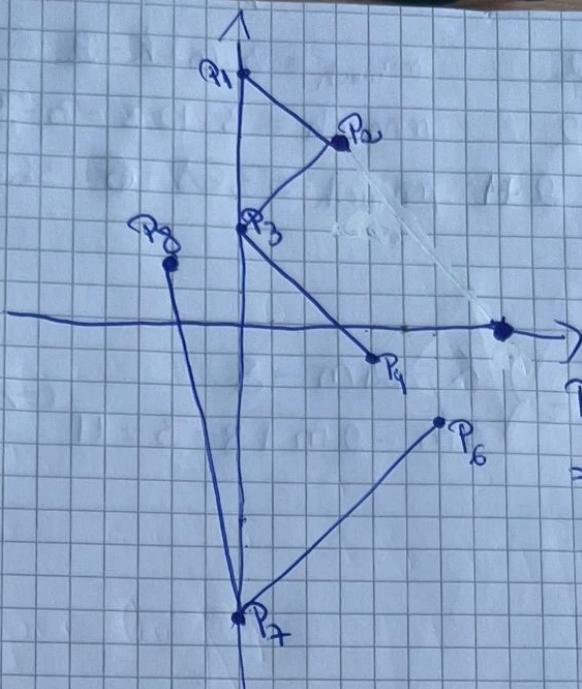


3.8



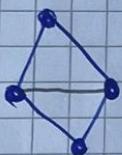
$$P_S = (5, 2)$$

$$P_g = (\beta + 1, 4)$$

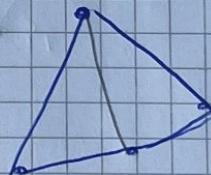
$P \in L \subset (-1, -3) \Rightarrow \beta$
 $\Rightarrow Y - \text{monoton}$

3.9

Caz I



Caz II A



Caz II B



4. Triangularea multimilor de puncte

$$A_0 = (10, 0)$$

$$A_1 = (11, 0)$$

:

$$A = (60, 0)$$

↓
51 elem

$$B_0 = (0, 30)$$

$$B_1 = (0, 31)$$

:

$$B = (0, 70)$$

↓
41 elem

$$C_0 = (0, 0)$$

$$C_1 = (-1, -1)$$

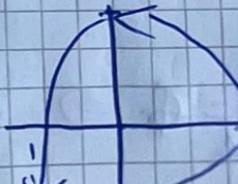
:

$$C = (-30, -30)$$

↓
31 elem

$$51 + 41 + 31 = 123 \text{ puncte} \Rightarrow m = 123$$

3 pct pe frontiera $\Rightarrow K = 3$



$$m_n = 2 \cdot m - K - 2$$

$$m_{\text{much}} = 3 \cdot m - K - 3$$

$$\begin{array}{r} 123 \\ 3 \\ \hline 369 \end{array}$$

$$m_{2n} = 2 \cdot 123 - 3 - 2$$

$$m_{\text{much}} = 3 \cdot 123 - 3 - 3$$

$$m_{2n+2} = 246 - 5 = 241$$

$$m_{\text{much}} = 369 - 6 = 363$$

4.2

$$\begin{cases} 2m - k - 2 = 6 \\ 3m - k - 3 = 11 \end{cases} \Leftrightarrow \begin{cases} k = 2m - 8 \\ 3m - 2m + 8 - 3 = 11 \end{cases} \Leftrightarrow \begin{cases} k = 2m - 8 \\ m + 5 = 11 \end{cases}$$

$$\Rightarrow \begin{cases} m = 6 \\ k = 12 - 8 \end{cases} \Leftrightarrow \begin{cases} m = 6 \\ k = 4 \end{cases}$$

4.3



I $\alpha - 1 \leq 1 \Rightarrow \alpha \in (-\infty, 2] \Rightarrow P_6$ se află în exterior sau pe laturi $\Rightarrow m = 6, k = 6 \Rightarrow$

$$\begin{cases} 4 \Delta \\ 9 \text{ muchii} \end{cases}$$

II $\alpha - 1 > 1$ și $\alpha - 1 \leq 7 \Rightarrow P_6$ se află în interiorul pentagonului $\Rightarrow m = 6, k = 5 \Rightarrow \begin{cases} 5 \Delta \\ 10 \text{ muchii} \end{cases}$

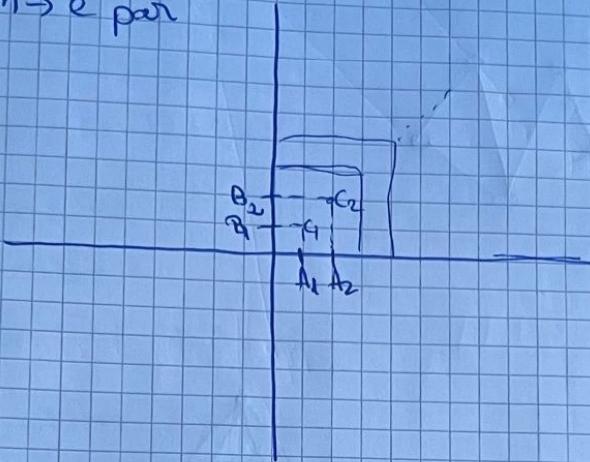
III $\alpha - 1 = 7 \Rightarrow \alpha \in \{7\} \Rightarrow P_6$ coincide cu $P_3 \Rightarrow m = 3, k = 5$

$$\Rightarrow \begin{cases} 3 \Delta \\ 7 \text{ muchii} \end{cases}$$

4537185

$N \geq -1 > 4$, $\Delta \in \{8, \infty\} \Rightarrow P_8$ este situat în ext \Rightarrow
 $m=6$ $k=6 \Rightarrow \begin{cases} 4 \\ 4 \\ 2 \end{cases}$ mediu

4.4 $m \rightarrow e$ par



$i=0 \dots m \Rightarrow i \leq m \Rightarrow \cancel{D=(m-i, i)} \dots$ cadrant

$$D = (0, m) \quad (i=m) = B(0, m)$$

$$D = (m, 0) \quad (i=0) = A(m, 0)$$

m par $\Rightarrow D = C$

$$N = 4m - 4$$

$$K = 3m - 2$$

$$\Delta = 2m - K - 2$$

$$\Delta = 2 \cdot (4m - 4) - 3m + 2 - 2$$

$$= 8m - 8 - 3m$$

$$= 5m - 8$$

$$\text{much} = 3m - K - 3$$

~~$$\text{much} = 3m -$$~~

$$\text{much} = 3(4m - 4) - 3m + 2 - 3$$

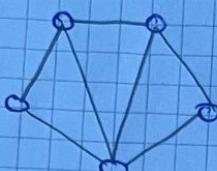
$$= 12m - 12 - 3m - 1$$

$$= 9m - 13$$

4.5

$$\begin{cases} 2m - k - 2 = 3 \\ 3m - k - 3 = 7 \end{cases} \Leftrightarrow \begin{cases} k = 2m - 5 \\ 3m - 2m + 5 - 3 - 7 = 0 \end{cases} \Leftrightarrow \begin{cases} k = 2m - 5 \\ m = 5 \end{cases}$$

$\Leftrightarrow \begin{cases} m = 5 \\ k = 5 \end{cases}$



4.6

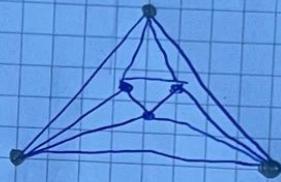
$$m = 6$$

$$mk = 12$$

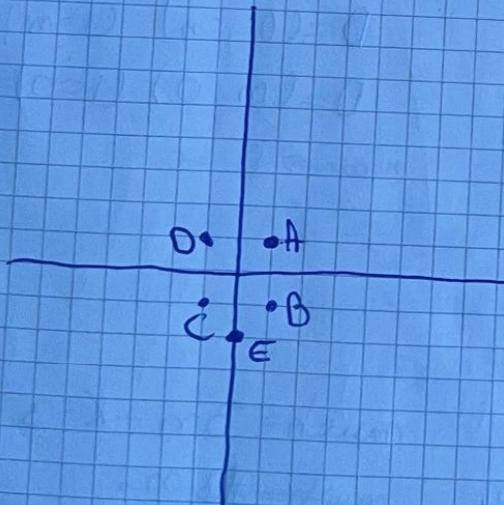
$$3m - k - 3 = 12$$

$$3 \cdot 6 - k = 15$$

$$k = 3$$



4.7



$$\text{I } \lambda \geq 1 \Rightarrow \begin{matrix} m=6 \\ k=6 \end{matrix}$$

$$\text{II } \lambda \in (-1, 1) \Rightarrow \begin{matrix} m=6 \\ k=5 \end{matrix}$$

$$\text{III } \lambda = -2 \Rightarrow \begin{matrix} m=5 \\ k=5 \end{matrix}$$

$$\text{IV } \lambda < -2 \Rightarrow \begin{matrix} m=6 \\ k=5 \end{matrix}$$

4.8 a) $3m - k - 3 = 6$

$$3m - k = 9$$

$$k = 3m - 9$$



Fie $m=4 \Rightarrow k = 3 \cdot 4 - 9$

$$k = 12 - 9 \Rightarrow k = 3$$

$$\Delta = 2 \cdot m - k - 2$$

$$\Delta = 2 \cdot 4 - 3 - 2$$

$$\Delta = 8 - 5 \Rightarrow \Delta = 3$$

b) Noi stim $k = 3m - 9$

$$m \geq 3 \quad I \quad m=3 \Rightarrow k=0 \quad N.O. \quad m \leq \frac{m(m-1)}{2}$$

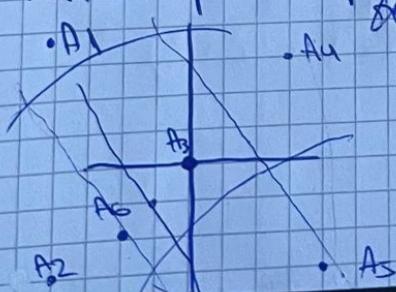
$$II \quad m=4 \Rightarrow k=3 \quad O.K. \quad 12 \leq m(m-1)$$

$$III \quad m=5 \Rightarrow N.O. \text{ mai convi}$$

Multimele admisite.

5. Diagramme Voronoi. Triangulare Delaunay

5.1 Nr. de somidle ale unui diagram Voronoi este egal cu
nr. de pct de pe frontiera acoperirii convexe ~~de date reale~~



\Rightarrow 5 muchii, fara A3 sunt 4 muchii

5.e Cum pot mi securiza calinarea \Rightarrow d. Veronoi

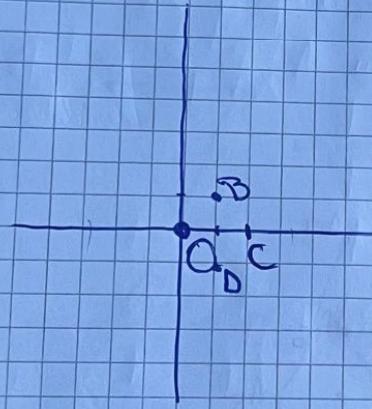
TAA

80

4 337 185 751127

5.3

$$A = (\lambda, 0)$$



$\lambda < 0 \Rightarrow A \in \text{im glara } OBCD$
 $\Rightarrow m=4$ (pt frontiera A, B, C, D)

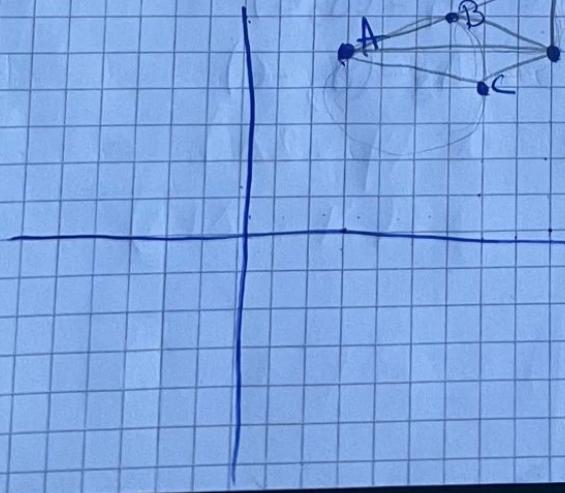
$\lambda = 0 \Rightarrow A = O \Rightarrow m=1$

$\lambda > 0 \Rightarrow A \in \text{ext lui } OBCD$

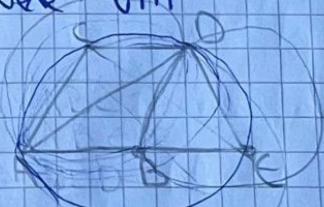
$m=4$ (pt frontiera O, B, D, A)

Nr mulțimi de tip semidreapta = nr de puncte de pe frontiera

5.4



acoperire convexă (m)



$\Delta ABD, \Delta ACD$ nu pot
 participa la realizarea
 acoperirii de lungaj

5.5

$i=0 \dots s$

$$A_0 = (1, 1)$$

$$A_1 = (2, 2)$$

$$A_2 = (3, 3)$$

$$\vdots$$

$$A_s = (s, s)$$

$$B_0 = (0, 0)$$

$$B_1 = (-1, 1)$$

$$\vdots$$

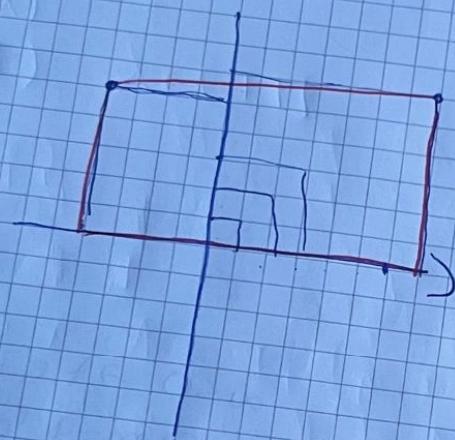
$$B_s = (-s, s)$$

$$C_0 = (0, 0)$$

$$C_1 = (0, 1)$$

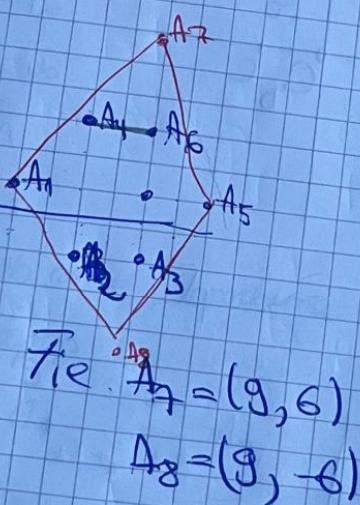
$$\vdots$$

$$C_s = (0, s)$$



$\Rightarrow 4$ semidivapte

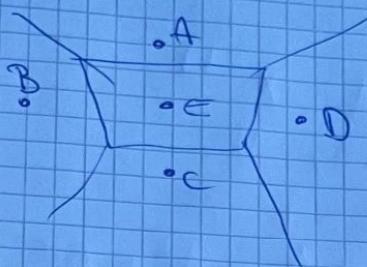
5.6



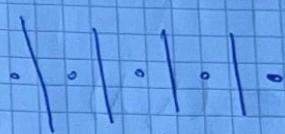
b)

5.7

a)



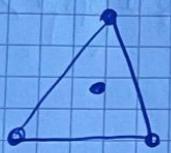
\Rightarrow 4 muchii de tip semidreptă

b) / Aceste n dresele de mai sus si P 

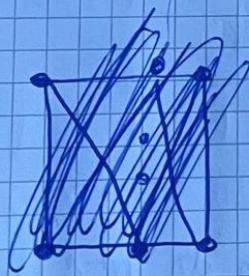
 4 muchii
de tip scără

5.8

Nr semidrepte = Nr puncte acoperite convexa



\Rightarrow 3 semidrepte



\Rightarrow 4 semidrepte

b)

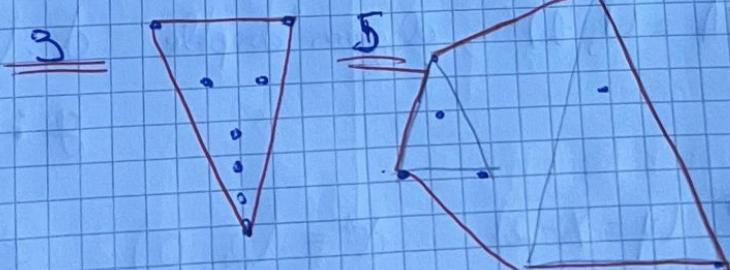
4 (a)

6

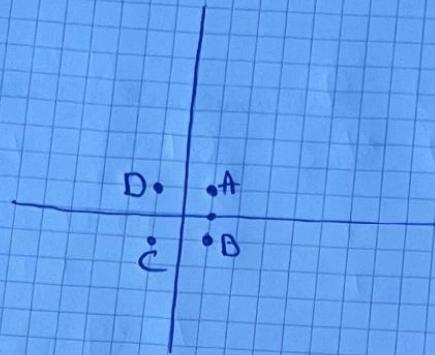


3

5



5.9



$$E = (\lambda, \lambda)$$

$$F = (1, \mu) \quad \lambda \in [-1, 1]$$

$$\text{Pt } \mu \in [-1, 1]$$

$$\text{Pt } \mu = -1 \Rightarrow F = B \Rightarrow 4 \text{ pct pe acop } \nabla \lambda$$

$$\mu = 1 \Rightarrow F = A \Rightarrow 4 \text{ pct pe acop } \nabla \lambda$$

$\mu \neq -1, 1$ pe tota pe acord cu A, B \Rightarrow 5 pct pe acop $\nabla \lambda$

$$\text{Pt } \lambda = -1 \Rightarrow E = C$$

$$\lambda = 1 \Rightarrow E = A$$

$\lambda \neq -1, 1$ \Rightarrow se află în interior

6. Dualitate

Pt un punct $p = (p_x, p_y)$ definim dreptă: $(y = p_x x - p_y)$
iar pt o dreaptă neverticală $d: (y = mx + b)$ definim punctul $d^*: (m, -b)$
(dualul lui d)

$$6.1 A = (1, 2), \text{ dreapta } d: (y = 2x)$$

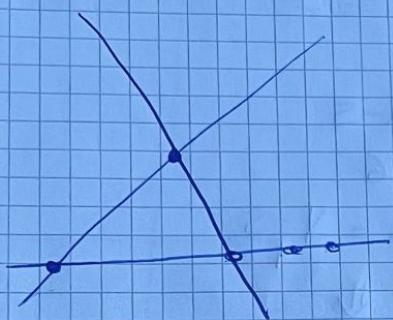
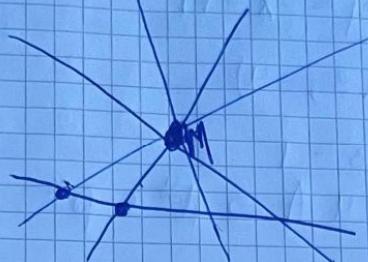
$$g: (y = -x + 3)$$

$$A^*: (y = x - 2) \quad d^*: (2, 0) \quad g^*: (-1, -3)$$

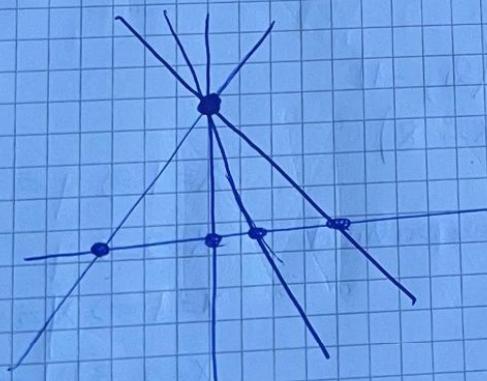
BS 751127

$$\frac{x-2}{1-2} = \frac{y-0}{3-0} \Rightarrow \frac{x-2}{-1} = \frac{y}{3} \Rightarrow y = x - 2$$

ii)



(configuratia completa (catena))



"Fie patru drepte care trec prin un acelasi punct M. Se alege doua dimpre de pe fiecare dimpre aceste doua drepte se considera cate un pct diferit de M si se considera dreapta determinata de cele doua pct"

Fie 4 puncte situate pe o aceeași dreaptă d. Se alege două dimpre de , sau fiecare dimpre aceste două puncte se consideră că e o dreaptă diferență de d și se consideră pct de intersecție al acestor două drepte

Fie 4 puncte drepte care ~~trec~~
~~tot~~ trec prin un acelasi punct.
Pe fiecare dreaptă se alege
căte un pct diferit de pct
comun, astfel că aceste patru
puncte nu sunt coliniare și
se consideră dreapta determinată
de aceste puncte .

TALK

6.2

$$p = (-1, 1) \Rightarrow p^*: (y = -x - 1)$$

$$d: (y = 3x + 4) \Rightarrow d^* = (3, -4)$$

$$l = 3 \cdot (-1) + 4 \quad -4 = -3 - 1$$

$$l = -3 + 4 \quad -4 = -4 \wedge \Rightarrow d^* \in p^*$$

$$l = l \wedge \Rightarrow p \in d$$

6.3

$$p_1 = (2, 5) \Rightarrow (p_1^*): y = 2x - 5$$

$$p_2 = (1, 6) \Rightarrow (p_2^*): y = x - 6$$

$$(p_1 p_2): \frac{x-2}{1-2} = \frac{y-5}{6-5} \Rightarrow \frac{x-2}{-1} = \frac{y-5}{1}$$



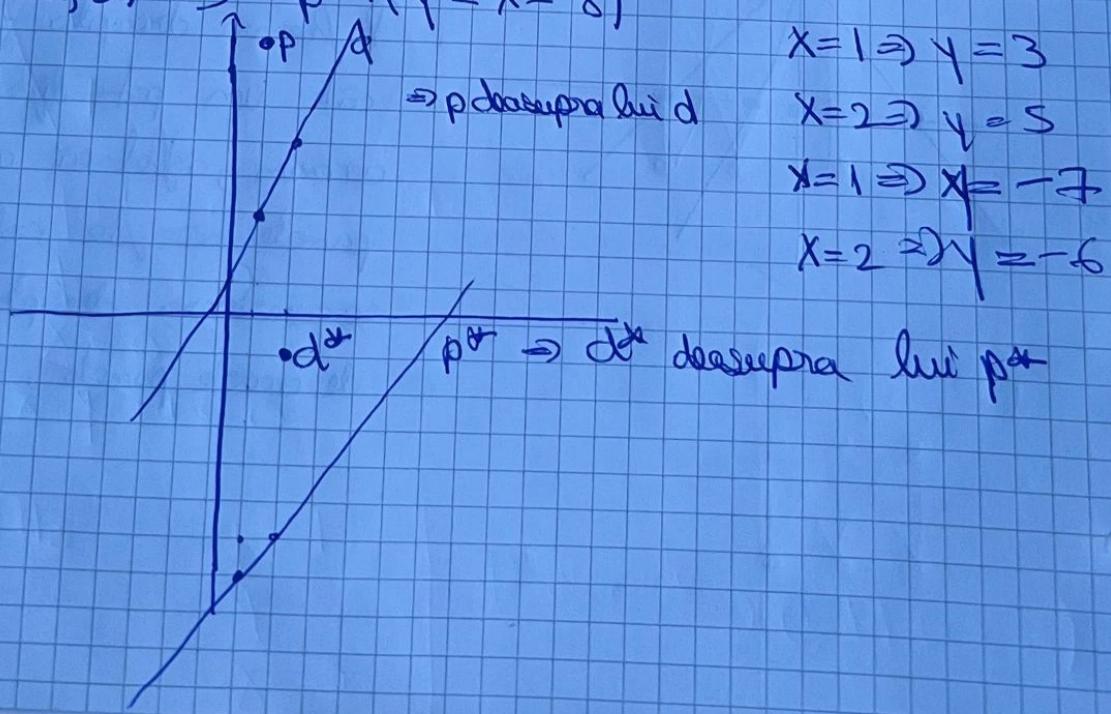
$$x - 2 = -y + 5$$

$$(p_1 p_2): y = -x + 7 \Rightarrow p_1 p_2^* = (-1, 7)$$

6.4

$$(d) - d: (y = 2x + 1) \Rightarrow d^* = (2, -1)$$

$$p = (1, 8) \Rightarrow p^*: (y = x - 8)$$



$$x=1 \Rightarrow y=3$$

$$x=2 \Rightarrow y=5$$

$$x=1 \Rightarrow x=-7$$

$$x=2 \Rightarrow y=-6$$

6.5 i) $d: (y = 2x - 3) \Rightarrow d^* = (2, 3)$

~~Fie~~ $P = x=1 \Rightarrow y=-1 \Rightarrow P(1, -1) \Rightarrow P^*: (y = x+1)$

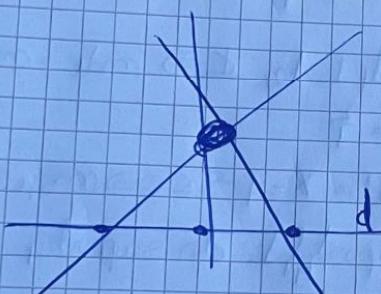
~~Q =~~ $x=2 \Rightarrow y=1 \Rightarrow Q(2, 1) \Rightarrow Q^*: (y = 2x-1)$

$x+1 = 2x-1$

$1+1 = 2x-x$

$x=2 \Rightarrow y=3 \Rightarrow \{d^*\} = P^* \cap Q^*$

ii)



Fie 3 drepte care trec prin acelasi pct M; pe fiecare dreapta se ia cate un pct (diferiti de M), astfel ca aceste pct sa fie coliniare

Fie 3 puncte situate pe acelasi dreapta d; dim se cere dimite aceste puncte se considera ~~ce~~ dreptele care se intersecteaza trec prin acelasi pct

6.6

$d: (x = y-1) \Rightarrow d: (y = x+1) \Rightarrow d^* = (1, -1)$

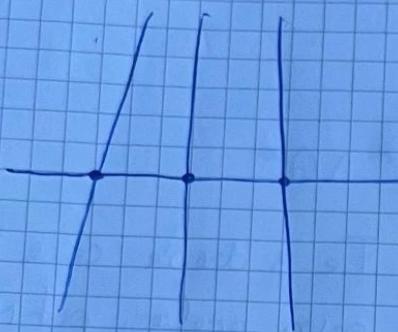
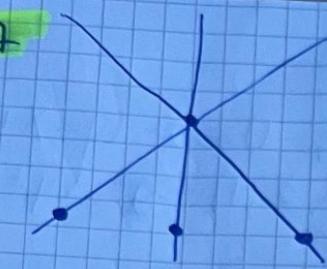
$x=1 \Rightarrow y=2 \Rightarrow P(1, 2) \Rightarrow P^*: (y = x-2)$

$x=2 \Rightarrow y=3 \Rightarrow Q(2, 3) \Rightarrow Q^*: (y = 2x-3)$

$x-2 = 2x-3$

$x=1 \Rightarrow y=-1 \quad (d^*) \Rightarrow d^* = P^* \cap Q^*$

6.7



Doi drepte care fac prim acelasi punct; pe aceeasi drepte se alege un punct, diferit de punctul comun al celor doi drepte

// mai repeta dualitatea, im special abs fundamentala

7. Intersecții de semiplane. Elemente de programare limitată

7.1 a)

$$H \cap H'$$

$$\begin{aligned} H: x+y-3 &\leq 0 \Rightarrow m_H = -1 \text{ și } d_1 \perp d_2 \quad (\Rightarrow m_{d_1} \cdot m_{d_2} = -1) \\ H': -2x+y+1 &\leq 0 \Rightarrow m_{H'} = 2 \end{aligned}$$

$$x+y-3 = -2x+y+1$$

$$3x = 4$$

$$3x-4 = x-y-5$$

$$2x+y+1=0$$

$$y = -2x-1$$

$$\text{Fie } H'' = x-y-5 \leq 0 \Rightarrow m_{H''} = 1$$

dr suport_{H'} \perp dr suport_{H''}

b) $H_1: -y+1 \leq 0 \quad \text{de } \mathbb{R}$

$$H_2: y-5 \leq 0$$

$$H_3: -x \leq 0$$

$$H_4: x-y+a \leq 0$$

$$H_1 \cap H_2 \cap H_3 \Rightarrow \begin{cases} y = 1 \\ y = 5 \\ x = 0 \end{cases}$$

y relevantă $\rightarrow (0, 1) \text{ și } (0, 5)$