SERII TAYLOR - CURS 5

① $f(x)=\sqrt{x+1}$, $a=0$, $n=3$

$f(x)=(x+1)^{\frac{1}{2}}$ $\qquad f(0)=1$

$f'(x)=\frac{1}{2}(x+1)^{-\frac{1}{2}}$ $\qquad f'(0)=\frac{1}{2}$

$f''(x)=\frac{-1}{4}(x+1)^{-\frac{3}{2}}$ $\qquad f''(0)=-\frac{1}{4}$

$f'''(x)=\frac{3}{8}(x+1)^{-\frac{5}{2}}$ $\qquad f'''(0)=\frac{3}{8}$

$$T_3=f(0)+\frac{f'(0)}{1!}x+\frac{f''(0)}{2!}\cdot x^2+\frac{f'''(0)}{3!}x^3$$

$$T_3=1+\frac{1}{2}x-\frac{1}{8}x^2+\frac{1}{16}x^3$$

② $f(x)=tg\,x$, $a=0$, $n=3$

$f'(x)=1+tg^2x$ $\qquad f'(0)=1$

$f''(x)=2tgx\,(tgx)'=2tgx(1+tg^2x)$ $\qquad f''(0)=0$

$f'''(x)=2(1+tg^2x)^2+2tgx\cdot 2tgx(1+tg^2x)$

$=(2+2tg^2x+4tg^2x)(1+tg^2x)$

$=(2+6tg^2x)(1+tg^2x)$ $\qquad f'''(0)=2$

$$T_3=f(0)+\frac{f'(0)}{1!}x+\frac{f''(0)}{2!}x^2+\frac{f'''(0)}{3!}x^3$$

$$T_3=0+x+0+\frac{1}{3}x^3=x+\frac{1}{3}x^3.$$

Polinomul Taylor

$$T_n(x)=f(a)+\frac{x-a}{1!}f'(a)+\frac{(x-a)^2}{2!}f''(a)+\dots+\frac{(x-a)^n}{n!}\cdot f^{(n)}(a)$$

$= 1 \Rightarrow R = 1.$

③ $f(x) = \arcsin x$, $a=0$, $n=2$

$f'(x) = \dfrac{1}{\sqrt{1-x^2}}$ $\qquad f'(0) = 1$

$f''(x) = \dfrac{-\dfrac{1}{2\sqrt{1-x^2}} \cdot (-2x)}{(1-x^2)} = \dfrac{x}{(1-x^2)\sqrt{1-x^2}}$ $\qquad f''(0)=0$

$T_2(x) = f(0) + \dfrac{f'(0)}{1!} x + \dfrac{f''(0)}{2!} x^2$

$T_2(x) = 0 + 1x + 0 \Rightarrow T_2(x) = x.$

④ $f(x) = x^2 - 2x + 5$, $a=0$, $n=3$

$f'(x) = 2x - 2$ $\qquad f'(0) = -2$

$f''(x) = 2$ $\qquad f''(0) = 2$

$f'''(x) = 0$ $\qquad f'''(0) = 0.$

$T_3(x) = f(0) + \dfrac{f'(0)}{1!} x + \dfrac{f''(0)}{2!} x^2 + \dfrac{f'''(0)}{3!} x^3$

$T_3(x) = 5 - 2x + x^2$

⑤ $f(x) = x^{\frac{3}{2}}$, $a=1$, $n=4$

$f'(x) = \frac{3}{2} x^{\frac{1}{2}}$ $\qquad f'(1) = \frac{3}{2}$

$f''(x) = \frac{3}{4} x^{-\frac{1}{2}}$ $\qquad f''(1) = \frac{3}{4}$

$f'''(x) = -\frac{3}{8} x^{-\frac{3}{2}}$ $\qquad f'''(1) = -\frac{3}{8}$

$f^{IV}(x) = \frac{9}{16} x^{-\frac{5}{2}}$ $\qquad f^{IV}(1) = \frac{9}{16}$

$$T_4(x) = f(1) + \frac{f'(1)}{1!}(x-1) + \frac{f''(1)}{2!}(x-1)^2 + \frac{f'''(1)}{3!}(x-1)^3 + \frac{f^{IV}(1)}{4!}(x-1)^4$$

$$T_4(x) = 1 + \frac{3}{2}(x-1) + \frac{3}{8}(x-1)^2 - \frac{1}{16}(x-1)^3 + \frac{\cancel{9}^{3}}{16 \cdot 2 \cdot \cancel{3} \cdot 4}(x-1)^4$$

⑥ $f(x) = \sin x$, $a = \frac{\pi}{2}$, $n=4$

$f'(x) = \cos x$ $\quad f'(\frac{\pi}{2}) = 0$

$f''(x) = -\sin x$ $\quad f''(\frac{\pi}{2}) = -1$

$f'''(x) = -\cos x$ $\quad f'''(\frac{\pi}{2}) = 0$

$f^{IV}(x) = \sin x$ $\quad f^{IV}(\frac{\pi}{2}) = 1$

$$T_4(x) = f(\frac{\pi}{2}) + \frac{f'(\frac{\pi}{2})}{1!}(x - \frac{\pi}{2}) + \frac{f''(\frac{\pi}{2})}{2!}(x - \frac{\pi}{2})^2 + \frac{f'''(\frac{\pi}{4})}{3!}(x - \frac{\pi}{2})^3 + \frac{f^{IV}(\frac{\pi}{4})}{4!}(x - \frac{\pi}{2})^4$$

$$T_4(x) = 1 + 0 - \frac{1}{2!}(x - \frac{\pi}{2})^2 + 0 + \frac{1}{24}(x - \frac{\pi}{2})^4$$

(7) $f(x)=\sqrt{x}$, $a=100$, $n=3$

$f'(x)=\frac{1}{2}x^{-\frac{1}{2}}$ $\quad f'(100)=\frac{1}{2}\cdot\frac{1}{\sqrt{100}}=\frac{1}{20}$

$f''(x)=-\frac{1}{4}x^{-\frac{3}{2}}$ $\quad f''(100)=-\frac{1}{4}\cdot\frac{1}{1000}=-\frac{1}{4000}$

$f'''(x)=\frac{3}{8}x^{-\frac{5}{2}}$ $\quad f'''(100)=\frac{3}{8}\cdot\frac{1}{100000}=\frac{3}{800000}$

$$T_3(x)=f(100)+\frac{f'(100)}{1!}(x-100)+\frac{f''(100)}{2!}(x-100)^2+\frac{f'''(100)}{3!}(x-100)^3$$

$$T_3(x)=10+\frac{1}{20}(x-100)+\frac{-1}{8000}(x-100)^2+\frac{1}{1600000}(x-100)^3$$

(8) $f(x)=\frac{1}{(x-4)^2}$, $a=5$, $n=5$

$f'(x)=-\frac{2(x-4)}{(x-4)^4}=-\frac{2}{(x-4)^3}$ $\quad f'(5)=-2$

$f''(x)=\frac{6}{(x-4)^4}$ $\quad \Rightarrow f''(5)=6.$

$f'''(x)=-\frac{24}{(x-4)^5}$ $\quad \Rightarrow f'''(5)=24$

$f^{IV}(x)=\frac{120}{(x-4)^6}$ $\quad \Rightarrow f^{IV}(5)=120$

$f^{V}(x)=-\frac{720}{(x-4)^7}$ $\quad \Rightarrow f^{V}(5)=-720$

$$T_5(x)=1-2(x-5)+3(x-5)^2-\frac{24}{6}(x-5)^3+\frac{120}{24}(x-5)^4-\frac{720}{120}(x-5)^5$$

$$T_5(x)=1-2(x-5)+3(x-5)^2-4(x-5)^3+5(x-5)^4-6(x-5)^5.$$

⑨ $f(x) = tg\,x$, $a = \frac{\pi}{4}$, $n = 4$

$f'(x) = 1 + tg^2 x \Rightarrow f'(\frac{\pi}{4}) = 2$

$f''(x) = 2\,tg\,x(1 + tg^2 x) \Rightarrow f''(\frac{\pi}{4}) = 4$

$f'''(x) = (2 + 6\,tg^2 x)(1 + tg^2 x) \Rightarrow f'''(\frac{\pi}{4}) = 16$

$f^{IV}(x) = 12\,tg\,x \cdot (1 + tg^2 x)^2 + (2 + 6\,tg^2 x) \cdot 2\,tg\,x(1 + tg^2 x)$

$\Rightarrow f^{IV}(\frac{\pi}{4}) = 12 \cdot 2^2 + 8 \cdot 2 \cdot 2 = 48 + 32 = 80$

$T_4(x) = 1 + \frac{2}{1!} \cdot (x - \frac{\pi}{4}) + \frac{4}{2!}(x - \frac{\pi}{4})^2 + \frac{16}{3!}(x - \frac{\pi}{4})^3 +$

$+ \frac{80}{4!}(x - \frac{\pi}{4})^4$

$T_4(x) = 1 + 2(x - \frac{\pi}{4}) + 2(x - \frac{\pi}{4})^2 + \frac{8}{3}(x - \frac{\pi}{4})^3 +$

$\frac{10}{3}(x - \frac{\pi}{4})^4.$

⑩ $f(x) = e^x \cdot \sin x$, $a = 0$, $n = 4$.

$f'(x) = e^x(\sin x + \cos x) \Rightarrow f'(0) = 1$

$f''(x) = e^x(\cancel{\sin x} + \cos x + \cos x - \cancel{\sin x}) \Rightarrow f''(0) = 2$

$f'''(x) = e^x(2\cos x - 2\sin x) \Rightarrow f'''(0) = 2$

$f^{IV}(x) = e^x(\cancel{2\cos x} - 2\sin x - 2\sin x - \cancel{2\cos x}) \Rightarrow f^{IV}(0) = 0$

$T_4(x) = \frac{f'(0)}{1}x + \frac{f''(0)}{2}x^2 + \frac{f'''(0)}{6}x^3 + \frac{f^{IV}(0)}{24}x^4.$

$= x + x^2 + \frac{1}{3}x^3$

(11) $(1+y)^{\alpha} = 1 + \sum_{n\geq 1} \frac{\alpha(\alpha-1)(\alpha-2)\dots(\alpha-n+1)}{n!} y^n.$

$f(x) = \frac{1}{1-x}$

$f^{(n)}(x) = \frac{(-1)^n \cdot n!\,(-1)^n}{(1-x)^{n+1}}$ $\qquad f^{(n)}(0) = n!$

$f(x) = \sum_{n\geq 0} \frac{n!}{n!} x^n = \sum_{n\geq 0} x^n$

$a_n = 1 \Rightarrow L = \lim \left| \frac{a_{n+1}}{a_n} \right| = 1 \Rightarrow R = \frac{1}{L} = 1.$

$\Rightarrow$ Multj. de conv. $(-1;1)$.

(12) $f(x) = \ln(x+1)$

$f' = \frac{1}{x+1}$

$f'' = -\frac{1}{(x+1)^2}$

$f^{(n)}(x) = \frac{(-1)^{n-1}(n-1)!}{(x+1)^n} \Rightarrow f^{(n)}(0) = (-1)^{n-1}(n-1)!$

$f(x) = \sum \frac{(-1)^{n-1}(n-1)!}{n!} x^n = \sum \underbrace{\frac{(-1)^{n-1}}{n}}_{a_n} \cdot x^n$

$\lim \left| \frac{a_{n+1}}{a_n} \right| = \frac{n}{n+1} \to 1. \Rightarrow R = 1 \Rightarrow$ Multj de conv $(-1,1)$

(18) $f(x) = e^{-x}$

$f^{(n)}(x) = (-1)^n e^{-x} \Rightarrow f^{(n)}(0) = (-1)^n$

$$f(x) = \sum_{n\geq 0} \underbrace{\frac{(-1)^n}{n!}}_{a_n} x^n$$

$$L = \lim \left|\frac{a_{n+1}}{a_n}\right| = \lim \frac{n!}{(n+1)!} = \frac{1}{n+1} \to 0$$

$R = \infty \Rightarrow M = \mathbb{R}$ mult. de conv.

(19) $f(x) = \sin^2 x \Rightarrow f'(x) = 2\sin x \cos x = \sin 2x$

$$f^{(n)}(x) = 2^{n-1} \sin\left(2x + \frac{(n-1)\pi}{2}\right) \Rightarrow f^{(n)}(0) =$$

$$= 2^{n-1} \sin \frac{(n-1)\pi}{2}, \; n \geq 1.$$

$$f(x) = \sum_{n\geq 1} \frac{1}{n!} \cdot \sin \frac{(n-1)\pi}{2} \cdot 2^{n-1} \cdot x^n =$$

$$\frac{1}{1!} \sin 0 \cdot 2^0 x^0 + \frac{1}{2!} \sin \frac{\pi}{2} \cdot 2^1 x^2 +$$

$$\frac{1}{3!} \sin \pi \cdot 2^2 x^3 + \dots = \frac{1}{2!} \cdot 2x^2 - \frac{1}{4!} 2^3 \cdot x^4 \dots$$

$$= \sum \underbrace{(-1)^{n+1} \cdot \frac{1}{2\cdot(2n)!}}_{a_n} \underbrace{(2x)^n}_{y} \Rightarrow L = \lim \left|\frac{a_{n+1}}{a_n}\right| =$$

$$\frac{1}{2(2n+2)!} \cdot \frac{2(2n)!}{1} = \frac{1}{(2n+1)(2n+2)}$$

$\Rightarrow M = \mathbb{R}$ mult de conv. $\Rightarrow L = 0 \Rightarrow R = \infty \Rightarrow y \in (-\infty, \infty)$

$\Rightarrow 2x \in (-\infty, \infty)$

(17) $f(x) = \sin 2x$

$$f^{(n)}(x) = 2^n \sin\left(2x + \frac{n\pi}{2}\right) \Rightarrow f^{(n)}(0) = 2^n \sin \frac{n\pi}{2}$$

$$\Rightarrow f(x) = \sum_{n\geq 0} \frac{2^n \sin \frac{n\pi}{2}}{n!} x^n = \sum \frac{\sin \frac{n\pi}{2}}{n!} \cdot 2^n x^n$$

$$= \frac{\sin 0}{0!}(2x)^0 + \frac{\sin \frac{\pi}{2}}{1!} \cdot (2x)^1 + \frac{\sin \pi}{2!} \cdot (2x)^2 +$$

$$\frac{\sin \frac{3\pi}{2}}{3!}(2x)^3 + \frac{\sin 2\pi}{4!}(2x)^4 + \frac{\sin \frac{5\pi}{2}}{5!}(2x)^5 + \dots =$$

$$= \frac{1}{1!}(2x)^0 + \frac{-1}{3!}(2x)^3 - \frac{1}{5!}(2x)^5 + \dots =$$

$$\sum_{n\geq 0} \frac{(-1)^n}{(2n+1)!}(2x)^{2n+1} = \sum_{n\geq 0} \underbrace{(-1)^n \cdot \frac{2^{2n+1}}{(2n+1)!}}_{a_n} x^{2n+1}$$

$$L = \lim \left| \frac{a_{n+1}}{a_n} \right| = \lim \frac{2^{2n+3}}{(2n+3)!} \cdot \frac{(2n+1)!}{2^{2n+1}} =$$

$$\frac{4}{(2n+2)(2n+3)} = 0 \Rightarrow R = \infty \Rightarrow M = \mathbb{R}$$

mult. de conv.

(15) $f(x) = \sin x$

$f^{(n)}(x) = \sin\left(x + \frac{n\pi}{2}\right) \Rightarrow f^{(n)}(0) = \sin\frac{n\pi}{2}$

$$f(x) = \sum_{n\geq 0} \frac{\sin\frac{n\pi}{2}}{n!} x^n =$$

$$\frac{\sin 0}{0!} x^0 + \frac{\sin\frac{\pi}{2}}{1!} x^1 + \frac{\sin\pi}{2!} x^2 + \frac{\sin\frac{3\pi}{2}}{3!} x^3 +$$

$$\frac{\sin 2\pi}{4!} x^4 + \dots = \frac{1}{1!} x - \frac{1}{3!} x^3 + \frac{1}{5!} x^5 + \dots =$$

$$\sum \underbrace{(-1)^n \frac{1}{(2n+1)!}}_{a_n} x^{2n+1}$$

$$L = \lim \left|\frac{a_{n+1}}{a_n}\right| = \frac{(2n+1)!}{(2n+3)!} = \frac{1}{(2n+2)(2n+3)} \to 0$$

$R = \infty \Rightarrow$ mult. de conv. $\mathbb{R}$

(16) $f(x) = \operatorname{arctg} x$

$$f' = \frac{1}{1+x^2} = (1+x^2)^{-1} = 1 + \sum_{n\geq 1} \frac{(-1)(-2)\dots(-n)}{n!} x^{2n} =$$

serie binomiale

$\alpha = -1,\ y = x^2$

$$= \sum_{n\geq 0} (-1)^n x^{2n} \Big| \int$$

$$\Rightarrow f(x) = \sum_{n\geq 0} \underbrace{(-1)^n \frac{x^{2n+1}}{2n+1}} + C. \quad (\text{pt } x=0 \Rightarrow C=0).$$

$M = (-1, 1)$

$$\Rightarrow L = \lim \left|a_{n+1}/a_n\right| = \frac{2n+1}{2n+3} \to 1 \Rightarrow R = 1 \Rightarrow$$

(13) $f(x) = e^x$

$f'(x) = e^x$

$f^{(n)}(x) = e^x \Rightarrow f^{(n)}(0) = 1.$

$$f(x) = \sum_{n\geq 0} \underbrace{\frac{1}{n!}}_{a_n} x^n$$

$$\Rightarrow L = \lim \left| \frac{a_{n+1}}{a_n} \right| = \frac{n!}{(n+1)!} = \frac{1}{n+1} \to 0.$$

$$R = \frac{1}{0} = \infty \Rightarrow$$

Multi de conv. $= \mathbb{R}$.

(14) $f(x) = \cos x$

$f'(x) = \sin x$

$f''(x) = \cos x$

$f'''(x) = -\sin x$

E in cors ! la derivate ugiale

$$\Rightarrow f^{(n)}(x) = \cos\left(x + \frac{n\pi}{2}\right).$$

$$\Rightarrow f^{(n)}(0) = \cos\frac{n\pi}{2}.$$

$$\Rightarrow f(x) = \sum_{n\geq 0} \frac{\cos\frac{n\pi}{2}}{n!} x^n = \frac{\cos 0}{0!} x^0 + \frac{\cos\frac{\pi}{2}}{1!} x +$$

$$\frac{\cos\pi}{2!} x^2 + \frac{\cos\frac{3\pi}{2}}{3!} x^3 + \frac{\cos 2\pi}{4!} x^4 + \dots$$

$$= 1 - \frac{1}{2!}x^2 + \frac{1}{4!}x^4 - \frac{1}{6}x^6 + \dots = \sum \underbrace{(-1)^n \frac{1}{(2n)!}}_{a_n} x^{2n}$$

$$L = \lim \left| \frac{a_{n+1}}{a_n} \right| = \frac{(2n)!}{(2n+2)!} = \frac{1}{(2n+1)(2n+2)} \to 0.$$

$R = \infty \Rightarrow$ Multi de conv. $= \mathbb{R}$

(20) $f(x) = \arcsin x$

$$f'(x) = \frac{1}{\sqrt{1-x^2}} = \left(1+(-x^2)\right)^{-\frac{1}{2}} = 1 + \sum_{n\geq 1} \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\cdots\left(-\frac{1}{2}-n+1\right)}{n!}$$

$$(-x^2)^n = \sum_{n\geq 0} \frac{1\cdot 3\cdot\ldots\cdot(2n+1)}{2^n\cdot n!} x^{2n}$$

(serie binomiale pt. $\alpha = -\frac{1}{2}$, $y = -x^2$.)

$$f'(x) = \sum_{n\geq 0} \frac{1\cdot 3\cdot\ldots\cdot(2n+1)}{2^n\cdot n!} x^{2n} \quad \Big| \int$$

$$f(x) = \sum \frac{1\cdot 3\cdot 5\cdot\ldots\cdot(2n-1)}{2^n\cdot n!} x^{2n+1} + C.$$

(pt. $x=0 \Rightarrow C=0$) $\Rightarrow$

$$f(x) = \sum_{n\geq 0} \underbrace{\frac{1\cdot 3\cdot 5\cdot\ldots\cdot(2n-1)}{2^n\cdot n!}}_{a_n} x^{2n+1}$$

$$L = \lim\left|\frac{a_{n+1}}{a_n}\right| = \lim \frac{1\cdot 3\cdot\ldots\cdot(2n+1)}{2^{n+1}(n+1)!} \cdot \frac{2^n\cdot n!}{1\cdot 3\cdot\ldots(2n-1)} =$$

$$= \lim \frac{(2n+1)}{2\cdot(n+1)} = 1 \Rightarrow R = 1 \Rightarrow M = (-1,1)$$

mult. de conv.

(1) (21) $f(x) = x^3 - 2x^2 - 5x - 2$; $x+4$.

Dezv. în serie de puteri ale lui $(x+4)$ și mult de conv.

grad $f = 3 \Rightarrow f^{(n)} = 0$ pt. $n \geq 4 \Rightarrow$ nu se dezv. în serie de puteri.

Det. pol. Taylor pt. $a = -4$.

$f(-4) = -64 - 2 \cdot 16 + 20 - 2 = -64 - 32 + 18 =$

$-96 + 18 = -78$

$f'(x) = 3x^2 - 4x - 5 \Rightarrow f'(-4) = 48 + 16 - 5 = 48 + 11 = 59$

$f''(x) = 6x - 4 \Rightarrow f''(-4) = -24 - 4 = -28$

$f'''(x) = 6 \Rightarrow f'''(-4) = 6.$

$$\Rightarrow f(x) = \sum_{n=0}^{3} \frac{f^{(k)}(-4)}{k!} (x+4)^k =$$

$$-78 + 59(x+4) - \frac{28}{2!}(x+4)^2 + \frac{6}{3!}(x+4)^3 =$$

$$-78 + 59(x+4) - 14(x+4)^2 + (x+4)^3.$$

(22) $f(x) = \ln x \quad ; (x-1)$

$a = 1$

$f'(x) = \frac{1}{x} \Rightarrow f^{(n)}(x) = \frac{(-1)^{n-1}(n-1)!}{x^n}$

$f^{(n)}(1) = (-1)^{n-1}(n-1)!$

$$f(x) = \underbrace{f(1)}_{=0} + \sum_{n\geq 1} \frac{(-1)^{n-1}}{n!}(x-1)^n \cdot \cancel{(n-1)!}$$

$$f(x) = \sum_{n\geq 1} \underbrace{\frac{(-1)^{n-1}}{n}}_{a_n} \cdot \underbrace{(x-1)^n}_{y}$$

$L = \lim \left|\frac{a_{n+1}}{a_n}\right| = \lim \frac{n}{n+1} = 1 \Rightarrow R = 1. \Rightarrow M_y = (-1, 1)$

$-1 < x-1 < 1 \mid +1 \Rightarrow 0 < x < 2 \Rightarrow M_x = (0, 2)$

mult. de conv.

(23) $f(x) = \frac{1}{x^2} \quad ; (x+1)$

$f'(x) = -\frac{2}{x^3} \Rightarrow f^{(n)}(x) = (-1)^n (n+1)! \cdot x^{-(n+2)}$

$f''(x) = +\frac{2 \cdot 3}{x^4}$

$f^{(n)}(-1) = (n+1)!$

mult. de conv.

$M_x = (-2; 0)$

$$f(x) = \sum_{n\geq 0} \frac{(n+1)!}{n!}(x+1)^n = \sum_{n\geq 0} \underbrace{(n+1)}_{a_n} \underbrace{(x+1)^n}_{y}$$

$L = \lim \left|\frac{a_{n+1}}{a_n}\right| = \frac{n+1}{n} \to 1 \Rightarrow R = 1. \Rightarrow M_y = (-1, 1)$

$-1 < x+1 < 1 \mid -1$

$-2 < x < 0$

(1) (24) $f(x) = \dfrac{1}{x^2+3x+2}$; $(x+4)$

$\Downarrow$

$a=-4.$

$$f(x) = \frac{1}{(x+1)(x+2)} = \frac{A}{x+1} + \frac{B}{x+2}$$

$$A(x+2)+B(x+1)=1 \quad \Rightarrow \quad \begin{array}{l} A+B=0 \\ 2A+B=1 \\ \hline A=1 \ ; \ B=-1. \end{array}$$

$$f(x) = \frac{1}{x+1} - \frac{1}{x+2}$$

$$f^{(n)}(x) = \frac{(-1)^n n!}{(x+1)^{n+1}} - \frac{(-1)^n \cdot n!}{(x+2)^{n+1}}$$

$$f^{(n)}(-4) = \frac{(-1)^n n!}{(-3)^{n+1}} - \frac{(-1)^n \cdot n!}{(-2)^{n+1}}$$

$$f(x) = \sum_{n\geq 0} \frac{(-1)^n \cancel{n!}}{\cancel{n!}\,(-3)^{n+1}} \cdot (x+4)^n - \sum \frac{(-1)^n \cancel{n!}}{\cancel{n!}\,(-2)^{n+1}} (x+4)^n$$

$$= \sum \frac{(-1)^n}{(+3)^{n+1} \cdot (-1)^{n+1}} \cdot (x+4)^n - \sum \frac{(-1)^n}{(-1)^{n+1} \cdot 2^{n+1}} (x+4)^n =$$

$$= -\sum_{n\geq 0} \left(\frac{1}{3}\right)^{n+1} \cdot (x+4)^n + \sum \left(\frac{1}{2}\right)^{n+1} (x+4)^n =$$

$$= \sum \underbrace{\left[\left(\frac{1}{2}\right)^{n+1} - \left(\frac{1}{3}\right)^{n+1}\right]}_{a_n} \underbrace{(x+4)}_{y}{}^n =$$

$$L = \lim \left| \frac{a_{n+1}}{a_n} \right| = \lim \frac{\left(\frac{1}{2}\right)^{n+1} - \left(\frac{1}{3}\right)^{n+1}}{\left(\frac{1}{2}\right)^{n} - \left(\frac{1}{3}\right)^{n}} =$$

$$\frac{\left(\frac{1}{2}\right)^{n+1}\left[1-\left(\frac{2}{3}\right)^{n+1}\right]}{\left(\frac{1}{2}\right)^{n}\left[1-\left(\frac{2}{3}\right)^{n}\right]} = \frac{1}{2} \Rightarrow R = 2 \Rightarrow M_y = (-2;2)$$

$$-2 < x+4 < 2 \quad | -4 \Rightarrow -6 < x < -2$$

$$M_x = (-6;-2).$$

(25) $f(x) = \sqrt{x}$, $x+2$

$a = -2$

$$f' = \frac{1}{2\sqrt{x}}$$

Dom. de def pt. f este $[0,\infty)$.

Cum $a = -2 \notin [0,\infty)$

$\Rightarrow$ nu se poate dezvolta într-o vecinătate a lui $a = -2$

(26) $f(x) = \cos x$, $\left(x - \frac{\pi}{2}\right)$

$\Downarrow$ $\qquad\qquad\qquad\qquad \Downarrow$

$\qquad\qquad\qquad\qquad a = \frac{\pi}{2}$

$$f^{(n)}(x) = \cos\left(x + \frac{n\pi}{2}\right) \Rightarrow f^{(n)}\left(\frac{\pi}{2}\right) = \cos\frac{(n+1)\pi}{2}.$$

$$f(x) = \sum_{n\geq 0} \frac{\cos\frac{(n+1)\pi}{2}}{n!}\left(x - \frac{\pi}{2}\right)^n =$$

$$= \frac{\cos\frac{\pi}{2}}{0!}\left(x-\frac{\pi}{2}\right)^0 + \frac{\cos\pi}{1!}\left(x-\frac{\pi}{2}\right)^1 + \frac{\cos\frac{3\pi}{2}}{2!}\left(x-\frac{\pi}{2}\right)^2 + \cdots$$

$$-\frac{1}{1!}\left(x-\frac{\pi}{2}\right)^1 + \frac{1}{3!}\left(x-\frac{\pi}{2}\right)^3 + \cdots$$

$=$

$$= \sum_{n\geq 0} \underbrace{\frac{(-1)^n}{(2n+1)!}}_{a_n} \underbrace{\left(x - \frac{\pi}{2}\right)}_{y}^{2n+1}.$$

$$L = \lim\left|\frac{a_{n+1}}{a_n}\right| = \lim \frac{(2n+1)!}{(2n+3)!} = \frac{1}{(2n+2)(2n+3)} \to 0.$$

$\Rightarrow R = \infty \Rightarrow M_y = \mathbb{R}$

$-\infty < x - \frac{\pi}{2} < \infty \Rightarrow x \in \mathbb{R}$

$M_x = \mathbb{R}$.

mult. de conv.

(28) sin 32

$$\sin x = \sum_{n\geq 0} (-1)^n \frac{x^{2n+1}}{(2n+1)!} = \frac{x}{1!} - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots$$

$x = 32^\circ = \frac{8\pi}{45} \simeq 0{,}5 \Rightarrow$ serie alternantă

C. Leibniz $\quad a_n = \frac{1}{(2n+1)!} \cdot \left(\frac{8\pi}{45}\right)^{2n+1} \to 0 \Rightarrow \sum (-1)^n a_n$ conv.

$\Rightarrow$ are suma S și det. n minim pt. care

$$|S - S_n| < a_{n+1} \leq \frac{1}{10^4} = 0{,}0001$$

$n=0 \Rightarrow a_0 = \left(\frac{8\pi}{45}\right)^1 = 0{,}55 < 0{,}0001$ (F)

$n=1 \Rightarrow a_1 = \frac{1}{6} \cdot \left(\frac{8\pi}{45}\right)^3 = 0{,}0277$ (F)

$n=2 \Rightarrow a_2 = \frac{1}{5!} \cdot \left(\frac{8\pi}{45}\right)^5 = 0{,}0004$ (F)

$n=3 \Rightarrow a_3 = \frac{1}{7!} \cdot \left(\quad\right)^7 = 0{,}000003$ (A)

$n+1 = 3 \Rightarrow n = 2.$

$$-a_3 < S - S_2 < a_3$$

$$-a_3 + S_2 < S < a_3 + S_2$$

$$S_2 = a_0 - a_1 + a_2 = 0{,}55 - 0{,}0277 + 0{,}0004$$
$$= 0{,}5227$$

$$-0{,}000003 + 0{,}5227 < S < 0{,}000003 + 0{,}5227 = \underline{0{,}522703}$$

$\underline{0{,}522697}$