Homework Test

Date limite : A rendre au plus tard le **Samedi 01 Avril 2023 à 18 heures** précises.

A rendre : Vous devrez envoyer votre travail en un seul fichier pdf à l'adresse ***papaousmane6c@gmail.com***.
Il faut mentionner vos nom et prénom(s) dans la première page et nommer le fichier comme suit
Nom-Prenom-DevoirMaths.pdf

NB :
- Le travail est personnel.
- Il n'est pas autorisé de discuter avec d'autres personnes.
- Il n'est pas autorisé d'envoyer votre devoir en plusieurs fichiers.
- Vous pouvez répondre en français ou en anglais (ou dans les deux!).
- Lire et respecter les instructions!

Exercice 1. Let M be the matrix

$$M = \begin{pmatrix} -2 & 0 & 1 \\ -2 & 0 & 1 \\ -4 & 0 & 2 \end{pmatrix}$$

  (a) Compute the eigenvalues and the eigenvectors of M.

  (b) Is M diagonalizable?

Exercice 2. Let M

$$M = \begin{pmatrix} 0 & 1 & 2 \\ 2 & 3 & 2 \\ 1 & 3 & 0 \end{pmatrix}$$

  (a) Determine the eigenvalues and the eigenvectors of M over the field $\mathbb{R}$.

  (b) Say, justifying the answer, if there exist a matrix $P \in M_3(\mathbb{R})$ and a diagonal matrix $D \in M_3(\mathbb{R})$ such that $M = PDP^{-1}$

  (c) Determine a possible choice of such P and D.

Exercice 3. Let A

$$A = \begin{pmatrix} 3 & 2 & -2 \\ -1 & 1 & 0 \\ 1 & 1 & 0 \end{pmatrix}$$

  1. is A diagonalizable?

  2. Triangularize the matrix A.

  3. Determine the Jordan decomposition.

Exercice 4. :
In each of the following problems, find the $A = LU$ factorization of the coefficient matrix, and solve the corresponding linear systems $Ax = b$.

  (a)

$$A = \begin{pmatrix} 1 & 3 \\ 3 & 2 \end{pmatrix}; \qquad b = \begin{pmatrix} -1 \\ 1 \end{pmatrix}; \qquad b = \begin{pmatrix} 2 \\ 5 \end{pmatrix}$$

(b)

$$A = \begin{pmatrix} 1 & 0 & -1 \\ 2 & -1 & 0 \\ -1 & 0 & 1 \end{pmatrix}; \qquad b = \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}$$

(c)

$$A = \begin{pmatrix} -1 & 1 & -1 \\ 1 & 1 & 1 \\ -1 & 1 & 2 \end{pmatrix}; \qquad b = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$$

Exercice 5. :

Frank speaks French and German; Musa speaks English, French and Italian; Omar speaks English, Italian, and Spanish; Assia speaks all the languages the others speak except French; and no one speaks any other language. Make a matrix $A = a_{ij}$ with rows representing the four people mentioned and columns representing the languages they speak. Put $a_{ij} = 1$ if person i speaks language j and $a_{ij} = 0$ otherwise. Explain the significance of the matrices AA^t and $A^t A$.

Exercice 6. A company makes bags and suitcases in two different factories. The table below gives the cost, in thousands of dollars, of manufacturing each product in each factory :

	Factory 1	Factory 2
Bags	135	150
Suitcases	627	681

(a) Knowing that the personnel costs represent 2/3 of the total cost, find the matrix that gives the personnel cost of each product in each factory.

(b) Find the matrix that gives the cost of the material of each product in each factory, assuming that, in addition to the personnel and material costs, there is a cost of 20.000 dollars for each product in each factory.

Exercice 7. Application :

Give Python code with numpy/scipy to find all results in exercises 1, 2, 3, 4, 5 and 6.