Supplementary Material

1 Engineering Optimization Problems

1.1 CBD (Cantilever Beam Design)

Minimize:

$$f(X) = 0.0624 \sum_{i=1}^{5} x_i$$

Subject to:

$$g_1(X) = \frac{61}{x_1^3} + \frac{37}{x_2^3} + \frac{19}{x_3^3} + \frac{7}{x_4^3} + \frac{1}{x_5^3} - 1 \leq 0$$

Variable bounds:

$$x_1, x_2, x_3, x_4, x_5 \in [0.01, 100]$$

1.2 CBHD (Corrugated Bulkhead Design)

Minimize:

$$f(X) = \frac{5.885(x_1 + x_3)x_4}{x_1 + \sqrt{|x_3^2 - x_2^2|}}$$

Subject to:

$$g_1(X) = -x_2 x_4 (2x_1/5 + x_3/6) + 8.94(x_1 + \sqrt{|x_3^2 - x_2^2|}) \leq 0$$
$$g_2(X) = -x_2^2 x_4 (x_1/5 + x_3/12) + 2.2(8.94(x_1 + \sqrt{|x_3^2 - x_2^2|}))^{4/3} \leq 0$$
$$g_3(X) = 0.0156x_1 - x_4 + 0.15 \leq 0$$
$$g_4(X) = 0.0156x_3 - x_4 + 0.15 \leq 0$$
$$g_5(X) = -x_4 + 1.05 \leq 0$$
$$g_6(X) = x_2 - x_3 \leq 0$$

Variable bounds:

$$x_1, x_2, x_3 \in [0, 100]$$
$$x_4 \in [0, 5]$$

1.3 HNO (Himmelblau's Nonlinear Optimization)

Minimize:

$$f(X) = 5.3578547x_3^2 + 0.8356891x_1x_5 + 37.293239x_1 - 40792.141$$

Subject to:

$$g_1(X) = 85.334407 + 0.0056858x_2x_5 + 0.0006262x_1x_4 - 0.0022053x_3x_5 - 92 \leq 0$$
$$g_2(X) = -85.334407 - 0.0056858x_2x_5 - 0.0006262x_1x_4 + 0.0022053x_3x_5 \leq 0$$
$$g_3(X) = 80.51249 + 0.0071317x_2x_5 + 0.0029955x_1x_2 + 0.0021813x_3^2 - 110 \leq 0$$
$$g_4(X) = -80.51249 - 0.0071317x_2x_5 - 0.0029955x_1x_2 - 0.0021813x_3^2 + 90 \leq 0$$
$$g_5(X) = 9.300961 + 0.0047026x_3x_5 + 0.0012547x_1x_3 + 0.0019085x_3x_4 - 25 \leq 0$$
$$g_6(X) = -9.300961 - 0.0047026x_3x_5 - 0.0012547x_1x_3 - 0.0019085x_3x_4 + 20 \leq 0$$

Variable bounds:

$$x_1 \in [78, 102]$$
$$x_2 \in [33, 45]$$
$$x_3, x_4, x_5 \in [27, 45]$$

1.4 IBD (I-Beam Design)

Minimize:

$$f(X) = \frac{5000}{x_3(x_2 - 2x_4)^3/12 + x_1x_4^3/6 + 2x_1x_4((x_2 - x_4)/2)^2}$$

Subject to:

$$g_1(X) = 2x_1x_4 + x_3(x_2 - 2x_4) - 300 \leq 0$$
$$g_2(X) = \frac{180000x_2}{x_3(x_2 - 2x_4)^3 + 2x_1x_4(4x_4^2 + 3x_2(x_2 - 2x_4))} + \frac{15000x_1}{x_3^3(x_2 - 2x_4) + 2x_3x_4^3} - 16 \leq 0$$

Variable bounds:

$$x_1 \in [10, 50]$$
$$x_2 \in [10, 80]$$
$$x_3, x_4 \in [0.9, 5]$$

1.5 PVD (Pressure Vessel Design)

Minimize:

$$f(X) = 0.6224x_1x_3x_4 + 1.7781x_2x_3^2 + 3.1661x_1^2x_4 + 19.84x_1^2x_3$$

Subject to:

$$
\begin{aligned}
g_1(X) &= -x_1 + 0.0193x_3 \leq 0 \\
g_2(X) &= -x_2 + 0.00954x_3 \leq 0 \\
g_3(X) &= -\pi x_3^2 x_4 - \frac{4}{3}\pi x_3^3 + 1296000 \leq 0 \\
g_4(X) &= x_4 - 240 \leq 0
\end{aligned}
$$

Variable bounds:

$$
\begin{aligned}
x_1, x_2 &\in \{1 \cdot 0.0625, 2 \cdot 0.0625, \ldots 99 \cdot 0.0625\} \\
x_3, x_4 &\in [10, 200]
\end{aligned}
$$

1.6 SRD (Speed Reducer Design)

Minimize:

$$
\begin{aligned}
f(X) = 0.7854x_1x_2^2(3.3333x_3^2 + 14.9334x_3 - 43.0934) - 1.508x_1(x_6^2 + x_7^2) + \\
7.4777(x_6^3 + x_7^3) + 0.7854(x_4x_6^2 + x_5x_7^2)
\end{aligned}
$$

Subject to:

$$
\begin{aligned}
g_1(X) &= \frac{27}{x_1x_2^2x_3} - 1 \leq 0 \\
g_2(X) &= \frac{397.5}{x_1x_2^2x_3^2} - 1 \leq 0 \\
g_3(X) &= \frac{1.93x_4^3}{x_2x_3x_6^4} - 1 \leq 0 \\
g_4(X) &= \frac{1.93x_5^3}{x_2x_3x_7^4} - 1 \leq 0 \\
g_5(X) &= \frac{\sqrt{(745x_4/(x_2x_3))^2 + 16.9 \times 10^6}}{110x_6^3} - 1 \leq 0 \\
g_6(X) &= \frac{\sqrt{(745x_5/(x_2x_3))^2 + 157.5 \times 10^6}}{85x_7^3} - 1 \leq 0 \\
g_7(X) &= \frac{x_2x_3}{40} - 1 \leq 0 \\
g_8(X) &= \frac{5x_2}{x_1} - 1 \leq 0 \\
g_9(X) &= \frac{x_1}{12x_2} - 1 \leq 0 \\
g_{10}(X) &= \frac{1.5x_6 + 1.9}{x_4} - 1 \leq 0 \\
g_{11}(X) &= \frac{1.1x_7 + 1.9}{x_5} - 1 \leq 0
\end{aligned}
$$

Variable bounds:

$$\begin{aligned} x_1 &\in [2.6, 3.6] \\ x_2 &\in [0.7, 0.8] \\ x_3 &\in \{17, 18, \ldots, 29\} \\ x_4, x_5 &\in [7.3, 8.3] \\ x_6 &\in [2.9, 3.9] \\ x_7 &\in [5.0, 5.5] \end{aligned}$$

1.7 TBTD (Three Bar Truss Design)

Minimize:

$$f(X) = (2\sqrt{2}x_1 + x_2) \times 100$$

Subject to:

$$\begin{aligned} g_1(X) &= \frac{\sqrt{2}x_1 + x_2}{\sqrt{2}x_1^2 + 2x_1x_2} \times 2 - 2 \leq 0 \\ g_2(X) &= \frac{x_2}{\sqrt{2}x_1^2 + 2x_1x_2} \times 2 - 2 \leq 0 \\ g_3(X) &= \frac{1}{x_1 + \sqrt{2}x_2} \times 2 - 2 \leq 0 \end{aligned}$$

Variable bounds:

$$x_1, x_2 \in [0, 1]$$

1.8 TCD (Tubular Column Design)

Minimize:

$$f(X) = 9.8x_1x_2 + 2x_1$$

Subject to:

$$\begin{aligned} g_1(X) &= \frac{P}{\pi x_1 x_2 \sigma_y} - 1 \leq 0 \\ g_2(X) &= \frac{8PL^2}{\pi^3 E x_1 x_2 (x_1^2 + x_2^2)} - 1 \leq 0 \\ g_3(X) &= \frac{2}{x_1} - 1 \leq 0 \\ g_4(X) &= \frac{x_1}{14} - 1 \leq 0 \\ g_5(X) &= \frac{0.2}{x_2} - 1 \leq 0 \\ g_6(X) &= \frac{x_2}{0.8} - 1 \leq 0 \end{aligned}$$

where $P = 2500$ kgf, $\sigma_y = 500$ kgf/cm^2, $L = 250$ cm, $E = 8.5 \times 10^5$ kgf/cm^2
Variable bounds:

$$x_1 \in [2, 14]$$
$$x_2 \in [0.2, 0.8]$$

1.9 TCSD (Tension/Compression Spring Design)

Minimize:

$$f(X) = x_1^2 x_2 (x_3 + 2)$$

Subject to:

$$g_1(X) = 1 - \frac{x_2^3 x_3}{71785 x_1^4} \leq 0$$
$$g_2(X) = \frac{4x_2^2 - x_1 x_2}{12566(x_2 x_1^3 - x_1^4)} + \frac{1}{5108 x_1^2} - 1 \leq 0$$
$$g_3(X) = 1 - \frac{140.45 x_1}{x_2^2 x_3} \leq 0$$
$$g_4(X) = \frac{x_1 + x_2}{1.5} - 1 \leq 0$$

Variable bounds:

$$x_1 \in [0.05, 2.00]$$
$$x_2 \in [0.25, 1.30]$$
$$x_3 \in [2.00, 15.0]$$

2 Feasible Domain

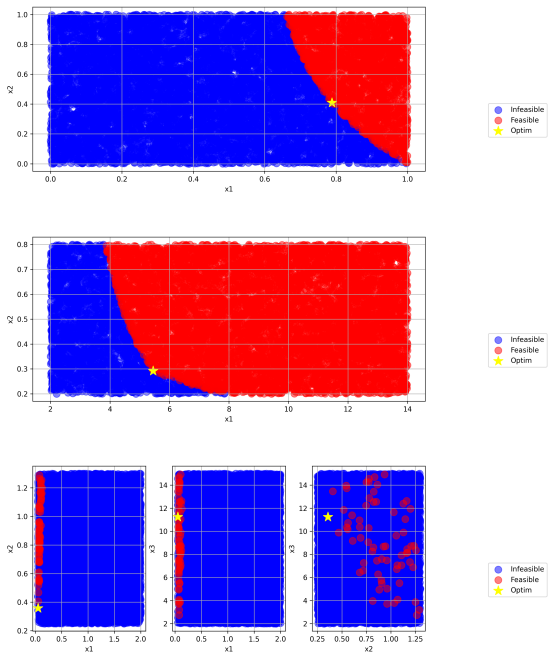

Figure 1: Feasible domain estimation for TBTD, Feasible proportion: 21.66%; Feasible domain estimatio for TCD, Feasible proportion: 76.9%; Feasible domain estimatio for TCSD, Feasible proportion: 0.68% (from top to bottom)

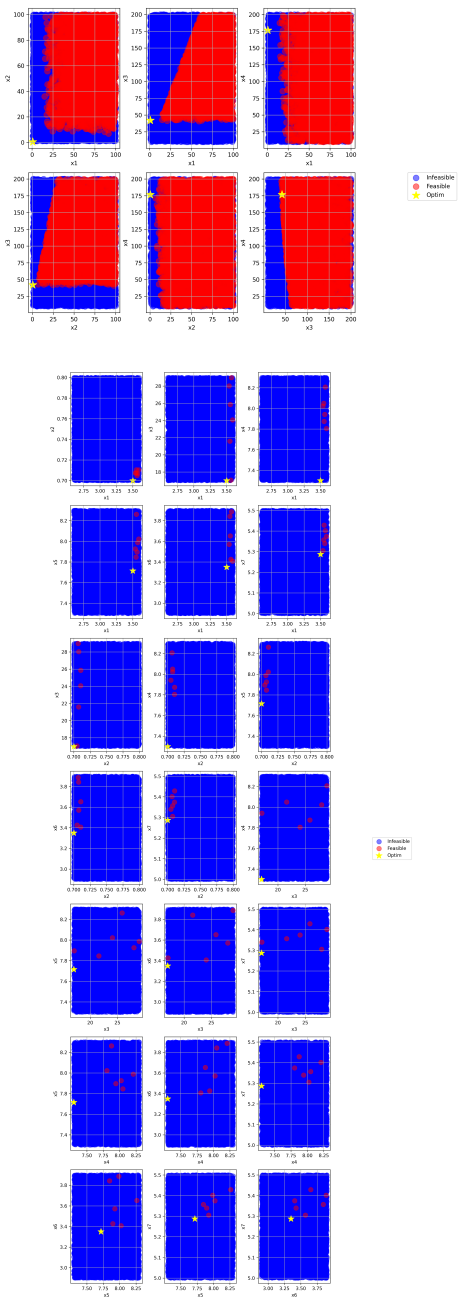

Figure 2: Feasible domain estimation for PVD, Feasible proportion: 40.66%; Feasible domain estimatio for SRD, Feasible proportion: 0.06%(from top to bottom)