Introduction to Artificial Intelligence - A3
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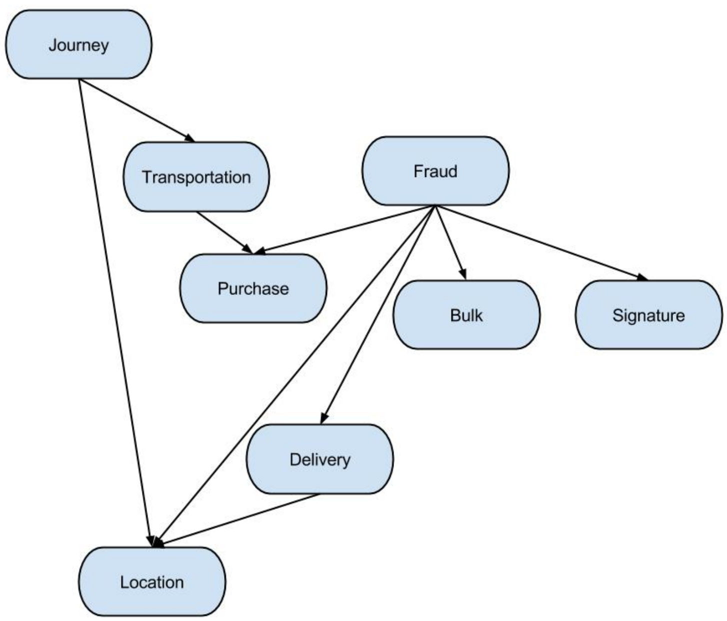
Question 1

a) A drawing of the network. Make sure that the arcs correspond to the dependencies described above. [5 marks].



b) Conditional probability tables for the various variables. [10 marks].
c) For each variable, a brief (1-3 sentences) description or justification for the selected values in the CPT. [5 marks].

P(Journey)

0.1

From prior experience, though, that 10% of credit card transactions occur while the cardholder is travelling.

P(Fraud)

0.01

Based on prior experience, 1% of all attempted transactions are known to be fraudulent.

P(Signature | Fraud)

Fraud\ Signature	Good	Bad
Yes	0.90	0.10
No	0.95	0.05

Provided signatures are generally accurate, but they are more likely to be off when the card is being used fraudulently.

P(Bulk | Fraud)

Fraud\ Bulk	Yes	No
Yes	0.5	0.5
No	0.2	0.8

Stolen credit card are likely to use to purchase bulk order or really expensive item. Credit card user will occasionally bulk groceries purchase.

P(Delivery | Fraud)

Fraud\ Delivery	Yes	No
Yes	0.8	0.2
No	0.1	0.9

People usually use credit card for day to day transactions. Online shopping is a rather rare. On the other hand, credit card thefts tend to make online purchases, because this will not require them to create a fake credit card.

P(Transportation | Journey)

Journey \ Transportation	Yes	No
Yes	0.9	0.1
No	0.2	0.8

Transportation expenses are more likely to occur when the cardholder is on a journey.

P(Transportation)	P(Fraud)	P(Purchase = electronics | Transportation, Fraud)	P(Purchase = groceries | Transportation, Fraud)	P(Purchase = misc | Transportation, Fraud)
Yes	**Yes**	0.6	0.05	0.35
Yes	**No**	0.05	0.35	0.6
No	**Yes**	0.8	0.01	0.19
No	**No**	0.1	0.6	0.3

Electronic purchases are more likely to be fraudulent, as they are more valuable. On the other hand, groceries are relatively cheap and is unlikely to be purchased in a fraudulent transaction.

Journey	Fraud	Delivery	P(Location = Near | Journey, Fraud, Delivery)	P(Location = Far | Journey, Fraud, Delivery)
Yes	**Yes**	**Yes**	0.1	0.9
Yes	**Yes**	**No**	0.3	0.7
Yes	**No**	**Yes**	0.1	0.9
Yes	**No**	**No**	0.1	0.9
No	**Yes**	**Yes**	0.2	0.8
No	**Yes**	**No**	0.3	0.7
No	**No**	**Yes**	0.2	0.8
No	**No**	**No**	0.9	0.1

Usual credit card activity is local and it is very unlikely to have vendor location that is far away. When the credit cardholder is on a journey or the order is for delivery or it’s a fraudulent transaction, probability of vendor location is far away increases.

Question 2

a) For each variable, state whether it is an evidence variable (something we observe), query (something we want to know), or neither (something that helps build a good network). [4 marks]

Variable	**Type**
Signature	Evidence
Bulk	Evidence
Purchase	Evidence
Delivery	Evidence
Location	Evidence
Journey	Neither
Transportation	Evidence
Fraud	Query

b) Is Delivery independent of Journey, given no other information? [1 mark].

- Yes, since Journey and Delivery are D-separated according to

c) Is Signature independent of Bulk, given the value of Fraud? [1 mark].

- Yes.

Question 3

a) Is Smoking independent of Pollution, given Positive X-ray? [1 mark].

- Smoking and Pollution are dependent given Positive X-ray

b) Is Smoking independent of Dypsnoea, given Bronchitis? [1 mark].

- Smoking and Dypsnoea are dependent given Bronchitis

c) What is the value of P(Bronchitis=true | Pollution=false, Smoking=true, Dypsnoea=true)? [2 marks].

Pollution - P
Smoking - S
Lung Cancer - L
Bronchitis - B
Positive X-Ray - X
Dypsnoea - D
True - T
False - F

Query:
P(B=T | P=F, S=T, D=T)
= P(b, -p, s, d) / P(-p, s, d)

P(B, -p, s, d) = $\sum_{L,X} P(-p, s, L, B, X, d)$

$$= \sum_L \sum_X P(-p, s, L, B, X, d)$$

$$= \sum_L \sum_X P(-p)P(s)P(L|-p,s)P(B|s)P(X|L)P(d|L,B)$$

$$= P(-p)\, P(s)\, P(B|s) \sum_L P(L|-p,s)\, P(d|L,B) \sum_X P(X|L)$$

Let $P(-p)\, P(s)\, P(B|s)$ be c_1(B)

$$= c_1 \sum_L P(L|-p,s)\, P(d|L,B)P(x|L) + c_1 \sum_L P(L|-p,s)\, P(d|L,B)P(-x|L)$$

$$= c_1 [\sum_L P(L|-p,s)\, P(d|L,B)P(x|L) + \sum_L P(L|-p,s)\, P(d|L,B)P(-x|L)]$$

Let $f_1(B)$ be $\sum_L P(L|-p,s)\,P(d|L,B)P(x|L)$

$$\sum_L P(L|-p,s)\,P(d|L,B)P(x|L)$$
$$= P(l|-p,s)\,P(d|l,B)P(x|l) + P(-l|-p,s)\,P(d|-l,B)P(x|-l)$$
$$= (0.03)P(d|l,B)(0.98) + (1-0.03)P(d|-l,B)(0.05)$$

$$c_1(B) = P(-p)\,P(s)\,P(B|s)$$
$$= (1-0.1)\,(0.2)\,P(B|s)$$
$$= (0.18)\,P(B|s)$$

Let $f_2(B)$ be $\sum_L P(L|-p,s)\,P(d|L,B)P(-x|L)$

$$\sum_L P(L|-p,s)\,P(d|L,B)P(-x|L)$$
$$= P(l|-p,s)\,P(d|l,B)P(-x|l) + P(-l|-p,s)\,P(d|-l,B)P(-x|-l)$$
$$= (0.03)\,P(d|l,B)(1-0.98) + (1-0.03)\,P(d|-l,B)(1-0.05)$$
$$= (0.03)\,P(d|l,B)(0.02) + (0.97)\,P(d|-l,B)(0.95)$$

$$P(b,-p,\ s,d) = c_1(b)\,(f_1(b) + f_2(b))$$
$$= (0.18)P(b|\ s)\,(f_1(b) + f_2(b))$$
$$= (0.18)(0.6)\,((0.03)P(d|l,b)(0.98) + (1-0.03)P(d|-l,b)(0.05) + f_2(b))$$
$$= (0.108)\,((0.03)(0.9)(0.98) + (0.97)(0.7)(0.05) + (0.03)\,P(d|l,b)(0.02) + (0.97)\,P(d|-l,b)(0.95))$$
$$= (0.108)\,((0.02646) + (0.03395) + (0.03)(0.9)(0.02) + (0.97)\,(0.7)(0.95))$$
$$= (0.108)\,((0.06041) + (0.00054) + (0.64505)) = 0.076248$$

$$P(-b,-p,\ s,d) = c_1(B)\,(f_1(-b) + f_2(-b))$$
$$= (0.18)P(-b|\ s)\,(f_1(-b) + f_2(-b))$$
$$= (0.18)(1-0.6)\,((0.03)P(d|l,-b)(0.98) + (1-0.03)P(d|-l,-b)(0.05) + f_2(-b))$$
$$= (0.18)(0.4)\,((0.03)(0.8)(0.98) + (0.97)(0.1)(0.05) + (0.03)\,P(d|l,-b)(0.02) + (0.97)\,P(d|-l,-b)(0.95))$$
$$= (0.18)(0.4)\,((0.03)(0.8)(0.98) + (0.97)(0.1)(0.05) + (0.03)(0.8)(0.02) + (0.97)\,(0.1)(0.95))$$
$$= (0.072)\,((0.02352) + (0.00485) + (0.00048) + (0.09215))$$
$$= (0.072)\,(0.121) = 0.008712$$

$$P(-p,\ s,d) = P(-b,-p,s,d) + P(b,-p,s,d) = 0.076248 + 0.008712 = 0.08496$$
$$P(b\mid -p,\ s,d) = \frac{P(b,-p,s,d)}{P(-p,s,d)} = \frac{0.076248}{0.008496} = 0.897457627$$

d) What is the value of P(Lung Cancer=true | Pollution=false, Smoking=true, Dypsnoea=true)? [2 marks].

Query:

$$P(l \mid -p, s, d) = \frac{P(l,-p,s,d)}{P(-p,s,d)}$$

From c)

$$P(l \mid -p, s, d) = \frac{P(l,-p,s,d)}{0.08496}$$

$$P(l, -p, s, d) = \sum_{B,X} P(-p, s, l, B, X, d)$$

$$= \sum_{B,X} P(l \mid -p, s) P(-p) P(s) P(d \mid l, B) P(X \mid l) P(B \mid s)$$

$$= P(l \mid -p, s) P(-p) P(s) \sum_{B} P(d \mid l, B)\, P(B \mid s)$$

Let $P(l \mid -p, s) P(-p) P(s)$ be c_3

$c_3 = P(l \mid -p, s)\, P(-p)\, P(s)$

$= (0.03)(1 - 0.1)\,(0.2) = 0.0054$

$$P(l, -p, s, d) = c_3 \sum_{B} P(d \mid l, B) P(B \mid s)$$

$= c_3[\, P(d \mid l, b)\, P(b \mid s) + P(d \mid l, -b)\, P(-b \mid s)]$

$= c_3[\, (0.9)\,(0.6) + (0.8)\,(1 - 0.6)]$

$= (0.0054)\,[\,(0.54) + (0.32)] = 0.004644$

$$P(l \mid -p, s, d) = \frac{P(l,-p,s,d)}{0.08496} = \frac{0.004644}{0.08496} = 0.54661016$$

e) What is the value of P(Lung Cancer=true | Pollution=false, Smoking=false, Positive X-ray=true, Dypsnoea=false)? [2 marks].

$P(l \mid -p, -s, x, -d) \;=\; \frac{P(l,-p,-s,x,d)}{P(-p,-s,x,-d)}$

$P(l, -p, -s, x, -d) \;=\; \sum_B P(l, -p, -s, B, x, -d)$

$\sum_B P(l, -p, -s, B, x, -d) = \sum_B P(-p)P(-s)P(l| -p, -s)P(-d \mid l, B)P(x|l)P(B| -s)$

$= P(-p)P(-s)P(l| -p, -s)P(x|l)\sum_B P(-d \mid l, B)P(B| -s)$

Let $P(-p)P(-s)P(l| -p, -s)P(x|l)$ be c_4

$c_4 \;=\; (1-0.1)(1-0.2)(0.001)(0.98) \;=\; 0.0007056$

$P(l, -p, -s, x, -d) \;= c_4\,[P(-d \mid l, b)P(b| -s) \;+\; P(-d \mid l, -b)P(-b| -s)]$

$= c_4\,[P(-d \mid l, b)P(b| -s) \;+\; P(-d \mid l, -b)P(-b| -s)]$

$= c_4\,[(1-0.9)(0.3) \;+ (1-0.8)(1-0.3)]$

$= (0.0007056)\,[(0.03) \;+ (0.14)] \;= 0.000119952$

$P(-l, -p, -s, x, -d) = \;\sum_B P(-l, -p, -s, B, x, -d)$

$\sum_B P(-l, -p, -s, B, x, -d) \;= \sum_B P(-p)P(-s)P(-l| -p, -s)P(-d \mid -l, B)P(x| -l)P(B| -s)$

$= P(-p)P(-s)P(-l| -p, -s)P(x| -l)\sum_B P(-d \mid -l, B)P(B| -s)$

Let $P(-p)P(-s)P(-l| -p, -s)P(x| -l)$ be c_5

$c_5 \;=\; (1-0.1)(1-0.2)(1-0.001)(0.05) \;= 0.035964$

$P(-l, -p, -s, x, -d) \;= c_5 \sum_B P(-d \mid -l, B)P(B| -s)$

$= c_5[P(-d \mid -l, b)P(b| -s) \;+\; P(-d \mid -l, -b)P(-b| -s)]$

$= c_5[(1-0.7)(0.3) \;+\; (1-0.1)(1-0.3)] \;=\; (0.035964)\,(0.09 \;+ 0.63) \;=\; 0.02589408$

$P(, -p, -s, x, -d) \;=\; P(-l, -p, -s, x, -d) \;+\; P(l, -p, -s, x, -d)$

$= 0.000119952 \;+\; 0.02589408 \;=\; 0.026014032$

$P(l \mid -p, -s, x, -d) \;=\; \frac{0.000119952}{0.026014032} \;=\; 0.004611049$

f) What is the marginal probability that a x-ray is positive? That is, what is the value of P(Positive X-ray=true)? [2 marks].
Query P(x). Relevant: P, S, L

$$P(x)$$

$$= \sum_{P,S,L,B,D} P(P,S,L,B,x,D)$$

$$= \sum_{P,S,L,B,D} P(P)P(S)P(L|P,S)P(B|S)P(x|L)P(D|L,B)$$

$$= \sum_{L} P(x|L) \sum_{P} P(P) \sum_{S} P(S)P(L|P,S) \sum_{B} P(B|S) \sum_{D} P(D|L,B)$$

$$= \sum_{L} P(x|L) \sum_{P} P(P) \sum_{S} P(S)P(L|P,S)$$

$$= P(x|l)P(p)(P(s)P(l|p,s) + P(-s)P(l|p,-s)) + P(-p)(P(s)P(l|-p,s) + P(-s)P(l|-p,-s))$$
$$+ P(x|-l)P(p)(P(s)P(-l|p,s) + P(-s)P(-l|p,-s)) + P(-p)(P(s)P(-l|-p,s) + P(-s)P(-l|-p,-s))$$

$$= (0.98)(0.1)((0.2)(0.05) + (1-0.2)(0.02)) + (1-0.1)((0.2)(0.03) + (1-0.2)(0.001))$$
$$+ (0.05)(0.1)((0.2)(1-0.05) + (1-0.2)(1-0.02)) + (1-0.1)((0.2)(1-0.03) + (1-0.2)(1-0.001))$$

$$= 0.0581096$$

g) Describe a part of this Bayes Net that exhibits “explaining away”, i.e., knowing a symptom S increases the probability of its causes $C_1,...,C_k$ but, subsequently, increasing the probability of one of those causes C_i decreases the probability of the other causes as it explains symptom S. [4 marks]

The probability of having lung cancer given the symptom of Dypsnoea is 0.23229. The probability of having a lung cancer when Bronchitis is diagnosed as well decreases to 0.01614 because the the Bronchitis “Explanins away” the cause of Dypsnoea and reduce the probability of lung cancer.

Want to show:

$$P(l|\,d,b)\ <\ P(l\,|\,d)$$

$$P(l\,|\,d,b)\ =\ \frac{P(l,d,b)}{P(d,b)}$$
$$P(l\,|\,d)\ =\frac{P(l,d)}{P(d)}$$

$$P(l\,|\,d,b)\ =\ \frac{P(l,d,b)}{P(d,b)}\ =\ \frac{0.0040824}{0.2529072} = 0.01614188919$$
$$P(l\,|\,d)\ =\frac{P(l,d)}{P(d)}\ =\ \frac{0.0074296}{0.319836} = 0.232294050700$$

Finding $P(l,d,b)$, Relevant: L,D,B,P,S

$$P(l,d,b)\ =\ \sum_{P,S,X} P(P,S,l,b,X,d)$$

$$= \sum_{P,S,X} P(P)P(S)P(l|P,S)P(b|S)P(X|l)P(d|l,b)$$

$$= P(d|l,b)\ \sum_{P} P(P) \sum_{S} P(S)P(l|P,S)P(b|S)\ \sum_{X} P(X|l)$$

$$= (0.9) \sum_{P} P(P) \sum_{S} P(l|P,S)P(b|S)P(S)$$

$$= (0.9)[P(p) \sum_{S} P(l|p,S)P(b|S)P(S)\ +\ P(-p) \sum_{S} P(l|-p,S)P(b|S)P(S)]$$
$$=\ (0.9)(0.00108 +\ 0.003456) =\ 0.0040824$$

Side calculations

$$P(p)\sum_S P(l|p,S)P(b|S)P(S) = P(p)[P(l|p,s)P(b|s)P(s) + P(l|p,-s)P(b|-s)P(-s)]$$
$$= (0.1)[(0.05)(0.6)(0.2) + (0.02)(0.3)(1-0.2)] = 0.00108$$

$$P(-p)\sum_S P(l|-p,S)P(b|S)P(S) = P(-p)[P(l|-p,s)P(b|s)P(s) + P(l|-p,-s)P(b|-s)P(-s)]$$
$$= (1-0.1)[(0.03)(0.6)(0.2) + (0.001)(0.3)(1-0.2)] = 0.003456$$

Finding $P(d,b)$

$$P(d,b) = P(l,d,b) + P(-l,\ d,b) = 0.0040824 + P(-l,\ d,b) = 0.0040824 + 0.2488248 = 0.2529072$$

Finding $P(-l,d,b)$

$$P(-l,d,b) = \sum_{P,S,X} P(P,S,-l,b,X,d)$$

$$= \sum_{P,S,X} P(P)P(S)P(-l|P,S)P(b|S)P(X|-l)P(d|-l,b)$$

$$= P(d|-l,b)\sum_P P(P)\sum_S P(S)P(-l|P,S)P(b|S)\sum_X P(X|-l)$$

$$= (0.7)\sum_P P(P)\sum_S P(-l|P,S)P(b|S)P(S)$$

$$= (0.7)[P(p)\sum_S P(-l|p,S)P(b|S)P(S) + P(-p)\sum_S P(-l|-p,S)P(b|S)P(S)]$$
$$= (0.7)(0.03492 + 0.320544) = 0.2488248$$

Side Calculations:

$$P(p)\sum_S P(-l|p,S)P(b|S)P(S) = (0.1)[P(-l|p,s)P(b|s)P(s) + P(-l|p,-s)P(b|-s)P(-s)]$$
$$= (0.1)[(1-0.05)(0.6)(0.2) + (1-0.02)(0.3)(1-0.2)] = 0.03492$$

$$P(-p)\sum_S P(-l|-p,S)P(b|S)P(S)] = (1-0.1)[P(-l|-p,s)P(b|s)P(s) + P(-l|-p,-s)P(b|-s)P(-s)]$$
$$= (1-0.1)[(1-0.03)(0.6)(0.2) + (1-0.001)(0.3)(1-0.2)] = 0.320544$$

Finding $P(l,d) = \sum_{P,S,X,B} P(P,S,l,B,X,d)$

$$= \sum_P P(P) \sum_S P(S)P(l|P,S) \sum_B P(B|S)P(d|l,B) \sum_X P(X|l)$$

$$= \sum_P P(P) \sum_S P(S)P(l|P,S) \sum_B P(B|S)P(d|l,B)$$

Side calculation

$$\sum_B P(B|S)P(d|l,B)$$
$$= P(b|S)P(d|l,b) + P(-b|S)P(d|l,-b)$$
$$= P(b|S)(0.9) + P(-b|S)(0.8)$$

Resume:

$$P(l,d) = \sum_P P(P) \sum_S P(S)P(l|P,S)(P(b|S)(0.9) + P(-b|S)(0.8))$$

$$= \sum_P P(P)((0.172)P(l|P,s) + (0.664)P(l|P,-s))$$

Side Calculation:

$$\sum_S P(S)P(l|P,S)(P(b|S)(0.9) + P(-b|S)(0.8))$$
$$= P(s)P(l|P,s)(P(b|s)(0.9) + P(-b|s)(0.8)) + P(-s)P(l|P,-s)(P(b|-s)(0.9) + P(-b|-s)(0.8))$$
$$= (0.2)P(l|P,s)((0.6)(0.9) + (1-0.6)(0.8)) + (1-0.2)P(l|P,-s)((0.3)(0.9) + (1-0.3)(0.8))$$
$$= (0.2)P(l|P,s)((0.86) + (0.8)P(l|P,-s)(0.83))$$
$$= (0.172)P(l|P,s) + (0.1328)P(l|P,s)P(l|P,-s)$$

Resume:

$$= P(p)((0.172)P(l|p,s) + (0.1328)P(l|p,s)P(l|p,-s)) + P(-p)((0.172)P(l|-p,s) + (0.1328)P(l|-p,s)P(l|-p,-s))$$
$$= (0.1)((0.172)(0.05) + (0.1328)(0.05)(0.02)) + (1-0.1)((0.172)(0.03) + (0.1328)(0.03)(0.001))$$
$$= 0.0074296$$

Finding $P(d) = \sum_{P,S,L,B,X} P(P,S,L,B,X,d)$, Relevant: P, S, L, B, D

$$= \sum_{P,S,L,B,X} P(P)P(S)P(L|P,S)P(B|S)P(X|L)P(d|L,B)$$

$$= \sum_P P(P) \sum_S P(S) \sum_L P(L|P,S) \sum_B P(B|S)P(d|L,B) \sum_X P(X|L)$$

$$= \sum_P P(P) \sum_S P(S) \sum_L P(L|P,S) \sum_B P(B|S)P(d|L,B)$$

Side: $\sum_B P(B|S)P(d|L,B) = P(b|S)P(d|L,b) + P(-b|S)P(d|L,-b)$

$$P(d) = \sum_P P(P) \sum_S P(S) \sum_L P(L|P,S)[P(b|S)P(d|L,b) + P(-b|S)P(d|L,-b)]$$

Side:

$$\sum_L P(L|P,S)[P(b|S)P(d|L,b)+P(-b|S)P(d|L,-b)]$$
$$=P(l|P,S)[P(b|S)P(d|l,b)+P(-b|S)P(d|l,-b)]+P(-l|P,S)[P(b|S)P(d|-l,b)+P(-b|S)P(d|-l,-b)]$$
$$=P(l|P,S)[P(b|S)(0.9)+P(-b|S)(0.8)]+P(-l|P,S)[P(b|S)(0.7)+P(-b|S)(0.1)]$$

$$P(d)=\sum_P P(P)\sum_S P(S)[P(l|P,S)(P(b|S)(0.9)+P(-b|S)(0.8)]+P(S)[P(-l|P,S)(P(b|S)(0.7)+P(-b|S)(0.1)]$$

Side:

$$\sum_S P(S)(P(l|P,S)(P(b|S)(0.9)+P(-b|S)(0.8))+P(-l|P,S)(P(b|S)(0.7)+P(-b|S)(0.1)))$$
$$=P(s)(P(l|P,s)(P(b|s)(0.9)+P(-b|s)(0.8))+P(-l|P,s)(P(b|s)(0.7)+P(-b|s)(0.1)))$$
$$+P(-s)(P(l|P,-s)(P(b|-s)(0.9)+P(-b|-s)(0.8))+P(-l|P,-s)(P(b|-s)(0.7)+P(-b|-s)(0.1)))$$
$$=(0.2)(P(l|P,s)((0.6)(0.9)+(1-0.6)(0.8))+P(-l|P,s)((0.6)(0.7)+(1-0.6)(0.1)))$$
$$+(1-0.2)(P(l|P,-s)((0.3)(0.9)+(1-0.3)(0.8))+P(-l|P,-s)((0.3)(0.7)+(1-0.3)(0.1)))$$
$$=(0.2)(P(l|P,s)(0.86)+P(-l|P,s)(0.46))+(0.8)(P(l|P,-s)(0.83)+P(-l|P,-s)(0.28))$$
$$=(0.172)P(l|P,s)+(0.092)P(-l|P,s)+(0.664)P(l|P,-s)+(0.224)P(-l|P,-s)$$

$$P(d)=\sum_P P(P)((0.172)P(l|P,s)+(0.092)P(-l|P,s)+(0.664)P(l|P,-s)+(0.224)P(-l|P,-s))$$
$$=P(p)((0.172)P(l|p,s)+(0.092)P(-l|p,s)+(0.664)P(l|p,-s)+(0.224)P(-l|p,-s))$$
$$+P(-p)((0.172)P(l|-p,s)+(0.092)P(-l|-p,s)+(0.664)P(l|-p,-s)+(0.224)P(-l|-p,-s))$$

$$=(0.1)((0.172)(0.05)+(0.092)(1-0.05)+(0.664)(0.02)+(0.224)(1-0.02))$$
$$+(1-0.1)((0.172)(0.03)+(0.092)(1-0.03)+(0.664)(0.001)+(0.224)(1-0.001))$$

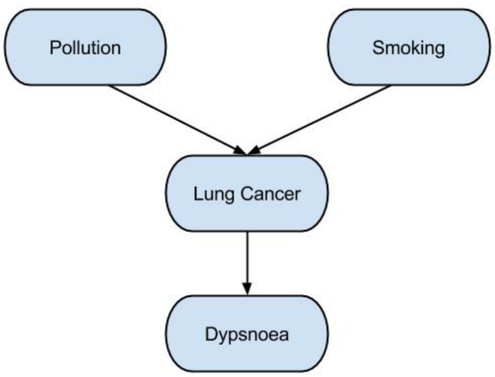
$$P(d)=0.03288+0.286956=0.319836$$

Question 4

Suppose that we do not have an X-ray machine and we are no longer interested in Bronchitis, i.e., the values of Positive X-ray and Bronchitis are always unknown to us. We want to alter the network so that it no longer contains those variables.

One temptation is to simply erase the two variables from our network. But this loses some of our prior information. Use marginalization (i.e. Variable Elimination) to remove the Positive X-ray and Bronchitis variables from the network, drawing a new network and updating the CPT of any affected variable. Eliminate the Positive X-ray variable first. [10 marks].

New Bayes' net:



Since Positive X-ray is conditionally independent of Dypsnoea, it can be removed from the Bayes' net without affecting the rest of the network. The new CPTs can be found by first joining all the variables to get $P(P, L, S, B, D, X)$. Variables are summed out to get individual probabilities that are required to find the CPTs.

CPTs:

s	P(s)
T	0.2
F	0.8

p	P(p)
T	0.1
F	0.9

l	P(d|l)
T	0.852018348623853
F	0.315154547655556

Variables are joint to get the following table

$P(B, P, S, L, D)$

d	b	p	s	l	P(b,p,s,l,d)
T	T	T	T	T	0.00054
T	T	T	T	F	0.00798
T	T	T	F	T	0.000432
T	T	T	F	F	0.016464
T	T	F	T	T	0.002916
T	T	F	T	F	0.073332
T	T	F	F	T	0.0001944
T	T	F	F	F	0.1510488
T	F	T	T	T	0.00032
T	F	T	T	F	0.00076
T	F	T	F	T	0.000896
T	F	T	F	F	0.005488
T	F	F	T	T	0.001728
T	F	F	T	F	0.006984
T	F	F	F	T	0.0004032
T	F	F	F	F	0.0503496
F	T	T	T	T	0.00006
F	T	T	T	F	0.00342
F	T	T	F	T	0.000048
F	T	T	F	F	0.007056
F	T	F	T	T	0.000324
F	T	F	T	F	0.031428
F	T	F	F	T	0.0000216
F	T	F	F	F	0.0647352
F	F	T	T	T	0.00008
F	F	T	T	F	0.00684
F	F	T	F	T	0.000224
F	F	T	F	F	0.049392
F	F	F	T	T	0.000432
F	F	F	T	F	0.062856
F	F	F	F	T	0.0001008
F	F	F	F	F	0.4531464

B is summed out to find P(D,P,S,L)

d	p	s	l	P(p,s,l,d)
T	T	T	T	0.00086
T	T	T	F	0.00874
T	T	F	T	0.001328
T	T	F	F	0.021952
T	F	T	T	0.004644
T	F	T	F	0.080316
T	F	F	T	0.0005976
T	F	F	F	0.2013984
F	T	T	T	0.00014
F	T	T	F	0.01026
F	T	F	T	0.000272
F	T	F	F	0.056448
F	F	T	T	0.000756
F	F	T	F	0.094284
F	F	F	T	0.0001224
F	F	F	F	0.5178816

P is then summed out to get P(D,S,L)

d	s	l	P(d,s,l)
T	T	T	0.005504
T	T	F	0.089056
T	F	T	0.0019256
T	F	F	0.2233504
F	T	T	0.000896
F	T	F	0.104544
F	F	T	0.0003944
F	F	F	0.5743296

Summing out S to get P(D,L)

d	l	P(d,l)
T	T	0.0074296
T	F	0.3124064
F	T	0.0012904
F	F	0.6788736

Sum out D to get P(L)

l	P(l)
T	0.00872
F	0.99128

Using the previous 2 tables to get P(D | L)

l	P(d|l)
T	0.852018348623853
F	0.315154547655556