MATLAB

Essential MATLAB for Scientists

Chap 5 : Matrices

Definition

Matrix is a rectangular array or table of numbers, symbols, or expressions, arranged in rows and columns

```
>> A=[1 2 3 4;5 6 7 8;9 10 11 12]

A =
```

$$\begin{array}{cc} & j \\ i & \left\{ \overbrace{\begin{matrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \end{matrix}} \right. \end{array}$$

A(i,j) where i and j are variables for indexes (index for row, index for column)

A(n,m)

Example: A(2,3)=

Notation

```
>> A=[1 2 3 4;5 6 7 8;9 10 11 12]

A =

     1     2     3     4
     5     6     7     8
     9    10    11    12
```

```
>> A(2,:)

ans =

     5     6     7     8
```

: is used to specify all values. In this case, at row 2, we will consider values from each column. Try with A(:,3)

Notation

```
>> A(:,:)

ans =

     1     2     3     4
     5     6     7     8
     9    10    11    12
```

Another way to write the same result is A(1:end,1:end) by specifying the range of rows and the range of columns (this is why abbreviation is :)

A(2,3:4)

A(1:2,2:end)

length(A) will give you the highest length between number of rows and number of columns (size)
A(1:2,2:length(A))

```
>> A(1:length(A),2:end)
Index in position 1 exceeds array bounds. Index must not exceed 3.
```

Operations

A(1,:)*2

The resulted vector will have the same size than A(1,:)

```
>> A(:,2)*2

ans =

     4
    12
    20
```

Exercise

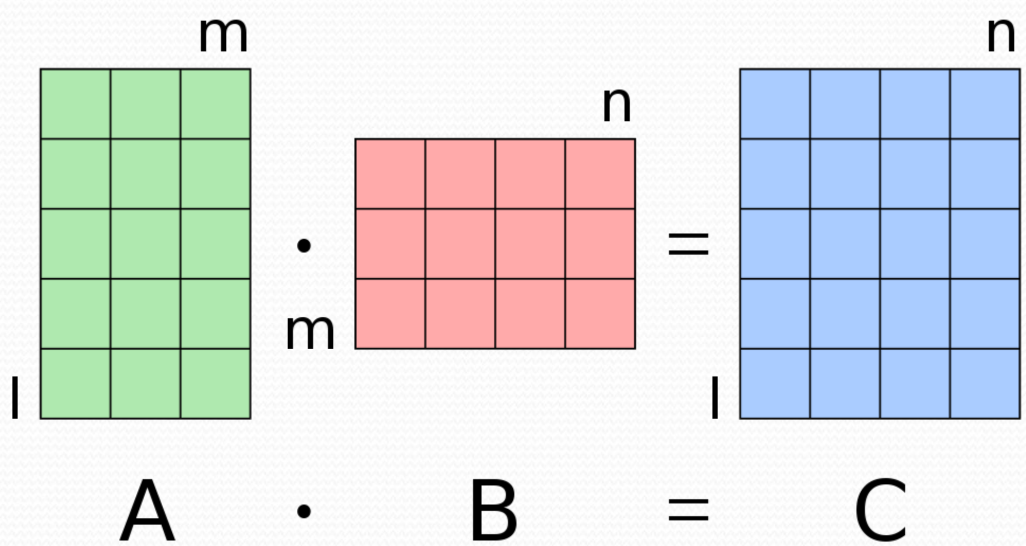
	T°C Day 1	T°C Day 2	T°C Day 3	T°C Day 4
Dubai	32	33	30	28
Abu Dhabi	30	27	31	29
Al Ain	28	27	30	26
Paris	15	17	14	16
Bordeaux	13	18	12	17

What is the average temperature at Al Ain?

What is the average temperature in France for Day2 ?

What is the average emirati temperature through 4 days?

Matrix multiplication



l is the number of rows in matrix A and in matrix C
m is the number of columns in matrix A, and the number of rows in matrix B
n is the number of columns in matrix B and in matrix C

A(l,m) * B(m,n) = C (l,n)

Matrix multiplication

$$c_{11}=a_{11}b_{11}+a_{12}b_{21}+a_{13}b_{31}+a_{14}b_{41}$$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \\ b_{41} & b_{42} & b_{43} \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \end{bmatrix}$$

2 x 4   4 x 3   2 x 3

$$c_{22}=a_{21}b_{12}+a_{22}b_{22}+a_{23}b_{32}+a_{24}b_{42}$$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \\ b_{41} & b_{42} & b_{43} \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \end{bmatrix}$$

Matrix multiplication

A=[1 2 3; 4 5 6]
B=[2 1 0;1 2 3]

```
A =

     1     2     3
     4     5     6


B =

     2     1     0
     1     2     3
```

A*B

Matrix multiplication

A=[1 2 3; 4 5 6]
B=[2 1 0;1 2 3]

```
A =

     1     2     3
     4     5     6


B =

     2     1     0
     1     2     3
```

A*B

Error

Matrix multiplication

A=[1 2 3; 4 5 6]
B=[2 1 0;1 2 3]

```
A =

     1     2     3
     4     5     6


B =

     2     1     0
     1     2     3
```

A.*B

```
ans =

     2     2     0
     4    10    18
```

Same size between A and B

Matrix multiplication

A=[1 4;2 5;3 6]
B=[2 1 0;1 2 3]

```
A =

     1     4
     2     5
     3     6


B =

     2     1     0
     1     2     3
```

A*B

Matrix multiplication

A=[1 4;2 5;3 6]
B=[2 1 0;1 2 3]

```
A =

     1     4
     2     5
     3     6


B =

     2     1     0
     1     2     3
```

A*B

```
ans =

     6     9    12
     9    12    15
    12    15    18
```

Matrix multiplication

A=[1 4;2 5;3 6]
B=[2 1 0;1 2 3]

```
A =

     1     4
     2     5
     3     6


B =

     2     1     0
     1     2     3
```

A.*B

Error

Matrix multiplication

A=[1 4;2 5;3 6]
B=[2 1 0;1 2 3]

```
A =

     1     4
     2     5
     3     6


B =

     2     1     0
     1     2     3
```

A.*B’

Transpose of a matrix

$$A = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}_{2 \times 3} \qquad A^T = \begin{bmatrix} a & d \\ b & e \\ c & f \end{bmatrix}_{3 \times 2}$$

size(A) is different of size(AT)

On Matlab:

AT=A'

```
>> A=[1 2 3; 4 5 6]

A =

     1     2     3
     4     5     6

>> A'

ans =

     1     4
     2     5
     3     6
```

Matrix multiplication

A'*B ≠ B*A'

A'=

1	4
2	5
3	6

```
B =

     2     1     0
     1     2     3
```

```
>> A'*B

ans =

     6     9    12
     9    12    15
    12    15    18
```

```
>> B*A'

ans =

     4    13
    14    32
```

Inverse of a matrix

For matrices, there is no such thing as division. You can add, subtract, and multiply matrices, but you cannot divide them.

We write A^{-1} instead of $1/A$ because we don't divide by a matrix!

When we multiply a number by its inverse we get 1:

$$8 \times 1/8 = 1$$

When we multiply a matrix by its inverse we get the Identity Matrix (which is like "1" for matrices):

$$A \times A^{-1} = I$$

Inverse of a matrix

The inverse of A is A-1 only when:

$A*A^{-1}=A^{-1}*A=I$

With I the matrix identity.

Matrix identity is a square (same number of rows and columns)

For example:

$$I= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Inverse of a matrix

In Matlab, Y = inv(X) computes the inverse of square matrix X .
1.X^(-1) is equivalent to inv(X)

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} = \frac{1}{ad-bc} \begin{pmatrix} d & -1 \\ -c & a \end{pmatrix}$$

determinant

adjoint matrix (In matlab, adjoint (A))

In matlab, det(X) returns the determinant of the square matrix X .

Inverse of a matrix

$$B = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

The **formula for the inverse** of a 3×3 matrix (Matrix B) is given as:

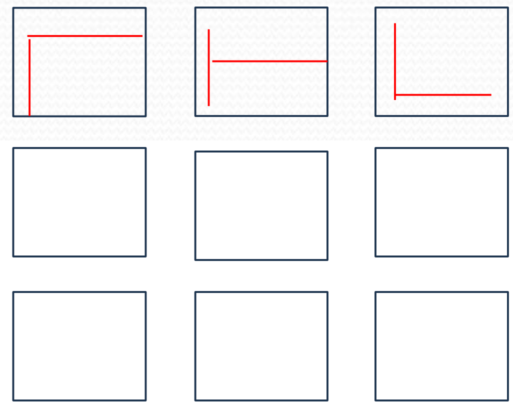
$$B^{-1} = \frac{1}{det(B)} \begin{bmatrix} (ei-fh) & -(bi-ch) & (bf-ce) \\ -(di-fg) & (ai-cg) & -(af-cd) \\ (dh-eg) & -(ah-bg) & (ae-bd) \end{bmatrix}$$

Where $det(B)$ *is the determinant of the* 3×3 *matrix given as:*

$$det(B) = a \begin{vmatrix} e & f \\ h & i \end{vmatrix} - b \begin{vmatrix} d & f \\ g & i \end{vmatrix} + c \begin{vmatrix} d & e \\ g & h \end{vmatrix}$$

$$det(B) = a(ei-fh)-b(di-fg)+c(dh-eg)$$

Inverse of a matrix



1st col, change row

$$B = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

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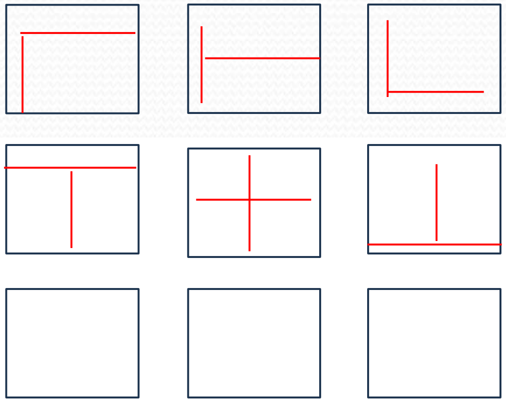
Inverse of a matrix



1st col, change row

2nd col, change row

$$B = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

The **formula for the inverse** of a 3×3 matrix (Matrix B) is given as:

$$B^{-1} = \frac{1}{det(B)} \begin{bmatrix} (ei-fh) & -(bi-ch) & (bf-ce) \\ -(di-fg) & (ai-cg) & -(af-cd) \\ (dh-eg) & -(ah-bg) & (ae-bd) \end{bmatrix}$$

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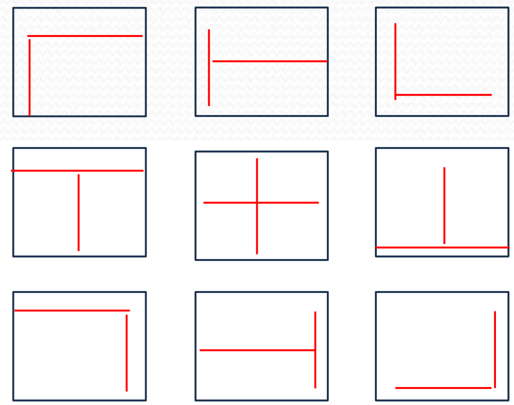
$$det(B) = a(ei-fh) - b(di-fg) + c(dh-eg)$$

Inverse of a matrix



1st col, change row

2nd col, change row

3rd col, change row

$$B = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

The **formula for the inverse** of a 3×3 matrix (Matrix B) is given as:

$$B^{-1} = \frac{1}{det(B)} \begin{bmatrix} (ei-fh) & -(bi-ch) & (bf-ce) \\ -(di-fg) & (ai-cg) & -(af-cd) \\ (dh-eg) & -(ah-bg) & (ae-bd) \end{bmatrix}$$

Where $det(B)$ *is the determinant of the* 3×3 *matrix given as:*

$$det(B) = a \begin{vmatrix} e & f \\ h & i \end{vmatrix} \quad -b \begin{vmatrix} d & f \\ g & i \end{vmatrix} \quad + c \begin{vmatrix} d & e \\ g & h \end{vmatrix}$$

$$det(B) = a(ei-fh)-b(di-fg) + c(dh-eg)$$

Inverse of a matrix

$$B = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 3 & -4 \\ -1 & 2 & 1 \end{bmatrix}$$

This is in fact a square matrix and let's check first if the determinant is 0 or not. Let's calcul
determinant:

Inverse of a matrix

$$B = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 3 & -4 \\ -1 & 2 & 1 \end{bmatrix}$$

This is in fact a square matrix and let's check first if the determinant is 0 or not. Let's calcul determinant:

$$det(B) = |B| = 1[(3)(1)-(-4)(2)]-2[(0)(1)-(-4)(-1)] + (-1)[(0)(2)-(3)(-1)]$$
$$= 1[3+8]-2[0-4] + (-1)[0+3]$$
$$= 1[11]-2[-4]-1[3]$$
$$= 11+8-3$$
$$= 16$$

The determinant isn't 0. Now, let's go ahead and calculate the **inverse** of matrix B.

Inverse of a matrix

$$B^{-1} = \frac{1}{det(B)} \begin{bmatrix} (ei-fh) & -(bi-ch) & (bf-ce) \\ -(di-fg) & (ai-cg) & -(af-cd) \\ (dh-eg) & -(ah-bg) & (ae-bd) \end{bmatrix}$$

Inverse of a matrix

$$B^{-1} = \frac{1}{det(B)} \begin{bmatrix} (ei-fh) & -(bi-ch) & (bf-ce) \\ -(di-fg) & (ai-cg) & -(af-cd) \\ (dh-eg) & -(ah-bg) & (ae-bd) \end{bmatrix}$$

$$B^{-1} = \frac{1}{16} \begin{bmatrix} 11 & -4 & -5 \\ 4 & 0 & 4 \\ 3 & -4 & 3 \end{bmatrix}$$

$$B^{-1} = \begin{bmatrix} \frac{11}{16} & -\frac{1}{4} & -\frac{5}{16} \\ \frac{1}{4} & 0 & \frac{1}{4} \\ \frac{3}{16} & -\frac{1}{4} & \frac{3}{16} \end{bmatrix}$$

System solver

$$\begin{cases} x + y + z = 6 \\ 2y + 5z = -4 \\ 2x + 5y - z = 27 \end{cases}$$

System solver

$$\begin{cases} x + y + z = 6 \\ 2y + 5z = -4 \\ 2x + 5y - z = 27 \end{cases}$$

$$\begin{pmatrix} 1 & 1 & 1 \\ 0 & 2 & 5 \\ 2 & 5 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 6 \\ -4 \\ 27 \end{pmatrix}$$

A X = B

$X = A^{-1} B$

We can find the values x, y and z (the X matrix) by multiplying the inverse of the A matrix by the B matrix

Exercise

$$\begin{cases} a1\ x+b1\ y=c1 \\ a2\ x+b2\ y=c2 \end{cases}$$

Different ways to solve the system and to program it:
-Exercise in Chap 3

Program in chap 3

Command window

```
x=[-5:1:5];
y=[-5:1:5];
a1=-2;b1=3;c1=8;
a2=4;b2=10;c2=16;
[xsolution,ysolution]=calculate_solutions(a1,b1,c1,a2,b2,c2,x,y)
```

Script file

```
function[xsolution,ysolution]=calculate_solutions(a1,b1,c1,a2,b2,c2,x,y)
for i=1:length(x)
        for j=1:length(y)
                res_1st_eq=a1*x(i)+b1*y(j);
                res_2nd_eq=a2*x(i)+b2*y(j);
                if res_1st_eq==c1 && res_2nd_eq==c2
                        xsolution=x(i);
                        ysolution=y(j);
                end
        end
end
end
```

Exercise

$$\begin{cases} a1\ x + b1\ y = c1 \\ a2\ x + b2\ y = c2 \end{cases}$$

Different ways to solve the system and to program it:
-Exercise in Chap 3
-2nd approach by elimination method and check if it is correct

Exercise

$$\begin{cases} a1\ x + b1\ y = c1 \\ a2\ x + b2\ y = c2 \end{cases}$$

Different ways to solve the system and to program it:

-Exercise in Chap 3

-2nd approach by elimination method and check if it is correct

y=(c2-(a2*c1/a1))*(1/(b2-(a2*b1/a1)))

x=(c1-b1*y)*(1/a1)

Exercise

$$\begin{cases} a1\,x + b1\,y = c1 \\ a2\,x + b2\,y = c2 \end{cases}$$

Different ways to solve the system and to program it:
-Exercise in Chap 3
-2nd approach by elimination method and check if it is correct

```
y=(c2-(a2*c1/a1))*(1/(b2-(a2*b1/a1)))
x=(c1-b1*y)*(1/a1)

If a1*x + b1*y==c1 & a2*x + b2*y==c2
        disp(x);disp(y);
end
```

Exercise

a1=-2;b1=3;c1=8;
a2=4;b2=10;c2=16;

$$\begin{cases} a1\,x + b1\,y = c1 \\ a2\,x + b2\,y = c2 \end{cases}$$

$$-2\,x + 3\,y = 8$$
$$4\,x + 10\,y = 16$$

Different ways to solve the system and to program it:
-Exercise in Chap 3
-2nd approach by elimination method and check if it is correct

```
y=(c2-(a2*c1/a1))*(1/(b2-(a2*b1/a1)))
x=(c1-b1*y)*(1/a1)

If a1*x + b1*y==c1 & a2*x + b2*y==c2
        disp(x);disp(y);
end
```

Exercise

$$\begin{cases} a1\ x + b1\ y = c1 \\ a2\ x + b2\ y = c2 \end{cases}$$

Different ways to solve the system and to program it:

-Exercise in Chap 3

-2nd approach by elimination method and check if it is correct

-3rd approach with matrix

Exercise

$$\begin{cases} a1\ x + b1\ y = c1 \\ a2\ x + b2\ y = c2 \end{cases}$$

Different ways to solve the system and to program it:

-Exercise in Chap 3

-2nd approach by elimination method and check if it is correct

-3rd approach with matrix

A\B

inv(A)*B

Exercise

$$\begin{cases} a1\,x + b1\,y = c1 \\ a2\,x + b2\,y = c2 \end{cases}$$

Different ways to solve the system and to program it:

-Exercise in Chap 3

-2nd approach by elimination method and check if it is correct

-3rd approach with matrix

-4th approach (function solve)

Exercise 2

A group took a trip on a bus, at 2$ per child and 3$ per adult for a total of 41$

They took the train back at 5$ per child and 10$ per adult for a total of 120$
How many children, and how many adults?

Exercise 2

$$\begin{cases} 2x + 3y = 41 \\ 5x + 10y = 120 \end{cases}$$

A=[2 3;5 10]
B=[41;120]

A\B

10 children and 7 adults

Exercise 3

Create a random matrix A with 20 rows and 10 columns in the range 0 to 5.

```
r=randi([1 20]);
c=randi([1 10]);
A(r,c)=1000
```

Create a program to find the value 1000, which cell (row and column) is it?

Command	Return
`a'`	Transpose of a
`find(a)`	Indices of all non-zero elements in a.
`fliplr(a)`	Matrix a, flipped horizontally
`flipud(a)`	Matrix a, flipped vertically.
`inv(a)`	Inverse of a
`min(a)`	Minimum-valued element of a. †
`max(a)`	Maximum-valued element of a. †
`numel(a)`	The number of elements of a.
`repmat(a,m,n)`	A matrix where matrix a is repeated in m rows and n columns
`reshape(a,m,n)`	Matrix a reshaped into m rows and n columns.
`size(a)`	The size of a (#rows, #columns, ...)
`sort(a)`	Vector a sorted into ascending order. †
`sum(a)`	Sum of elements of a. †
`unique(a)`	The list of unique elements of a in ascending order.

1. Create a cell array of 8 elements such that element *i* contains the identity matrix of size *i*,
   $i = 1; :::; 8$.

2. Create an example to demonstrate that the following equation holds;

$$(ABC)^T = C^T B^T A^T ,$$

where A, B and C are matrices of different sizes, and the product ABC is feasible.

To summarize

-Definition of a matrix

-Different indexes

-Matrix multiplication

-Transpose of a matrix

-Inverse of a matrix

-Different ways for solving system