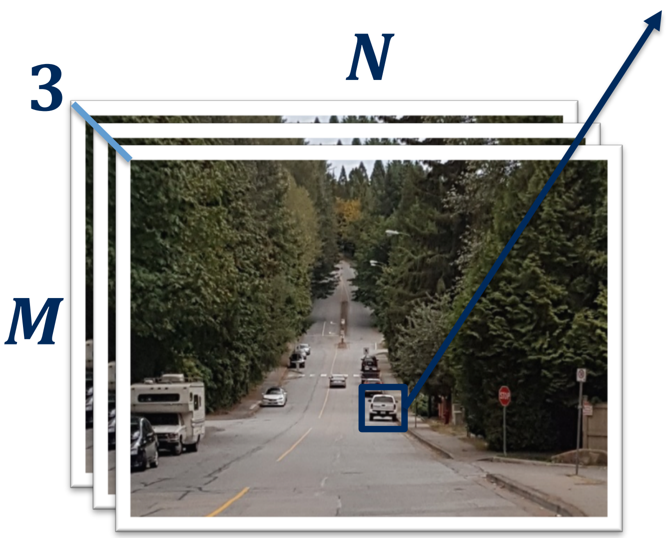
- **Image** coordinates to **Pixel** coordinates:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} \rightarrow \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \frac{1}{z}\begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

The Digital Image: Greyscale



The Digital Image: Color



u

v

128	127	127	23
127	255	127	25
41	41	0	0
42	43	0	0

R

124	4	144	153
4	4	12	12
4	4	4	4
1	1	0	0

G

0	1	1	2
255	255	2	2
255	255	23	62
233	233	56	52

B

depth

Summary

- 3D points in the world coordinate frame can be projected to 2D points in the image coordinate frame using projective geometry equations
- These equations rely on the camera intrinsic parameters, as well as its extrinsic location in the world coordinate frame
- A color camera image is represented digitally as an MxNx3 array of unsigned 8 bit or 16 bit integers

- **Next: Camera Calibration**