Time-Varying Formation Tracking for Second-Order Multi-Agent Systems Subjected to Switching Topologies With Application to Quadrotor Formation Flying

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***Abstract*—Time-varying formation tracking analysis and design problems for second-order Multi-Agent systems with switching interaction topologies are studied, where the states of the followers form a predefined time-varying formation while tracking the state of the leader. A formation tracking protocol is constructed based on the relative information of the neighboring agents. Necessary and sufficient conditions for Multi-Agent systems with switching interaction topologies to achieve time-varying formation tracking are proposed together with the formation tracking feasibility constraint based on the graph theory. An approach to design the formation tracking protocol is proposed by solving an algebraic Riccati equation, and the stability of the proposed approach is proved using the common Lyapunov stability theory. The obtained results are applied to solve the target enclosing problem of a multiquadrotor unmanned aerial vehicle (UAV) system consisting of one leader (target) quadrotor UAV and three follower quadrotor UAVs. A numerical simulation and an outdoor experiment are presented to demonstrate the effectiveness of the theoretical results.**

***Index Terms*—Formation tracking control, Multi-Agent system, switching interaction topology, target enclosing, unmanned aerial vehicle (UAV).**

I. Introduction

FORMATION control of Multi-Agent systems has attracted considerable research interest in recent years, and has found applications in a variety of areas, such as drag reduction [1], surveillance [2], telecommunication relay [3], and source seeking [4]. Inspired by biological formation behaviors in the natural world (see, e.g., [5]), formation control has been a research topic for more than 20 years in the scientific community, and several classic formation control strategies, which include leader–follower, virtual structure, and behavior based ones, have been proposed [6]–[11]. With the development of technologies on single unmanned aerial vehicle (UAV) and the increment of the demand for multiple UAVs working together to accomplish certain complex tasks, how to realize the desired formation for multi-UAV systems using those strategies has become the common concern for researchers from the control engineering and robotics communities [12].

Although leader–follower, virtual structure, and behavior based strategies were applied to study the formation control problems of multi-UAV systems in [13]–[23], it has been pointed out in [24] that these three strategies have their own weaknesses. For example, the leader–follower based strategy lacks of robustness due to that the failure of the explicit leader may destroy the whole formation, and the virtual structure based strategy is not fully distributed as it requires each UAV to track its own waypoints. Over the past five years, great advances have been made in consensus control of Multi-Agent systems, and numerous results have been derived (see, e.g., [25]–[30], and the references therein). In [31], consensus protocols were extended to deal with the formation control problems of second-order Multi-Agent systems and it is shown that leader–follower, virtual structure, and behavior based formation control strategies can be unified in the framework of the consensus based ones. More results on consensus based formation control can be found in [33]–[35].

It should be pointed out that in [32]–[35], only formation stabilization problems for multi-UAV systems were addressed. In many practical applications, such as the source seeking and target enclosing, forming the desired time-varying formation is only the first step for a multi-UAV system and the whole formation may also need to track the trajectory generated by the virtual/real leader to perform the tasks. In such scenarios, time-varying formation tracking problems arise, where a group of followers keep the desired time-varying formation while tracking the trajectory of the leader. Considering the fact that the interaction topology among the agents may be unreliable, it is more meaningful to study time-varying formation tracking problems for Multi-Agent systems with switching interaction topologies. Note that the formation tracking control for multi-UAV systems can be decoupled into an inner-loop control and an outer-loop

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control, where the dynamics of the outer loop can be modeled by double integrators (see, e.g., [13], [23], and [33]–[35]). To the best of our knowledge, time-varying formation tracking control for second-order Multi-Agent systems subjected to switching topologies is still open. Although time-invariant formation tracking problems and consensus tracking problems for double-integrator Multi-Agent systems with fixed or switching topologies were studied in [36]–[40], these previous results cannot be directly extended to solve the time-varying formation tracking problems as the time-varying varying formation may bring the derivative of the formation information to both the analysis and design. To this end, this paper studies the time-varying formation tracking control problems with switching topologies.

Compared with the previous relevant results, the contributions of this paper are threefold. First, the states of the followers are not only required to achieve the predefined time-varying formation but also need to track the state of the leader. In [32]–[35], only formation stabilization problems were studied and no explicit leader was considered. Moreover, the formations in [32] and [33] are time-invariant. Because the time-varying formation will bring the derivative of the formation information to both the analysis and design, the results on time-invariant formation tracking control or consensus tracking control of double-integrator Multi-Agent systems in [36]–[40] cannot be directly applied to solve the time-varying formation tracking problems in this paper. Second, the interaction topology among the agents can be switching. The criteria for the second-order Multi-Agent systems to achieve time-varying formation tracking are both necessary and sufficient. In [34], [36]–[38], and [41], the interaction topologies are restricted to be fixed. It has been pointed out in [42] that cooperative control problems for Multi-Agent systems with switching topologies are much complicated and challenging than the fixed cases. The approaches for dealing with the fixed topology cases cannot be extended to the switching topologies cases. Third, the obtained results can be applied to solve the target enclosing problems for multi-UAV systems, and practical experiment using four quadrotor UAVs is carried out in the outdoor environment. In [13], [17], [19], and [21]–[24], the proposed results were demonstrated by numerical simulations.

The rest of this paper is organized as follows. Preliminaries on graph theory and the problem formulation are given in Section II. Time-varying formation tracking analysis and protocol design problems are studied in Section III. An application to target enclosing of multiquadrotor UAV systems is presented for illustration in Section IV. Conclusions are drawn in Section V.

Throughout this paper, for simplicity of notation, let $0_{a\times b}$ represent the zero matrix with dimension $a \times b$. Denote by $\mathbf{1}_N$ a column vector of size N with 1 as its elements. Let I_N represent an identity matrix with dimension N and $\otimes$ be the Kronecker product.

II. Preliminaries and Problem Description

In this section, basic concepts on graph theory are introduced and the problem description is presented.

A. Basic Concepts on Graph Theory

Let $G = \{V(G), E(G), W(G)\}$ be a weighted directed graph with N vertices, where $V(G) = \{v_1, v_2, \ldots, v_N\}$, $E(G) \subseteq \{(v_i, v_j) : v_i, v_j \in V(G), i \neq j\}$, and $W = [w_{\text{ij}}] \in \mathbb{R}^{N\times N}$ represent the vertex set, the edge set, and the weighted adjacency matrix associated with G, respectively. Denoted by $e_{\text{ij}} = (v_i, v_j)$ the edge of G, where vertex v_i is called a neighbor of the vertex v_j. For any $i, j \in \{1, 2, \ldots, N\}, w_{\text{ij}} > 0$ if and only if $e_{\text{ji}} \in E$ and $w_{\text{ij}} = 0$ otherwise. An edge e_{ij} is called undirected if and only if $e_{\text{ij}}, e_{\text{ji}} \in E(G)$ and $w_{\text{ji}} = w_{\text{ij}}$. The neighbor set of the vertex v_i is denoted by $N_i = \{v_j \in V(G) : e_{\text{ji}} \in E(G)\}$. The in-degree of the vertex v_i is defined as $\deg_{\text{in}}(v_i) = \sum_{j=1}^{N} w_{\text{ij}}$. Let $D(G) = \text{diag}\{\deg_{\text{in}}(q_i), i = 1, 2, \ldots, N\}$ be the degree matrix of G. The Laplacian matrix of G is defined as $L = D(G) - W(G)$. A path from v_{i_1} to v_{i_r} is a sequence of ordered edges with the form of $e_{i_k i_{k+1}}$ $(k = 1, 2, \ldots, r-1)$. A graph G is said to have a spanning tree if there exists at least one vertex having a path to all the other vertices.

Note that the graph associated with the interaction topology of the Multi-Agent system in this paper can be switching. Let S denote all the possible graphs with an index set $\mathrm{I} \subset \mathbb{N}$, where $\mathbb{N}$ stands for the set of natural numbers. Let $\sigma(t) : [0, +\infty) \to \mathrm{I}$ be a switching signal whose value is the index of the graph at t. Define $w_{\sigma(t)}^{\text{ij}}$ $(i, j \in \{1, 2, \ldots, N\}$ to be the weight associated with the edge from vertices v_j to v_i. Denote by $G_{\sigma(t)}$ and $L_{\sigma(t)}$ the graph and corresponding Laplacian matrix at t, respectively. Let $N_{\sigma(t)}^{i}$ be the neighbor set of agent i at $\sigma(t)$. Throughout this paper, it is assumed that the admissible switching signal has a dwell time $T_d > 0$.

B. Problem Description

Consider a Multi-Agent system with N agents. The switching interaction topology among the N agents can be described by the graph $G_{\sigma(t)}$ with each agent being a vertex in $G_{\sigma(t)}$ and the interaction from agent i $(i \in \{1, 2, \ldots, N\})$ to agent j $(j \in F)$ is represented by the edge e_{ij}. An agent is called a leader if it has no neighbors and is called a follower if it has at least one neighbor. Assume that there exist one leader and $N-1$ followers. Let $F = \{2, 3, \ldots, N\}$ be the subscript set for the followers. The control objective is to let all the $N-1$ followers form a predefined time-varying formation while tracking the trajectory of the leader. The dynamics of the leader are described by

$$\begin{cases} \dot{x}_1(t) = v_1(t) \\ \dot{v}_1(t) = \alpha_x x_1(t) + \alpha_v v_1(t) \end{cases} \tag{1-1}$$

where $x_1(t) \in \mathbb{R}^n$ and $v_1(t) \in \mathbb{R}^n$ are the position and velocity vectors of the leader 1, respectively, with $n \geq 1$ being the dimension of the space, and α_x and α_v are known damping constants. The dynamics of the $N-1$ followers can be modeled by

$$\begin{cases} \dot{x}_i(t) = v_i(t) \\ \dot{v}_i(t) = \alpha_x x_i(t) + \alpha_v v_i(t) + u_i(t) \end{cases} \tag{1-2}$$

where $x_i(t) \in \mathbb{R}^n$, $v_i(t) \in \mathbb{R}^n$, and $u_i(t) \in \mathbb{R}^n$ are the position, velocity, and control input vectors of the follower i $(i \in F)$, respectively. In the following, for the sake of simplicity in description, let $n = 1$ if not otherwise specified. However, all the results hereafter can be directly extended to the higher dimensional case by using the Kronecker product.

The time-varying formation for the followers to form is specified by $h_F(t) = [h_2^T(t), h_3^T(t), \ldots, h_N^T(t)]^T$, where $h_i(t) = [h_{\mathrm{ix}}(t), h_{\mathrm{iv}}(t)]^T$ $(i \in F)$ is the piecewise continuously differentiable formation vector for follower i with $h_{\mathrm{ix}}(t)$ and $h_{\mathrm{iv}}(t)$ being the components of $h_i(t)$ corresponding to the position and velocity, respectively. Let $\xi_k(t) = [x_k(t), v_k(t)]^T$ $(k = 1, 2, \ldots, N)$.

Definition 1: Multi-Agent system (1) is said to achieve time-varying formation tracking if for any given bounded initial states

$$\lim_{t\to\infty} (\xi_i(t) - h_i(t) - \xi_1(t)) = 0_{2\times 1} (i \in F). \tag{2}$$

Remark 1: When the time-varying formation tracking is achieved by Multi-Agent system (1), the state of the leader may lie inside or outside the time-varying formation specified by $h_F(t)$. In the case, where $\lim_{t\to\infty} \sum_{i=2}^N h_i(t) = 0_{2\times 1}$, it can be obtained from (2) that $\lim_{t\to\infty} (\sum_{i=2}^N \xi_i(t)/(N-1) - \xi_1(t)) = 0_{2\times 1}$, which means that $\xi_1(t)$ lies in the center of the time-varying formation specified by $h_F(t)$. Therefore, by choosing $\lim_{t\to\infty} \sum_{i=2}^N h_i(t) = 0_{2\times 1}$. Definition 1 becomes the definitions for target enclosing or target pursuing. Therefore, target enclosing or target pursuing problems of Multi-Agent systems can be treated as special cases of the time-varying formation tracking problems discussed in this paper.

It is worth noting that $h_i(t)$ $(i \in F)$ is not only the global coordinate, but also the relative offset vector of $\xi_i(t)$ with respect to $\xi_1(t)$. To show the roles of $h_i(t), \xi_i(t)$ $(i \in F)$, and $\xi_1(t)$ more clearly, consider the following illustration example.

Illustration Example 1: Consider a Multi-Agent system with four followers and one leader, which means that $F = \{2, 3, 4, 5\}$. For simplicity, let the states of the four followers form a time-invariant square formation with edge $2d$ while enclosing the state of the leader in the state space. Since the formation is time-invariant, $h_F(t)$ can be rewritten as h_F. To specify the desired time-invariant square formation, one can choose $h_2 = [-d, d]^T$, $h_3 = [d, d]^T$, $h_4 = [d, -d]^T$, and $h_5 = [-d, -d]^T$. If (2) in Definition 1 is satisfied, one gets that h_i $(i \in F)$ represents the relative offset vector of $\xi_i(t)$ with respect to $\xi_1(t)$. Moreover, it follows from (2) that $\lim_{t\to\infty} ((\xi_i(t) - \xi_j(t)) - (h_i(t) - h_j(t))) = 0_{2\times 1}$ $(i, j \in F)$, which means that the two squares formed by h_i and $\xi_i(t)$ $(i \in F)$ are congruent with each other. Note that $\sum_{i=2}^5 h_i = 0_{2\times 1}$, which means $\xi_1(t)$ lies in the center of the square formed by $\xi_i(t)$ $(i \in F)$. Therefore, the desired formation enclosing is realized by the Multi-Agent system. Fig. 1 shows the intuitive geometric relationships among $\xi_i(t)$, $h_i(t)$ $(i \in F)$, and $\xi_1(t)$.

Consider the following time-varying formation tracking protocol for Multi-Agent system (1) with switching topologies:

$$\begin{aligned} u_i(t) = \; & K \sum_{j=2}^{N} w_{\sigma(t)}^{\mathrm{ij}} ((\xi_i(t) - h_i(t)) - (\xi_j(t) - h_j(t))) \\ & + K w_{\sigma(t)}^{i1} ((\xi_i(t) - h_i(t)) - \xi_1(t)) \\ & - \alpha h_i(t) + \dot{h}_{\mathrm{iv}}(t) \end{aligned} \tag{3}$$

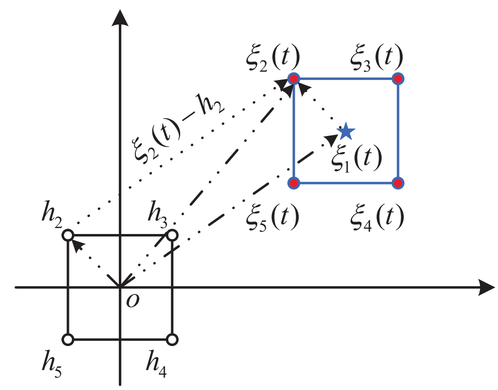

Fig. 1. Illustration example for square formation enclosing with five agents.

where $i \in F$, $\alpha = [\alpha_x, \alpha_v]$, and $K = [k_{11}, k_{12}]$ is a constant gain matrix. Let $\xi_F(t) = [\xi_2^T(t), \xi_3^T(t), \ldots, \xi_N^T(t)]^T$, $B_1 = [1, 0]^T$, and $B_2 = [0, 1]^T$. Considering the leader–follower topology structure, one gets that the Laplacian matrix $L_{\sigma(t)}$ has the following form:

$$L_{\sigma(t)} = \begin{bmatrix} 0_{1\times 1} & 0_{1\times(N-1)} \\ l_{\sigma(t)}^{\mathrm{lf}} & L_{\sigma(t)}^{\mathrm{ff}} \end{bmatrix} \tag{4}$$

where $l_{\sigma(t)}^{\mathrm{lf}} \in \mathbb{R}^{(N-1)\times 1}$ and $L_{\sigma(t)}^{\mathrm{ff}} \in \mathbb{R}^{(N-1)\times(N-1)}$ are associated with the interactions from the leader to the followers and the interactions among the followers, respectively. Under protocol (3), the closed-loop dynamics of Multi-Agent system (1) can be written in a compact form as

$$\begin{cases} \dot{\xi}_1(t) = (B_1 B_2^T + B_2 \alpha) \xi_1(t) \\ \dot{\xi}_F(t) = (I_{N-1} \otimes (B_1 B_2^T + B_2 \alpha) + L_{\sigma(t)}^{\mathrm{ff}} \otimes B_2 K) \xi_F(t) \\ + (l_{\sigma(t)}^{\mathrm{lf}} \otimes B_2 K) \xi_1(t) + (I_{N-1} \otimes B_2 B_2^T) \dot{h}_F(t) \\ - (L_{\sigma(t)}^{\mathrm{ff}} \otimes B_2 K - I_{N-1} \otimes B_2 \alpha) h_F(t). \end{cases} \tag{5}$$

Remark 2: From (1-1) and (1-2), one sees that the second-order model consists of each agent's position and velocity feedback terms, which can also be named the damping terms. The role of the damping terms is to assign the motion modes of the leader by placing the eigenvalues of $B_1 B_2^T + B_2 \alpha$ at the desired locations in the complex plane. Since $(B_1 B_2^T, B_2)$ is controllable, one can always find the desired damping constant α for any given motion modes. Note that the dynamics of the leader has no external control inputs. Therefore, for the given a_x, a_v, and initial states, the trajectory of the leader can be calculated previously. However, for each of the follower, the trajectory of the leader is unknown. Moreover, the second-order model described by (1-1) and (1-2) is more general than the well-known double-integrator one (see [12] for more details). By choosing $\alpha = 0$, the model described by (1-1) and (1-2) becomes the double-integrator ones in [31], [36], [38], and [41].

This paper mainly focuses on the following three problems for Multi-Agent system (1) under protocol (3) with switching topologies:

1) under what conditions the time-varying formation tracking can be achieved;

2) how to design the protocol (3) to achieve the time-varying formation tracking; and
3) how to demonstrate the effectiveness of the obtained theoretical results on a practical multiquadrotor UAV system.

III. Time-Varying Formation Tracking Analysis and Protocol Design

In this section, both the time-varying formation tracking analysis and design problems for Multi-Agent system (5) with switching topologies are investigated. Necessary and sufficient conditions for Multi-Agent system (5) to achieve time-varying formation tracking are presented. An approach to design the protocol (3) is proposed.

Assumption 1: For each of the possible interaction topology $G_{\sigma(t)}$ and any follower i $(i \in F)$, there exists at least a path from the leader 1 to the follower i. Moreover, the interactions among the $N-1$ followers are undirected.

If Assumption 1 is satisfied, then one gets that $G_{\sigma(t)}$ contains a spanning tree rooted at the leader 1. The following lemma reveals some important properties of the Laplacian matrix.

Lemma 1 ([44]): If $G_{\sigma(t)}$ contains a spanning tree, then $L_{\sigma(t)}$ has a simple zero eigenvalue with the associated right eigenvector $\mathbf{1}_N$, and all the other $N-1$ eigenvalues have positive real parts.

Assumption 1 reveals that $L^{\mathrm{ff}}_{\sigma(t)}$ is a symmetric matrix, which means that all the eigenvalues of $L^{\mathrm{ff}}_{\sigma(t)}$ are real. From (4), one sees that $L_{\sigma(t)}$ has at least one zero eigenvalue and all the other $N-1$ eigenvalues are the same as those of $L^{\mathrm{ff}}_{\sigma(t)}$. Therefore, based on Lemma 1, the following conclusion can be obtained directly.

Lemma 2: If Assumption 1 is satisfied, then $L_{\sigma(t)}$ has a simple zero eigenvalue with the associated right eigenvector $\mathbf{1}_N$, and all the other $N-1$ eigenvalues are positive and are the same as those of $L^{\mathrm{ff}}_{\sigma(t)}$.

Now we are ready to present the analysis results.

Theorem 1: Multi-Agent system (1) with switching topologies achieves time-varying formation tracking under protocol (3) if and only if for any $i \in F$, the formation tracking feasibility condition $\lim_{t\to\infty}(h_{\mathrm{iv}}(t) - \dot{h}_{\mathrm{ix}}(t)) = 0$ is satisfied and the switched linear system described by

$$\dot{\phi}(t) = \left(I_{N-1} \otimes (B_1 B_2^T + B_2\alpha) + L^{\mathrm{ff}}_{\sigma(t)} \otimes B_2 K\right)\phi(t) \quad (6)$$

is asymptotically stable, where $\phi(t)$ is the state of the system described by (6).

Proof: Let $\xi(t) = [\xi_1^T, \xi_F^T]^T$. Multi-Agent system (5) can be rewritten as

$$\begin{aligned}\dot{\xi}(t) = &\begin{bmatrix} B_1 B_2^T + B_2\alpha & 0_{2\times(2N-2)} \\ 0_{(2N-2)\times 2} & I_{N-1}\otimes(B_1B_2^T + B_2\alpha)\end{bmatrix}\xi(t) \\ &+ (L_{\sigma(t)}\otimes B_2K)\xi(t) + \begin{bmatrix} 0_{2\times(2N-2)} \\ I_{N-1}\otimes B_2B_2^T\end{bmatrix}\dot{h}_F(t) \\ &- \begin{bmatrix} 0_{2\times(2N-2)} \\ L^{\mathrm{ff}}_{\sigma(t)}\otimes B_2K + I_{N-1}\otimes B_2\alpha\end{bmatrix} h_F(t). \end{aligned} \quad (7)$$

Define $\psi_i(t) = \xi_i(t) - h_i(t)$ $(i \in F)$, $\psi_F(t) = [\psi_2^T(t), \psi_3^T(t), \ldots, \psi_N^T(t)]^T$ and $\psi(t) = [\xi_1^T, \psi_F^T]^T$. Then, one has that $\psi(t) = \xi(t) - [0_{(2N-2)\times 2}, I_{2N-2}]^T h_F(t)$ and

$$\xi(t) = \psi(t) + \begin{bmatrix} 0_{2\times(2N-2)} \\ I_{2N-2}\end{bmatrix} h_F(t). \quad (8)$$

Substituting (8) into (7) yields

$$\begin{aligned}\dot{\psi}(t) = &\begin{bmatrix} B_1 B_2^T + B_2\alpha & 0_{2\times(2N-2)} \\ 0_{(2N-2)\times 2} & I_{N-1}\otimes(B_1B_2^T + B_2\alpha)\end{bmatrix}\psi(t) \\ &- \begin{bmatrix} 0_{2\times(2N-2)} \\ L^{\mathrm{ff}}_{\sigma(t)}\otimes B_2K + I_{N-1}\otimes B_2\alpha\end{bmatrix} h_F(t) \\ &+ (L_{\sigma(t)}\otimes B_2K)\psi(t) + \begin{bmatrix} 0_{2\times(2N-2)} \\ I_{N-1}\otimes B_2B_2^T\end{bmatrix}\dot{h}_F(t) \\ &- \begin{bmatrix} 0_{2\times(2N-2)} \\ I_{(2N-2)}\end{bmatrix}\dot{h}_F(t). \end{aligned} \quad (9)$$

From (4), one obtains that

$$\left(L_{\sigma(t)}\otimes B_2K\right)\begin{bmatrix} 0_{2\times(2N-2)} \\ I_{(2N-2)}\end{bmatrix} = \begin{bmatrix} 0_{2\times(2N-2)} \\ L^{\mathrm{ff}}_{\sigma(t)}\otimes B_2K\end{bmatrix}. \quad (10)$$

Recalling that $B_1 = [1,0]^T$ and $B_2 = [0,1]^T$, one has that

$$\begin{bmatrix} 0_{2\times(2N-2)} \\ I_{N-1}\otimes B_2B_2^T\end{bmatrix} - \begin{bmatrix} 0_{2\times(2N-2)} \\ I_{(2N-2)}\end{bmatrix} = \begin{bmatrix} 0_{2\times(2N-2)} \\ I_{N-1}\otimes B_1B_1^T\end{bmatrix}. \quad (11)$$

Substituting (10) and (11) into (9) leads to

$$\begin{aligned}\dot{\psi}(t) = &\begin{bmatrix} B_1 B_2^T + B_2\alpha & 0_{2\times(2N-2)} \\ 0_{(2N-2)\times 2} & I_{N-1}\otimes(B_1B_2^T + B_2\alpha)\end{bmatrix}\psi(t) \\ &+ (L_{\sigma(t)}\otimes B_2K)\psi(t) - \begin{bmatrix} 0_{2\times(2N-2)} \\ I_{N-1}\otimes B_1B_1^T\end{bmatrix}\dot{h}_F(t) \\ &+ \begin{bmatrix} 0_{2\times(2N-2)} \\ I_{N-1}\otimes B_1B_2^T\end{bmatrix} h_F(t). \end{aligned} \quad (12)$$

Let $T = \begin{bmatrix} 1 & 0_{1\times(N-1)} \\ \mathbf{1}_{N-1} & I_{N-1}\end{bmatrix}$. Then, one gets $T^{-1} = \begin{bmatrix} 1 & 0_{1\times(N-1)} \\ -\mathbf{1}_{N-1} & I_{N-1}\end{bmatrix}$. From Lemma 2, one has $L_{\sigma(t)}\mathbf{1}_N = 0_{N\times 1}$, which means that

$$l^{\mathrm{lf}}_{\sigma(t)} + L^{\mathrm{ff}}_{\sigma(t)}\mathbf{1}_{N-1} = 0_{(N-1)\times 1}. \quad (13)$$

It follows from (13) that

$$T^{-1}L_{\sigma(t)}T = \begin{bmatrix} 0_{1\times 1} & 0_{1\times(N-1)} \\ 0_{(N-1)\times 1} & L^{\mathrm{ff}}_{\sigma(t)}\end{bmatrix}. \quad (14)$$

Define $\varsigma(t) = (I_{N-1}\otimes I_2)\psi_F(t) - (\mathbf{1}_{N-1}\otimes I_2)\xi_1(t)$ and $\zeta(t) = [\xi_1^T(t), \varsigma^T(t)]^T$. Then, one gets that $\zeta(t) = (T^{-1}\otimes I_2)\psi(t)$ and

$$\psi(t) = (T\otimes I_2)\zeta(t). \quad (15)$$

Substituting (15) into (12) and premultiplying both sides of (12) by $T^{-1} \otimes I_2$, one has

$$\dot{\zeta}(t) = \begin{bmatrix} B_1 B_2^T + B_2\alpha & 0_{2\times(2N-2)} \\ 0_{(2N-2)\times 2} & I_{N-1} \otimes (B_1 B_2^T + B_2\alpha) \end{bmatrix} \zeta(t) + \left(\begin{bmatrix} 0_{1\times 1} & 0_{1\times(N-1)} \\ 0_{(N-1)\times 1} & L^{\mathrm{ff}}_{\sigma(t)} \end{bmatrix} \otimes B_2 K \right) \zeta(t) + \begin{bmatrix} 0_{2\times(2N-2)} \\ I_{N-1} \otimes B_1 B_2^T \end{bmatrix} h_F(t) - \begin{bmatrix} 0_{2\times(2N-2)} \\ I_{N-1} \otimes B_1 B_1^T \end{bmatrix} \dot{h}_F(t). \tag{16}$$

Note that $\zeta(t) = [\xi_1^T(t), \varsigma^T(t)]^T$. It follows from (16) that

$$\dot{\xi}_1(t) = (B_1 B_2^T + B_2\alpha)\xi_1(t) \tag{17}$$

$$\dot{\varsigma}(t) = (I_{N-1} \otimes (B_1 B_2^T + B_2\alpha) + L^{\mathrm{ff}}_{\sigma(t)} \otimes B_2 K)\varsigma(t) + (I_{N-1} \otimes B_1 B_2^T) h_F(t) - (I_{N-1} \otimes B_1 B_1^T)\dot{h}_F(t). \tag{18}$$

Let

$$\psi_T(t) = (T \otimes I_2)\,[\xi_1^T, 0_{1\times(2N-2)}]^T \tag{19}$$

$$\psi_{\bar{T}}(t) = (T \otimes I_2)\,[0_{1\times 2}, \varsigma^T(t)]^T. \tag{20}$$

Note that $[\xi_1^T(t), 0_{1\times(2N-2)}]^T = e_1 \otimes \xi_1(t)$, where $e_1 \in \mathbb{R}^N$ with 1 as its first entry and 0 elsewhere. It can be obtained from (19) that

$$\psi_T(t) = Te_1 \otimes \xi_1(t) = \mathbf{1}_N \otimes \xi_1(t). \tag{21}$$

It follows from (15), (19), and (20) that

$$\psi(t) = \psi_T(t) + \psi_{\bar{T}}(t) \tag{22}$$

and $\psi_T(t)$ and $\psi_{\bar{T}}(t)$ are linearly independent of each other. From (21) and (22), one gets that

$$\psi_{\bar{T}}(t) = \xi(t) - \begin{bmatrix} 0_{2\times(2N-2)} \\ I_{2N-2} \end{bmatrix} h_F(t) - \mathbf{1}_N \otimes \xi_1(t). \tag{23}$$

That is, we have

$$\psi_{\bar{T}}(t) = \begin{bmatrix} 0_{2\times 1} \\ \xi_F(t) - h_F(t) - \mathbf{1}_{N-1} \otimes \xi_1(t) \end{bmatrix}. \tag{24}$$

It holds from (24) that Multi-Agent system (1) with switching interaction topologies achieves time-varying formation tracking under protocol (3) if and only if

$$\lim_{t\to\infty} \psi_{\bar{T}}(t) = 0_{2N\times 1}. \tag{25}$$

From (20) and the fact that $T \otimes I_2$ is nonsingular, one obtains that (25) is equivalent to

$$\lim_{t\to\infty} \varsigma(t) = 0_{(2N-2)\times 1}. \tag{26}$$

That is, $\varsigma(t)$ represents the time-varying formation tracking error. From (18), it holds that $\varsigma(t)$ converges to zero if and only if the system described by

$$\dot{\phi}(t) = \left(I_{N-1} \otimes (B_1 B_2^T + B_2\alpha) + L^{\mathrm{ff}}_{\sigma(t)} \otimes B_2 K \right) \phi(t) \tag{27}$$

is asymptotically stable and

$$\lim_{t\to\infty} ((I_{N-1} \otimes B_1 B_2^T) h_F(t) - (I_{N-1} \otimes B_1 B_1^T)\dot{h}_F(t)) = 0. \tag{28}$$

It can be verified that (28) is equivalent to

$$\lim_{t\to\infty} \left(B_1 B_2^T h_i(t) - B_1 B_1^T \dot{h}_i(t) \right) = 0_{2\times 1} (i \in F). \tag{29}$$

That is, we have

$$\lim_{t\to\infty} \left(h_{\mathrm{iv}}(t) - \dot{h}_{\mathrm{ix}}(t) \right) = 0 (i \in F). \tag{30}$$

From (27) and (30), the conclusion of Theorem 1 can be obtained. This completes the proof.

Remark 3: From Theorem 1, one sees that for Multi-Agent system (1) with switching interaction topologies under protocol (3), not all the formation tracking can be realized. The derivative of the position component of the desired formation vector for the follower should be equal to the velocity component of the desired formation vector eventually, which is determined by the second-order dynamics described by (1). In [36]–[40], consensus tracking problems for second-order Multi-Agent systems were studied. However, the results in [36]–[40] cannot be extended to deal with the problem in this paper since time-varying formation tracking feasibility problems were not considered. From (1-1), one sees that the leader in this paper has no external control inputs, which means that the trajectory of the leader is only determined by its dynamics and initial states. Moreover, the leader and follower have the same dynamics and the desired time-varying formation is required to be compatible with the dynamics of the agent. Considering these facts, the precise time-varying formation tracking can be realized using the proposed protocol (3) in this paper. It should be pointed out that if the leader has external control inputs, it may be difficult to realize precise formation tracking without the leader's acceleration measurement. In the case, where $\lim_{t\to\infty} \sum_{i=2}^{N} h_i(t) = 0$, necessary and sufficient conditions for second-order Multi-Agent systems to achieve target enclosing can be obtained directly from Theorem 1.

Let $\lambda^i_{\sigma(t)}$ $(i = 1, 2, \ldots, N)$ be the eigenvalues of the Laplacian matrix $L_{\sigma(t)}$. Without loss of generality, it is assumed that $\lambda^1_{\sigma(t)} \le \lambda^2_{\sigma(t)} \le \cdots \le \lambda^N_{\sigma(t)}$. From Lemma 2, one gets that $\lambda^1_{\sigma(t)} = 0$ and $\lambda^2_{\sigma(t)} > 0$. Define $\lambda_{\min} = \min\{\lambda^i_{\sigma(t)} (\forall \sigma(t) \in \mathrm{I}; i = 2, 3, \ldots, N)\}$. Recalling that $L^{\mathrm{ff}}_{\sigma(t)}$ is symmetric, there exists an orthogonal matrix $U_{\sigma(t)} \in \mathbb{R}^{(N-1)\times(N-1)}$ such that

$$U^T_{\sigma(t)} L^{\mathrm{ff}}_{\sigma(t)} U_{\sigma(t)} = \Lambda^{\mathrm{ff}}_{\sigma(t)} = \mathrm{diag}\{\lambda^2_{\sigma(t)}, \lambda^3_{\sigma(t)}, \ldots, \lambda^N_{\sigma(t)}\}. \tag{31}$$

Based on Theorem 1, an approach to design the protocol (3) is proposed in the following theorem.

Theorem 2: If $\lim_{t\to\infty}(h_{\mathrm{iv}}(t) - \dot{h}_{\mathrm{ix}}(t)) = 0$ $(i \in F)$ holds, Multi-Agent system (1) with switching interaction topologies achieves time-varying formation tracking under protocol (3) by $K = -\delta\lambda^{-1}_{\min} R^{-1} B_2^T P$, where $\delta > 0.5$ is a given constant and P is the positive solution to the following algebraic Riccati equation:

$$P(B_1B_2^T + B_2\alpha) + (B_1B_2^T + B_2\alpha)^T P - PB_2R^{-1}B_2^T P + Q = 0 \quad (32)$$

where $R = R^T > 0$ is any given constant matrix and $Q = D^T D \geq 0$ with $(D, B_1B_2^T)$ detectable.

Proof: Consider the stability of the switched linear system (6). Construct the following Lyapunov candidate function:

$$V(t) = \phi^T(t)(I_{N-1} \otimes P)\phi(t). \quad (33)$$

Equation (33) can be rewritten as

$$V(t) = \phi^T(t)(U_{\sigma(t)} \otimes I_2)(I_{N-1} \otimes P)(U_{\sigma(t)}^T \otimes I_2)\phi(t). \quad (34)$$

Since $V(t)$ is continuously differentiable, $(U_{\sigma(t)}^T \otimes I_2)\phi(t)$ is continuously differentiable. Let $(U_{\sigma(t)}^T \otimes I_2)\phi(t) = \bar{\phi}(t) = [\bar{\phi}_2^T(t), \bar{\phi}_3^T(t), \ldots, \bar{\phi}_N^T(t)]^T$. Taking the derivative of $V(t)$ along the trajectory of system (6) and substituting $(U_{\sigma(t)}^T \otimes I_2)\phi(t) = \bar{\phi}(t)$ and $U_{\sigma(t)}^T L_{\sigma(t)}^{\mathrm{ff}} U_{\sigma(t)} = \Lambda_{\sigma(t)}^{\mathrm{ff}}$ into the derivative gives

$$\begin{aligned}\dot{V}(t) = &\ \bar{\varphi}^T(t)(I_{N-1} \otimes (P(B_1B_2^T + B_2\alpha)))\bar{\varphi}(t)\\ &+ \bar{\varphi}^T(t)(I_{N-1} \otimes ((B_1B_2^T + B_2\alpha)^T P))\bar{\varphi}(t)\\ &+ \bar{\varphi}^T(t)(\Lambda_{\sigma(t)}^{\mathrm{ff}} \otimes ((B_2K)^T P + PB_2K))\bar{\varphi}(t). \quad (35)\end{aligned}$$

Substituting $K = -\delta\lambda_{\min}^{-1}R^{-1}B_2^T P$ and $P(B_1B_2^T + B_2\alpha) + (B_1B_2^T + B_2\alpha)^T P = PB_2R^{-1}B_2^T P - Q$ into (35) leads to

$$\begin{aligned}\dot{V}(t) = &\ \bar{\phi}^T(t)\left(I_{N-1} \otimes \left(PB_2R^{-1}B_2^T P - Q\right)\right)\bar{\phi}(t)\\ &+ \bar{\phi}^T(t)(\Lambda_{\sigma(t)}^{\mathrm{ff}} \otimes (-2\delta\lambda_{\min}^{-1}PB_2R^{-1}B_2^T P))\bar{\phi}(t). \quad (36)\end{aligned}$$

From (31) and (36), one has

$$\begin{aligned}\dot{V}(t) = &\sum_{i=2}^{N} \bar{\varphi}_i^T(t)((1 - 2\delta\lambda_{\sigma(t)}^i\lambda_{\min}^{-1})PB_2R^{-1}B_2^T P)\bar{\varphi}_i(t)\\ &- \sum_{i=2}^{N} \bar{\varphi}_i^T(t)Q\bar{\varphi}_i(t). \quad (37)\end{aligned}$$

It can be verified that

$$1 - 2\delta\lambda_{\sigma(t)}^i\lambda_{\min}^{-1} < 0. \quad (38)$$

From (37) and (38), it holds that

$$\dot{V}(t) \leq 0. \quad (39)$$

Note that the dwell time $T_d > 0$. From (37), $\dot{V}(t) \equiv 0$ if and only if $\bar{\phi}_i(t) \equiv 0_{2\times 1}$ $(i = 2, 3, \ldots, N)$, which means that $\phi(t) \equiv 0_{(2N-2)\times 1}$ as $U_{\sigma(t)}^T \otimes I_2$ is nonsingular. Therefore, the switched linear system described by (6) is asymptotically stable. Since $\lim_{t\to\infty}(h_{\mathrm{iv}}(t) - \dot{h}_{\mathrm{ix}}(t)) = 0$ $(i \in F)$ is satisfied, it holds from the proof of Theorem 1 that Multi-Agent system (1) with switching interaction topologies achieves time-varying formation tracking. The conclusions of Theorem 2 can be obtained. ■

Remark 4: Theorem 2 presents an approach to determine the gain matrix K of protocol (3) by using the minimum nonzero eigenvalue of all the possible Laplacian matrices and solving the algebraic Riccati equation (32). From the proof of Theorem 1, if the formation tracking feasibility condition is satisfied, the stabilization of the time-varying formation tracking for Multi-Agent system (5) is equivalent to the stabilization of the switched linear system described by (6). To deal with the influence of switching topologies on time-varying formation tracking, the stability of the designed protocol (3) is proved based on the common Lyapunov function theory. The existence of the desired K can be guaranteed due to that $(B_1B_2^T, B_2)$ is stabilizable. It should be pointed out that the conclusions of Theorems 1 and 2 are suitable for all the topologies satisfying Assumption 1. In the case, where $h_F(t) \equiv 0$ and $\alpha = 0$, the obtained results can be applied to solve the consensus tracking problems for double-integrator Multi-Agent systems with switching topologies.

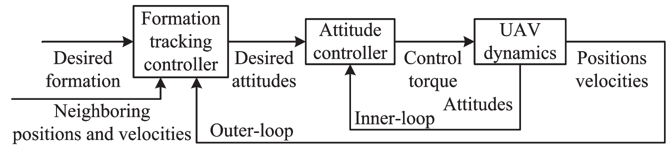

Fig. 2. Two-loop formation tracking control scheme for the multiquadrotor UAV system.

IV. Application to Target Enclosing of Quadrotors

In this section, the theoretical results are applied to deal with the target enclosing problem of a multiquadrotor UAV system. Both numerical and experimental results are given to demonstrate the effectiveness of the results obtained in the previous sections.

The dynamics of a quadrotor UAV can be classified into the trajectory dynamics and the attitude dynamics, where the time constants for the trajectory dynamics are much larger than the ones for the attitude dynamics [19]. Therefore, the formation tracking control for a multiquadrotor UAV system can be decoupled into an inner-loop control and an outer-loop control, where the inner-loop controller stabilizes the attitudes and the outer-loop controller drives the UAV toward the desired position [35]. Fig. 2 shows the two-loop control scheme for the multiquadrotor UAV system. Because the formation tracking discussed in this paper is mainly concerned with the positions and velocities, the dynamics of the leader and follower quadrotor UAVs in the outer loop can be approximately modeled by (1-1) and (1-2), respectively (see, e.g., [34], [35], [45], and [46] for more details). In this configuration, the obtained results in the previous sections can be applied to solve the time-varying formation tracking control problems of the multiquadrotor UAV systems.

In this application, three followers and one leader are involved in the multiquadrotor UAV system. Fig. 3 shows the time-varying formation tracking experimental platform which includes one ground control station (GCS) and four quadrotors with flight control system (FCS). The tip-to-tip wingspan and the weight of each quadrotor are 65 cm and 1400 g, respectively. The payload of each quadrotor is 200 g and the maximum flight time is about 10 min. The FCS is developed based on a user-programmable DSP. Three one-axis gyroscopes, a three-axis

Fig. 3. Quadrotor formation tracking experimental platform.

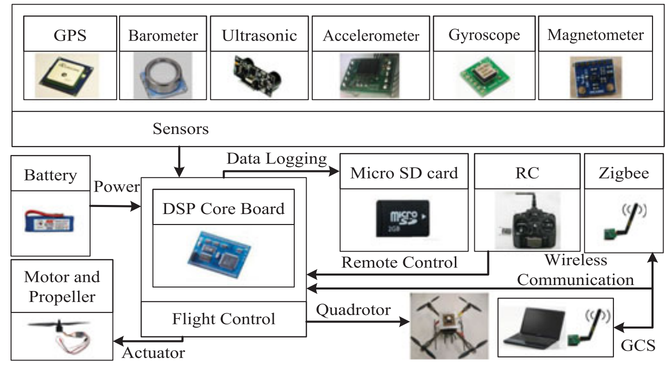

Fig. 4. Hardware structure of the quadrotor UAV system.

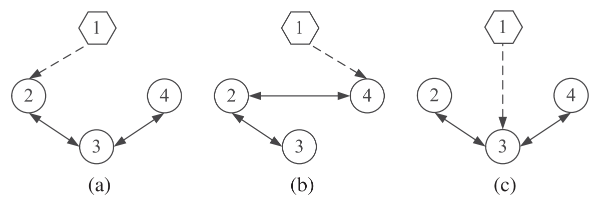

Fig. 5. Switching interaction topologies.

TABLE I
DESCRIPTIONS OF THE MAIN COMPONENTS USED IN THE QUADROTOR UAV SYSTEM

Components	Part numbers	Key features
DSP	TMS320F28335	Working frequency: 135 MHz; CPU: 32-bit
GPS	UBloxLEA6H	Resolution: ± 2 m; conversion rate: 10 Hz
Ultrasonic	URM37V4.2	Resolution: ± 1 cm; range: 10 m
Barometer	MS5803	Resolution: ± 0.012 mbar; range: 10–1300 mbar
Magnetometer	HMC5983	Resolution:± 2 mGs; range: 8 Gs
Gyroscope	ADXRS610	Resolution: ± 6 mV/°/s; range: ± 300°/s
Accelerometer	HQ7001	Resolution: 16-bit AD; range: ± 6 g
Zigbee	DTK1605H	Max conversion rate: 115 200 b/s; transmission range: 1600 m
Frame	XAircraft 650 Value	Weight: 880 g; diameter: 550 mm
Electronic speed controller	Xaircraft BLC-10A	Max control rate: 500 Hz; rated current: 10 A
Motor	XAircraft 2215	Outer diameter: 22 mm; bearing length: 15 mm
Propeller	XAircraft 1045	Diameter: 10 inch; thread pitch: 4.5 inch
Battery	3S-5AH-30C	Voltage: 11.1 V; capacity: 5000 mAh

magnetometer, and a three-axis accelerometer are employed by the FCS to estimate the attitude and acceleration of each quadrotor. The position and velocity in the horizontal plane (XY plane) of each quadrotor are measured by the global positioning system (GPS) module. An ultrasonic range finder and a barometer are used to measure the height of each quadrotor for the cases flying near and far from the ground, respectively. A micro SD card is used to record the key flight parameters onboard. To deal with the emergency situation, the remote controller is kept for each quadrotor. The wireless communication network among quadrotors and the GCS are built by Zigbee modules, where the Zigbee module on each quadrotor works as the router and the one on the GCS works as the coordinator. Using the Zigbee network, control commands can be sent to a specified quadrotor or broadcasted to all the quadrotors and the required information for constructing the formation tracking protocol can be transferred between any two quadrotors. Moreover, each quadrotor sends its states to the GCS every 1 s for the use of monitoring. Table I shows the part numbers and key features of the main components used in the quadrotor UAV system. Fig. 4 shows the hardware structure of the quadrotor UAV system.

The multiquadrotor UAV system is required to perform a target enclosing task in the horizontal XY plane, that is, $n = 2$. Let quadrotor 1 be the leader, which is also known as the target. Let quadrotors 2, 3, and 4 be the followers. The three followers are required to enclose the target using a circular formation. Note that the height and the yaw angle of each quadrotor are not involved in the formation tracking and can be controlled separately [47]. The heights of all the quadrotors are controlled to be 1.5 m using the proportional–integral–derivative (PID) controller with 6, 0.5, and 6 as the coefficients for the proportional, integral, and derivative terms, respectively. The yaw angles of the four quadrotors are, respectively, controlled to be 3.53°, -13.79°, -63.86°, and 12.19° by using the PID controller with 1, \$0.1, and 0.4 as the coefficients for the proportional, integral and derivative terms, respectively. The inner loop for the roll and pitch angles is controlled by the proportional–derivative (PD) controller (see [48] for more details) with 8 and 3 as the coefficients for the proportional and derivative terms, respectively, at a frequency of 500 Hz. The outer loop for the trajectory in the XY plane is controlled by the formation tracking controller (3) at a frequency of 10 Hz. To calculate the relative positions and velocities with respect to its neighbors, each quadrotor first obtains the neighboring absolute positions, velocities, and formation information via the communication network, and then derives the desired values via subtractions. In the case, where $n = 2$, $\xi_i(t)$ and $u_i(t)$ can be rewritten as $\xi_i(t) = [x_{\rm iX}, v_{\rm iX}, x_{\rm iY}, v_{\rm iY}]^T$, $x_i(t) = [x_{\rm iX}, x_{\rm iY}]^T$, $v_i(t) = [v_{\rm iX}, v_{\rm iY}]^T$ $(i = 1, 2, 3, 4)$, $h_k(t) = [h_{\rm kxX}, h_{\rm kvX}, h_{\rm kxY}, h_{\rm kvY}]^T$, and $u_k(t) = [u_{\rm kX}, u_{\rm kY}]^T$ $(k = 2, 3, 4)$.

Due to the limit of the flying space, the motion modes of the target are set to be stable by specifying the eigenvalues of $B_1 B_2^T + B_2\alpha$ at $-0.6 + 0.8j$ and $-0.6 - 0.8j$ ($j^2 = -1$) with the damping constants $\alpha_x = -1$ and $\alpha_v = -1.2$, which means that the final position of the leader is stationary. The target enclosing is implemented in the following two steps. In the first step, during $t \in [0\,\text{s}, 30\,\text{s})$, the three followers approach the target with a specified formation described by

$$h_2(t) = \begin{bmatrix} f_x(-9.14, 1, t) \\ f_v(-9.14, 1, t) \\ f_x(-15.6, 1, t) \\ f_v(-15.6, 1, t) \end{bmatrix}, h_3(t) = \begin{bmatrix} f_x(-3.26, 2, t) \\ f_v(-3.26, 2, t) \\ f_x(-14.26, 2, t) \\ f_v(-14.26, 2, t) \end{bmatrix}$$

$$h_4(t) = \begin{bmatrix} f_x(15.91, 3, t) \\ f_v(15.91, 3, t) \\ f_x(-13.3, 3, t) \\ f_v(-13.3, 3, t) \end{bmatrix}$$

where

$$\begin{aligned} f_x(d_x, k, t) = {} & d_x + (10\cos(k/3) + d_x)\,\text{sat} \\ & \times \left(\frac{t}{|10\cos(k/3) + d_x|}\right) \\ l f_v(d_v, k, t) = {} & \text{sign}\left(\frac{1}{|10\cos(k/3) + d_v|}\right) \\ & \times \left|1 - \text{sign}\left(\text{sat}\left(\frac{t}{|10\cos(k/3) + d_v|}\right)\right)\right| \\ \text{sat}(\vartheta(t)) = {} & \begin{cases} \text{sign}(\vartheta(t)), |\vartheta(t)| \geq 1 \\ \vartheta(t), \quad \text{else.} \end{cases} \end{aligned}$$

In the second step, during $t \in [30\,\text{s}, 225\,\text{s}]$, the three followers keep a circular formation around the target, where the circular formation is specified by

$$h_i(t) = \begin{bmatrix} 10\cos(0.1t + 2\pi(i-2)/3) \\ -\sin(0.1t + 2\pi(i-2)/3) \\ 10\sin(0.1t + 2\pi(i-2)/3) \\ \cos(0.1t + 2\pi(i-2)/3) \end{bmatrix}, i = 2, 3, 4.$$

During $t \in [0\,\text{s}, 225\,\text{s}]$, the interaction topology among the four quadrotors is randomly chosen from G_1, G_2, and G_3, as shown in Fig. 5, with $T_d > 10\,\text{s}$.

From the expression of $h_F(t)$, one sees that the desired formation for the followers is time-varying. It should be pointed out that the formation specified by $h_F(t)$ in this application cannot be realized by introducing the offsets to the protocols in [36]–[40]. It can be verified that the time-varying formation tracking feasibility condition is satisfied. Choose $\delta = 0.6$, $Q = 0.3I_4$, and $R = I_4$. From Theorem 2, one can obtain $K = I_2 \otimes [-0.4246, -0.6705]$. Choose the initial states of the target and the three followers as $x_1(t) = [1.45, -0.13, 1.41, -0.53]^T$, $x_2(t) = [-9.41, -0.01, 15.6, 0.05]^T$, $x_3(t) = [-3.26, -0.24, -14.26, 0.26]^T$, and $x_4(t) = [15.91, -0.05, -13.3, -0.05]^T$. Fig. 6 illustrates the switching signal in both simulation and experiment within $t = 225\,\text{s}$. Figs. 7 and 8 show the position and velocity trajectories of the four quadrotors in the simulation and experiment, respectively, where the initial positions of the four quadrotors are marked by circles and the final positions

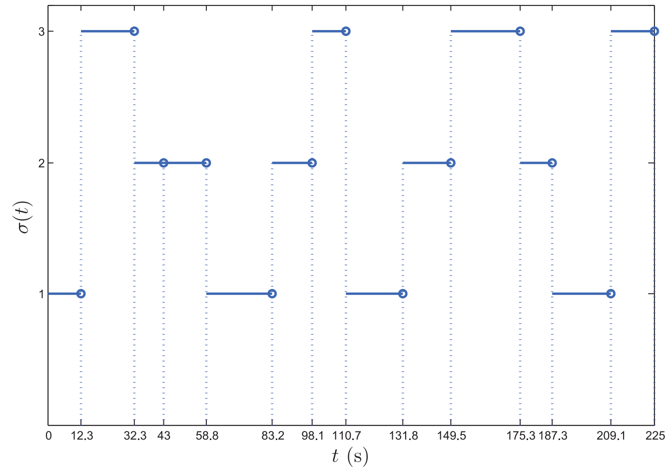

Fig. 6. Switching signal for the topology in the simulation and experiment.

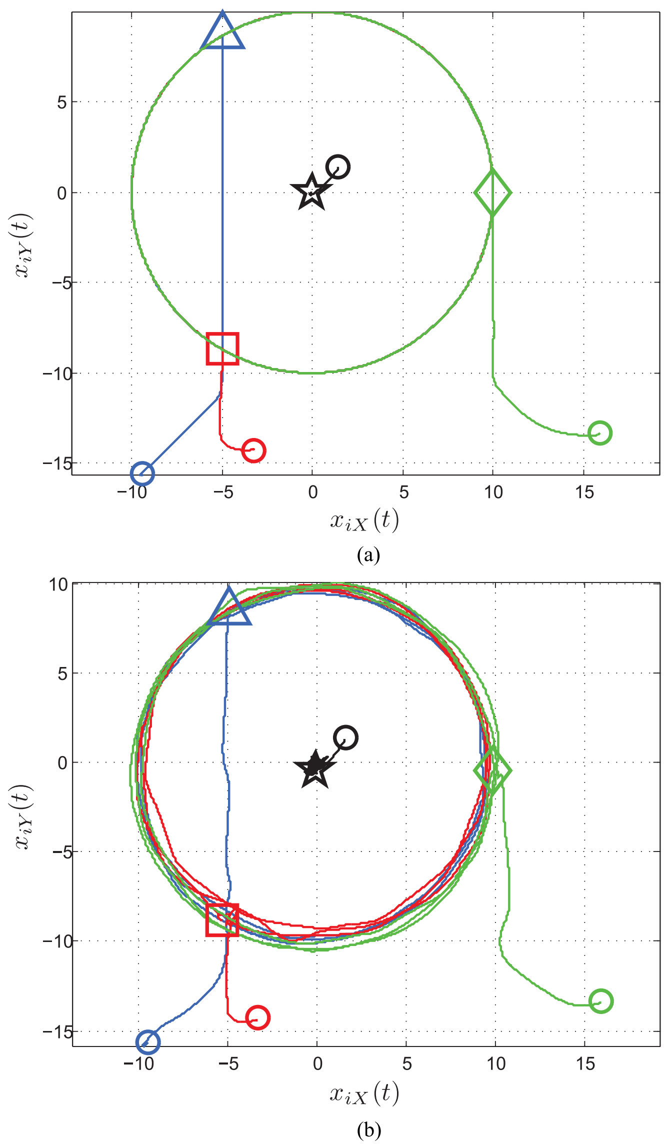

Fig. 7. Position trajectories of the four quadrotors within $t = 225\,\text{s}$.

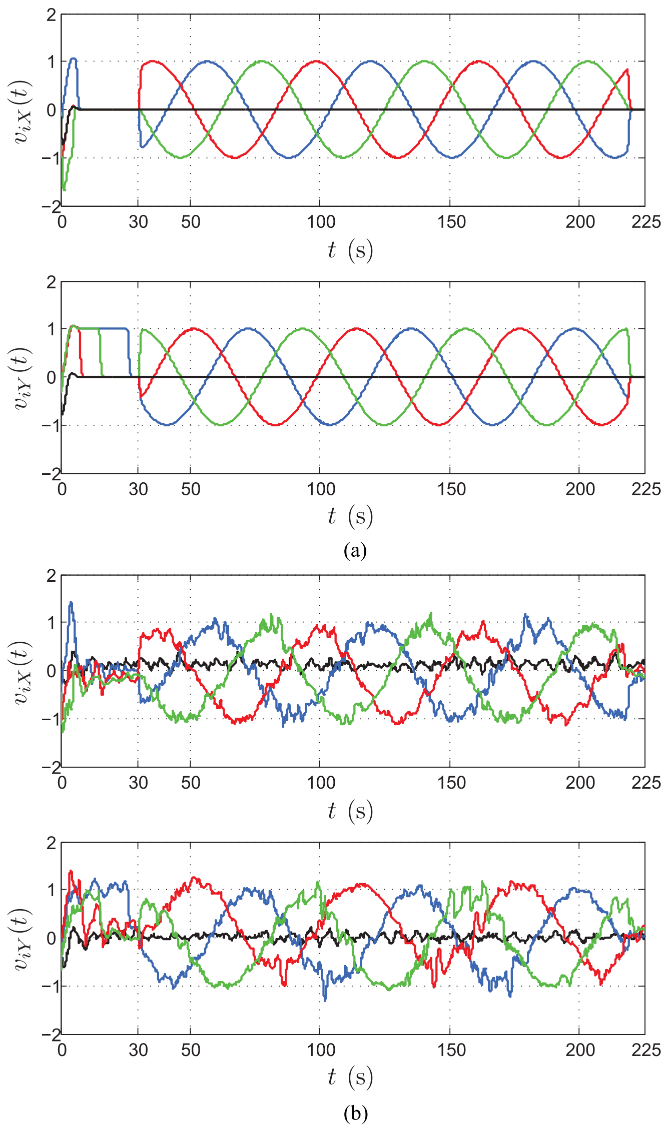

Fig. 8. Velocity trajectories of the four quadrotors within $t = 225$ s.

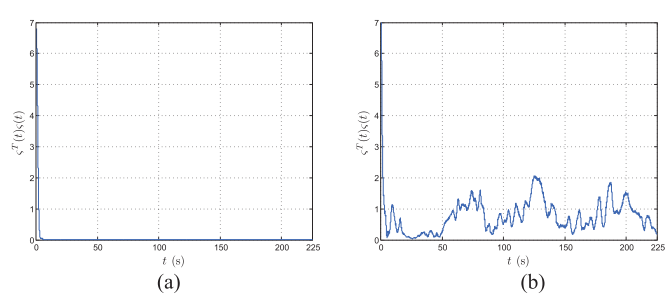

Fig. 9. Curves of the formation tracking error within $t = 225$ s.

are denoted by diamonds, triangles, squares, and pentagrams, respectively. Fig. 9 depicts the curves of the time-varying formation tracking error in both the simulation and experiment within $t = 225$ s. Fig. 10 shows a captured image of four quadrotors in target enclosing. From Figs. 6 to 10, one sees that the three follower quadrotors enclose the target quadrotor using the desired

Fig. 10. Captured image of the four quadrotors in target enclosing.

time-varying formation under the influence of the switching interaction topologies in both the simulation and experiment. It should be pointed out that due to the sensor errors and external disturbances, etc., there exist certain control errors in the figures for the experiment, which is reasonable for practical applications. The video of the experiment can be found at http://v.youku.com/v_show/id_XMTQxMjM5ODM4MA or https://youtu.be/uDQmfr7TTyI.

V. Conclusion

Time-varying formation tracking control problems for second-order Multi-Agent systems with switching interaction topologies were studied. A formation tracking protocol was constructed based on the relative information of neighboring agents. Necessary and sufficient conditions for Multi-Agent systems with switching interaction topologies to achieve time-varying formation tracking were presented. An approach to design the formation tracking protocol was proposed by solving an algebraic Riccati equation. The stability of the proposed approach was proved using the common Lyapunov stability theory. The obtained results were applied to solve the target enclosing problem of a multiquadrotor system consisting of one leader (target) quadrotor UAV and three follower quadrotor UAVs. A future research direction is to extend the results in this paper to the case where the switching interaction topologies among followers can be directed. Another interesting topic for future study is to consider the effects of external control inputs for the leader and various disturbances on the time-varying formation tracking control.

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